Space Flight Mechanics
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Lecture – 93
Interception in Coplanar orbit

Welcome to lecture 93; last time we have looked into the interplanetary transfer. This time we will look into land review problem and that it is basically intersection of two vehicles or you want to dock 2 vehicles. So first we have to bring them near to each other. So what are the processes involved in that, that we are going to discuss today. To mathematical aspect basically will look into.

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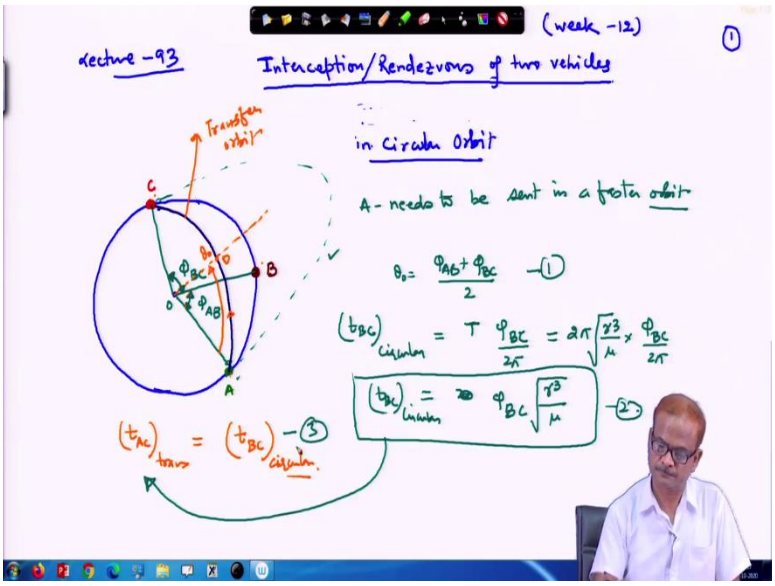

The intersection of Rendezvous of 2 vehicles in circular orbit; so, this is in circular orbit. Let say this is a circular orbit about the earth; it can be any planet but let us assume that is to be the earth. This is planet A and we have another planet B here and it is intended that A catches B at the point C. And they may not be necessarily equally spaced they can be anywhere. I will make the figure that way only. Let us say the point is here.

This is the point C. These points we write as O. So, this will be OB and this is OC. This angle is ϕ_{AB} and this angle we write as ϕ_{BC}. So, these are the angles to be covered and let us assume that; so for sending touching B at point C by the satellite A this will involve sending the satellite

A in faster orbit. So A needs to be sent in faster orbit. So previously we cannot send it in some orbit like this because this will take much more time to go through this orbit.

So time of travel will become even larger. So one option to this is; that is we sent it in a faster orbit and that orbital may look like something like this. And let us say that this is the perigee point. So this bisects AC into two parts. So here this is the transfer orbit and satellite here needs to be sent along this. So, that by the time B reached to C ok A reaches from A to C along the transfer trajectory.

$$(t_{AC})_{trans} = (t_{BC})_{Circular}$$

So t_{AC} belongs to transfer trajectory this will be equal to t_{BC} along the circular trajectory this is the requirement. And once we fulfill this requirement our job is done. So, this point let us say this point D system hyperbolic arbitrary plan or maybe elliptical it depends on the situation, but it will emerge as the ones you work out at that time it will emerge which orbit is required. So beforehand you cannot decide it. So we assume that the perigee is located here in this point and it is a symmetrical point with respect to A and C.

Now the location of this from this point we assume this angle to be θ_0, this is θ_0. This angle is ϕ_{BC} and this angle is; this whole angle from here to here this is ϕ_{BC}. So we will have θ_0 equal to $\phi_{AB} + \phi_{BC}$ divided by 2. Now t_{BC} the time taken in the circular orbit this will be equal to the time period in the circular orbit, which we can write as t time period in the circular orbit.

