Space Flight Mechanics
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Lecture – 96
Interception in non-coplanar orbit (Contd.,)

Welcome to lecture 96, we have discussed about the non-coplanar orbit manoeuvre. So in that context last time we saw that one orbit is the polar one another one is the equatorial orbit and satellite in equatorial orbit it has to go and meets satellite in the polar orbit at certain position. So in that context we worked out the complete problem but towards the end because of their lack of time, I could not discuss some of the points.

So let us look into those points will wind up and then will do the same type of another problem so that the things get concluded.

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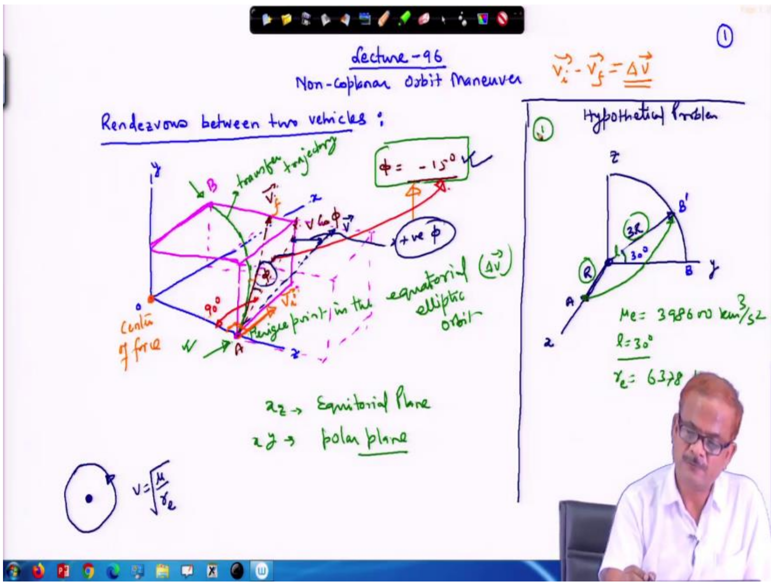

If you look into this particular problem what we have discussed last time so there was a satellite at A and it has to go and catch up the satellite at B. And the orbit in which it has to go so that orbit is like this, it is non-coplanar manoeuvre. So you are going from equatorial plane which is xz plan, xz is the equatorial plane and xy is the polar plane. So that satellite has to be sent from point A to point B and the corresponding trajectory shown in green.

This is a transfer trajectory. So last time what we have looked into that the flight path angle turns out to be - 15° . So this -15° here it is located you can look from this place. This is the perigee point in the equatorial elliptic orbit and at this point the impulse ΔV is given to ΔV is provided here in this place. So the initial velocity was along this direction. We will you some other colour.

Initial velocity along direction this is V i and this is your final velocity. $V_i - V_f$ this gives you ΔV and this ΔV can we need to break along the three axis X, Y and Z to get how much impulse is required along all the three axes. So as you can see from this figure because this ϕ is negative. So this ϕ goes inside. This is going inside. This is negative ϕ because this is the centre of force and this angle then becomes 90° in this plane.

$$\vec{V_i} = \vec{V_f} - \Delta\vec{V}$$

So, therefore you can see that from or either the angle between this and this; this is not within this plane. As we draw more lines and it gets complicated. I will use another colour. The angle between this and this dotted line dotted brown line and this blue line. It is a 90°. You can see that with respect to the radius vector the ϕ angle here it will turn out to be negative. So, this ϕ angle this is negative shown here in this place.

On the other hand, if the same angle it comes outside that means instead of going inside if it is somewhere here in this place, it is going like this V direction is here in this place so then this ϕ this gets a positive value so this ϕ is then positive. While this ϕ which I have shown here in brown. So this ϕ is negative. So I am encircling here this ϕ is negative. So this way you have to visualise it.

Where the things are going? How velocity vector is located with respect to the initial velocity vector and also with respect to the X, Y and Z reference frame. So that you can get the three components along the X, Y and Z reference frame. And thereafter things we have calculated that how much impulse will be required along all the three axes. Today we do one more same type of problem.

So the only difference here it lies in the figure. It is a little high it is a hypothetical problem. So here in this problem let us consider that XYZ. This is the reference frame we are having and mass of the earth we will assume to be concentrated or just at the centre of the earth and so that its surface is not there. And why I am telling like this because I am taking orbit which we called the bridging orbit means if this is the surface of the earth.

