**Space Flight Mechanics**
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**Module No # 17**
**Lecture No # 86**
**Trajectory Transfer (Contd.)**

Welcome to eighty sixth lecture. We have been discussing about the Hohmann transfer bi-elliptic transfer and we looked into different aspect which one is better.

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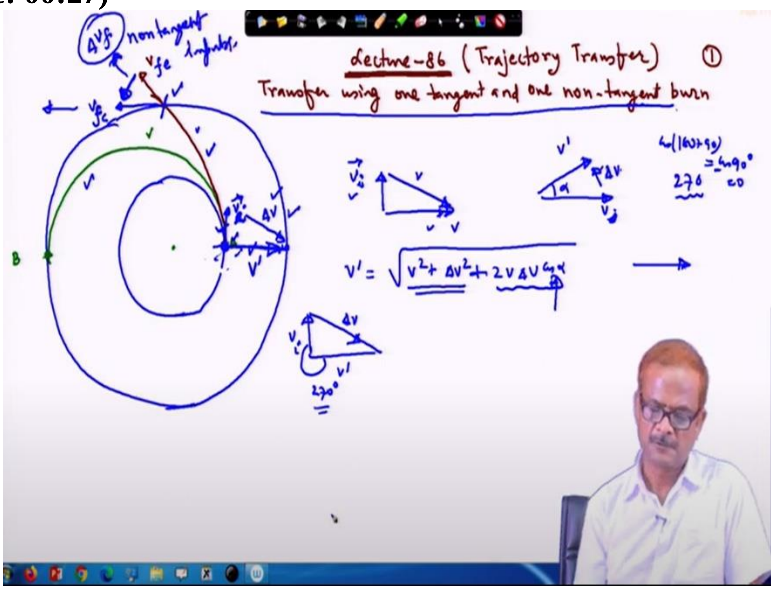



Ok now today here in this lecture we will do this 1 tangent and 1 non tangent one why? Because the Hohmann transfer it takes a longer time for transferring from one orbit to another orbit. Let us say I have this inner orbit and there is another outer orbit and I have to go from inner to outer or outer to inner orbit. So in the Hohmann transfer we will follow the trajectory like this so going from point A to point B.

So total flight time then is increased instead of doing this if I suppose if I carry this maneuver instead of going like this if it is done like from here it starts the same point you are starting but; and 1 tangent burn you are giving. And there after the orbit goes like this. And then the velocity here will be in this direction and the circular velocity will be in the final orbit this is the v fc and this is v f in the elliptic orbit.

So you require this much of impulse to change from this elliptic orbit to this one. So there by we reduce the travel time. So time to travel from this point to this point it gets shortened. Can we go from this point to this point directly? So you can see that the velocity here in this direction at v a this is the velocity or let us write this as v i and we have to go here in this direction. So how much impulse will be required?

So one vi is here in this direction ok and this is your v here in this direction velocity you have to change. So going with certain velocity here in this direction to change the velocity from this direction to this direction you require impulse like this. But is it feasible? Let us look through the equation what we have written as this is v i and this is say v prime so or either simply we write as v and this is delta v.

$$v' = \sqrt{v^2 + \Delta v^2 - 2\ v\ \Delta v\ cos\alpha}$$

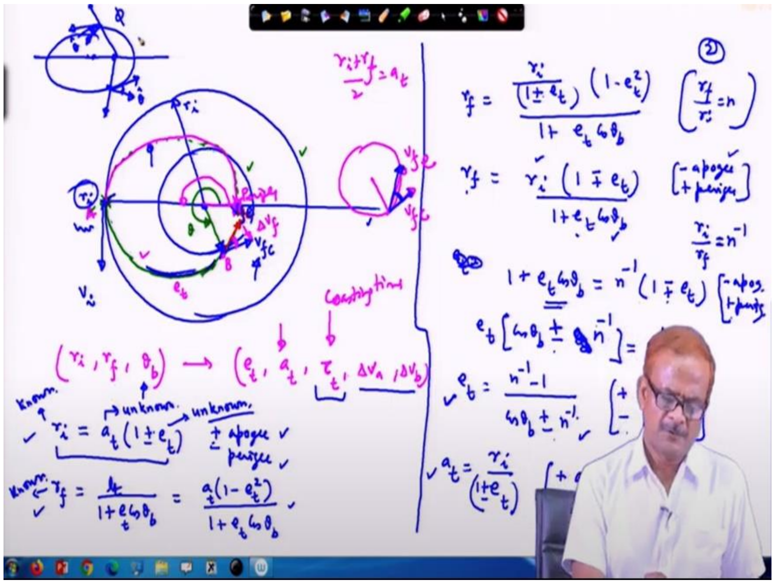
So v prime will be v square + delta v square - 2 v delta v and this angle is alpha then Cos alpha under root + 2v times delta v Cos alpha. So if v is like this and you want to change it in the perpendicular direction. So with v it is making here 270 degree. If we put here Cos alpha = 270 degree so that means it is a 0. Cos (180 + 90) = - Cos 90 and this will be equal to 0. So this term drops out and we are left with this.

