Space Flight Mechanics
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Lecture – 95
Interception in non-coplanar orbit

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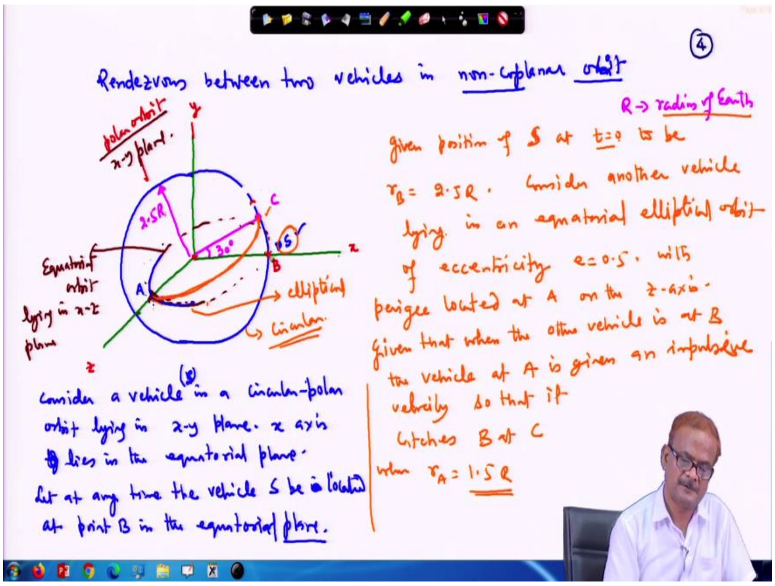

Welcome to lecture 95. So ah already we were discussing about the intersection in non coplanar orbit so this is the figure I have shown here and I have stated the problem also in the last lecture. So we are going to work out this lecture and from this already is A basic concept I have given. The same concept is going to be applied to this problem.

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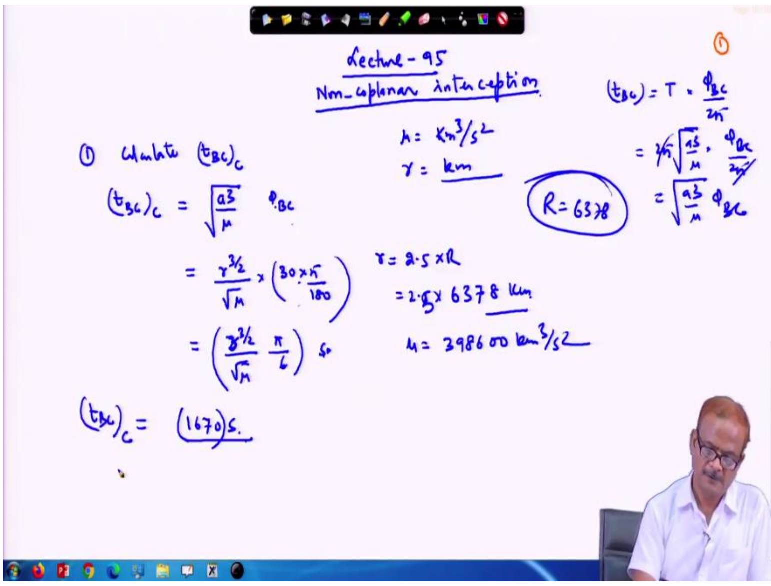

So the first step we calculate the time required to go from this point to this point. So this is t_{BC} in the circular orbit. This is A case of circular orbit. This can be also A elliptical orbit the polar orbit can also be elliptical. To calculate t_{BC} in the circular orbit t_{BC} in the circular orbit this will be time period. So already we have written this particular part a^3 divided by μ under root times Φ_{BC} from B to C that angle we need to use here.

