Aircraft Structures - 1
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Lecture No -22
Statically Indeterminate Structures

So, welcome back to aircraft structures one course this is Professor Anup Ghosh from Aerospace Engineering department of IIT kharagpur. We are at the last lecture of fourth week or module 4 in series the lecture number is 22, we will solve a few problem of statically indeterminate structures. In this problem solving will be you following you may say the complementary energy method stationary problem or say Castigliano's theorem method or and we will solve using dummy load method and we also will be solving one example with Rayleigh-Ritz method.

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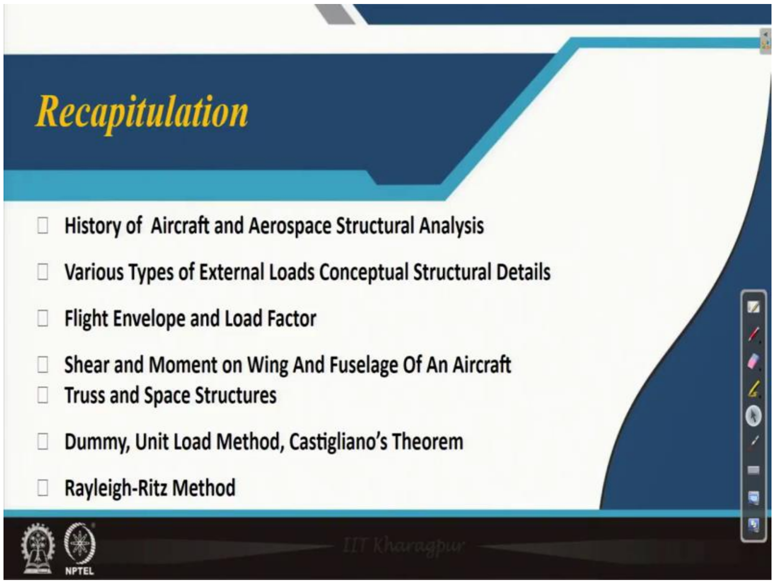

So all these methods are already introduced to you and we will will see how those methods are applied for problem solving in indeterminate structures. As a recapitulation already we have covered many things history of aircraft and solid mechanics or structural analysis, various types of external loads experienced by the aircraft structures, flight envelope, the value of n how it varies the load factor. We have considered we have seen how the moment varies on wing or on fuselage because of the load for a typical example we have solved.

We in energy methods we have solved various methods as it is listed during this week in the last few lectures that is dummy load method, unit load method, Castigliano's theorem and Rayleigh-Ritz method.

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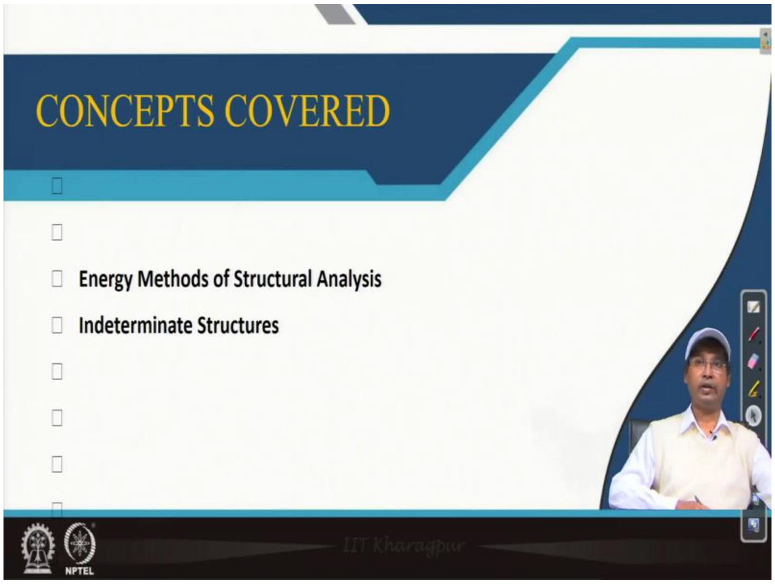

So, now we will proceed further to solve the indeterminate structures.