And then it has to do proportionally divided into by the corresponding angle. So this will be ϕ_{BC} divided by 2π . From this place we get t is equal to 2π AQ by μ this is a circular orbit. So, this r remains constant for this. So we can write this r^3 divided by μ times ϕ_{BC} divided by 2π . So this ϕ gets reduced to ϕ_{BC} times r^3 divided by μ under toot.

$$\theta_o = \frac{\phi_{AB}+\phi_{BC}}{2}$$

$$(t_{BC})_{Circular} = T\frac{\phi_{BC}}{2\pi} = 2\pi\sqrt{\frac{r^3}{\mu}} \times \frac{\phi_{BC}}{2\pi}$$

$$(t_{BC})_{Circular} = \phi_{BC}\sqrt{\frac{r^3}{\mu}}$$

And we are to match this time with this time in that transfer orbit this is required, it is r^3. If this is matched then you got the correct solution.

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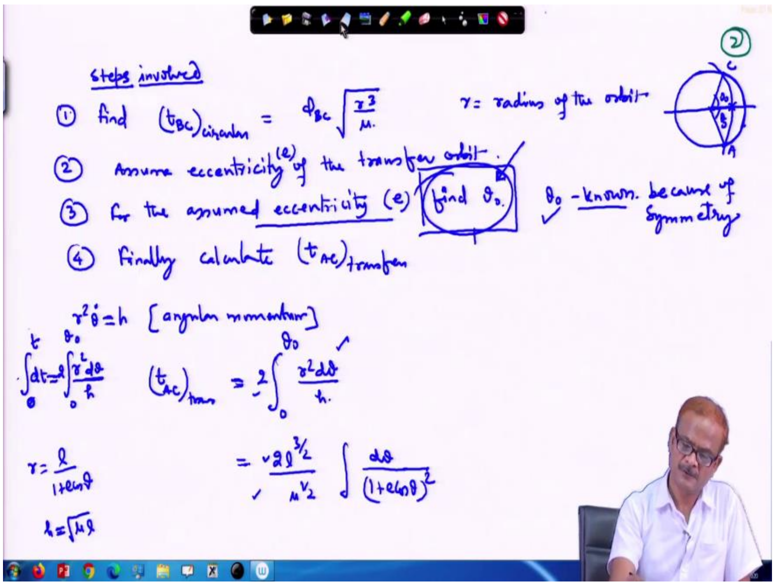

So, now we have to find out what that hyperbolic orbit or elliptic orbit will be in which I have to send the satellite. So that it catches B at C. The steps involved to find t_{BC} in the circular orbit this already have done. This is ϕ_{BC} times r^3 divided by μ where r is the radius of the orbit assume eccentricity of the transfer orbit. For the assumed eccentricity is e what is required that we need to know the θ_0. Find the θ_0.

Then finally calculate ah time involved A to C in the transfer order t_{AC} in the transfer orbit. So these are the steps involved and equation that we need to use is the same as we have done earlier. So we have $r^2\dot{\theta}$ into h; this is the angular momentum equation. And from there, we have written dθ by dt, so r^2 dθ divided by h and this we can write as dt and this is the thing that we are looking for.

$$r^2\dot{\theta} = h$$

$$(t_{AC})_{trans} = 2\int_0^{\theta_0} \frac{r^2 d\theta}{h}$$

So this will be two times 0 to θ_0, this is the halfway. So either from this place to this place or either from this place to this place because this is a perigee so the perigee we can measure the distance. So twice of that the time taken will be this and this is θ_0 and this is also θ_0. So, left hand side we write time 0 to; or simply this side what will do? We will keep it 0 to t and this side we will write 2 to θ_0.

$$= \frac{2l^{\frac{3}{2}}}{\mu V_2}\int \frac{d\theta}{(1+e\cos\theta)^2}\ ;\ \int_0^t dt = \int_0^{\theta_0} \frac{r^2 d\theta}{h}, r = \frac{l}{1+e\cos\theta}, h = \sqrt{\mu l}$$

So this will give us t_{AC} transfer time $r^2 d\theta$ by h_0 to θ_0 times 2 and this quantity already we have evaluated for the ellipse and for hyperbola and all others. This is what we can write where r we are replacing by 1 + e cos θ and h we have written in terms of μ times l under root, so you get this equation. And these 2 comes from the above. Once we have done this now we require absolute equation.