So Φ_{BC} this angle we can indicate this as Φ_{BC} this is ϕ_{BC} so Φ_{BC} by 360 ° or 2π write here in degree so accordingly we have to manage. And A is here 2.5 hour so this becomes r to the power 3 by 2 times μ under root and here 30° + 360°. What we are doing that t_{BC} is quantity will be 3 times ϕ_{BC} divided by 2π so this is 2π a^3 by μ under root and Φ_{BC} by 2π ; 2π 2π cancels out this.

$$(t_{BC})_C = \sqrt{\frac{a^3}{\mu}}\ \phi_{BC}$$

So Φ_{BC} we have to write in radians. So this we should not write here. If we are writing this way for this part should not come. And Φ_{BC} once we have expressed this way so Φ_{BC} should enter as radian. 30° times Π by 180 ° so r to the power 3 by 2 divided by μ under root and 5 by 6 this much of seconds. So, we are going to use unit of μ in kilometre^3 per Second^2.

$$= \frac{r^{\frac{3}{2}}}{\sqrt{\mu}} \times 30\ \times \frac{\pi}{180}$$

$$(t_{BC})_C = 1670\ s$$

R unit will be using as kilometre this will come in seconds. So unit will be in terms of seconds. So if we insert the value now here r is equal to 2.5 times R, so 2.5 and R is 6378 kilometre. I have taken here the radius of earth is 6378 which is the equatorial radius. If we insert these values here in this place we get here and μ we have taken here as 398600 kilometre^3 per Second Square.

$$\text{Where, } r = 2.5\ R = 2.5\ \times 6378\ km$$

$$\mu = 398600\,\frac{km^3}{s^2}$$

So insert these values and then you get this has 1670 seconds. So this is time in the circular orbit.

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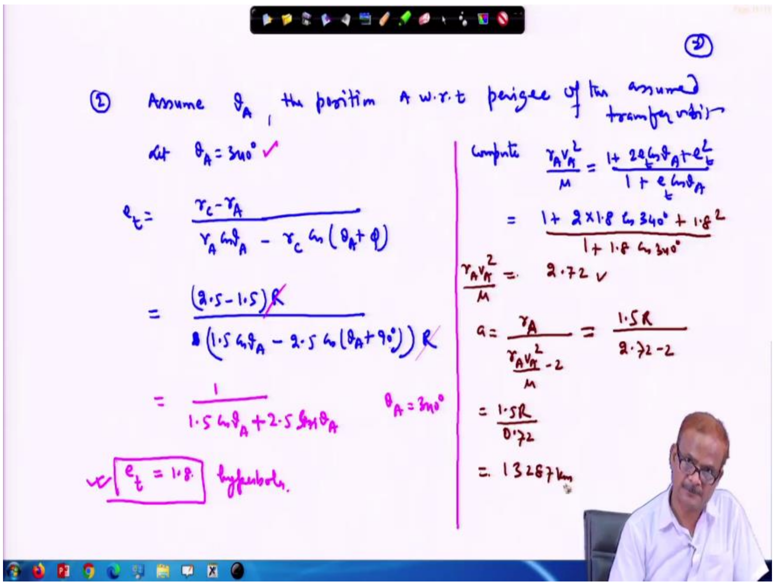

Now the second state is assumed θ_A. And this records little bit skill or experienced visualisation and so which is the position of A with respect to perigee of the assumed transfer orbit. Let θ_A is equal to 340°. Then we use this expression it is equal to r_C - r_A divided by r_A cos θ_A cos θ_A + Φ. You can assume any other value there is no problem in that.

Then r_C we have to insert r_{AC} we have to insert all these things we need to insert in this. So in the transfer orbit now you can see that this is A transfer orbit shown by the orange line and r A is 1.5R and r_C is 2.5 R. So r_A and r_C is known to us. So we can utilise those information 2.5 - 1.5 R divided by 1.5 cos θ_A - 2.5 cos θ_A + Φ, Φ here in this case the angle from this pink line to this green line this is 90°.