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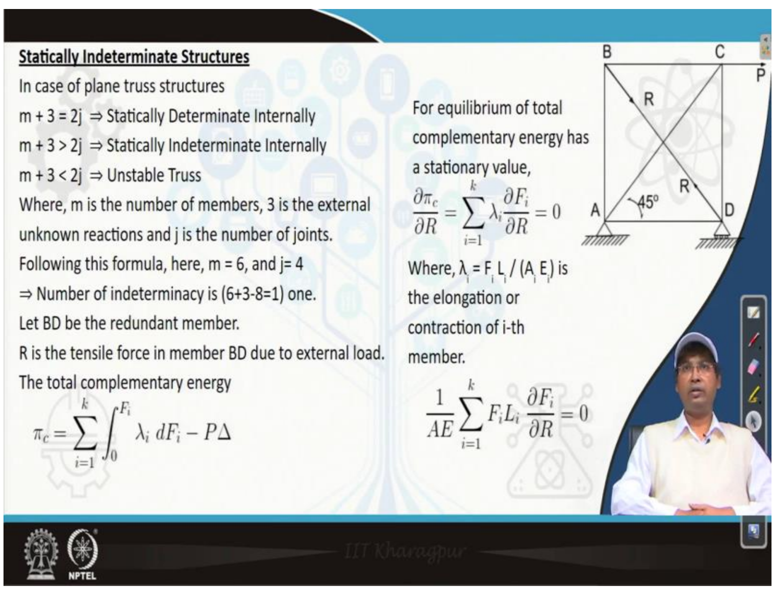

In indeterminate structures our first example is a very, very simple truss example this truss example is first thing we should see that it is in an indeterminate structure and how it is indeterminate and how can we take care of the indeterminancy. Statically determinate structure this truss there are two truss members this thing is repeated I think twice even then there is no harm in repeating because this very easy questions are frequently asked m + 3 equals to 2 j is the statically determinate internally m is the number of members 3 is the reactions external reactions and 2j number of joints multiplied by 2.

If it is greater than the net indeterminate if it is less it is unstable structure. So, where m is the number of members 3 is the externally unknown reactions and j is the number of joints. Following this formula here m is equals to 6 j is equals to 4, 1 2 3 4 5 6, 1 2 3 4 number of indeterminancy is 6 + 3 - 8 is equals to 1, 3 is because it is a plane truss we have equations 3 equations equilibrium equations and then it gives us that there is only one internal indeterminacy.

So in that sense if we remove either this diagonal or this diagonal this structure remains stable but it becomes determinate structure. Somebody may ask why not this or this maybe this one is possible to remove but this one if we remove that then also it is possible but it depends on your ease of work how do you want to proceed. So, in this particular example we will we will remove this diagonal and we will assume that it is a tension force acting in that member of magnitude R.

Let BD be the redundant member R is the tensile force in the member BD due to external R is the tensile force in the member BD due to external load. The total complementary energy PI c as we have done many times that is individual force and the variation of the force, if we integrate that lambda i dF i from 0 to a F i equals to 1 to k minus this is the external energy induced into it so that complementary energy what we have is for equilibrium of total complementary energy has a stationary value.

If we consider with respect to the R variation and we get the equation this way and where lambda is equals real equation of or contraction of ith member due to the external load and that gives us the equation in this form 1 by AE i equals to 1 to k F i L i del F i del R equals to 0.

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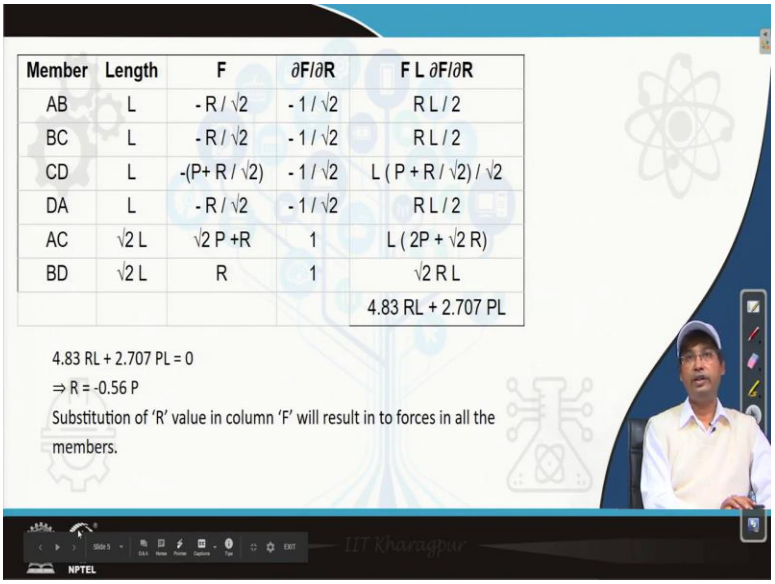

