Space Flight Mechanics
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Lecture – 97
Interception in non-coplanar orbit (Contd.,)

Welcome to lecture 97, so we have been doing the transfer manoeuvre in the non coplanar orbit. We will complete the problem what we have been doing so let us finish it.

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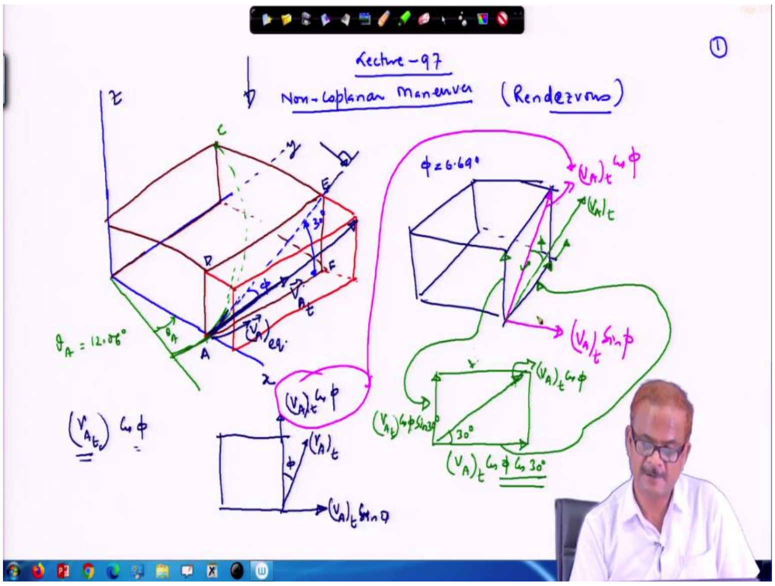

As usual the transfer orbit is lying here, but because of the perigee location which is located at on the backside. This is θ A this is equal to 12. 06°. So this velocity vector at this point in the transfer orbit it lies outside where its aligned we can make from that figure. So velocity vector which is here not going towards this directions going towards this corner. So from the previous figure this is the point A and then point D we have written here D we have written E and F.

This angle is your ϕ angle here in this case and ϕ is given to be 6.69° as we have got in the last calculation. And angle between this and this, this is 30° which is the inclination of the orbit. Your orbit is going to starting from this place and it is going like this and reaching to the point C. But the velocity at this point is along this direction as I have shown here. So this is the velocity vector direction this is your V in the transfer order.

In the actual orbit this which was the equatorial orbit. So there at V_A we can show this as V_A in the equatorial orbit. So this is different so if you look a projection in this here in this direction we have taken X in this case Y and Z here. So the projection in the XY plane we can write it. So first of all, we need to take it V_A t it is a component in the A, D, E, F that plane we have to take.

So $(V_A)_t$ component in that plane will be given by cos ϕ. This quantity multiplied by cos ϕ if we look for the projection from look from the top. So this figure will appear like this. Look from the top but perpendicular to this line. Perpendicular to this line if we look that means to the inclined plane if you look in the perpendicular manner so your velocity is it will appear in this direction this is the ϕ angle V_A transfer orbit.

Immediately you can realise that along the direction the X Direction this $(V_A)_t$, sin ϕ and along this direction this is $(V_A)_t$, cos ϕ. So now the situation runs down to the following condition. This is very important to realise where the velocity vector is lying. This component now I am showing along this direction. This is your quantity the vector along like this. So this is $(V_A)_t$ cos ϕ.

And another vector it is aligned $(V_A)_t$ sin ϕ you can realise that what will be the resultant. Obviously, the resultant will lie along this direction which is $(V_A)_t$ which is making with this ϕ angle. So, what we need to do? We need to take component of this. So taking this plane and along this direction, then you have $(V_A)_t$ Cos ϕ and this is 30°.And this becomes $(V_A)_t$ cos ϕ times cos 30°.