So on this, itself if there is a satellite moving in free space not on the touching with this. And you know that the corresponding velocity in the circular orbit will be given by μ by r under root. So here r we have to replace by r e which is the radius of the earth. And the whole assumed to be concentrated on the centre. So this we call as grazing orbit show let us that this is the X direction, Y direction and this is the Z direction.

$$V = \sqrt{\frac{\mu}{r_e}}$$

And the problems which state like this. From the previous problem we have little bit of difference here that this radius is 3r and the other radius is just r it is lying here. So your point A is now at located at a distance r from the centre. And then the satellite has to go and catch up. Initially this satellite is here and this will go to the B′ or let us say B′ or C′ whatever it is.

And this angle is given to 30°. Your satellite has to go and catch up. The satellite is in electrical orbit it as to go and catch up this here in this place. So, again the same kind of figure here but only thing is that your distances are different. So here this angle let us this is latitude l equal to 30° this is given. Previous problem I am just recalls the previous problem. And radius of the earth we are taken to be 6378 kilometres.

And μ earth we will take it as 398600 kilometres cubic per second square. We can start working with this. First let us; what we have done last time in the last two lectures. We are going to follow the same step.

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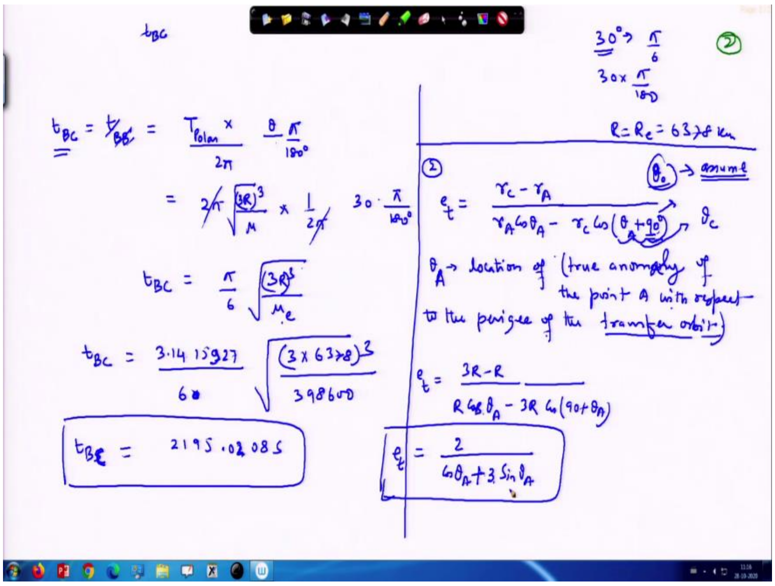

So, going from time from B to B′ or the same thing last time we have written as BC. This time will be the time period in the orbit. This orbit is circular orbit so how much time it will take to go from this place to this place. So, t BC′ or BC let us make it t only to be consistent with the last time. So, t_{BC} so the time period in the polar orbit and this as to be divided by the corresponding 2Π and multiplied by corresponding the angle to which the satellite is going from B to C.

So that this is θ divided by Π so this is θ divided by 180° this will get converted into θis given in 30° this we have to convert into Radian so this will be Π by 6 Radians what we will do that is multiplied by Π by 180°. This gets converted into the time taken to go from B to C. So in the polar orbit this is 2Π 3r whole cube which is the radius of the polar orbit divided by μ.

$$t_{BC} = \frac{T_{Polar}}{2\pi} \times \theta \frac{\pi}{180°} = 2\pi\sqrt{\frac{(3R)^3}{\mu}} \times \frac{1}{2\pi} \times 30\frac{\pi}{180°}$$

And then this 1 by 2Π and then θis Π by180°. So θwill replace why 30 ° this, this cancels out and we get here Π by 6 times 3r cube. So inserting the corresponding values we know this quantity r is simply r_e equal to 63780 kilometres. So if we insert those values and this is μ earth so inserting these values 0 to 08 seconds. The time required to move from point B to C in the circular orbit.

$$t_{BC} = \frac{\pi}{6}\sqrt{\frac{(3R)^3}{\mu_e}} = \frac{3.1415927}{6}\sqrt{\frac{(3\times 6378)^3}{398600}}$$

$$t_{BC} = 2195.0208\, s$$

Now, we need to determine the transfer orbit in which orbit we want to send it? And the time taken to go from point A to point C, so this is to A to C, t_{AC} this must be equal to t_{BC}. So if this happens then the Rendezvous will take place. For calculating this, the next step we have to calculate the eccentricity of the transfer orbit and this we have done $r_C - r_A$ divided by r_A cos θ_A.

$$e_t = \frac{r_C - r_A}{r_A \, Cos \, \theta_A - r_C Cos(\theta_A + 90°)}$$

This Π is not the; we may use some other notation here maybe not to confuse with the flight path angle which we are quite often using it. We will write it as θ_0. So here θ_0 this is the location of all the true anomaly of the point A with respect to perigee of the transfer orbit and assuming θ_0 this is important because based on this only e_t can be calculated and this we have to assume.