So, this is 90 ° here and then r we have to take it outside. So this R and this R we can cancel out. This leaves us with 1 divided by 1 + 1.5 cos θ_A - 2.5 cos θ_A + 90 become sin θ_A. And this θ plus; because this is in the second quadrant, we replace it by plus here in this place Cos θ_A + 90 °. So this e_t and θ_A is here 340°. So e_t will turn out to be 1.8.

$$\theta_A = 340°$$

$$e_t = \frac{r_c - r_A}{r_A \text{Cos } \theta_A - r_c \text{ Cos}(\theta_A + \phi)} = \frac{(2.5-1.5)R}{2(1.5 \text{ Cos } \theta_A - 2.5 \text{ Cos } (\theta_A + 90°))R}$$

$$= \frac{1}{1.5 \text{ Cos}\theta_A + 2.5 \text{ Sin}\theta_A}$$

$$e_t = 1.8$$

So that means for the assumed θ_A the orbit comes out to be A hyperbola. Now compute $r_A v_A^2$ by μ this we need to work out. And this quantity is $1 + 2 e \cos \theta_A + e^2$ divided by $1 + e \cos\theta_A$. So you can observe here that 2e is already given this is e_t is 1.8 and Cos θ_A this is 340°. But for here if you go and look here in this figure cos θ_A itself locate; you are locating this perigee you are locating this line with respect to perigee of the transfer orbit.

Perigee of the transfer orbit ah sorry the point which you are at which you are launching that you are deciding with respect to the perigee of the transfer orbit. So here let us say as it is shown here. It will be difficult to show here in this figure, but I will try let us say this is the perigee of the transfer orbit. From this place to this place, we have to go all the way from here and come to this place this gives you 340° then.

$$\frac{(r_A V_A^2)}{\mu} = \frac{1+e_t \text{Cos}\theta_A + e_t^2}{1+e_t \text{Cos}\theta_A}$$

That means the location at which you are launching your satellite is location with respect to the perigee of the transfer orbit is 340°. Perigee is lying ahead of the perigee of the transfer orbit lying ahead of the other point A here in this case. So we insert all these values 1.8^2 divided by 1 + 1.8 cos 340°. Then this gives you $r_A v_A^2$ divided by μ equal to 2.72.

$$= \frac{1+2 \times 1.8 \text{ Cos } 340° + 1.8^2}{1+1.8 \text{ Cos } 340°}$$

$$\frac{(r_A V_A^2)}{\mu} = 2.72$$

And then from here we can get A equal to r A divided by $r_A V_A^2$ divided μ -2 your r A is nothing but 1.5 R and this quantity already we have derived here 2.72 - 2 so this turns out to be 1.5 R divided by 0.72 inserting the value for the R, which is the radius of the earth this gives 13287 kilometres.

$$a = \frac{r_A}{\frac{(r_A V_A^2)}{\mu} - 2} = \frac{1.5 \text{ R}}{2.72 - 2}$$

$$= \frac{1.5\text{R}}{0.72} = 13287 \text{ km}$$

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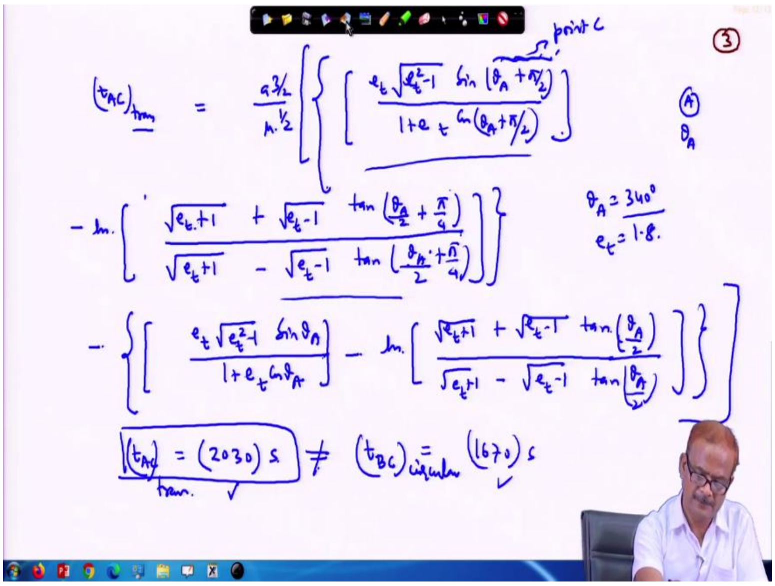

