Space Flight Mechanics

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Module No # 16
Lecture No # 79
Trajectory Transfer

Welcome to lecture 79 today so we are starting for the tenth week and we are going to discuss about the trajectory transfer. So the trajectory transfer it can be co planer it can be non-coplanar it can be interplanetary it can be on the same planet. So we are first going to discuss all this aspects what is mean by trajectory transfer and where do we apply okay.

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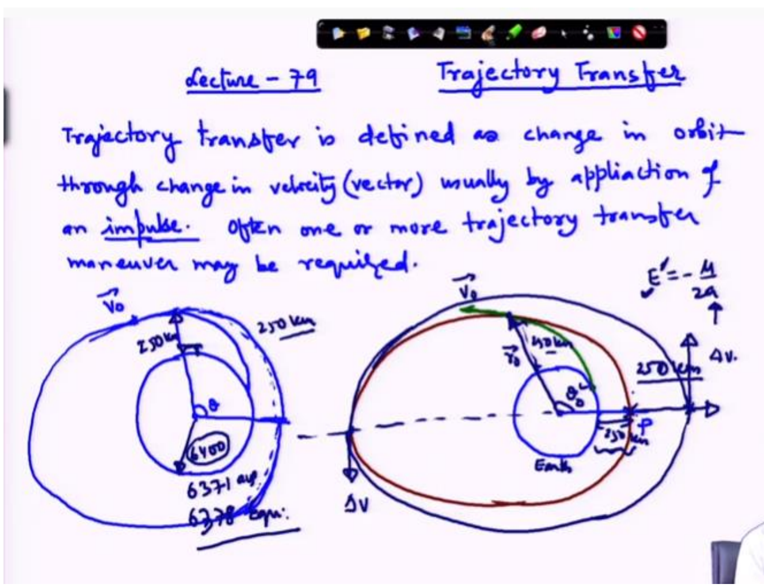

So let us start with trajectory transfer is defined as changed in orbit through change in velocity (vector) usually by application of an impulse. Often 1 or more trajectory transfer maneuver may be required sometimes it may be 2 sometimes it may be more. So in general we will see that minimum of 2 is required but just say you want to throw a satellite out of a satellite orbit into the space getting it lost there is a one impulse can be enough in that case.

But in general if you are doing the earth worm satellite maneuver the minimum of 2 impulse per 1 trajectory transfer let us say required. So as we have discussed earlier that once we long the satellite so we inject into the orbit somewhere, here $\vec{v_0}$is the velocity vector imparted to this say this is the perigee position the true anomaly will be measured from this place. And the satellite will coast to complete the orbit it will go all this way and then come back like this.

It is not a good figure but I can make another one so here in this case if I take this as an ellipse and at the center at the focus of an ellipse I have heard and this is the perigee position and somewhere from this station the satellite is been launched. So it is injected here in the orbit here in this place so at this point it will become tangent to this orbit in this is your theta the true anomaly.

So this is injection point $\overrightarrow{v_0}$ is the initial velocity and $\overrightarrow{r_0}$ is the radius (05:15) so already this issues we have discussed earlier. So once you have injected this is the orbit in which the satellite is it this is the very distorted figure remember. Because the earth radius is 6400 kilometers and we have written is 6371 this is the average value or 6378 the equatorial radius we have used. So say roughly the 6400 kilometers so if you are launching the satellite in 250 kilometers orbit at the altitude so this distance is your from here to here 250 kilometers.

So it will be a various small thing it will looks something like this if you look on the scale so it will be very close to the surface. But for clarity I have expanded it so this is the earth so once the satellite is injected so this is not the orbit in which you are looking this is the altitude say 250 kilometers or maybe at this point your altitude is 250 kilometers. At the perigee position it will be the least okay so either here or either here whatever it is.

So that we have to compute where theperigee will lie and that depends on the theta value theta 0 that we have to estimate and we have done it earlier in the orbit determination process. So thereafter as earlier also we have discussed that this perigee needs to be raised 250 kilometer if it remains in this place. So slowly this orbit will decay it will start decaying and the orbit will move into the atmosphere as it loses energy E prime we have written as $\mu/2a$.

So as you E' is the total energy per unit mass so as E decays so E will become smaller and smaller. So that it becomes more and more negative so the first maneuver required say this is elliptical in nature. So the first maneuver they require will be here in this place you impart ΔV, V is velocity here, so that this is raised. See immediately the lowest altitude has to be raised okay so if you impart velocity it will go up from here this position will remain unchanged and this position will come here in this place.

