Space Flight Mechanics
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Lecture – 91
Interplanetary Mission

Welcome to lecture 91 we have discussed last time about the interplanetary transfer. In that context we looked into how the Hohmann transfer will take from one planet to another planet and thereafter we also looked into the; we have to get out of the influence of the sphere of earth out of the influence of sphere of influence what we call SOI sphere of influence of earth. We need to get out of that.

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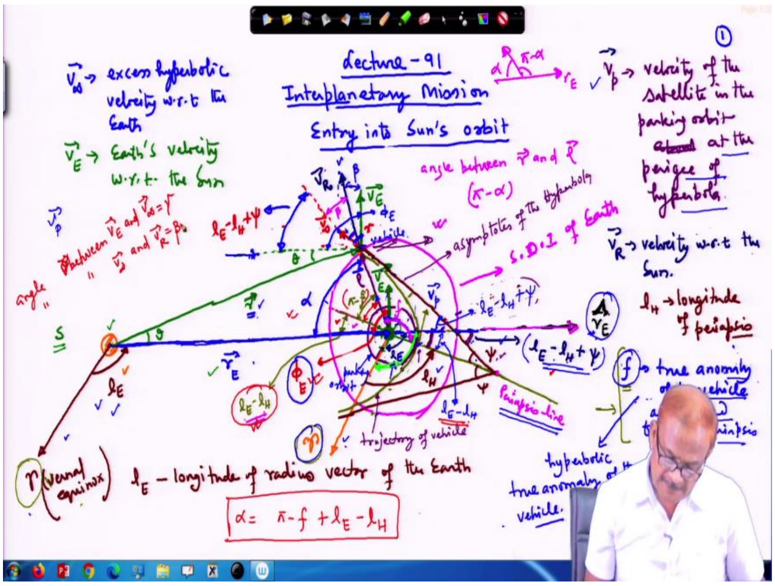

So getting out of that it requires certain velocity to be given at the perigee. Ok that velocity we are imparting here. So, that Velocity will be imparted here at the perigee position extra velocity. In that context we have I have made this figure kept it ready for the reference purpose. To see here on the left hand side leftmost side this γ is which is the horn of the perigee ok and this is the direction for the vernal Equinox.

The same thing is also shown here line coming out from the centre of the earth. The figure remains the same as we have drawn in the last class. So the green line here this green line. What is the shown here as a circle this represents your Earth. Let us say that Earth is just represented by this green wall at this point. So I will even remove this particular part so that our representation is much more clear.

So green wall shown here this is the Earth and on the left hand side here in this place. This is your sun has written here r_E is the radius vector of the earth. So here this is the radius vector. This is shown till this point till the centre of the earth. So the radius vector will goes all the way from this point to this one. Similarly, small r is the radius vector of the vehicle so vehicle is located here.

This angle is l_E which is the longitude of re so this is being measured from the vernal Equinox. Similarly if we look here in this place I have written here l_E and this γ is also shown here in this place, so angle measured from this place as shown by this l_E. As you can see here this angle is locate the unit vector r_E or the just line l_E which is drawn so it locates this with respect to the vernal Equinox So this l_E and this l_E both are same that means this and this they are parallel both are towards the vernal Equinox.

Beside this I have shown here the l_H it is the perigee position a line of periapsis the axis line I have written for the last time I have shown there in the hyperbola. If we give impulse so this is the velocity at the perigee. If this is the impulse given here in this point, we $V_P – V_C$ where C is the velocity in the V_P. So V_P already I have written here velocity of the satellite in the parking orbit at the perigee of the hyperbola.

This line is your parking orbit. So, in the parking; orbit at this point where the periapsis is located. So here we give an impulse parking orbit and hyperbola perigee they are touching each other they are touching. So the impulse is given at this point that make the velocity of the satellite V_P and thereafter it goes here and reaches the sphere of influence where I have seen this as a vehicle.