$$(t_{AC})_{tran} = \left[\frac{e_t\sqrt{e_t^2-1}\,\text{Sin}(\theta_A)}{1+e_t\text{Cos}\,(\theta_A)}\right] - \ln\left(\frac{\sqrt{e_t+1}+\sqrt{e_t-1}\tan(\frac{\theta_A}{2})}{\sqrt{e_t+1}-\sqrt{e_t-1}\tan(\frac{\theta_A}{2})}\right)$$

$$(t_{AC})_{trans} = 2030\text{ s } \neq (t_{BC})_{Circular} = 1670\text{ s}$$

13287 kilometres so this is the semi major axis of the transfer orbit. Once we have got this reminder of the transfer orbit now, we can work out. So compute in the; calculate the t_{AC} transfer orbit this we need to work out. So we have the equation in the hyperbolic orbit. Already we have got e equal to 1.8 this represents a hyperbolic orbit. And therefore, the expression for the hyperbolic orbit we have to pick up.

$$(t_{AC})_{tran} = \frac{2\,a^{\frac{3}{2}}}{\mu^{\frac{1}{2}}}\left[\frac{e_t\sqrt{e_t^2-1}\,\text{Sin}(\theta_A+\frac{\pi}{2})}{1+e_t\text{Cos}\,(\theta_A+\frac{\pi}{2})}\right] - \ln\left(\frac{\sqrt{e_t+1}+\sqrt{e_t-1}\tan(\frac{\theta_A}{2}+\frac{\pi}{4})}{\sqrt{e_t+1}-\sqrt{e_t-1}\tan(\frac{\theta_A}{2}+\frac{\pi}{4})}\right)$$

So e_t times v $e_t{}^2 - 1$ under root sin $\theta_A + \pi$ by 2 $\theta_A + \pi$ by 2 where we are locating? We are locating this point C this is referring to point C divided by 1 + e_t cos $\theta_A + \pi$ by 2 then minus ln then we have our $e_t + 1$. This quantity will be θ_A divided by 2 and Π by 2 half of this whole angle this becomes Π by 4. So, this minus e_t at the point A this is corresponding to point C this whole thing right now we have written this is corresponding to point C.

$$\theta_A = 340°$$

$$e_t = 1.8$$

It is $e_t{}^2 - 1$ now we are writing for point A tan θ_A divided by 2 now it is no longer Π by 4 will appear here. Because for the θ_A once we are writing for the point A, so it is just the angular position is θ_A so here θ_A by 2 appears. And then $e_t + e_t - 1$ tan θ_A divided by 2 this we need to compute. θ A already we are presumed to be 340 degree. This value we need to insert here in this place and work it out.

And e_t we have got as 1.8. From here if we insert all these values t_{AC} turns out to be 2030 seconds, which is not equal to t_{BC} in the circular orbit. This is in the transfer orbit so that means and this side was 1670 second. This means this is taking more time in the transfer orbit in this Orange orbit going from A to C taking more time than the time taken from B to C. So then what we need to do?

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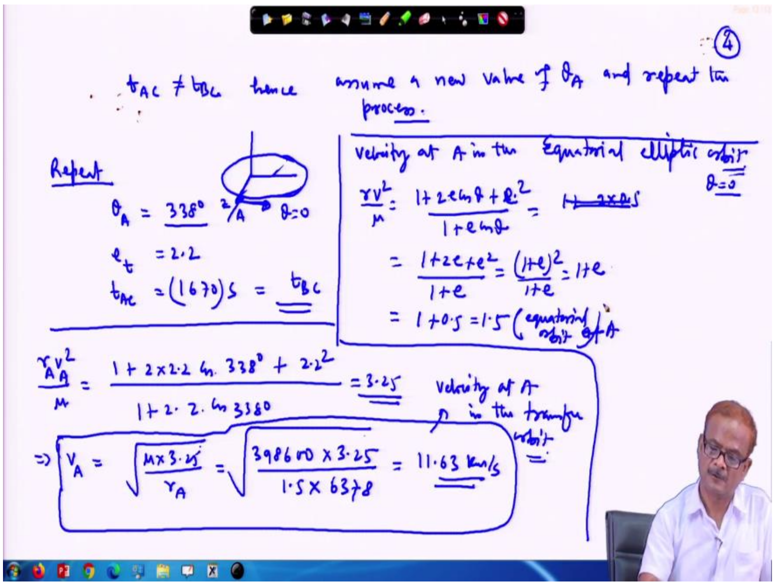