So with that understanding following the procedure of tables solving this type of problems truss problems we are quite familiar with the process what we will do? We will try to find out the deflection in that particular or the equation in that particular member as it is given in the previous page. So, the component F del F del R and F L del F del R all these components are calculated and what do we have is that this is this loads you can easily find out.

I have not elaborated those loads how do you find out removing considering the member force R and external force P better you solve the problem that portion is kept here if we solve the problem it becomes that for AB which is of length L is minus R by root 2 and it goes sequentially for different members there is no point of reading each and every members member forces but the point to note that it is this column is derivative partial derivative with respect to the R.

So this gives minus 1 by root 2 - 1 by root 2 - 1 by root 2 - 1 by root 2 this is 1 because R is coefficient 1 here also it is 1 and then we do this multiplication and do the summation of all these columns and that leads to that 4.83 RL + 2.707 PL is equals to 0 and from there we get the value of R again one more job is left for you to do. You can have one more column here if we substitute the value of R you can easily find out the forces in this member.

So the redundant force is first found out that is minus of 0.56 P we assume tension it has become a compression because it is minus. So, substituting sun of r value in column F will result in to the force forces in all the members. Hope you will complete this problem let us proceed for the next problem to solve.

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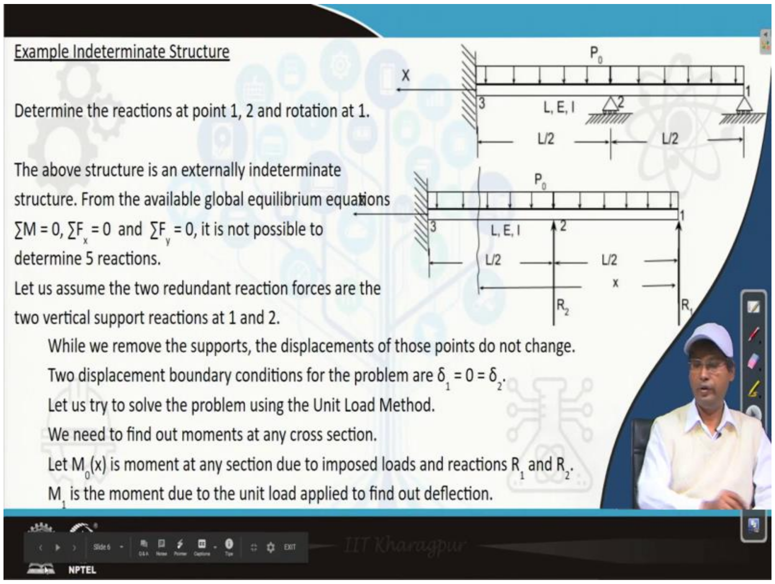

This is an one more example of indeterminate structure here we will be following unit load method. It is a cantilever beam but there are two supports at one at an two at one at and two which is uniformly under uniformly distributed load P 0. This one and two are equidistant apart that is L by 2 and L by 2 and since there are roller supports only vertical reactions are to be considered. So, the total number of unknowns here are 5, 3 here though we do not have any force in the x direction but we have that equation we have that unknowns.

If we have some inclined force then they definitely x will become the unknown. So, there are 5 unknowns and the indeterminacy if we see in that sense the number of indeterminacy is 2 because we have 5 unknowns. The above structure is an externally indeterminate structure externally please note it previous things whatever we were discussing all those were internally indeterminate structure. This is externally because from support point of view from stability or external equilibrium point of view it has more than the necessary support.