And this component that means the component along this direction. This component along this direction will appear like this is $(V_A)_t$ cos ϕ times sin 30° and while this is cos 30°.This one corresponds to this particular one. So what this direction, so this way, you know along the three directions how much impulse is required. And once what will be the final velocity along these three axis's once we know this impulse can be calculated.

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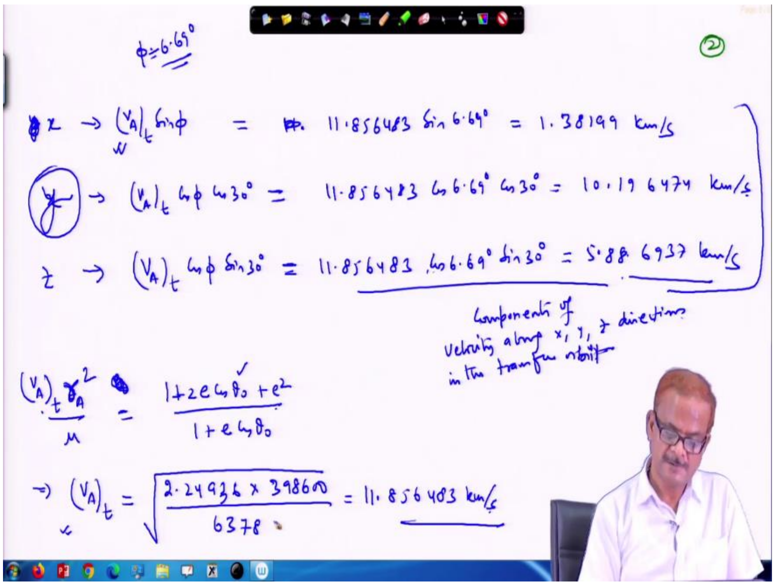

So, this problem is complicated by not by the concept but rather by the calculation that you have to compute so much. Therefore $(V_A)_t$ sin ϕ this will be along the Y direction. And along the X direction we have $(V_A)_t$ X direction is; this is the X direction we have taken this is phi. This direction we have taken as X and the off side it is going as; this is inclined plane. So this is corresponding to the inclined plane but this is along the X direction.

So therefore, we should write here it is X rather than ϕ and along the X direction. This is along the Y direction and this is along the Z direction. So Y direction is $(V_A)_t$ cos ϕ cos 30° this is along the Y direction and similarly $(V_A)_t$ cos ϕ and sin 30° and this value is ϕ we know and this is 6.69°.So if we insert this we can get all these values.

This turns out to be 11 $(V_A)_t$ this quantity we need to really work it out. Till now we have not return it. So we will do that calculation first $(V_A)_t$ in the transfer orbit. This is eccentricity of the transfer orbit we can apply this equation. Already we have done this. The final calculation we have but still we can calculate again V_A in the transfer orbit r^2 divided by μ equal to 1 + 2e cos θ_0 + e^2 divided by 1 + e cos θ_0.

$$\frac{(V_A) r_A^2}{\mu} = \frac{1 + e_t \mathrm{Cos}\theta_0 + e_t^2}{1 + e_t \mathrm{Cos}\theta_0}$$

$$(V_A)_t = \sqrt{\frac{2.24936 \times 398600}{6378}} = 11.856483\,\frac{\mathrm{km}}{\mathrm{s}}$$

And all these values we have already used to compute. So the final value we are computed from this quantity. So if you reorganise it V_A will be 2.24936 into μ divided by r_A under root.

So this is whatever we have written here so it comes from that place. So instead of wasting time we can simply right here in $(V_A)_t$ in the transfer orbit this will be 2.24936 into 398600 this is μ and r is 6378 under root 11.856483 kilometre per second.

