Aircraft Structures - 1
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Lecture No -25
Stress Transformation and Principal Stress

So, welcome back to aircraft structures one course this is Professor Anup Ghosh from Aerospace Engineering Department, IIT Kharagpur. We are in the 5th week lectures this is in sequence 25th lecture. Today we will learn about principal stress in two dimension in three dimension some are easily introduced, some are introduced with mathematical concept with tensor calculus. So, whatever you understand it is good otherwise i would suggest you to refer some advanced books as it is listed there or maybe some advanced books on tensor calculus.

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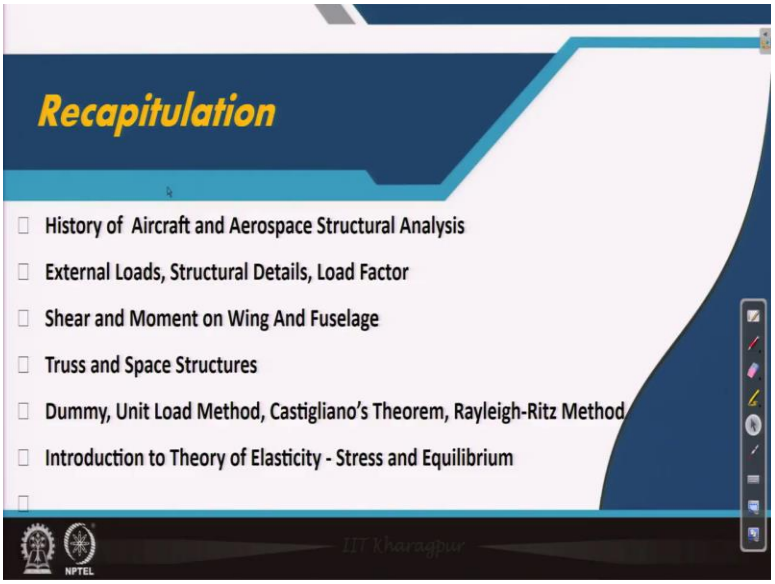

So with that we will step forward for the recapitulation slide that is as usual history of aircraft and aerospace structural analysis or solid mechanics. Then external loads structural details load factors all those things we have seen. How it affects how do external loads come those are experienced by different structural parts then shear and movement effects on fuselage and wing. How 3d truss is used in aircraft structures one big example is landing gear other examples are may be tailboom of helicopter.

Or there are many others these are two visible distinct places where we see. So, like that we have also covered different energy methods to find out deflection how deflection is important in probably last or last to last lecture I have tried to tell you in detail. Introduction of theory of elasticity already you ha you are introduced with theory of elasticity stress and equilibrium equations we have done in our last lectures.

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And today we will be covering some topics which encompasses the stress transformation will solve a small problem. We will try to derive the principal stresses we will try to have the idea of principal stresses. So, that idea is important and that plays a big role why stress is important principle stress is important principal stress is important in the sense that is the highest amplitude of stress acting inside the structure because of different loading condition.

And that may directly govern to failures like say if it is a tensile failure or a compressive crushing failure. Those are the stresses which is since those are maximum in amount in magnitude those leads to that type of failure. So, for a combined stressed body where three dimensional stress is acting. In practical actually all the structures whatever we see are experiencing almost all the components of a stresses like starting from sigma 1 sigma 11 sigma 22 sigma 33 to normal stresses to shear stresses like sigma 12 sigma 23 or sigma 12 23 and 31.

So all stresses may act so in combination of all stresses which one and which direction it becomes maximum that is important to study we need to find out depending on that structure is designed. So, that is the reason it is very, very important those are having some important properties and we will observe those properties.

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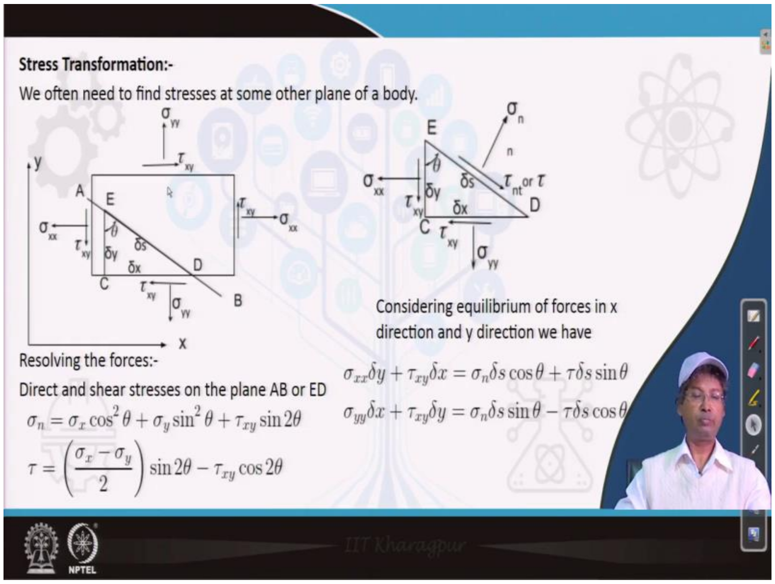

So, let us move to the first topic first topic is a stress transformation.I stress transformation what we do is a simple way how a stress is transformed this is probably covered in your mechanics in some courses and if it is not with some skip some detail we will try to cover now. So, what we are we have in our first figure we often need a to find stresses at some other plane of a body.

