Space Flight Mechanics
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Lecture – 94
Interception in Coplanar orbit (Contd.,)

Welcome to lecture 94 now we are going to look into the intersection problem between two vehicles in elliptical orbit. Already we have done it for the circular orbit.

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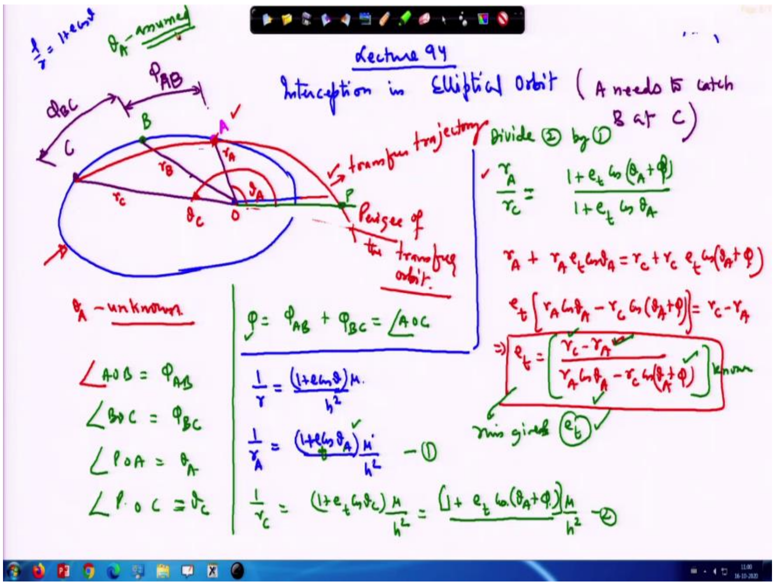

So let us start with, so here circular or elliptical orbit is given 1 satellite is located at$_{A}$ and another satellite is located at B and it is required that A touches B at C. A needs to catch B at C, so angles are given in here in this case. And these angles not equal and it may appear from the figure that it is equal but it is not equal. So this as usual we will show this as; let us show it outside this angle is ϕ_{BC} angle between this and this, this is ϕ_{AB}.

And this is ϕ_{BC}. Now we need to send the satellite and faster orbit so let us say this is the orbit that we choose. And for this orbit the perigee lies here in this position perigee of the transfer orbit which we can show here as θ or this is the location of this we can write this as θ_{A}. And similarly, the location of this from this place from this perigee this will be θ_{C}.

We are not worried about that θ_B because θ_B does not lie on the transfer trajectory. So this is to transfer trajectory. So we are not worried about this. So here θ_A is not known to us, so θ_A this is unknown and we need to work out this quantity. So will write here the angles this point is O and perigee as it appears from this place that it is lying along this ground line but that is not the truth.

I can make it little look different. Let us say this is like this, this is the perigee point. So the angle AOB this is ϕ_{AB}, angle BOC is equal to ϕ_{BC} as we have named it and angle POA is equal to θ_A angle POC is equal to θ_C. So these are angle we are taking we will write ϕ equal to ϕ_{AB} + ϕ_{BC} is equal to angle AOC. Now we need to recall what I have done earlier for 2 non-tangent bond.

$$\angle AOB = \phi_{AB}$$
$$\angle BOC = \phi_{BC}$$
$$\angle POA = \theta_A$$
$$\angle POC = \theta_C$$
$$\phi = \phi_{AB} + \phi_{BC} = \angle AOC$$

How the satellite orbit is to be decided? The transfer trajectory is to be decided this we have done earlier. So that information we need to use here in this place. So let us write 1 by r equal to 1 + e cos θ this is the expression 1 by r we write as 1 + e cos θ divided by l so this l can be written in terms of h^2 time μ h^2 divided by μ so μ goes to numerator.

$$\frac{1}{r} = \frac{(1+e\mathrm{Cos}\theta)\mu}{h^2}$$

Following this equation then we can write r_A equal to 1 by e cos θ_A e times cos θ_A times μ by h^2. And here one should remember that while we are using this equation and θ here we are writing. So θ_A and θ_C this quantity are defined only along the transfer trajectory. So here we have it should be e t, this is e t. Similarly, we will have 1 by r_C 1 + e t cos θ_C μ by h^2 and this quantity is nothing but 1 + e t cos θ_C, cos θ_C is; If you look for the angle θ_C is θ_A + ϕ.