So all the angles we have to describe here and then this green line the light green line based on the light green line this is the trajectory of hyperbola. This is the trajectory of vehicle. Satellite is coming out of the sphere of influence here in this point and this dark line these are the asymptotes of the hyperbola. You can say as indicated this is measured with respect to as we have done in the conic system.

So, this is the angles Ψ here of the asymptotes measured with the periapsis line. Now this angle; the angles shown here this one where I am drawing and this line it goes here this is $l_E - l_H$ because the angle for the l_E you are measuring from; l_E has been measured all the way from line of periapsis. I will have to draw one more line. I will draw a light line for this. So you are measuring l_E from this place to this place. So this is your indicating l_E. So this is l_E. So immediately you can see that this angle will be $l_E - l_H$. Then therefore this being the external angles this one this becomes $l_E - l_H + \Psi$.

The Ψ plus this angle added together they become $l_E - l_H + \Psi$ to the opposite angle here in this place has shown by this particular segment the angle here. This is l_E minus. We have shown this part here as $l_E - l_H$ and removing this from this place for clarity purpose. So this angle which is the vortex angle then this becomes $l_E - l_H + \Psi$. As it is written here; this is a true anomaly of the vehicle as measured from the periapsis so f is shown here in the pink.

You see here this is the pink line shown here in this place. Ok. So this is your f. So this is f. Then ϕ is the angle between the r_E vector here and the V vector as shown by Orange line Orange or red red line ok it shown by the red line here this angle this is your ϕ. Ok all the angles need to be written carefully. So you can see that the this is your $\phi - f$ angle as shown here. So this angle is is Φ - f I am showing again by red line.

So, angle $l_E - l_H$ and therefore this is also this $l_E - l_H$ because it is a vertex angle and therefore the A angle, which is shown here is α will be equal to $l_E - l_H + \pi$ - f so α equal to $\pi - f + l_E - l_H$ angle between vehicle V angle between V and V_∞ we write this as γ and angle between V_∞ and V are derived as β. So these are the nomenclature here π - f carefully representing all the angles.

$$\alpha = \pi - f + l_E - l_H$$

Based on this now; we can start writing the equations for the various angles and the relations. Figure looks very complicated but it is not very complicated. It just because of rewriting and drawing various line it is appearing little complex.

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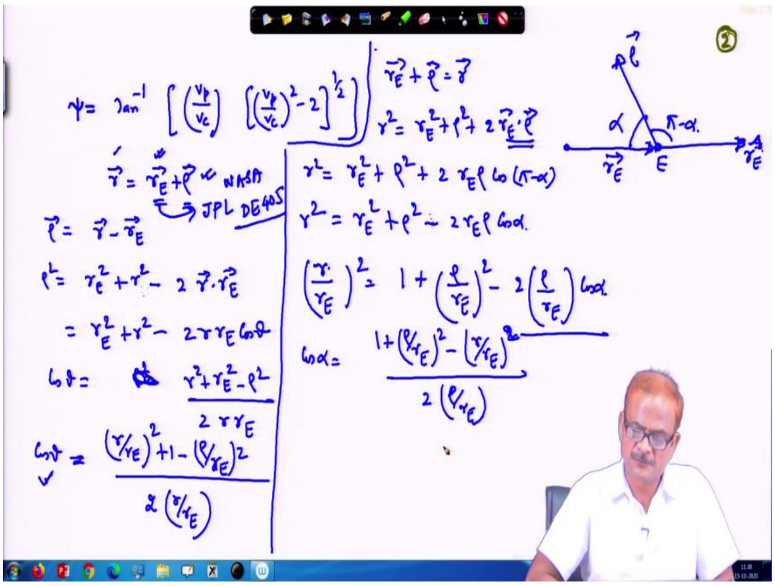