From the available global equilibrium equations sum of M equals to 0, sum of x F x equals to 0 sum of F y equals to 0 is not it is not possible to determine 5 reactions. Let us assume the two redundant reaction forces are the two vertical support reactions at 1 and 2. These are the two redundant forces, so redundant density is that indeterminate and redundancy this term is also used sometime.

Redundant supports are those supports if we remove those supports after removal of the support salsa structures remains externally stable. While we remove the supports the displacements of those points do not change that has to be maintained. Two displacement boundary conditions for the problems are delta 1 and delta 2, delta 1 is here delta 2 is here is equals to 0. Let us try to solve the problem using in unit load method.

We need to find out moments at any cross section also because sorry we need to find out moment at any cross section. So, let M 0 x is moment at any section due to imposed loads and reactions R 1 and R 2 are considered there. So, this is what M 0 is calculated while we are considering that R 1 is present R 2 are present instead of considering those as supports we are saying that those are some loads support reaction loads are applied and we are supposed to find out the M 0 x, M naught x.

And M 1 is the moment due to the unit load applied to find out the deflection let us see how do we proceed with help of figure.

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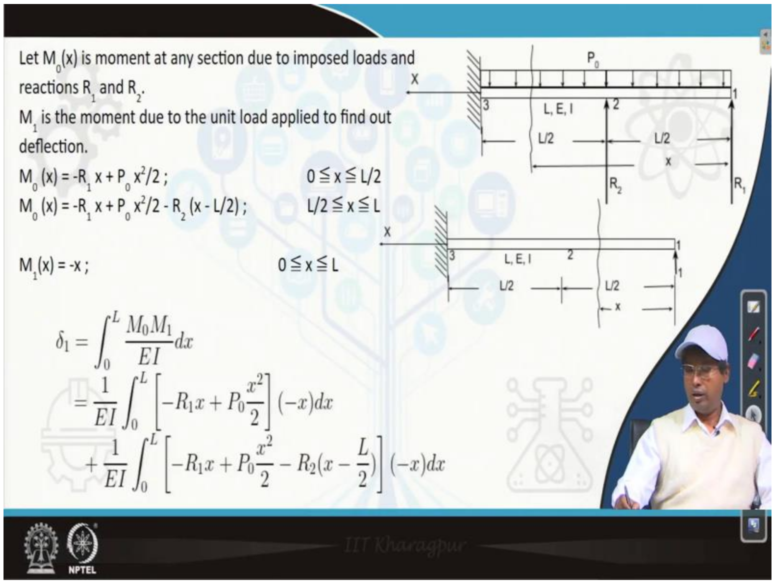

As we have mentioned in the previous one M 0 x and M 1 x already we have mentioned. So, we if we refer to this figure if we consider a section here the moment is equals to R 1 multiplied by x that is what is there minus of it and it is considered this way minus and the other way the P 0 the uniformly distributed load comes in this direction and that is considered plus so P 0 x square by 2 is that load, so it is from here to here we calculate the moment.

And then for the M 0 x the limit for here from here to here that is L by 2 to L we have one more force R 2 that is what this is added by this R 2 but here the effective length is different x - L by 2 is put there. So, moment is found out for R 1 and R 2 we need to find out moment for unit loads and we need to form equations and we need to solve. There are two unknowns R 1 and R 2, so we need to at least have two equations to solve it.

So this is the first case we are considering all other loads are removed only unit load one is applied at the tip and because of that moment is very simple it is minus x wherever you go it is minus x increases with x from 0 to L and then it is M 0 M 1 with two limits it is put I can meticulously put that this is brought down here minus x is there and then again minus x and this is what down here and we need to carry out the integration hope this simple integration you can carry out easily.