$$\cos(180 + 90) = -cos(90) = 0$$

So this way if we are trying to go from this place to this place ok one mistake is there that we should correct what we have shown here this is the angle Cos alpha is between ok no. we have done it correctly this is ok this is v prime and this is v i we have written this is delta v ok delta v so that this is the resulted prime ok this is fine. So this is the resultant velocity and this is initial velocity and this is the impulse ok.

So if we add this 2 vectors this is v i and if we add the impulse here delta v so we are getting this as v prime. So the angles from here to here this is 270 degree this is fine ok. So the shortest possible time is that I give a large impulse at here in this point very large impulse so that it directly moves from this point to this point but giving a large impulse is not possible very large impulse it is not possible.

So it is not a practical thing .ok you cannot design rocket like that it gives you infinite impulse and then directly move from one place here to here. So therefore to reduce the sometimes it may be required that instead of going in a Hohmann transfer you give 1 tangent burn here and give 1 non-tangent burn here. This is why non tangent because this delta v here delta v f required here this is not along v fc it is not along this direction okay and therefore this is called non tangent burn. Non tangent. So this is what we are going to do in this and again the written materials I will provide you for this particular part and we quickly do this part here. **(Refer Time: 06:50)**



Say initially this is the initial one let us write this as the vi in the circular orbit and we are trying to go in this. So we are following a first a tangent bond here we are following the tangent burn and it goes like this coasting going like this and then so if we start from this place we can look that the true anomaly of the transfer orbit is this is theta. And initial point this happens to be this 2 are the circular orbit the same way you can also do for the elliptical orbit.

So we start from this point given tangent burn here and then the velocity vector is directed along this direction this is in the v fe while in the circular orbit the velocity vector is directed tangent to the orbit which is v fc. So we need impulse here in this direction this is delta in the opposite direction rather in this direction we require impulse delta v f to put here in the inner orbit ok. So what are the steps involved that we are going to write.

So here in this particular case we can state the problem like this r initial is given r final is given and theta let us say this point is B. And theta B is given that we have to go and get into the inner orbit at the point B. And from there then we need to work out how much impulse total impulse is required so in that context we need the eccentricity of the transfer orbit this e t is required. And then we also require the semi major axis of the transfer orbit.

And also we require the t or tow whatever you say this is the coasting time going from this place to this place A to B this is point A and then we also need delta v a and delta v b we need to work out all these thing.

$$(r_i , r_f , \theta_b) \rightarrow (e_t , a_t , \tau_t , \Delta v_a , \Delta v_b)$$

Here in this case e is directly not available we cannot write r i + r f /2 = a transfer because this is only one the orbit is touching each other. Both the; this transfer orbit is touching both the inner and the outer orbit at that time only we can do this.

$$\frac{r_i + r_f}{2} = a_t$$

So here in this case we have to go through some other route. We start with this r initial at this point equal to r a transfer orbit times 1 + - e transfer orbit plus is for if we start the manual here in this case this happens to be apogee so for this is for of apogee if the maneuver is starting at and minus is for perigee. So that your r i r initial the r initial is here this point this radius is r i. So your distance to this point is r i r initial equal to a transfer times 1 + e t where e is the eccentricity of the transfer orbit which is shown in green.