So V_A is known once V_A is known this can be computed so this becomes 11.856483 sin 6.69°. Generally, this quantity cos 6.69° into cos 30° this turns out to be 10.196474 kilometre per second. Remember this is along the Y direction. And along the Z direction then we have sin 6 point so this is cos 6.69° times sin 30°.This quantity turns out to be 5.886937 kilometre per second.

$$x \;\rightarrow (V_A)_t \text{ Sin } \phi = 11.8564 \text{ Sin } 6.69° = 1.38199 \frac{\text{km}}{\text{s}}$$

$$y \rightarrow (V_A)_t \text{ Cos } \phi \text{ Cos } 30° = 11.856483 \text{ Cos } 6.69° \text{ Sin } 30° = 10.1964 \frac{\text{km}}{\text{s}}$$

$$z \;\rightarrow (V_A)_t = \text{Cos } \phi \text{ Sin } 30° = 11.8564 \text{ Cos } 6.69° \text{ Sin } 30° = 5.8869 \frac{\text{km}}{\text{s}}$$

So these are the three components. These are the components of the velocity along the X, Y and Z direction at the point A. So these are components of velocity along X, Y, Z direction in the transfer orbit. Once we have done this we need the velocity in the equatorial elliptical orbit.

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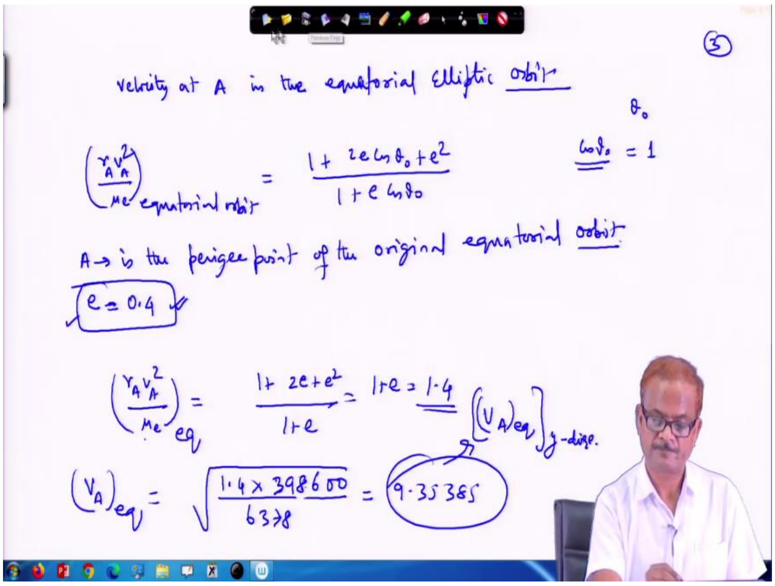

So, velocity at A in the equatorial elliptic orbit we can utilise the semi equation to compute this only thing this will be in the elliptical orbit. And for the equatorial orbit we apply the same formula 2e cos θ_0 e^2 1 + cos θ_0. Here in this case for cos θ_0 is equal to 1 because as per our previous problem if you remember it is the perigee point of the transfer orbit, perigee point of the original orbit.

So A is the perigee point of the original equatorial orbit. And eccentricity now at this stage you can assume anything because this has not been used anywhere. So let us assume this to be e equal to 0.4. Ok anything you can take because this is a problem. So anything we can consider. So e equal to 0.4 if we considered this value then r_A V_A^2 divided by μ earth in the equatorial orbit this is 1 + 2e + e^2 divided by 1 + e.

$$\left(\frac{(V_A)r_A^2}{\mu}\right)_{Eq} = \frac{1+e_t Cos\theta_0+e_t^2}{1+e_t Cos\theta_0} = \frac{1+2\,e+e^2}{1+e} = 1.4$$

So this gets reduced to 1 + e equal to 1.4 and therefore V_A in the equatorial orbit will be 1.4, r A is 6378 kilometres. It will come in the denominator 6378 and μ e it will go in the numerator which is 398600 and take the under root of this. So V_A turns out to be 9.35385 and what this quantity is nothing but V_A in the equatorial orbit along the Y direction.