So this way your orbit gen it will look like something this way so this is the change orbit now this has come to this place then you may live to raise this one so then you give impulse here in this point by ΔV. So this point will be raised so this way you can circularize the orbit or you can make it elliptical you can increase the altitude or you can decrease the altitude it depends on the your requirement.

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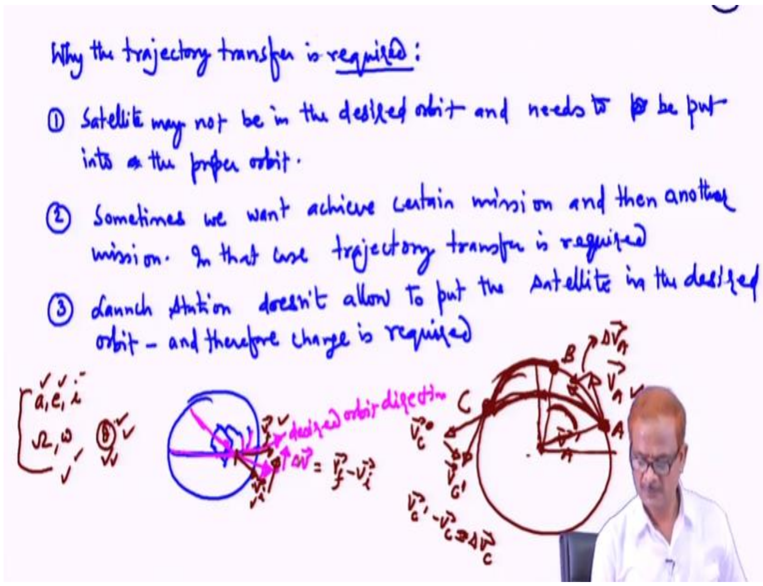

So these issues we have discussed earlier also so in general why the trajectory transfer is required? Reason can be the orbit in which you have launched this is not desired orbit to satellite may not be in the desired orbit and it is requires maneuver needs to be transferred or needs to be put into the proper orbit. Sometimes we want to achieve certain mission and then another mission. In that case trajectory transfer is required not only this one but also the launch station does not allow to put the satellite in the desired orbit and therefore change is required.

And this is especially happens in this case of India if you see the India is located here and here the Bangladesh and other countries are there and your star is here in this place okay. The satellite is you want to launch a satellite in the equatorial orbit say you wanted gio-stationary satellite and this is the equatorial orbit but your launch station position is somewhere half of this okay. Thereafter you are also interested in launching the satellite so this launch station grounded station position it does not allow to put into directly the this equatorial orbit.

So first what we will do we will launch in certain other direction and also you have to say whether countries so in the case of rocket sales it does not fall over the populated area. So say you are launching along this direction and so this is not a proper direction you need here in this direction. So this is the desired direction desired orbit and therefore from this orbit bringing it to this orbit we require some extra impulse to be given.

So if you give extra impulse here in this point okay so here initial velocity is given here in this direction this is V_i and this isV_f. So this is $\Delta V = V_f - V_i$ this is the impulse required so if you give impulse like this here in this direction okay. So this velocity vector will change to this velocity vector and thereby your satellite orbit will change. Okay so this way you will not only you can change the, a, e, i etc., but also the nodal angle can be changed the argument of perigee can be changed and obviously θ is the true anomaly.

So this is also done in the same orbit you can change the value ofthe θ. So in that case see in the some circular orbit that is one satellite here Aand another satellite B is here okay. And if you want to catch the satellite so basically docking is required.So and the orbit has to remain the same so what will do that as it is moving from this place to this place comes to point C. So send it in some faster orbit so that it goes and mix here so initial velocity vector is here $\vec{V}_A$ and we need to give a impulse here.

So that the velocity becomes tangent to this orbit the velocity will be here in this direction $\overrightarrow{V_A}'$. So this is the necessary impulse then $\overrightarrow{\Delta V_A}$ and similarly here the velocity vector at C is $\vec{V}_C$ and this is going here in this direction to $\vec{V}_C{}'$okay. Or we can say that $\vec{V}_C$ is the velocity here so $\vec{V}_C{}'$ is the change so $\vec{V}_C{}' - \vec{V}_C$ this is $\overrightarrow{\Delta V_C}$ is the impulse required here in this place okay.

Then hereby this satellite will be transferred from this place to the satellite B position. So B will by the time it will move from this position to this position and the same time it has to Ahas to move from here to the C. So 2 impulse one impulse is required here okay and one impulse is required here so minimum 2 impulse these are required. So depending on the your requirement and which parameter or this element or vital element you are trying to effect so thereby what trying to effect here in this case you are trying to effect θ.