So this stress transformation is brought fast to be before the principle stresses because the principal stress is obtained from the stress transformation. Unless we have the concept of stress transformation it is difficult to imagine where the principal stress is acting. So, in this particular case again my aim will be to discuss more on concepts like how does it act where it acts what are the phenomena those things I will also try to cover mathematics whatever is written on slide and the way it is done.

But some of the stuffs as I say are probably will not come into your understanding as and when it is said you need to put some time to understand those you need to put consult some other books to understand those. I would suggest if you are planning for higher studies you should go through those and try to understand. Again we are going away from today's topic, so let us start today's topic. What is the stress transformation? The need is that to find stress at some other plane.

We can imagine some element in books in many times this element is shown as a rectangle sorry a square but it is not necessarily to be square. So, it is considered as a rectangle here and that rectangle because of the external loads is experiencing possible all stresses we are not considering the third dimension. Third dimension which is towards the board or away from the board it is considered that it is a two dimensional case where a plane is under these stresses.

Possible stress components are as we see the normal stress is shown as sigma xx sigma yy this is balancing one and we also have tau xy. Last class we have established that there is also always a complementary shear stress so these are the complementary components. So, once we have these two pair they are also must have these two pair these two this pair should come along with this pair. So, now the question as we have started that we need to find out stress say on this inclined plane inside like this element that inclined plane is denoted by AB.

And we are separating out that here once we separate it out there we do not know the stress condition at this plane that is the reason we have put that sigma n is acting normal to that plane and there is one more component that is tau nt or along the plane. So, this idea already we have that even if there are other components in two dimensional cases all those components may sum up to these two components one which are orthogonal to each other 90 degree to each other sigma n and tau nt.

Sometimes nt is drop it is directly said as tau. So, other components as it is shown here those are brought here only thing is that it is given a finite dimension a small dimension that is a very small delta x in this direction delta y in this direction and it is delta s in this direction. So, if we try to consider the equilibrium x direction that is what if this is the force amount this is acting in this direction tau xy del del x is acting in this direction.

This is considered the other direction sigma n ok theta is not shown. So, this is theta while this is theta so if this is theta then we have that delta n cos theta is the component and delta s is the area considering unit width of the element and the other tau is having sin theta component. So, we get this equation in this direction. Similarly in the vertical direction if we consider we have sigma yy dx the area we have tau xy this tau xy acting on delta y and sigma n sine component it is acting upward whereas this is acting downward its on the other side of the equal sign so it is having minus sign and it is in this form. So it is acting in vertical direction.

Now if we solve these two equation we can easily get this equation and this equation solving I have skipped intentionally so it you can easily do because it is already covered otherwise I can give you a simple idea to do it. If we multiply this equation with cos and this equation with sine this with cos this with sine and add it up this portion will cancel and we will get the expression of sigma n and this delta y by delta s which is nothing but the cos.

So other cos will come from here no we are multiplying with the cos that makes the cos square theta. So, similar way we can find we can find out that sigma n equals to sigma x cos square theta plus sigma y sin square theta plus tau x y sin two theta and the way I have said if you follow the same way and some simple subtraction of the two equations will lead to the expression of tau or shear on a plane which is at theta angle with vertical.

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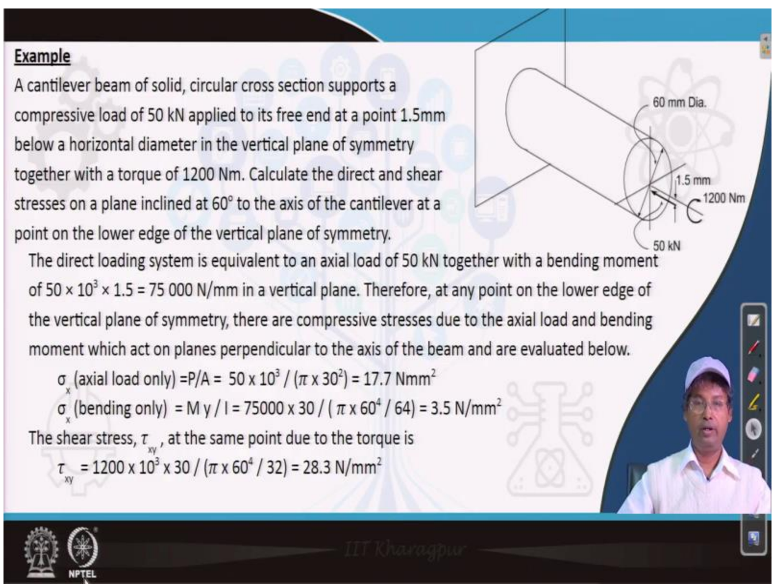