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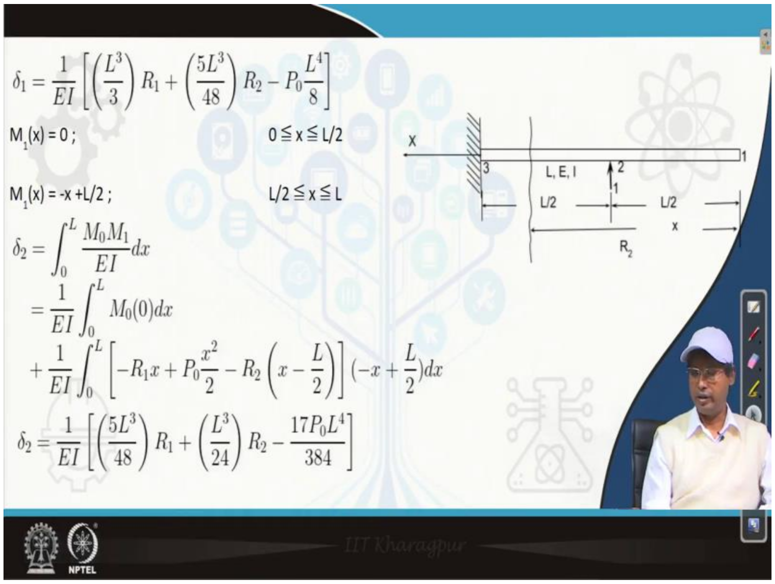

So, if we carry out the integration we get the first equation involving R 1 and R 2 delta 1 is equals to 0 we can put here or we can put later in this process delta this equation is equated to 0 later. We need to have one more equation. So, for that this is the second figure here all other loads are removed only unit load is applied at position two and for that this is there is no moment that is what it is equals to 0 and M 1 x is equals to - x + L by 2.

So that is what is the moment here and as usual this equation is put in this equation to find out delta 2 integrated from 0 to L this first portion is 0 there is nothing so this portion becomes 0 only remaining portion is this one and we get the delta 2 is equals to this value.

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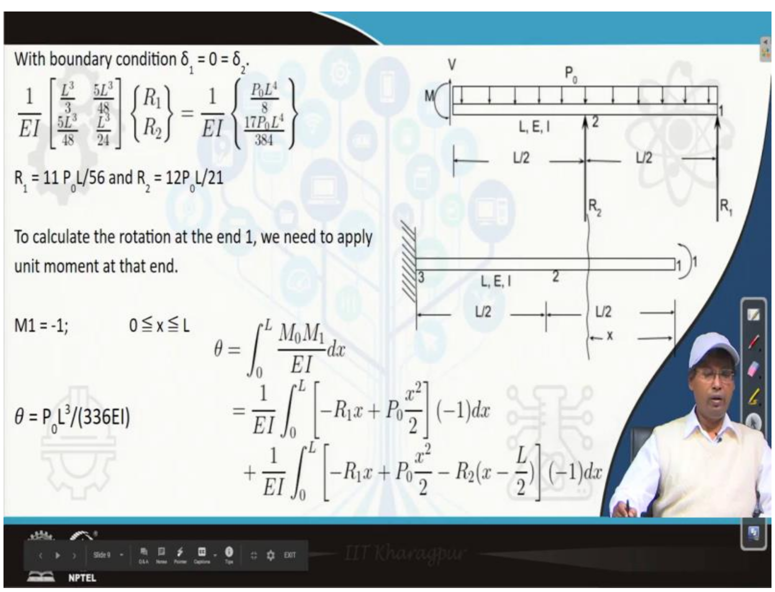

So, now we put the boundary condition delta 1 and delta 2 equals to 0 it is written in matrix form you may not write the matrix forms you can simply solve the equation and if you solve the equations we get that R 1 is equals to e11 P 0 L by 56 and R 2 equals to 12 P 0 L by 21. To calculate the rotation at the end one we need to apply that this much is solved so the unknown forces are known R 1 and R 2 are known.

But one more question has been asked in this problem to solve what is the rotation here it is from may be some deflection. So it may be a following a deflection line something like that the rotation here has been also asked what is that value. So, let us try to find out that for that what we need to do is that we need to apply a unit moment as it is shown here. And for that moment similarly it is minus 1 for unit moment and we put those values and find out the integration and we get the solution theta is equals to p 0 L cube by divided by 336 EI.

So it is quite easy way to find out deflections for determinate and indeterminate structure the indeterminate unit load method you can easily learn that well and apply it for further problem solving. Let us move forward to the next example.