$$r_i = a_t(1 \pm e_t), \ \begin{Bmatrix} -perigee \\ +apogee \end{Bmatrix}$$

Similarly r final this will be r final is here in this point ok. And that we can write in terms of transfer orbit so 1 + e Cos theta here b we have written so we will use this notation theta b. And this quantity a times 1 - e transfer square 1 + this is also e transfer because both are cutting here circular orbit and the elliptic orbit which is the transfer orbit they are crossing each other this quantity must be the same.

$$r_f = \frac{l_t}{1 + e_t \, cos\theta_b} = \frac{a(1 - e_t^2)}{1 + e_t cos\theta_b}$$

So we have got r f and r i equation also we have written like this where a is unknown and e t is also unknown here. Both these quantities are unknown. But r i this quantity is known here r f is known. In this equation e t and a t both are unknown. So the same thing it will applies here l t of the transfer orbit. So therefore r f it becomes we can replace this a t from this place. So this we can write as r i / 1 + - e t times 1 -e t square / 1 + e t Cos theta b.

$$r_f = \frac{\frac{r_i}{(1 \pm e_t)}(1 - e_t^2)}{1 + e_t\, cos\theta_b}$$

This can be written as 1 - + e t minus for apogee and plus for perigee. This is because of the division ok. Here the upper one was apogee and lower one for perigee. So once we divide with plus sign so we get minus sign here we corresponds to apogee ok.

$$r_f = \frac{r_i\,(1 \pm e_t)}{1 + e_t\, cos\theta_b}, \quad \begin{Bmatrix} -perigee \\ +apogee \end{Bmatrix}$$

So this is your final radius 1 + e t Cos theta b, r f is known, r i is known theta b is known only this is e t is not known. So e t we can calculate from this place so this gives you e t equal to if we solve it so what I will do that to simplify I will write here r i / r f = n inverse.

$$\frac{r_i}{r_f} = n^{-1}\ , \quad \left\{\frac{r_f}{r_i} = n\right\}$$

$$1 + e_t \cos\ \theta_b = n^{-1}(1 \pm e_t)$$

$$e_t[\cos\theta_b \pm n^{-1}] = n^{-1} - 1, \qquad \begin{Bmatrix} -perigee \\ +apogee \end{Bmatrix}$$

$$e_t = \frac{n^{-1} - 1}{\cos\theta_b \pm n^{-1}}\ , \quad \begin{Bmatrix} -perigee \\ +apogee \end{Bmatrix}$$

$$a_t = \frac{r_i}{(1 \pm e_t)}\ , \qquad \begin{Bmatrix} -perigee \\ +apogee \end{Bmatrix}$$

Always we are using the symbol r f / ri = n. So ri / r f will be n inverse. So this gets simplified 1 + e t Cos theta b this = n inverse 1 - + e t. And then combine the terms which is in terms of e t ok. So e t we will take it outside so e t Cos theta b and from this place then get you as plus minus where this is minus for apogee and plus for perigee e t Cos theta b and minus sign on this side gets plus. Plus sign get minus.

So this is e t times n inverse so e t we are taking outside so n inverse we put it like this this is n inverse -1. And therefore e t = n inverse -1 / Cos theta b + -n inverse where plus sign is for apogee again the sign reversal (16:46) takes place and this is for perigee. So this way e t is known once e t is known therefore a t can be calculated a t = ri / 1 + - e t where plus is for apogee and minus is for perigee.

So this way e t is known a t is known for the transfer orbit. So this exercise we need to do. And we have to take care that here in the denominator the quantity does not become 0 if that happens then you can immediately see that your e t becomes infinite ok. So this is not a defined case. So we have to take care of that particular thing ok.