So if you draw figure on a reduced scale the things will be clear to you in which orbit what should be the θ_0. We have looked that this value was perhaps 340° we assumed in the beginning. But this time it will turn out to be different. So then inserting all the values here r_C is 3R – R_A is R here R cos θ_A. Here 1 connection is required this θ_0 we are writing as the true anomaly of the point A.

Rather we have written as θ_A this is what. θ_A is the location of the true anomaly of the point A with respect to the perigee of the transfer orbit. So this we write as θ_A and the path here which appears this is nothing but the location let us say that perigee is lying here of the transfer orbit this is the perigee of the transfer orbit. From this perigee this location then this becomes θ_A.

That is the location of A and what will be the location of C this will be θ_C. So θ_C can be written as θ_A + 90° here in this case. This is not θ_0 the notation I have been using but rather they should be 90°. So, this part is basically your θ_C. Same definition location of the point C with respect to the perigee of the transfer orbit. So this cos θ_A - R3 this is 3R cos 90 + θ_A.

$$e_t = \frac{3R - R}{R \, Cos\theta_A - 3R \, Cos \, (90 + \theta_A)}$$

$$e_t = \frac{2}{Cos\theta_A + 3 \, Sin\theta_A}$$

And this will get reduced to R and R will cancel out so 3 - 1 this becomes 2 and here cos θ_A - 3 and Cos θ. So this will become + sin θ_A. So this is your e_t and now depending on the assumed value of θ_A you will get the value for the e_t.

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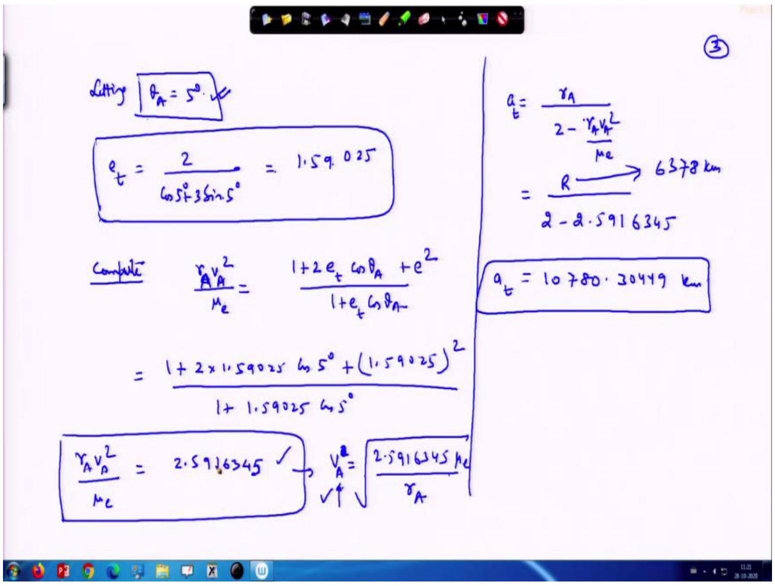

So letting θ_A to be 5°; so this gives you e t equal to 2 to divide cos 5 ° + 3 sin 5 ° + so this is the value for the e_t. Actually, for solving this problem I programmed and just using the programs and I found a the suitable but the initial value I guessed depending on the figure. You can draw the figure and you can try to guess where the perigee of the transfer orbit will lie.

$$\theta_A = 5°$$

$$e_t = \frac{2}{Cos\ 5° + 3Sin\ 5°} = 1.59025$$

This is just a guess. And there after I have done everything using the program written in Fortran. So now compute r_A v_A^2 divided by μ_e. So from here your r_A is known μ what is known. So the velocity at the point A in the transfer orbit will be known to you, e_t is 1.590259 cos θ is 5° and this result is 2.5916345. Once we have got this value for the semi major axis of the transfer orbit can be calculated which is r divided by 2 – r_A v_A divided by;

$$\frac{(r_A V_A^2)}{\mu_e} = \frac{1+e_t Cos\theta_A + e_t^2}{1+e_t Cos\theta_A} = \frac{1+2\times1.59025\ Cos\ 5° + (1.59025)^2}{1+1.59025\ Cos\ 5°}$$

$$\frac{(r_A V_A^2)}{\mu_e} = 2.5916345$$

So r_A is the point where we have the distance R from the centre of the earth 2 - this quantity 2.5916345 and this quantity is nothing but 6378 kilometres. So, once we insert these values. So e_t turns out to be 10780.30449 kilometre. So this is the semi major axis of the transfer orbit. Also, from here the v_A^2 this becomes 2.5916345 μ earth divided by r_A and this we get the under root.