$$\frac{1}{r_A} = \frac{(1+e\ \mathrm{Cos}\theta_A)\mu}{h^2}$$
$$\frac{1}{r_c} = \frac{(1+e_t\mathrm{Cos}\ \theta_c)\mu}{h^2} = \frac{\left(1+e_t\mathrm{Cos}(\theta_A+\phi)\right)\mu}{h^2}$$

As we have defined here in this place times μ divided by h^2 or simply we can write in terms of l it does not matter. So r_A by r_C we divide the second let us say this is 1 and this is 2 so divide 2 by 1 and this will yield r_A by r_C equal to 1 + e t cos θ 1 + ϕ divided by 1 + e t cos θ_A. Now we will look into this place r_A is the quantity which is known to us because this is defined by also where it is located with respect to the in the original trajectory.

$$\frac{r_A}{r_C} = \frac{1+e_t \text{Cos}\,(\theta_A+\phi)}{1+e_t \text{Cos}\,\theta_A}$$

In this trajectory where A is located, this is known to us because that angle is given. Position of that will be known to you. So r_B will also be known and r_C will also be known to you. So these angles are known then therefore it can be determined where they are located. So we utilise all this information for solving this problem and therefore from here we can write if we arrange. This is r_A + r_A e t θ r_C e t cos θ_A + ϕ. And separating out e t r_A times cos θ_A – r_C times cos θ_A + ϕ is equal to $r_C - r_A$.

$$r_A + r_A\rho_t \text{Cos}\,\theta_A = \; r_c + r_c e_t\, \text{Cos}(\theta_A + \phi)$$

$$\rho_t[r_A\, \text{Cos}\theta_A - r_c\, \text{Cos}\,(\theta_A + \phi)] = r_c - r_A$$

$$\rho_t = \left[\frac{r_c - r_A}{r_A \text{Cos}\theta_A - r_c\, \text{Cos}(\theta_A+\phi)}\right]$$

This implies e t is equal to θ_A + ϕ now look here in this equation it is important to discuss this r_A is known. This r_A is known this r_C is known θ_A we are assuming ϕ is known. All these quantities on the right-hand side in the situation is known. So, this side is known and therefore this helps you, so this gives e t. Now we can proceed to work. So, once it is known and we have assumed already the position θ_A.

θ_A is assumed therefore it is known and rest others r_v^2 by μ as we were discussing earlier. So it can be computed.

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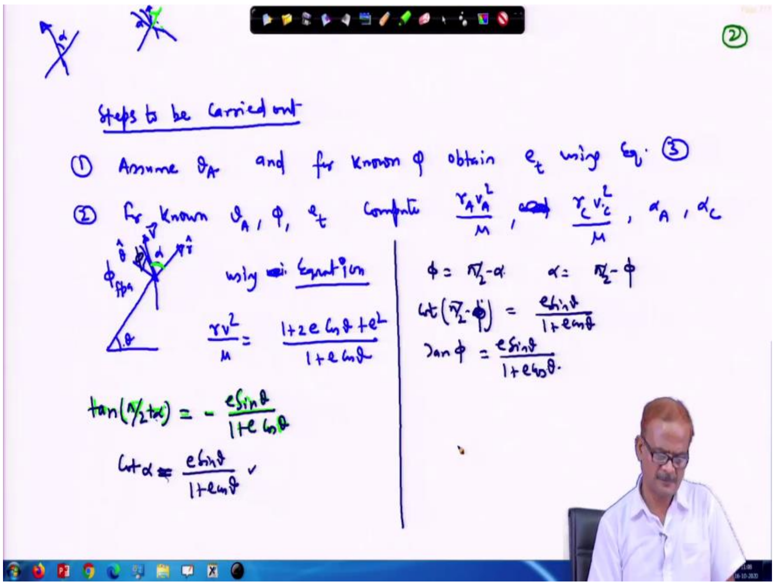

This is 1 and 2 and this let us say this equation is 3. Now steps to be carried out assumed θ_A and for known ϕ, ϕ is known as it is written earlier and for known ϕ obtain e t using equation 3. This is the first step. Now for known θ_A already assumed so θ_A, ϕ and e t compute $r_A V_A^2$ divided μ and are $r_A V_C^2$ divided by μ and the corresponding angle where it is located.