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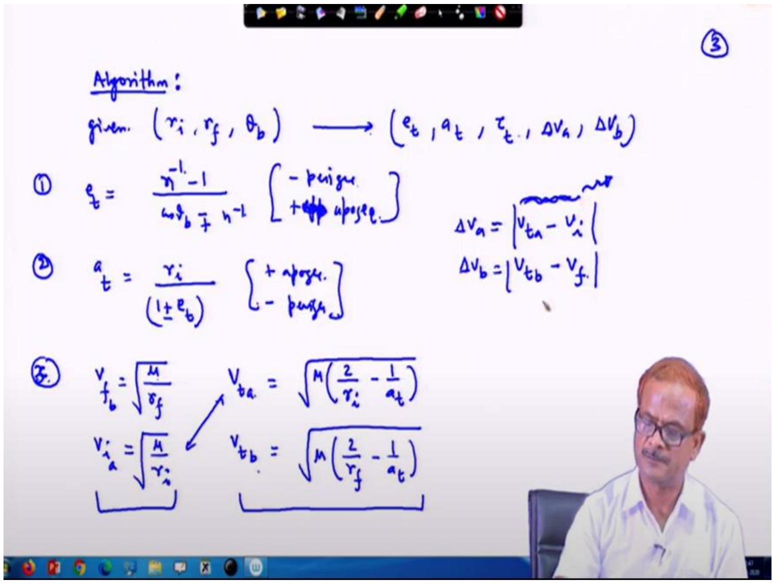


Once this is done then we write the algorithm here so given r initial r final and theta b for the where you have to capture in the second orbit find e t a t tou transfer or simply tou t and then delta v a and delta vb.

Algorithm:

$$given\ (r_i\,, r_f\,, \theta_b) \rightarrow (e_t\,, a_t\,, \tau_t\,, \Delta v_a\,, \Delta v_b\,)$$

①

$$e_t = \frac{n^{-1} - 1}{cos\theta_b \pm\ n^{-1}}, \qquad \begin{Bmatrix} -perigee \\ +apogee \end{Bmatrix}$$

②

$$a_t = \frac{r_i}{1 \pm e_t}\,, \qquad \begin{Bmatrix} -perigee \\ +apogee \end{Bmatrix}$$

③

$$v_{f_b} = \sqrt{\frac{\mu}{r_f}} \quad , \quad v_{ta} = \sqrt{\mu\left(\frac{2}{r_i} - \frac{1}{a_t}\right)}$$

$$v_{i_a} = \sqrt{\frac{\mu}{r_i}} \quad , \quad v_{tb} = \sqrt{\mu\left(\frac{2}{r_f} - \frac{1}{a_t}\right)}$$

$$\Delta v_a = |v_{ta} - v_i|$$

$$\Delta v_b = |v_{tb} - v_f|$$

So e transfer we already we have written this is n inverse -1 as we have written here n inverse -1 Cos theta b - + n inverse minus for perigee e transfer minus for perigee and plus for apogee.

minus for perigee and plus for apogee. Here this minus sign is for perigee and plus for apogee. So this is the first step the second one the at we have to get so this we have written as r initial divided by 1 + e t divide 1 + - e t and plus sign is for apogee and minus for perigee. Then the third step calculate v final which will be equal to mu / r final under root v initial is mu / r initial under root and in the transfer orbit v t at this point where you are starting.

At a here so v transfer orbit at a, this will be mu times 2 / r initial - 1 / a transfer under root ok and v tb as per it appears here v tb this is mu times 2 / r f - 1 / a t under root. So from here you have the 2 impulses required it is known to us. delta v a we can write as v ta -vi and delta v b = v tb- v f. So what we need to do that ok we have to take care of the sign also. So now look into this in the initial orbit what is the velocity and the transfer orbitWhat is the velocity? here this particular part. This is corresponding to a here this is corresponding to b . this is corresponding to b.ok So once you write it like this, there after you need not worry about which one is positive negative just take the magnitude of this. After calculating the quantity and the sign of this will tell you whether you have to give a impulse to reduce the velocity or to increase the velocity. That will be visible from this point ok. So I am not going into all those details you can check it.

Similarly at the final point once we are going here in this point so whether the velocity required is here in this direction the change in velocity or here in this direction that can be calculated depending on the sign. This is I am leaving up to you to work it out. There after what remains is delta v a delta v b this is time to coast from this place to this place. For this you need in the transfer orbit first we need to calculate what the time is taken to come from this point to this point ok.

Taking this is the orbit and this is the perigee position here perigee at lying here in this point ok. And from here then the, it starts what is the time taken to come to this point. And then calculate the time to come to from this point again from starting this point to point b and then subtract it so you will get the time. Otherwise various other ways are there so already we have discussed this in the and you might have done the problem using the Kepler's equation.