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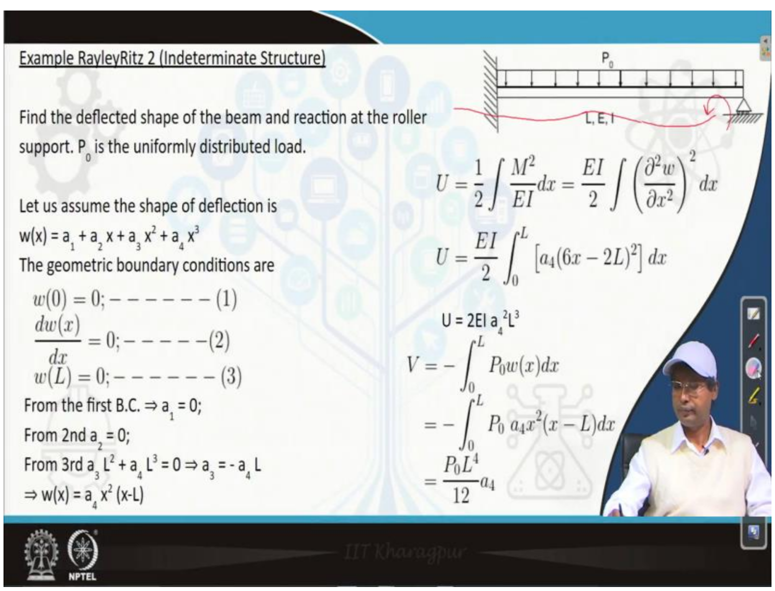

This is again in one indeterminate problem and we will use the Rayleigh-Ritz method to solve this. Here as we have already discussed in the previous example the a priori assumed function displacement function was very close to the exact one that is the reason with only one i equals to

1 or considering only the first term we get a very, very close solution. Let us see what happens in this particular case. So, let us try to solve the problem. This is a propped cantilever beam and under uniformly distributed load find the deflected shape of the beam and the reaction at the roller support.

P 0 is uniformly distributed load. Let us assume the shape of the deflection is w x equals to a 1 + a 2 x + a 3 x square + a 4 x square, so it is assumed that the reflection what it is shown here is of will be following this function so this depends on how accurately we assume this function the accurate the solution we get. So, let us see the geometrical the geometric boundary conditions are displacement is 0 at 0 here slope is 0 definitely slope is 0 as I have also tried to draw the slope zero here and also displacement at this point is also equals to 0.

First the boundary condition this boundary condition if we implement that gives us that a 1 is equals to 0 because all others are going zero, a 1 is for second boundary condition if we implement that is the derivative. So, this also will become 0 this is already 0 that gives us this will become 1 so this a 2 will become 0 and then the third one gives us a relation between a 3 and a 4 and that we have a 3 is equals to - a 4 L.

So if you substitute those values our modified equation becomes w x equals to a 4 x square multiplied by x – l. So, with that consideration we try to find out let us try to find out the total potential energy and the total potential energy becomes; to do that the U and V we need to find out U is found out by integration 0 to l M square by EI and this M is put here a double derivative of w partial derivative of w and that value we get.

This value hope this small steps you can easily do and after integration we get that U is equals to twice EI a 4 square L cube and energy due to the externally applied load P 0 - 0 to l integration we do P 0 w x as usual in the previous case also you have done and if we carry out that integration we get that P 0 L to the power 4 by 12 multiplied by a 4. Now we are supposed to add these two and suppose to consider a variation with respect to a 4.