Now we have start working with the angles. So already last time we have derived this relationship Ψ is equal to tan inverse V_P by V_C times square - 2 whole power 1 by 2 this is the relation last time we have got. Also, we can observe from the previous figure that this $\vec{r}$ this vector will be nothing but r will be $r_E + \rho$ where ρ is the vector till the vehicle. As shown here this is a $\vec{\rho}$ this is your $\vec{\rho}$.

$$\psi = \mathrm{Tan}^{-1}\left[\left(\frac{V_p}{V_c}\right)\left[\left(\frac{V_p}{V_c}\right)^2 - 2\right]^{\frac{1}{2}}\right]$$

$$\vec{r} = \overrightarrow{r_E} + \vec{\rho}$$

$$\vec{\rho} = \vec{r} - \overrightarrow{r_E}$$

$$\rho^2 = r_E^2 + r^2 - 2\vec{r}\,.\overrightarrow{r_E}\,\mathrm{Cos}\theta$$

So r is equal to $r_E + \rho$ and therefore the ρ can be written as ρ^2 equal to $r_E{}^2 + R^2$ so $r_E{}^2 + R^2 - 2R\, r_E$ cos θ. So here θ is the angle between angle between r and r_E and therefore cos θ becomes ρ^2 cos θ equal to $\rho^2 + r_E{}^2 - \rho^2$ divided by 2 times r r_E and if you look here in this cos θ, we have to determine also this cos θ and then r we know.

$$\mathrm{Cos}\theta = \frac{r^2 + r_E^2 - \rho^2}{2r\, r_E}$$

$$\mathrm{Cos}\ \theta = \frac{\left(\frac{r}{r_E}\right)^2 + 1 - \left(\frac{\rho}{r_E}\right)^2}{2\left(\frac{r}{r_E}\right)}$$

r is known from r_E and ρ . ρ is known to us because this we get from estimation orbit determination problem. And the r earth which is shown here this is no known from JPL, Jet propulsion laboratory. This is NASA JPL developmental ephemeris 405. Special program is here. So from there you get it the program is written in FORTRAN obviously it is not so easy to understand that also. You need certain experienced there after you can work on all these things.

So therefore, the cos θ can be determined there after we have also $r_E + \rho$ is equal to r and this we can and writers r^2 equal to $r_E^2 + \rho^2 + 2\ r_E\ \rho$ times cos. So the angle between r_E and angle between r_E and ρ is; now if I will draw the figure here in this place, this is your true vector and b. Vector is going here in this direction. And this is a $\hat{r}_E$.

$$\vec{r} = \overrightarrow{r_E} + \vec{\rho}$$

$$r^2 = r_E^2 + \rho^2 + 2\,\overrightarrow{r_E}.\vec{\rho}$$

$$r^2 = r_E^2 + \rho^2 + 2\,r_E\rho\cos(\pi - \alpha)$$

$$r^2 = r_E^2 + \rho^2 - 2\,r_E\rho\cos\alpha$$

So r_E again is coming from the sun to the earth is located here. This angle you are taken as α and therefore this angle is $\pi - \alpha$. So the angle between the r_E and the ρ vector is $\pi - \alpha$ and this gets give save $r_E{}^2\ \rho^2 + 2\ r_E\ \rho$ times here this will be minus and therefore from this place r by r_E^2 ρ by $r_E{}^2$ - 2 cos α. So therefore, from this place we can get the angle cos α which will be equal to 1 + ρ by $r_E{}^2$ – whole square divided by 2 ρ by r_E.

$$\left(\frac{r}{r_E}\right)^2 = 1 + \left(\frac{\rho}{r_E}\right)^2 - 2\left(\frac{\rho}{r_E}\right)\cos\alpha$$

$$\cos\alpha = \frac{1+\left(\frac{\rho}{r_E}\right)^2 - 2\left(\frac{\rho}{r_E}\right)^2}{2\left(\frac{\rho}{r_E}\right)}$$

Cos θ is known α is known as all; this thing because the ρ and r_E all these things are known and therefore this angle become known.