$$(V_A)_{eq} = \sqrt{\frac{1.4 \times 398600}{6378}} = 9.35385$$

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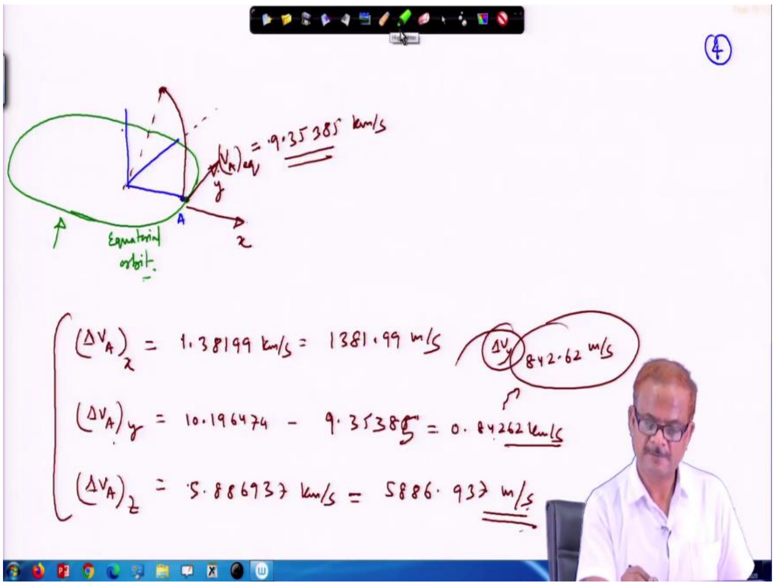

Now you can understand that in the equatorial orbit at this point there is a point A in the original elliptical orbit. So this is a perigee point for the original orbit and it may look something like this. So this is your equatorial orbit. Because this is your perigee point and this angle is measured from perigee for the transfer orbit this was different. This was not zero remember θ_0 equal to 0 in this case 0°.

In the case of transfer orbit, we assumed it and whatever we are assuming this is given basically. This orbit it is known to you. In this orbit it is moving and from there then you are to transfer to the polar orbit. You have to take it from this place to this. This is the point to which the orbit as to reach. Along this direction the velocity V_A in the equatorial orbit which is the Y direction here in this case.

And we have taken X along this direction. So this turns out to be 9.35385 kilometre per second. So we can immediately see that the impulse required along different direction V at A along the X direction. ΔV_A along the Y direction and ΔV_A along the Z direction we can compute from all the information that we have gathered. So, ΔV_A along the Y direction what will be first we have to go back and look into what was the value corresponding value here.

So along the Y Direction This is 10.196474 kilometre per second and in this orbit how much this is actually along the Y Direction this is 9.35385 kilometre per second. We will subtract we will get the corresponding result this turns out to be 0.84262 kilometre per second. That means this value is 842.62 metre per second. These much of impulse ΔV along the Y direction we need to give.

$$(\Delta V_A)_x = 1.38199\frac{km}{s} = 1381.99\frac{km}{s}$$

$$(\Delta V_A)_y = 10.196474 - 9.35385 = 0.84262\frac{km}{s}$$

Along the x direction because there was no initial velocity along the X direction that remains whatever we have calculated earlier which is 1.38199 kilometre per second 1381.99 metre per second. And along the Z direction also we have calculated which was 5.886937 kilometre per second because there is no velocity in the original orbit around the directions. So this is the impulse required to this becomes 5886.937 metre per second.

$$(\Delta V_A)_z = 5.886937\frac{km}{s} = 5886.937\frac{km}{s}$$

So these are the impulse required along the three directions. So this completes this problem and we have a look that how this; the complete process takes place. So our co-planar manoeuvre this part is over and non co-planar manoeuvre we have finished it. So, here in this; while solving this problem. Sometimes you may require propagation of the orbit. This thing I have not covered.

So quickly I will try to cover that and maybe I will consider that as a next lecture. So thank you very much in the next lecture I will record. Thank you.