We need to send it in further faster orbit. So, we need to send into the for the faster orbit. So from here what we have got t_{AC} this is not equal to t_{BC} and assume A new value of θ A and repeat the process. So in the repeat process θ is present to be 378 ° so corresponding e_t turns out to be 2.2 and t_{AC} this turns out to be 1670 seconds this equal to t_{BC} so job is done.

Now once this is done then we need to compute the impulse required and the other things. So here $r_A V_A^2$ divided by μ corresponding to this will be 1 + 2 times 2.2 cos 338 ° + 2.2 whole square and divided by 2.2 cos 338°. From here we can get the velocity at v_A to be equal to μ times 3.2 divided by r_A under root 398600 times 3.25 and r_A is 1.5 r.

$$\theta_A = 338°; e_t = 2.2; t_{AC} = 1670\ s = t_{BC}$$

$$\frac{(r_A V_A^2)}{\mu} = \frac{1+2\times2.2\ \text{Cos}\ 338°+2.2^2}{1+\ 2\times2.2\ \text{Cos}\ 338°} = 3.25$$

So r is here 6378 and this gives 11.63 kilometre per second. So this is the velocity at A in the transfer orbit. Velocity at A in the equatorial elliptical orbit, how this will be given? So we use the simplification $r_v{}^2$ divided by μ is equal to 1 + 2e cos θ + e ^{2} divided by 1 + e cos θ but here in this case e is equal to 0.5 and Cos θ will be; θ here will be 0 because you are taking the transfer orbit, the transfer orbit is going like this.

$$V_A = \sqrt{\frac{\mu \times 3.25}{r_A}} = \sqrt{\frac{398600 \times 3.25}{1.5 \times 6378}} = 11.63 \, \frac{km}{s}$$

And the actual orbit; remove the transfer orbit we are discussing about the equatorial orbit. This is the equatorial orbit and this is the perigee position. So it is a given that it is a line on the perigee. Here on the z-axis the perigee line. At this position θ becomes 0. So we want to insert θ is equal to 0 so this is simple 2e or we can write like this 1 + 2e + e ^{2} divided by 1 + e and this is 1 + e whole square divided by 1 + r equal to 1 + e.

$$\frac{r_A V_A^2}{\mu} = \frac{1+2\,e\;\mathrm{Cos}\,\theta + e^2}{1 + e\cos\theta} = \frac{1+2e+e^2}{1+e} = \frac{(1+e)^2}{1+e} = 1 + e = 1 + 0.5 = 1.5$$

And the actual orbit this is given to be 0.5 so this is 1.5. So $r_v{}^2$ by μ is known to us for the; this is for equatorial orbit at A.