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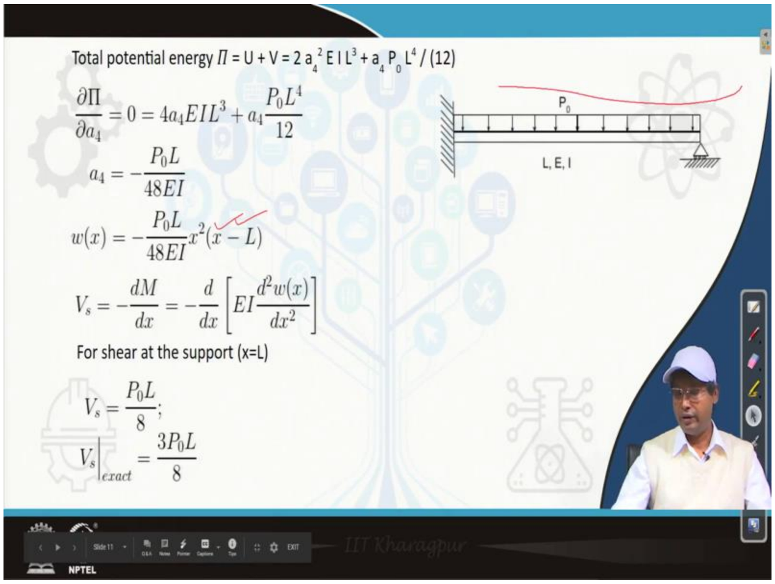

So to do that what we have done the total potential energy U + V is calculated here and then as I have said that variation with respect to a 4 or partial derivative of total potential energy has been found out and with that variation what we have is that if we apply that simple differentiation partial differentiation if we do it leads to that a 4 equals to minus p0 l by 48 e i and w x becomes - P 0 L by 48 EI x square multiplied by x – L.

And the shear force if we can are able to find out at this that is nothing but the reaction here that is why the shear force has been has been found out which is nothing but negative of del M del x here dM dx, so if we carry out that without M is equals to double derivative of the w function at x equals to L this steps you please carry out it is considered here x equals to L if we put we get that shear force here.

The shear force or the reaction here is equals to P 0L by 8. But please note that it is not very close to the exact solution. This solution is 3 P 0 L by 8 that we can easily find out by unit load method or any other energy method for indeterminate structure solution. Why it is not matching that is the reason thing I have tried to discuss it depends on the experience of the engineer who is analyzing it.

How close do you assume the function with respect to the displacement. If you look back this function assumption finally this one with boundary condition no way describes this displacement closely and that is the reason we are not able to find out the exact solution in this way. So, either will have to consider the function properly or will have to think of doing experiments to get some variable or may be longer polynomials will have to take we need to consider other boundary conditions to find out those solutions and we will have to proceed.

So with all these example we would like to conclude but I have a suggestion to the students that I have solved many problems with different methods. So, you can apply all the methods to all the problems and try your solution and check whether the solution is satisfactorily working for the other methods or not. With that and note let us conclude today's lectures and we come to the end.

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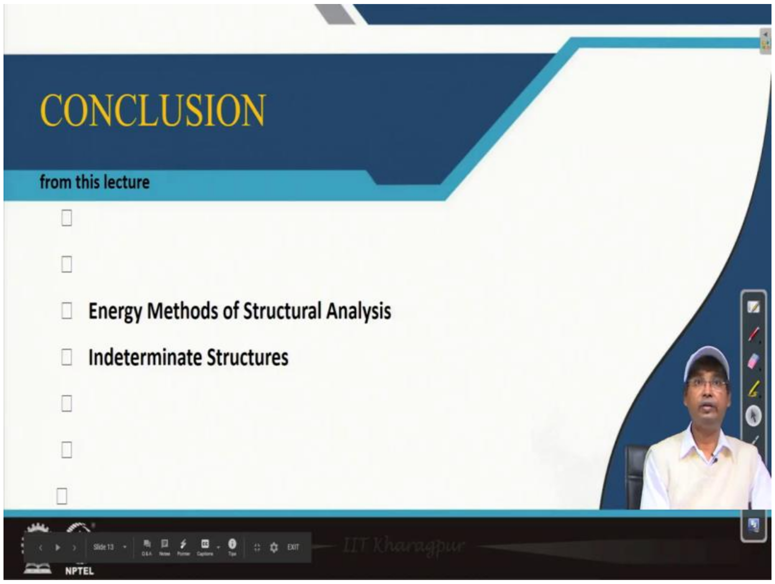

With energy method and inter and also in this lecture we have solved indeterminate problems. So, thank you for attending this lecture and we will start the theory of elasticity portion in our next week lecture, thank you for attending.