This is circular orbit you are just trying to effect here in this case the true anomaly can be measured from any point. But if it is circular orbit and both the satellites are in the same orbit so their angle will be maintained all the times this angle from here to here this will always be maintained. So we have to grab this angle and catch it so for catching at this is the strategy okay so we have seen that while doing this we can affect θ.

Similarly a, e, i etc., all this things can be changed and this is not a very big problem whatever we have covered already using those concept we will be able to do it. So some historical details and other things I will supply you in the write of that I am providing we will go for more of the technical issues.

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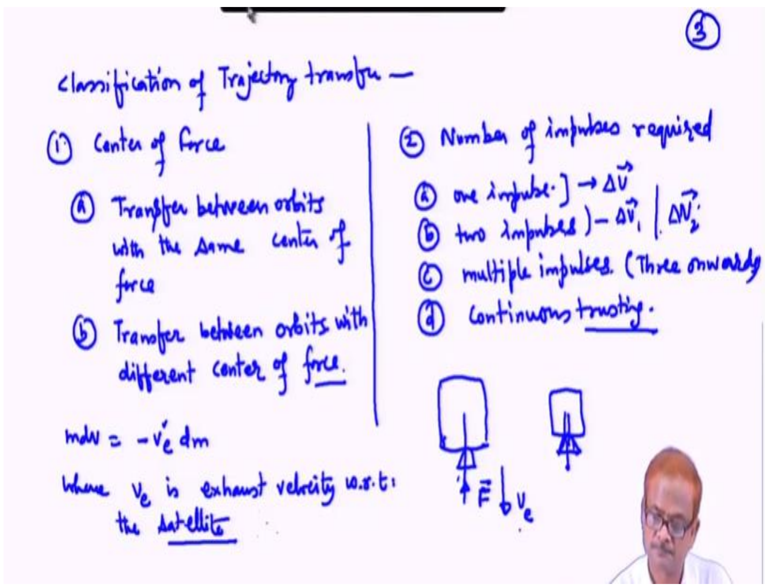

Classification of trajectory transfers so we can classify into different category one is based on the center of force. The other one is based on number of impulses is required transfer between orbit with the same center of force this is what we have discussing here. Here the center remains the same and your transferring either from this point to from this point to this point okay. Or it can be another orbit you want to set up another orbit say rather than you have to establish another orbit like this.

So this can be another case but here the center of force that remains same so transfer between orbits with the center of force. And another will be inter planetary or like you are going to the moon so this transfer from moon center of force to another center of force so transfer number of impulses.

So minimum one impulse required so one impulse this is the minimum 2 impulse in multiple or 3 onwards 2 impulses multiple impulses and then the continuous trusting.

Okay so we have all this cases so impulse means you at one point were giving change in velocity ΔV okay. So at 2 different points you are giving impulses ΔV_1 and ΔV_2. So rocket is fired onward rocket on the satellite so if you have the satellite so onward rocket is there. So this is fire for a short duration and the thrust (21:22) this will provide the necessary thrust and this equation is governed by the rocket equation for this.

So if it is in the space where atmosphere is not there so simply we can write $m\,dV = -V_e$ times dm where V_e is the exhaust velocity with respect to the satellite. And this can be derived this is simple to derive it and remember here the quantity already we have put a sign here minus and V_e is a positive quantity here. Though it is an opposite direction and this is with respect to the satellite so this is not the absolute velocity.

This is relative velocity and this is the velocity that you will be knowing the exhaust velocity and generally it is available to you. Okay and it depends on the construction of the satellite and this nozzle and your thrusting system.

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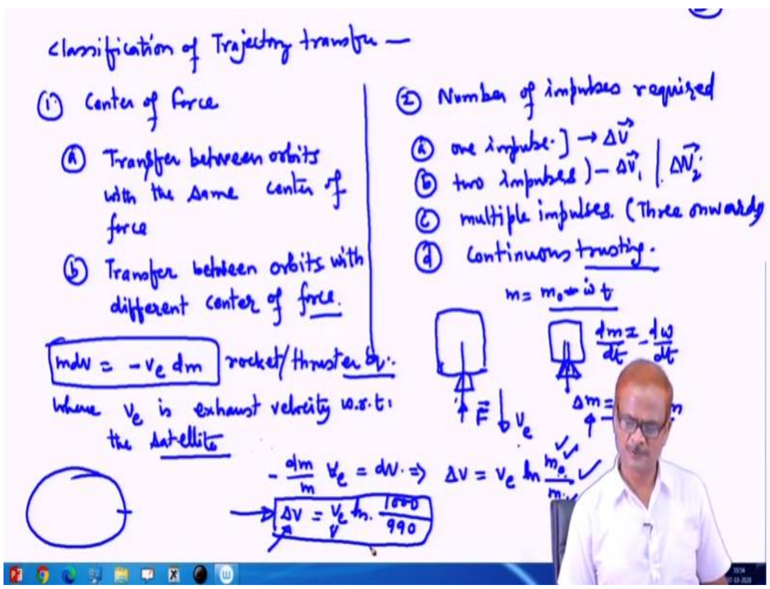