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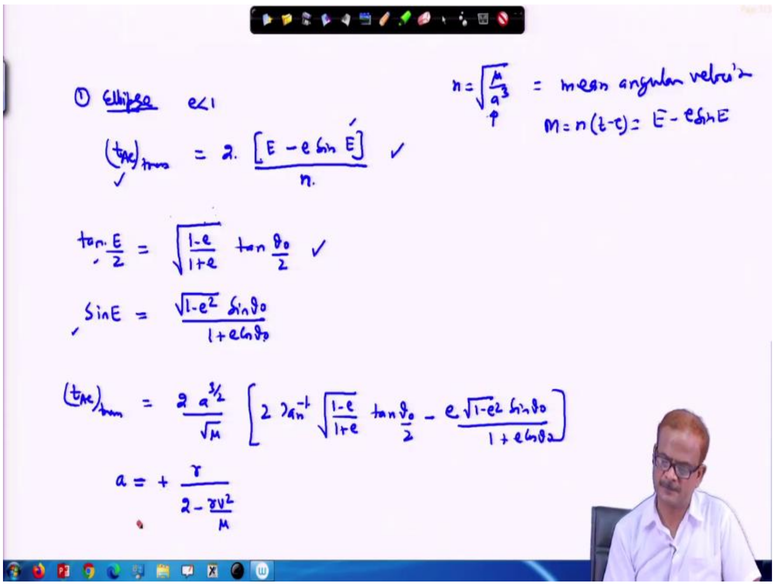

So already we have all the equation we will list here for the case, ellipse. Where e is less than 1, so we have got the solution as t_{AC} transfer will be equal to 2 times E - e sin E divided by n where n is nothing but μ by a^3 under root. So mean angular velocity in this case it is a constant mean sorry in this case we have, for the transfer eclipse this is not a mean angular velocity remains constant.

$$(t_{AC})_{trans} = 2\frac{[E - e\ \mathrm{Sin}\ E]}{n}$$

If the angular velocity itself is constant; so n equal to mean angular velocity; so we are using equation which is here n t – τ equal to E – e sin E. And E – e sin E this can be further written in terms of E and sin E and so let me list that also. So, that we refresh all these things. So tan E by 2 this quantity we have 1 - e by 1 + e under root tan θ_0 divided by 2 this is the equation we have used.

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\theta_0}{2}$$

And also the sin E is equal to 1 - e^2 under root times sin θ_0 divided by 1 + e cos θ_0. So these are the things that we have derived earlier we need not work out again and again. So, sin E is known from this place. E will be known from this place. Therefore, if we have assumed the value of E and then we calculate θ_0 and thereafter then we can work with this equation either if the case of ellipse.

Similarly, for the case of parabola equation we have written. So the t AC transfer time this will be two times; it can be reduced to 2 times a^3 by a to the power 3 by 2 divided by μ because n is in denominator. So you can see that a^3 it will become a^3 by μ under root a^3 to the power 3 by 2 divided by μ under root and this times 2 times tan inverse this quantity we are using 1 - 3 divided by 1 + 3 under root and θ_0 divided by 2 – e times sin E.

$$\text{SinE} = \frac{\sqrt{1-e^2}\,\text{Sin}\,\theta_o}{1+e\,\text{Cos}\,\theta_o}$$

$$(t_{AC})_{trans} = \frac{2a^{\frac{3}{2}}}{\sqrt{\mu}}\left[2\,\text{Tan}^{-1}\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta_o}{2} - \frac{e\sqrt{1-e^2}\,\text{Sin}\,\theta_o}{1+e\,\text{Cos}\theta_o}\right]$$

$$a = \frac{r}{2-r\frac{v^2}{\mu}}$$

So e times 1 – e^2 sin θ_0 divided by 1 + e cos θ_0. So this is the equation that we need to use and for this transfer orbit a will be given by plus minus r by 2 - rv^2 divided by μ. So for the ellipse we will have only the plus sign here. If it is hyperbola, then this place will get replaced with a minus sign.