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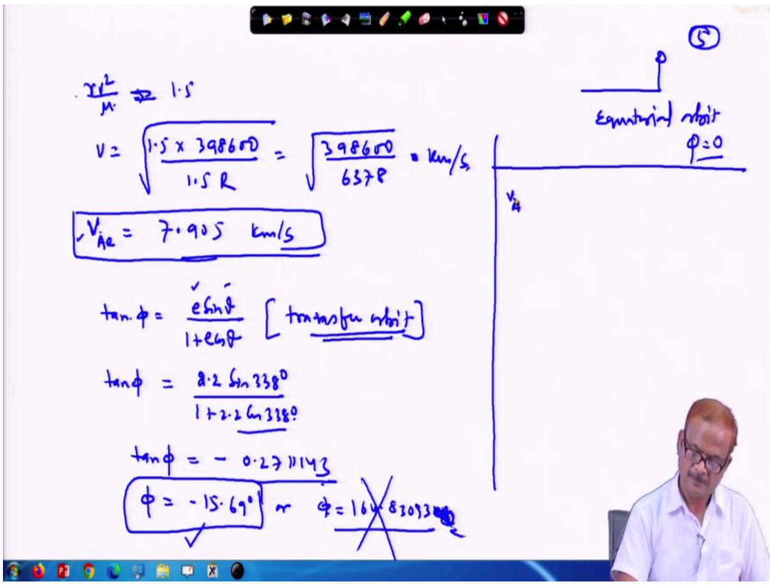

So, quickly now once the $r_v{}^2$ by μ is known, so we can calculate v from this place this quantity is 1.5 and therefore v will be 1.5 times 398600 divided by r this is 1.5 R under root so this is 398600 divided by 6378 under root kilometre per second. So this turns out to be 7.905 kilometre per second. So, that we have got velocity at point A in the equatorial orbit. So this we can write as in the equatorial orbit.

$$\frac{r_A V_A^2}{\mu} = 1.5$$

$$V = \sqrt{\frac{1.5 \times 398600}{1.5\ R}} = \sqrt{\frac{398600}{6378}}\frac{km}{s}$$

$$V_{AC} = 7.905\frac{km}{s}$$

And this part we have got in the transfer orbit v_{AC} this is v_{AC}. Now once we have got the velocity but these 2 things are not in the same direction one is inclined orbit another one is equatorial orbit then we need to calculate impulse along the all the three axes ok. We need to calculate the impulse along all the three axes. So here in this case tan ϕ we have to calculate which we have written as e sin θ by 1 + cos θ.

$$\tan\phi = \frac{e\ \mathrm{Sin}\ \theta}{1 + e\cos\theta}$$

This gives us this is for the transfer orbit we are going to calculate transfer orbit. For the equatorial orbit we know here the θ is 0, ϕ will be 0 here in that place that means it is just; this is the radius vector so velocity is in this direction. So ϕ here in this case is 0. So for equatorial orbit ϕ is equal to 0. We are calculating here only for the transfer orbit. So tan ϕ equal to the corresponding e we need to place your; input here in this place.

So if we insert all those values this is 2.2 we have calculated and sin 338° divided by 1 + 2.2 cos 338°. So this turns out to be; this will give A negative value of this so tan ϕ - 0.2711143 this implies ϕ equal to -15.69 °. It will be A negative one or Φ equal to 164.830934182, something like this. So we remove other quantity. And we know that the ϕ value it will be in range between + - 90°.

$$\tan\phi = \frac{2.2 \mathrm{Sin}\ 338°}{1 + 2.2\ \mathrm{Cos}\ 338°}$$

$$\tan\phi = -0.2711143$$

$$\phi = -15.69°$$

So we discard this and we keep this value and then I write the final values put here in this place and the rest of the discussion I will do in the next lecture. So the impulse required.

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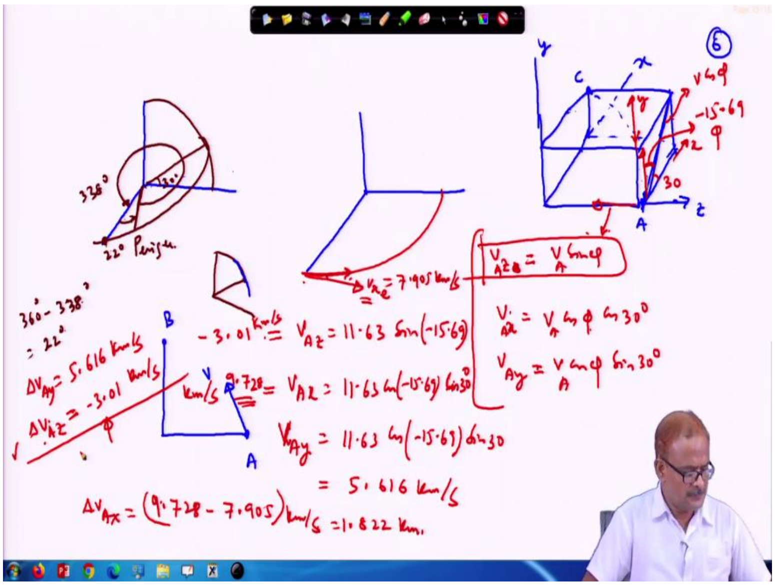