So I will just do that derivation before that let us complete this part so if this equation is given you can take this to with the rocket equation or the thruster equation in this space. In atmosphere this will get modified by the ambient (23:25) pressure and the exhaust pressure. So in that case we are not taking into a counter okay so if I try to integrate it, $\frac{dm}{m}$ with minus sign here we can write this as and multiplied by $V_e$equal to dV.

And once we integrate we can write $\Delta V = V_e$ times $ln\frac{m_A}{m_B}$ so you are starting the thrusting or simply A or B we can write 1 and 2. Okay $\frac{m_0}{m}$, m_0 is the initial mass and m is the final mass. Where $m = m_0$ minus the burning rate either plus or minus sign whichever you want to use we can use it. So let us say the burning rate I indicate it by $\dot{w}$ times t and therefore from here we will have

$$\frac{dm}{dt} = -\frac{dw}{dt}.$$

So you can see that if your initial mass of the satellite was $1000kg$ and after trusting for some time you are getting say, some $10kg$ or fuel is burn. So 990 and these are logarithmic phase this that exhaust velocity is given to you so this is the impulse provided. So wherever whatever the impulse is provided so if we know this quantity ΔV, V_e is known to us and m_0 is known to us and therefore m can be calculated.

And once m is calculated so the amount of fuel burned will be $m_0 - m$ so, this is the amount of fuel bond. So, for completing the maneuver suppose you want to keep the satellite in the orbit for 10 years so, during 10 years' time satellite will not remain in the same orbit. As we have done the things in the general perturbation method okay that is not going to remain in the same orbit it will keep changing over a period of time.

Then if it changes then we need to thrust it and put it back into the same orbit so periodically this orbit maintenance will be done. And in that periodic maintenance so say you are doing periodic maintenance say after 1 month. So after 1 month how much fuel is required this for your propulsion system how much the propulsion liquid or the solid which is required that can be computed using this equation.

So for 1 maneuver this is the requirement okay thereafter your weight will reduce the satellite mass will be different. So that will act as your initial mass so if you are doing this maneuver after every 1 month so you will be able to compute how much the fuel will be burnt. So this is an iterative process of designing the satellite so in the beginning you need to assemble all the components. And you are not a of your even about a inertia of the satellite so those things are computed.

So many methods of doing it the theoretically also can be done on the catiayou(27:20) can do it or using your just experimental method this can be done. So once you know that these are the things going into the satellite so you can approximately guess that what will be the weight of the satellite okay. And from there then you can start designing if this is the weight of the satellite if I put this much of how much propellant will be required.

Now for housing that propellant the propellant covering is required that propulsion system the whole propulsion system in which you are putting the propellant or the fuel. So that has to be added to the satellite mass so it is not one shot process nobody can do in one shot. So it is iterative process it is done iteratively and as you refine you get more and more the precise. So after fewiteration using computer you will see that you are getting the nice result get converges.

So for as many maneuvers is required that many maneuver you can plan so depending on the propellant your satellite life depends okay. Once a propellant is lost so satellite will start deviating from the desired orbit and there after it gets lost okay. So this whole process therefore needs to be properly planned and coded on a computer and once this has been done then finally after the launch once is injected in the orbit we will see that almost its follows the whatever you have planned.

Because in the space most of the things are already defined what will be effect of the planets or the earth or the sun or the moon whatever all those things are known. So over a period of time how much your orbit will deviate that you will beknowing. And once you know that so everything can be worked out. But however we do not have that much of time to go into this and every detail of all this things. But with the basics that I will cover we will be work of this issue.