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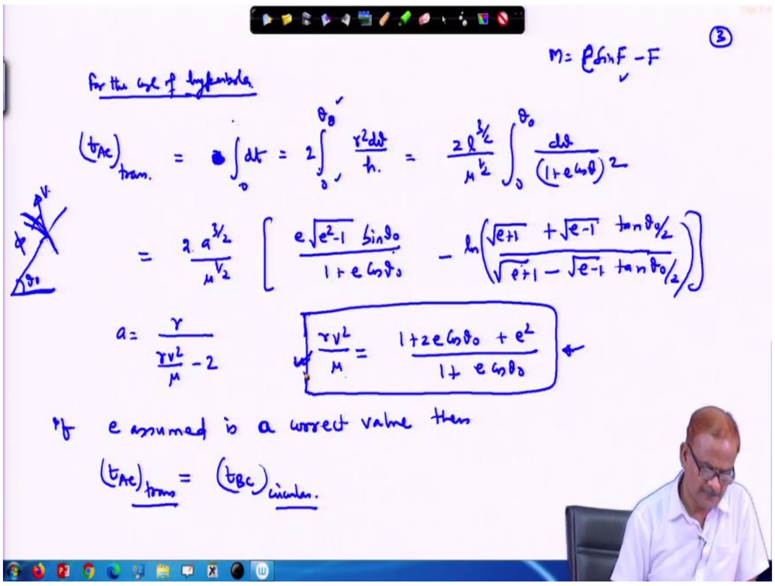

For the case of hyperbola t_{AC} transfer similarly this will be 2 dt or simply we write here dt_0 to t ok we can write like this dt not to confuse anything and on the right-hand side, whatever we have written earlier that we need to do the 2 times 0 to θ_0 $r^2 d\theta$ divided by h. As in the previous case and this gives us the equation, we have already derived this while discussing the Kepler's equation.

$$(t_{AC})_{tran} = 2\int_0^{\theta_o} \frac{r^2 d\theta}{h} = \frac{2\, l^{\frac{3}{2}}}{\mu^{\frac{1}{2}}} \int_0^{\theta_o} \frac{d\theta}{(1+e\cos\theta)^2}$$

So you can refer back to that portion. For the hyperbola we have the Kepler's equation M equal to e sin F - F this is what we have written. So instead of capital E we have here capital F -1n e + 1 under root + e – 1 under root sin θ divide by 2. So this is integration from 0 to 2 then we get this expression. Where a is equal to r divided by rv^2 divided by μ -2. And also the relationship we have derived earlier rv^2divided by μ is equal to 1 + 2 e cos θ_0 + e^2 divided by 1 + e cos θ_0.

$$= \frac{2\, a^{\frac{3}{2}}}{\mu^{\frac{1}{2}}} \left[\frac{e\sqrt{e^2-1}\,\sin\theta_o}{1+e\cos\theta_o}\right] - \ln\left(\frac{\sqrt{e+1}+\sqrt{e-1}\tan\frac{\theta}{2}}{\sqrt{e+1}-\sqrt{e-1}\tan\frac{\theta_o}{2}}\right)$$

$$a = \frac{r}{r\frac{v^2}{\mu}-2}$$

$$\frac{rv^2}{\mu} = \frac{1+2e\cos\theta_o+e^2}{1+e\cos\theta_o}$$

This we have done while doing the orbit determination problem before that we did it ok, so we derived all these quantities. So these 2 are the standard equation that we need to work out. So if e is assumed is a correct value then t_{AC} transfer will be equal to t_{BC} along a circular orbit otherwise it will not be true. And while working we have to careful that this is the; we written this equation in terms of the true anomaly.

$$(t_{AC})_{trans} = (t_{BC})_{trans}$$

So the velocity vector is here. This is the orbit here. And tangent to this the velocity vector is there in this direction this is the ϕ angle. This is the radial direction. So this way we are getting this equation. Now we go ahead with completing all the steps.

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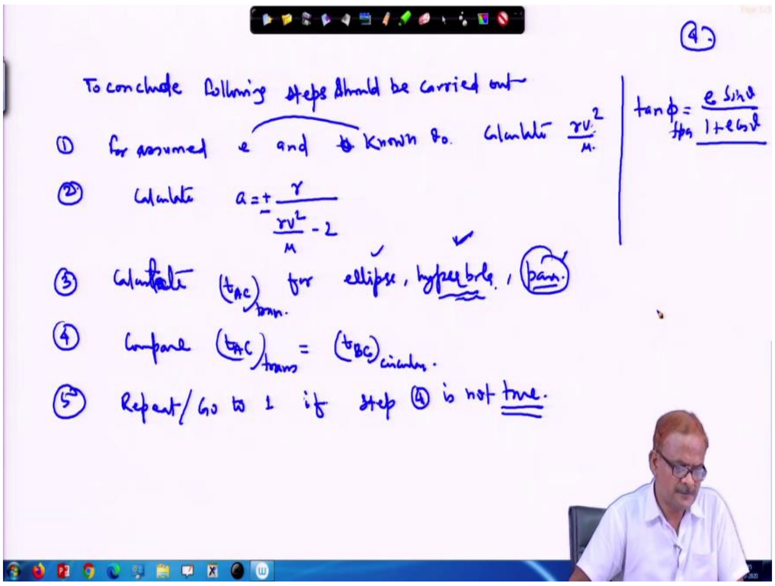