So with that small concept we are trying to solve a problem. In this the problem is shown on the figure it is also described in the writing. So, let us read the writing first, a cylindrical beam this is the cylindrical beam of circular cross section circular cross section supports a compressive load 50 Newton. This is the compressive load 50 Newton, 50 kilo Newton applied to its free end this section is the free end at a point 1.5 mm 1.5 mm below a horizontal diameter in the horizontal plane of symmetry.

This is the horizontal diameter and the horizontal plane of symmetry below that that is the reason from that point it is 1.5 mm below together with a torque of 1200 Newton meter the torque is 1200 Newton meter calculate the direct and shear stresses on a plane inclined at 60 degree to the axis of the cantilever is this one. At a point on the lower edge of the vertical plane of the symmetry so it is asked that at this point what is the shear stress on a plane inclined 60 degree to the axis.

So we need to find out that, so to find out that it is a combined bending and axial and torsion problem there are two loads external loads actually one is this axial load other is torsion. But since it is not acting on the axis of symmetry it will produce some moment so that moment we need to find out and there is a torsion and because of that the element here is stressed and what are the stresses and stress components that we need to find out.

So the direct loading system is equivalent to an axial load of 50 kilo Newton together with bending moment off that is what the bending moment comes here 50 that is the kilo 1.5 Newton millimeters that means 75000 Newton millimeter Newton per millimeter. I think it is a mistake it is not oblique it is Newton millimeter. This is Newton millimeter so with that note therefore at any point on the lower edge of the vertical plane of symmetry there are compressive stress due to the axial load that 50 kilo Newton.

And bending moment which act on planes perpendicular to the axis of the beam and are evaluated below so the sigma x direct P by A axial load only that is 50 kilo Newton divided by the area we get 17.7 Newton millimeter square and the other way bending stress M is the bending moment here y is the how far it is from the axis of symmetry this is 30 that is 30 is there is the y, i is the section moment of inertia of that circular section its put here and we get that 3.9 sorry 3.5 Newton per millimeter square.

Both are acting actually as a compressive load because we are talking about the lower point. So, with that condition we also have a shear stress. The shear stress tau xy at the same point due to the torque is this is the amount of torque and then the similar way we put the torsion formula and we get this is the value of polar moment of inertia of that particular section 30 is the radius and we get that 28.3. So we have normal stresses as well as shear stress.

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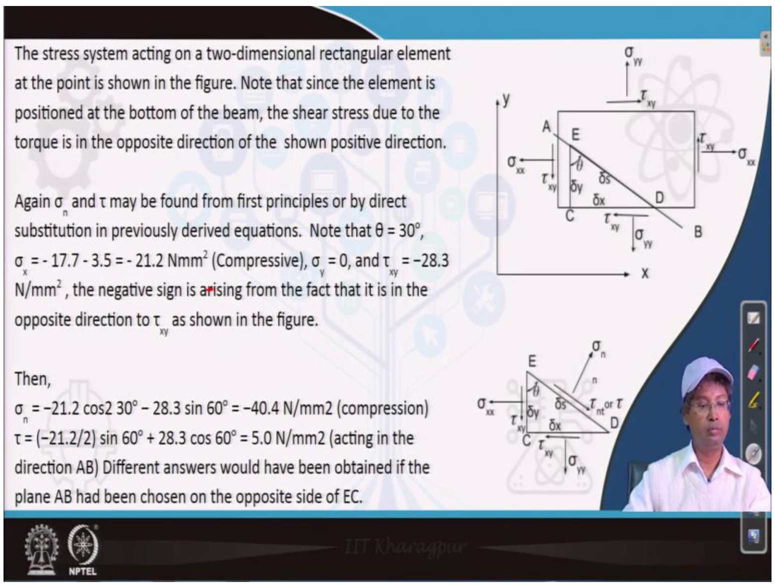

Now we observe that the stress system acting on a two dimensional rectangular element at the point is shown in the figure it is same figure we are considering we do not have any problem only it is said that this angle is 60 degree. The instead of specifying this angle in the problem this angle is specified. Note that since the element is positions at the bottom of the beam the shear stress due to the torque is in the opposite direction of the shown positive direction.