$$V_A^2 = \sqrt{2.5916345\,\frac{\mu_e}{r_A}}$$

$$a_t = \frac{r_A}{2-\frac{r_A V_A^2}{\mu_e}} = \frac{6378}{2-2.5916345}$$

$$a_t = 10780.30449\ km$$

So V_A will be available from this point and when this is required this quantity will be required once we are looking for how much impulse is to be given. And in which direction this is going to be? It is obvious from our previous figure you can see that this impulse has to be in whichever orbit you want to bring it. So the final velocity at point A the velocity at point A should be in the Plane of the transfer.

And plane of transfer here has been known from the previous problem that it is making 30° and here also we have in this problem also we are taking 30°. So the inclination of the transfer orbit to the here in this case xy plane we are aware of it. We have changed the notation here. It was z I have made it this point as x. This axis tag that I have changed and this does not matter you can choose any you can make it y or whatever you like you can do.

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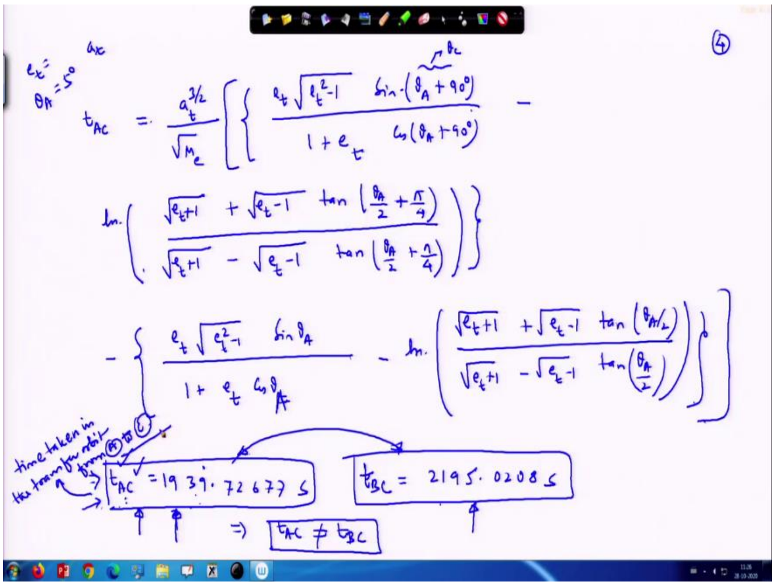

So with this information we need to now find out the transfer time. So, t_{AC} This expression again what we have used last time we will be using it e t square - 1 is not tell this is e sin θ_A plus this is the location of whatever it is going here this is your θ_C. So this is θ_A + 90° the transfer orbit of θ_A + 90° minus; so last time formula we are using it. This is for the final position and the quantity to be subtracted is for the initial position e_t a^2- 1 + sin θ_A cos θ_A - ln θ_A by 2.

$$t_{AC} = \frac{2\, a_t^{\frac{3}{2}}}{\mu_e^{\frac{1}{2}}}\left[\frac{e_t\sqrt{e_t^2-1}\,Sin(\theta_A+\frac{\pi}{2})}{1+e_tCos\,(\theta_A+\frac{\pi}{2})}\right] - ln\left(\frac{\sqrt{e_t+1}+\sqrt{e_t-1}\,tan\left(\frac{\theta_A}{2}+\frac{\pi}{4}\right)}{\sqrt{e_t+1}-\sqrt{e_t-1}\,tan(\frac{\theta_A}{2}+\frac{\pi}{4})}\right)$$

$$t_{AC} = \left[\frac{e_t\sqrt{e_t^2-1}\,Sin(\theta_A)}{1+e_tCos\,(\theta_A)}\right] - ln\left(\frac{\sqrt{e_t+1}+\sqrt{e_t-1}\,tan\left(\frac{\theta_A}{2}\right)}{\sqrt{e_t+1}-\sqrt{e_t-1}\,tan(\frac{\theta_A}{2})}\right)$$