Now one thing we need to note. See here inclined orbit is going like this and this inclination is 30°. This is 30°. But here we know the perigee position is located for the inclined orbit here perigee position which we have got as; from this place to going all the way and coming to this place this we have calculated as 338°. So that means 360 - 338 how much this will be for 40 + 2; 22°.

So, 22 ° it is a line ahead of this. This is 22°. This perigee but we have to give impulse in this point. So we will have to make figure in a little different way to understand this. So let us assume that this is the polar orbit and your orbit is minute little visualisation for this. This is the line z this is the line x and this is the line y and you are giving impulse at z so that it goes and touches at z.

This is A point C this is point A. So here Φ is given to be - 15° as we have calculated here - 15.69° that means this orbit is going to live little inside, if I have A line from this place to this place. Let me make this figure little properly so that if we look in the planner view says if I look here in the xz plane. So if I look in the xz plane, so this is point A and this is point B ok.

So your velocity vector projected on the xz plan it is like this in this direction. But the velocity vector is lying in A plane which is making 30° angle. Here this is going inside and making with this 15.69 this is negative angle on this side. And angle is 30° and this is your V vector. So V vector component along this direction. This will be let us say this is Φ so v sin ϕ along this direction.

And along this direction this will be v cos ϕ and this can be broken along 2 portions one along the z axis and one along the y axis and another along the here x-axis. So this is along the z axis and v_z equal to v sin ϕ and the x will be equal to; and this I am doing at point A. So v_x we will be v cos ϕ times cos 30° and v_{Ay} will we be v cos ϕ into sin 30°. So these are the equation that we need to use and insert the corresponding value.

So V_z will be v we have calculated at A how much is required and this we have already worked out V_{AC} is 11.6 3 so 11.6 3 into sin – 15.69 and v_{Ax} equal to 11.63 cos - 15.69 into Cos 30 ° and v_{Ay} is equal to 11.63 cos - 15.69 into sin 30°. This quantity will turn out to be this we already it is computed. This value I will insert here. So v_{Ax} this turns out to be 9.728 this is in kilometre per second.

$$V_{Az} = 11.63\ \text{Sin}\ (-15.69)\ \text{Sin}\ (30) = 2.72$$

$$V_{Ay} = 11.63 \cos(-15.69) \text{Sin}\ (30) = 5.616\,\frac{\text{km}}{\text{s}}$$

$$\Delta V_{Ax} = (9.728 - 7.905)\,\frac{\text{km}}{\text{s}} = \ \ 1.822\ \text{km}$$

V_{Ay} this turns out to be 5.616 kilometre per second and v_{Az} this is -3.01 kilometre per second. So, already we have velocity along the x axis along the x-axis because in the equatorial orbit here already your satellite is moving like this along the vx direction already your speed is given the actual one in the equatorial orbit this is 7.905 kilometre per second. So that means we need to change the velocity along the x direction by;

We will get this by subtracting from this place to ΔV_{Ax} will be 9.728 - 7.905 kilometre per second and this is 1.822 kilometre per second. Rest of the direction we do not have any velocity initially. So the corresponding velocity ΔV_{Ay} this becomes 5.616 kilometres per second and ΔV_{Az} this becomes -3.01 kilometre per second. So this is the impulse required. This is impulse is recorded along the y direction.

This says that this is to be applied along the negative z direction. So rest of the remaining things I will discuss in the next lecture we close your today. Thank you very much.