So I am not going into those details. So this way this problem is completed. Ok I have taken little short cut on this aspect ok. One more point that we need to discuss the left the left out one is the this angle. So this angle already if you remember I have discussed this point a lot because once we did the generalized trajectory transfer so at that time we have discussed this part. So this is your v fc and in this direction this is your v fe which is has got covered here this is v fe and this is v fc.

So v fc is tangent to the inner circle as shown here in this point. And v fe is going inside its penetrating inside. So how much this angle will be this can be calculated as we have done earlier. This happens to be the flight path angle. So flight path angle as you can see from this point in this case this turns out to be negative. Here once we come say if this is the ellipse and this is its focus center is here and then you are entering here in this point.

So at this point this is the radius vector perpendicular direction is here and v vector is lying like this. So this flight path angle this gets negative value this is the r direction. Here if we are going here in this direction so from here to here this is the theta direction theta cap direction this is your theta cap direction. And V direction is like this so here this is phi so this phi turns out has to be positive. Ok this issue, was raised in one of the question by some student related to some of the tutorial sheet. There I have replied to that.

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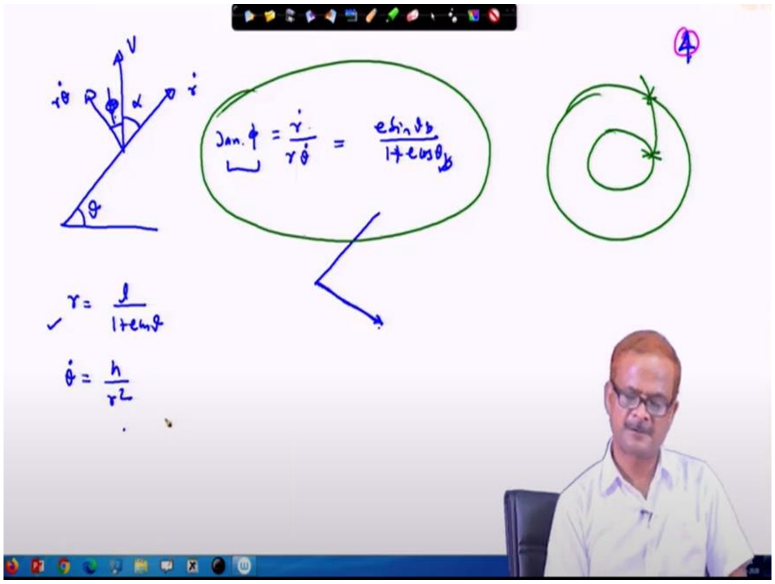

So here quickly we have look into this point and this is nothing but our recall from the previous session this is r theta direction. So r theta dot is here in this direction this is v and some where this is theta phi we have written here. So this is your flight path angle in which angle we have indicated it as alpha and this angle is your theta angle. So tan phi this is written as r dot / r dot theta dot and this we have derived it also ok.

$$\tan\phi = \frac{\dot{r}}{r\dot{\theta}}$$

$$r = \frac{l}{1 + e\cos\theta}$$

$$\dot{\theta} = \frac{h}{r^2}$$

$$\tan\phi = \frac{\dot{r}}{r\dot{\theta}} = \frac{e\sin\theta}{1 + e\cos\theta}$$

So tan phi ; r dot and from where we have got r = l / 1 + e Cos theta and directly from there we are getting r and theta dot is coming from h / r square. So using this we have written the result and this can be written as e sin theta / 1 + e Cos theta. So if you know the theta the true anomaly where in this case it is theta b, this is theta b. So here just you need to insert this theta b. So that gives you the phi value.

So you know the angle between this 2 vectors 1 vector is going like this from the focus 1 vector is along this direction or let us go into the previous figure this is your v fc here in this direction. And then this is your v here in this direction v fe ok. So delta f required here in this place delta v f that we can compute so this is the quantity here. And from this place your phi angle is known whether positive or negative that we have to consider.

And all other values we have already done so just look back into that lecture and from there you will get to know that how then we to proceed because there we have done for very generalized case we are going from this orbit to this orbit. So there also we are doing the non tangent burn and here also we are doing non tangent burn in both the places and based on that all the calculations are done so ok we stop here.