So we need to put the e_t which we have calculated on the previous page θ_A is 5°. So only θ_A and e_t, e_t also we have already calculated and μ earth is known to us. If we insert these values we will get the value for the θ_{AC}. So, θ_{AC} turns out to be 1939.72677 second. While t_{BC} this is t_{BC} calculated as 2195.0208 seconds this was t_{BC}. So this implies t_{AC} is not equal to t_{BC} difference is large.

$$t_{AC} = 1939.726775$$

$$t_{BC} = 2195.02005$$

$$t_{AC} \neq t_{BC}$$

Remember that if you have hyperbolic orbit is used in the hyperbolic orbit your velocity maybe something like 10 kilometres per second say as we will see later on. So if we look into the difference almost this is 206 second of difference in 200 second that will be a lot more difference. So what we need to do that t_{AC} is small that means you are going in a faster orbit and t_{BC} is large.

So I have to reduce the velocity little bit so that t_{AC} becomes 2195. So that means we have to go in a slower orbit. So this is time taken to go from point A to C in the transfer orbit time taken in the transfer orbit from A to C.

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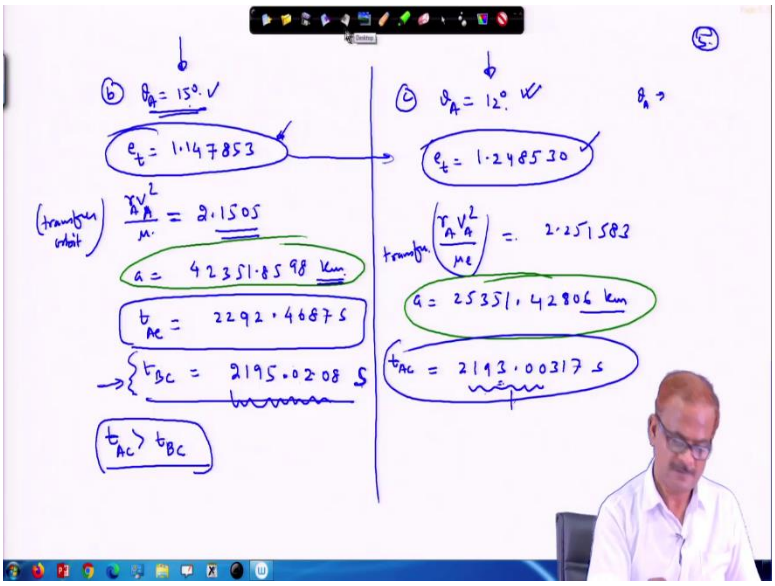

So in the next trial I made it θ_A is equal to 15°. So the corresponding value for the e_t then turns out to be because I programmed it. So it was easy for me to do this problem. And $r_A V_A^2$ this is for the transfer orbit remember. This turns out to be 2.1505. Semi major axis this turns out to be; so you can see that this eccentricity has been reduced. The reduction of the eccentricity earlier it was 1.5 something 1.59.

$$\theta_A = 15°$$

$$e_t = 1.147853$$

$$\frac{(r_A V_A^2)}{\mu_e} = 2.1505$$

Larger the eccentricity the fasters the orbit smaller the eccentricity that means you are moving toward the parabolic orbit so little away from the elliptical orbit. The orbit becomes slow. So A then turns out to be 42351.8598 kilometre and t_{AC} 2292.4687 second. And if we compare this with t_{BC}, t_{BC} is 2195.0208 second this is t_{BC} really here in this case t_{AC} is greater than t_{BC}.

$$a = 42351.85\ km$$

$$t_{AC} = 2292.46875\ \text{s}$$

$$t_{BC} = 2195.0208\ s \qquad \therefore t_{AC} > t_{BC}$$

That means in the orbit in which the satellite will go; now has become slow. So we have to make it fast. The next step the θ_A present to be equal 12°. So θ_A is equal to 12° remember because I have done it through program. So it is very easy for me to do this but it to do on calculator it takes time. Assuming θ_A is equal to 12° e_t turns out to be 1.248530.