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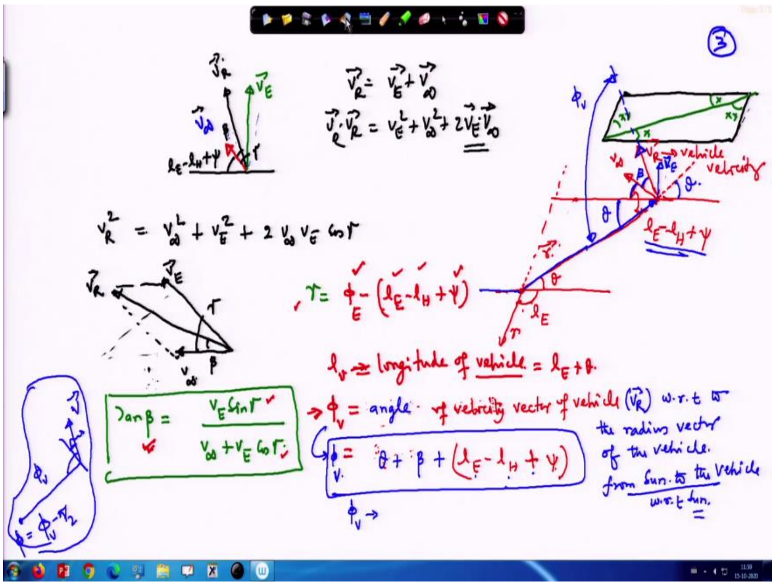

Then we go to the vehicle so it is shown that the V_∞ it is in this direction and V_∞ is shown like this. V_∞ is shown like this and then the V_E were shown by proper colour. This is the V Earth and V_R resultant is shown in black. So the resultant is V_R, this angle we have written as β and from here to here this we write as γ moreover an angle with the horizontal angle as shown here is $l_E - l_H + \psi$.

$$V_R^2 = V_\infty^2 + V_E^2 + 2\, V_\infty V_E \text{Cos}\gamma$$

This angle is $l_E - l_H + \psi$ so we will utilise all this information for solving the problem here. So V earth then this becomes $V_\infty{}^2 + V_E{}^2 + 2$ times V_∞ times Cos γ angle between $V_E + V_\infty$ by taking the dot product V_R dot V_R gives you $V_E{}^2 + V_\infty{}^2 + 2V_E$ time V_∞ this is dot. The angle between these two is γ therefore, it becomes cos γ and here this is square.

$$\overrightarrow{V_R} = \overrightarrow{V_E} + \overrightarrow{V_\infty}$$

$$\overrightarrow{V_\rho}\,\overrightarrow{V_R} = V_E^2 + V_\infty^2 + 2\,\overrightarrow{V_E}\,\overrightarrow{V_\infty}$$

Thereafter the angle γ now looks here in this place. This is the V_∞ and rather other way we draw the figure other way. This is V_∞ and with respect to this, this is your V earth the vector and V resultant is something like this. This is your V_R; this angle we have shown by β and this whole angle as γ. Remember this angle and this angle are not same are not send this angle is equal to this angle.

And this angle this is equal to this angle and this angle this is equal to this angle and there γ and β they are not the same. Ok so from this place we will have tan β equal to V_E sin γ divided by $V_\infty + V_E \cos\gamma$ where γ is angle already we have define. γ is angle as shown here in this figure this angle is; ok also look here this ϕ_E angle I have shown here in this place. Φ_E is the angle between the horizontal line, which is the r_E line parallel to r_E and the V is vector.

So this is ϕ, so from ϕ if I subtract this angle, so we get this as γ . So here we can write here γ is equal to $\phi_E - l_E - l_H + \Pi$ this is your γ angle. So again shifting this, this is your γ angle from here to here this whole thing is γ as shown here in this place. So, if we want to show it outside. So this is your from this place to this place this is the γ angle β is already shown.