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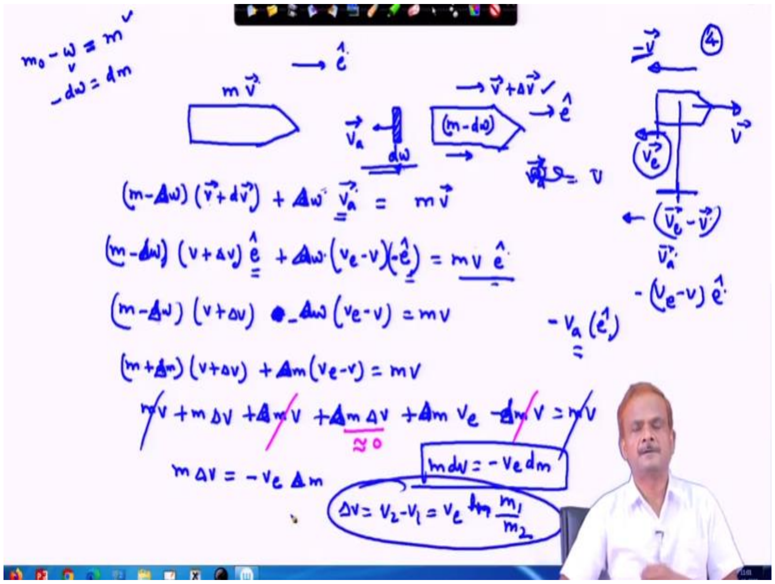

So quickly I will wind up the rocket equation say the initial mass of the rocket is m and $\vec{V}$ is the velocity and this we take as the $\hat{e}$ direction positive direction. Okay and thereafter out this a small mass dw is burnt out and then the mass of the rocket and it is velocity becomes $\vec{V} + \Delta V$. And this dw this is thrown out with absolute velocity $\overrightarrow{V_a}$, and this is a width dw so because of this just gets the reactive force here in this direction.

So we directly cannot apply the Newton's law and therefore we go through the conservation of linear momentum equation. So we have $(m - dw)$ times $V + dV$ and the momentum of this part so this is $\overrightarrow{dw}$. And then the momentum of this has to be added so V absolute is given so V absolute of this $\overrightarrow{dw}.\overrightarrow{V_a}$, and this must be equal to the initial momentum. Now $\overrightarrow{V_a}$ is the absolute velocity of this and the absolute velocity equation we can write as see for here we can show it like this with respect to the rocket $\overrightarrow{V_e}$ is the exhaust velocity.

Okay rocket itself is moving here in this direction with velocity $\vec{V}$ okay so if I impose the velocity is $-\vec{V}$ here in this direction. So this rocket becomes a stationary then with respect the inertial space what will be exhaust velocity. So this will be given by $\overrightarrow{V_e} - \vec{V}$ and here in this direction and this is your $\overrightarrow{V_a}$ so this we can insert here in this space and expand it also dw this we write as $V + \Delta V$ and $\hat{e}$ direction. Here in this $\hat{e}$ direction we are taking here and plus dw and $\overrightarrow{V_a}$ as we have written here this is $\overrightarrow{V_e} - \vec{V}$.

So this also we changing here itself $\overrightarrow{V_e}$ we have shown here in this direction so this is $\overrightarrow{V_e} - \vec{V}$ times $\hat{e}$. $-\vec{V}$ is here in this direction so we can write it this way we take minus sign okay. So $\overrightarrow{V_e} - \vec{V}$ times $\hat{e}$ with a minus $= mV$ times $\hat{e}$. So this is the initial velocity. So here the way we have written it is I could have also written as V_a times $\hat{e}$ with minus sign here in this place and then V_a simply replacing it by $V_e - V$.

Now expanded it so you can see that we can remove so only this quantity remains $(V + \Delta V) + dw(V_e - V)$ and here this here come with the minus sign $= mV$. Now if m_0 is the initial mass m is the mass at any time so this indicates or w is the mass burn so this is the positive quantity $w\ V$ we are taking. So the mass later on it becomes m and once we differentiate this we can see that dw will be equal to dm.

So dw can be replaced by $-dm$ so this becomes +, $(V + \Delta V)$ and this becomes $+dm$ times $(V_e - V)$ and we expand it

$$mV + m\,\Delta V + dmV + dm\Delta V + dmV_e - dmV = mV$$

This and this cancels out. And then $-dmV$ this drops out this is the quantity which is of second order so this is nearly 0 okay as compared to the other term. So therefore we drop them out drop this term and we get here $m\,\Delta V$ equal to $-V_e$ times this is Δm here we should have written Δw here in all the places it is okay Δw or dw directly whatever you want to write it is okay.

Okay so $-V_e$ times Δm and we should write the same thing in the once the Δt tends to 0 so we can write in terms of $mdV = -V_e$ times dm and this is the equation which leads you to $\Delta V =$

$$V_2 - V_1 = V_e\ ln\frac{m_1}{m_2}$$

So this is the thrusting equation so we stop here and in the lecture we will continue.