So transfer orbit you can see that now this value has gone up. The transfer orbit so that it becomes faster. So earlier $r_A V_A^2$ for the transfer orbit this quantity will be 2.251583 corresponding A turns out to be 25351.42806 kilometre. We can see the difference here in these two places. A small change in eccentricity that makes a large difference in the semi major axis once A is known so therefore t_{AC} can be computed using the equation this equation.

$$\theta_A = 12°$$

$$e_t = 1.248530$$

$$\frac{(r_A V_A^2)}{\mu_e} = 2.251583$$

$$a = 25351.42806\ km$$

$$t_{AC} = 2193.00317\ km$$

Only thing that you need to do is replace here θ and that place by 15° here in this case by 12°. So, t_{AC} turns out to be 2193.00317 second. Now you can compare with this value. So, little bit short of this so that means by now you might have realised that this orbit corresponding to θ equal to 12° is little faster. So, we have to make it little slow. So that means θ_A I need to choose little greater than 12°.

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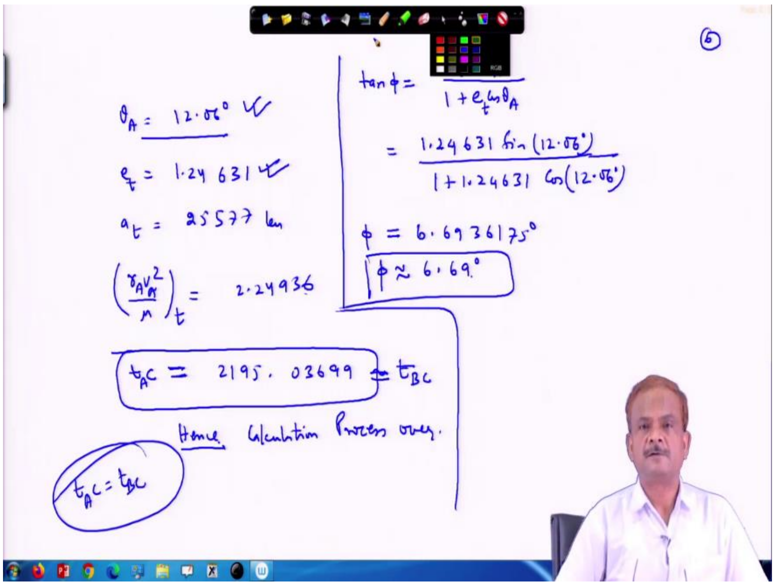

So, in the next trial we choose θ_A is equal to 12.06°. So corresponding e t will be turnout 21.254631 A transfer orbit 25577 and $r_A V_A^2$ divided by μ in the transfer orbit this turns out to be route to reach 2.24936 and t_{AC} the transfer time 2195.03699. So, it is very close to the t_{BC}.

So, this is almost close to t_{BC}. So hence calculation process over. What we need to do now is just to find out the impulse.

$$\theta_A = 12.06°$$

$$e_t = 1.24631$$

$$a_t = 25577\ km$$

$$\left(\frac{(r_A V_A^2)}{\mu_e}\right)_t = 2.24936$$

$$t_{AC} = 2195.03699\ \approx t_{BC}$$

How much impulse is required along the three axes? So we have got here t_{AC} equal to t_{BC}. So already I have drawn the figure so first of the need is to find out the impulse angle this flight path angle ϕ which we are writing as e sin θ_A recalling equation from the last lecture A cos θ_A and this is e transfer. So the flight path angle for the transfer orbit so 1.24 what we are computed this quantity this will go here 24631 and sin θ_A is this quantity 12.06 °.

$$tan\phi = \frac{e_t Sin\ \theta_A}{1+e_t Cos\theta_A} = \frac{1.24631\ Sin\ (12.06°)}{1+\ 1.24631\ Cos\ (12.06°)}$$

And this gets your ϕ is equal to 6.6936175° approximately this is 6.69 °. So this implies that the orbit now is lying along this direction. It is coming out of this plane. Here A is written and let us make this point as A D E and F. Because B is a point on the ground somewhere it is B pointed here. So this we can make it C. So it is going out of the plane A, F, C and D and it is on the right hand side as shown here.

So this is the situation now. Your transfer orbit is going inside like this and the velocity vector; so here because of the perigee position location your velocity direction is coming outside and it will be something like this. With this we are making this angle as ϕ. This is a positive angle 6.69 we have got it degree. This is your point C, this is the point F, this is the point A and this is point D.

It is going outside of the plane D, E, F, A towards the right and how much that angle is making with this plane. So with this plane this line it is making angle. So the angle between; if I extend this line and this line this is 30°. Because the transfer orbit inclination is 30°. So rest of the calculation it becomes easy through this graph. So I have made all that is a graph for you for the figure.

Using that figure you can work out the whole thing. So, rest of the things we do it in the next lecture. Thank you very much.