And where is the ϕ angle? Φ angle is the angle between this line and this line this is your Φ. This is $l_E - l_H + \Psi$ now you can get all the things here. So γ is equal to $\phi - l_E - l_H + \Psi$. So therefore, Ψ is known to us and perhaps we are written it ϕ or Ψ let me check. I have indicated here ϕ so here also I will write this is Φ_E. So this is your Φ. So Φ is known to us l_E l_H and Ψ all the unknown and therefore γ becomes known.

$$\gamma = \phi_E - (l_E - l_H + \psi)$$

So γ is known and therefore β can also be estimated. So, all the angles this way can be estimated. l_v we write as the longitude of vehicle which is nothing but $l_E + \theta$, $l_E + \theta$ angle between this and this, this is your l_v. So therefore, l_v is $l_E + \theta$ by vehicle is the longitude of vehicle this is ϕ_V longitude of vehicle is if we have written by l_v.

Here we have indicated this quantity we have to estimate so l_V will be ok already we have indicated this part will be equal to $l_E + \theta$ ok longitude of vehicle already we have indicated we need ϕ_v. So, ϕ_v here let us write it ϕ_v be defined as longitude of velocity vector of vehicle and how much this will be? ϕ_v now you can see here this is the velocity vector of the vehicle which is the velocity vector of the vehicle.

V_∞ is the excess hyperbolic velocity V_R is the longitude; V_R is the velocity vector of the vehicle. So we will have to write it on the next page maybe longitude of velocity vector of vehicle that means we are measuring this from the vernal Equinox this is γ this is the horizontal line and vehicle velocity say this is your V_∞ and this is V_R is your vehicle velocity. This angle is β angle is already we have written $l_E - l_H + \Psi$. And then others angle we have indicated as this is with respect to the horizontal.

And then from here this is your vehicle, this is your γ direction vernal Equinox direction show you here like this. Figure is little complicated what if we do it presently, we can do it this we are soon as l earth conditional and this angle we have shown as θ. Now we can write that angle. Longitude of this will be $l_E + \theta$ with respect to this line will write the longitude.

$$\phi_v = \theta + \beta + (l_E - l_H + \psi)$$

With respect to this how much this is going to make the angle. Ok so we have here this is a vector and B vector is let us look here in this problem we first write this as a $l_E - l_H + \Psi$ this is the angle which we have found here V_∞ that makes this angle V_∞ makes $l_E - l_H + \Psi$ this is the angle that it makes with the horizontal. I will clear it and write it properly. This angle we have written as $l_E - l_H + \Psi$. So with the horizontal this is making $l_E - l_H + \Psi$.

The V_R will be making angle $l_E - l_H + \Psi + \beta$. So, with the horizontal then that makes angle really $l_E - l_H + \Psi + \beta$. This is with respect to the horizontal with this line. So, with this then how much angle this it will make. This is the question. So Ψ we add θ to this $\theta + \beta + l_E - l_H + \Psi$ this gives us this longitude of the vehicle velocity. As we have written here ϕ_v.

All the angles we have to write here properly to work it out. ok so that is what we are doing here that once we are writing here ϕ_v equal to $\theta + \beta + l_E - l_H + \Psi$. This angle we have added which is $l_E - l_H + \beta$ we have added θ we have added. So with respect to this line where this V_R How much handle this is making? If we draw it like this, this is the angle here we are writing as ϕ_v.

So we should not rather write this as the longitude that the angle of the velocity vector or vehicle with respect to the radius vector. We should write it this way. The longitude is always measured with respect to the vernal Equinox that would not be a correct term. I will change this to ϕ the angle of velocity vector of vehicle which is V_R with respect to the radius vector of the vehicle from Sun to the vehicle this is again with respect to the sun.

Why this is important because this appears like something like if you remember in your description of the earth, the velocity vector B is given if this angle is we are writing as ϕ_A flight path angle immediately we can make out from this place which will be $\Phi_v - \Phi$ by 2 equal to ϕ

this way we can write. And therefore, once this angles are known so will be able to calculate all the heliocentric parameters of the vehicle.