So this is suppose θ and this is trajectory to this, this is B this we can say this is α and this is the flight path angle ϕ and this is B and this is r greater than $\hat{r}$, this is θ direction this is $\hat{\theta}$. So we need to calculate all these things α_A at the point A and α_C. Using equation which equation we have to use; already mentioned rv^2 divided by μ 1 + 2 e cos θ + e^2 divided by 1 + e cos θ this we need to use.

$$\frac{rv^2}{\mu} = \frac{1+2e\,\text{Cos}\theta+e^2}{1+e\cos\theta}$$

Here θ appears as the true anomaly. And here tan α we want to decide α want to write. So see α I have written from this point somewhere it may be also if this is v direction I have written α from here. α can be; let me draw little bit properly. So this is v direction this I have shown as A.

But α can be measured from the horizontal also. That means α I can take from this place to this place. But this I am not taking throughout perhaps as far as I remember I have used with respect to this, this I have decided as α angle. This I always taken as α angle, we have to be careful about this. So tan ϕ by 2 + α this will be given by -e sin θ look back into the equation we have derived e cos θ, -e sin θ divided by 1 + e cos θ.

$$\tan\left(\frac{\pi}{2}+\alpha\right) = -\frac{\text{e Sin }\theta}{1+\text{e Cos }\theta}$$

So from this place we get cot α equal to e sin θ divided by 1 + e cos θ. So cot α we have written like this so here ϕ equal to this angle is ϕ so ϕ equal to ϕ by 2 - α so cot ϕ by 2 – A. α becomes all ϕ by 2 - ϕ we write here in terms of ϕ ϕ by 2 - ϕ equal to e sin θ + 1 by e cos θ or equally we can write here tan ϕ as we have written earlier. So this equation needs to be used whenever required. Rest $\dot{r}$ and other things already I have written.

$$\text{Cot }\alpha = \frac{\text{e Sin }\theta}{1+\text{Cos }\theta}\,;\ \phi = \frac{\pi}{2}-\alpha$$

$$\text{Cot }\left(\frac{\pi}{2}-\phi\right) = \frac{\text{eSin }\theta}{1+\text{eCos}\theta}$$

$$\text{Tan}\phi = \frac{\text{eSin }\theta}{1+\text{eCos}\theta}$$

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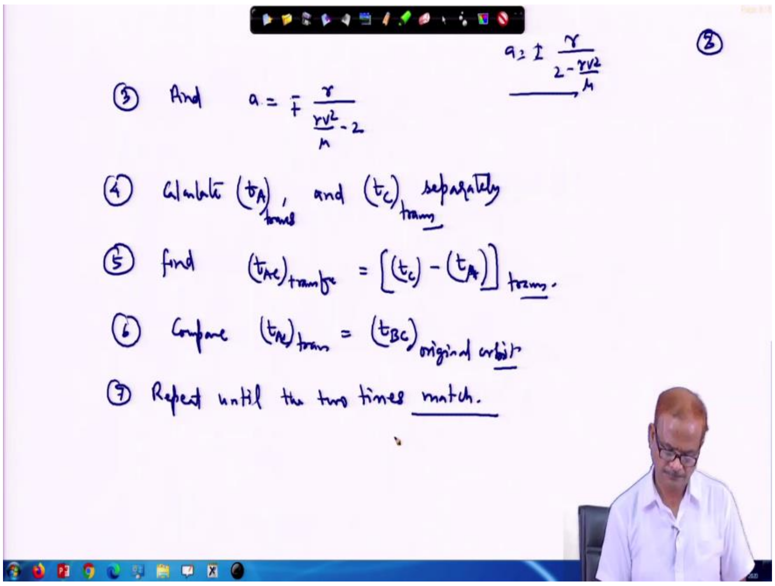

This is$_A$ step no number 2. Find a equal to plus minus r by rv^2 by μ -2 or 2 - whatever the way you write we can write here minus plus r by 2 - rv^2 divided by μ both can be written here. Once we write plus minus irrespective of either I write 2 first or rv^2 by μ first does not matter. So we have to choose it properly. Calculate t_A now here in this case because it is elliptical orbit we need to calculate t_A and t_C separately.