So to conclude, following steps should be carried out. For assumed e and known θ_0. θ_0 determine from this assumed e known θ_0 calculate rv^2 by μ. Calculate a equal to r divided by rv^2 divided by μ this will be plus minus 2 or 2 - rv^2 by μ. Once I put here plus minus accordingly you have to be careful while working with this.

$$a = \pm \frac{r}{\frac{rv^2}{\mu} - 2}$$

Then calculate t_{AC} in the transfer orbit for ellipse or hyperbola or it maybe even parabola but parabola coming it is a very rare. It may be either ellipse or hyperbola. Most of the time you have to go faster so hyperbola will be required, automatically it will emerge. So forth compare t_{AC} this is transfer orbit with t_{BC} in the circular orbit. Repeat slash go to 1, if step 4 is not true.

$$\tan\phi = \frac{e \text{ Sin } \theta}{1 + e \text{ Cos } \theta}$$

This implies that it is only through iterative process you will get the solution not otherwise. And also we can note that tan ϕ which is the flight path angle. Ok this equation we have written as e sin θ divided by 1 + e cos θ. We have derived earlier. So this also you will require this for solving the problem. So once we are done with this.

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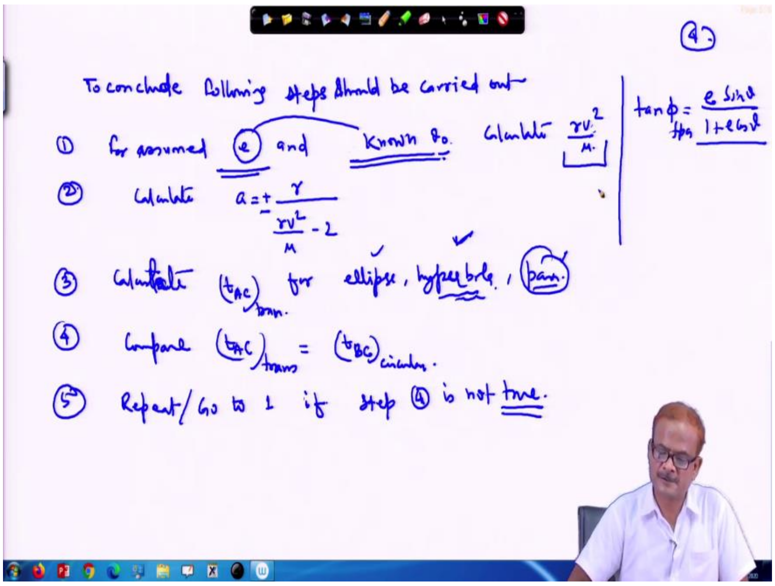

So in this process what is important that we have to assume e and for the assumed e from there we can get known θ_0. θ_0 is already calculated. Let me check those steps what I have written here. For the assumed eccentricity e this is not required always. This step for assumed eccentricity, find θ_0 this is not always required. In the circular case this θ_0 is known because of symmetry.

So the circular face because we are; for the circular face if you look the orbit, we can always do like this and this is points A and this is point C. So we can always divide like this θ_0 and this is θ_0. So θ_0 is known here in this case because A and C; the angle between this to this is A to C; angle is known to us so θ_0 is known here in this case. But in the elliptic if you have the original orbit in which the satellites are moving.

If it is an elliptic then in that case you need to find θ_0. So therefore, here in this case it is written here. For assumed e and known θ_0 it is not written that we have to work out θ_0. So for circular orbit, you are not required to work out θ_0. But for elliptical orbit you need to work out θ_0. And once you know the θ_0 rv^2 divided by μ this can be computed. Using this equation, we have written earlier this equation.

So we stop here and in next lecture we will start in which we do that Rendezvous problem in the elliptical orbit and thereafter we will take the non coplanar orbits and its intersection. Thank you very much.