So this nomenclature we can change so that we do not confuse with the actual long longitude which is measured with respect to the vernal Equinox.

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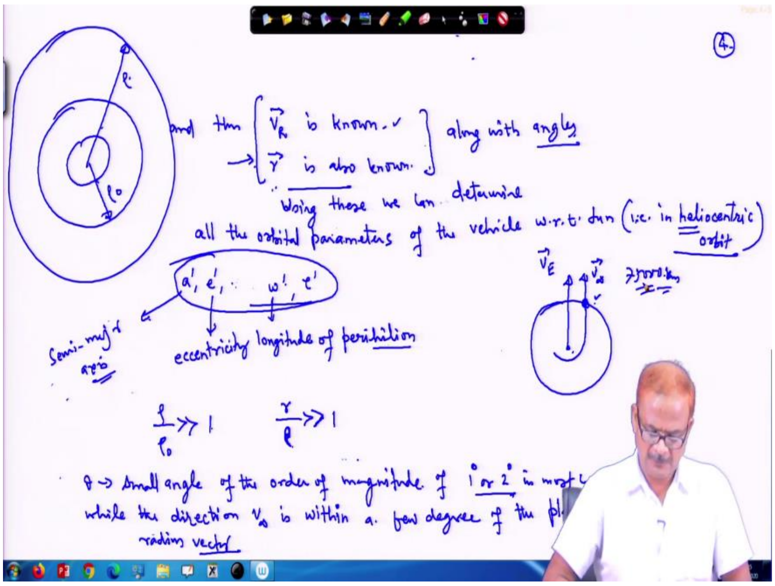

And thus V_R is known and r is also known along with the corresponding angles along with angles. Once this is known for you know in the orbit determination problem. We can know the orbit with respect to the sun. So using this we can determine all the orbital parameters of the vehicle with respect to the sun that is in your heliocentric orbit. So heliocentric orbit parameters will be able to determine from this place what those are? Those are $a', e', \Omega', \omega', \tau'$ time of perigee places all these things will be known to us. And this is what we are looking for.

Ω is not required here. This is not of concern to us because we are looking into the planet motion until and unless we refer to the inclination and Ω those are out of orbit parameters so those are not of interest to us. Only what are the interest to us are these four parameters a', e', ω', τ'. This is longitudinal of this called as a perihelion.

This is the eccentricity as usual. Why perihelion it is coming from the heliocentric because it is about the sun and this is semi major axis. Also, we should note that ρ by ρ_0, ρ_0 is the radius of the parking orbit as we have done last time. This is your parking orbit ρ_0 and outside this we have a sphere of influence who is radius we have written as rho. This is much greater than 1 and r by ρ this will also be much greater than 1.

As compare to the sphere of influence the distance of the vehicle from the sun this is very large. And therefore, θ this becomes small angle as shown in the previous figure small angle of the order of magnitude of 1° or 2° in most cases while the direction of V_∞ is within a few degree of the planet to centric radius vector. What is the Planet to centric radius vector? This is ρ.

So with ρ the V_∞ is making only few degree of angle. So ideally you should happen that once the launch is done. And if this is the velocity of the earth, so your after the launch V_∞ should come in the same direction, ideally, but this will not be the situation as shown in the previous figure. If this is a contrast situation then we have to take into account all the angles and worked out.

A small angle angular or the velocity estimation error in this place it results in thousands of kilometres like the 75,000 like that errors while it reaches the mass it is very sensitive. Part of this sensitivity analysis is also there but we are not going to all those things not already the school as suggested a lot. So we want to keep it short. While we do simulations, all these things will be obvious in the same.

If you are; already coded and in the computer then by giving a small error you can check that how much error creating. So we will stop here and will continue in the next lecture. Thank you.