$$a = \mp\frac{r}{\frac{rv^2}{\mu}-2}$$

And t_C we need to calculated separately. And from there point t_{AC} transfer and this is to be done in the transfer orbit, t_{AC} transfer will be t_C – t_A in the transfer orbit then compare t_{AC} transfer with t_{BC} in the actual orbit this is an original orbit. If this is equal your job is done, if not then we have to repeat from Step 1. Repeat until 2 times match. So this is the process involved.

$$(t_{AC})_{transfer} = [(t_c) - (t_A)]_{transfer}$$

Now based on this I will take the case of$_A$ non coplanar orbit problem. And because non coplanar orbit or planar orbit it does not matter much since the process is the same. So I will solve one problem based on this.

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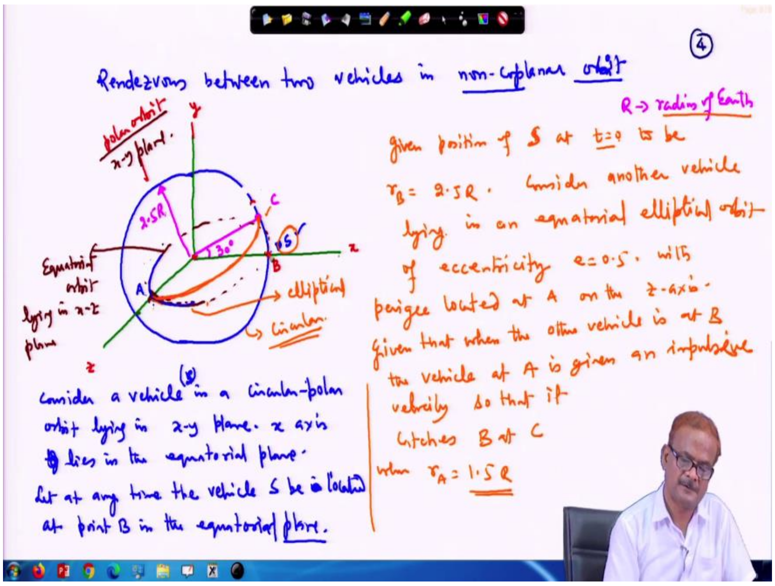

Rendezvous between two vehicles in non coplanar orbit, here in this case and I can compose this problem in multiple ways. So say this is your polar orbit. And here we write z this is x and this is y. This is point B and here is some point C this is point C. Latitude is 30° angle. Radius of the circle is 2.5 R Earth. So R Earth I will simply write it as 2.8 R is the radius of earth.

The problem it states like this. I will have to write the problem here. And let us consider another orbit whose perigee lies on the z axis and at A distance 1.5 hour. And from here; and this is originally an elliptical orbit lying in the xy plane. So this is analytical orbit lying in the xz plane. So this we called as equatorial orbit. This is your equatorial orbit lying in xz plane and this lying-in polar orbit in the xy plane.

Quickly I write the statement for this. Consider vehicle in a polar orbit in circular polar orbit lying in xy plane x axis lies in the equatorial plane. Consider a vehicle we name this vehicle let us name this vehicle as V. consider a vehicle V what will get confused? WE write this as satellites. So let at any instant and let at any time the vehicle S; be located at point B in the equatorial plane.

Your vehicle is lying here at point B and what is to be done? That another satellite which is moving in this tutorial orbit so here this is its perigee position and this position we are writing as a. So we have to decide an orbit such that if I give impulse along this orbit at point A so it goes and catches B at C. So, this is what is required it goes and catches B at C. So, given position of S at t is equal to 0 to be r_B equal to 2.5 R where R is the radius of earth. As written here.

Consider another vehicle lying in and equatorial elliptical orbit of eccentricity so this is$_A$ circular orbit. And this is your elliptical orbit of eccentricity is equal to 0.5 with perigee located at A on the z axis and given that when the other vehicle is at the vehicle at A is given and impulse velocity or impulsive velocity so that it catches B at C; where r_A is given to be 1.5R.

So see latitude is given to be 30 degrees so this is the information and you have to decide how much impulse need to be given at A such that the vehicle at A will go and catch the vehicle this is at the point C this is the first objective. We will continue in the next lecture. I have stated the problem. Thank you very much.