So the torque tau is acting in this direction tau is internal so it is acting in this direction again sigma n and tau may be found from first principle or by direct substitution in the previously derived equations so theta is 30 degree that is what i just now mention here the in the according to the problem it is mentioned that this is 60 degree figure is not conforming to that please consider that this is 60 degree that is the reason we have saying that theta is equals to 30 degree.

The total compressive stress we get as minus of 21.2 Newton here we have need a oblique per millimeter square compressive sigma y is 0 there is no force in the y direction tau xy is minus 28.3 Newton per millimeter square. The negative sign is arising from the fact that it is opposite direction to tau x y as shown in the figure. Now what we we do we simply substitute the values in the equations and we get that sigma n is equals to -40.4 Newton per millimeter square.

So it is minus that is the reason it is not in this direction it is in this direction and it is compressive in nature. And whatever tau we are getting is that five newton per millimeter square acting in the direction AB from this to this this direction is ok. Different answer would have been obtained if the plane AB had been chosen on the opposite side of EC if we chose choose a plane something like this or something like this then we will have a different answer so that is what is said the direction may change because it is on the other side plane which is 90 degree apart.

But anyway we have considered this way and the problem is solved so let us go forward with our next topic.

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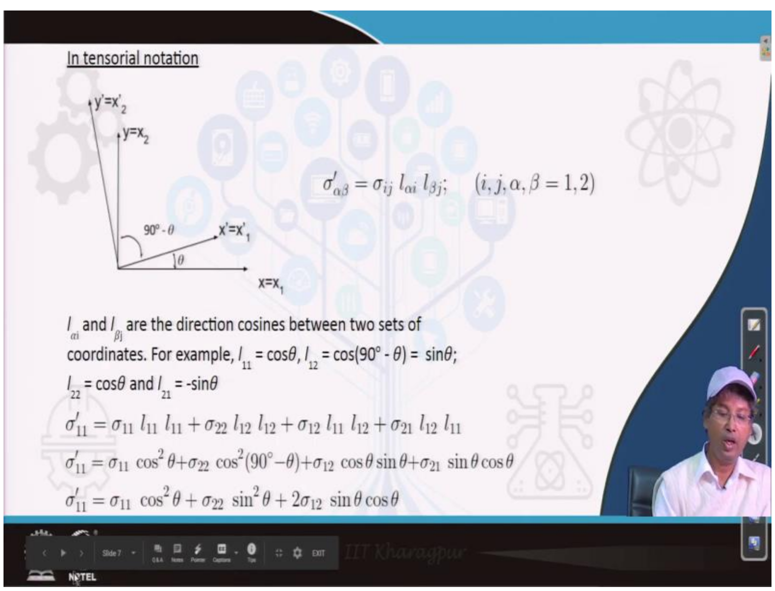

In this topic whatever what we have are introduced that stress transformation that stress transformation we have written in tensorial notation. We are considering that the prime axis which is given by alpha beta is the transferred axis system or the plane system on which we want the stress to be found out and the known system where the sigma ij is the stress condition and we have the direction cosines known as l alpha i and l beta i.

And since we are talking about two dimension that is the reason 12 is considered here but it is also applicable for three dimension one the same equation we will use this in different way written to derive the principal stress we will see that time. So, to understand this in a better way we have used some diagram theta 90 degree minus theta all these things are given alpha i and beta j are the direction cosines as it is given and as an example alpha 11 is cos theta alpha 12 is 90 degree minus theta equals to sin theta alpha 22 is cos theta and alpha 21 is minus sin theta.

Now if we consider as an example the 11 as the normal stress so i j is equals to 11 sorry alpha beta is equals to 11 so this is one i this is bit one j so if that is the case only i j to vary that is what 11 22 12 21 the variation of i j is considered and similar way whatever values are coming as 11 since these are both are one it is 11 11 here it is this is 2 to both 2 that is the reason these two are 2 here 12 that is the reason 11 12 here 21 21 is there. So, accordingly if we substitute the values from here we get the equation whatever we have got for sigma n.

So this is one good understanding and that this small tensorial expression can give us the coordinate transformation very easily and stress transformation to a different coordinate or to a different plane with respect to plane it is easy to understand. But actually we are imagining a different coordinate system which is inclined in that manner. In three dimensional in two dimension it is very easy to imagine in three dimension I would request you or suggest you to to form some access system like my three fingers.

And then you can have rotate this axis system I do not have two right hands that is the problem. So, you can make one more right angle system and put along with this and with respect to this center or maybe having a translation you can easily imagine the how it changes the access system from one to the other. So, keep it in mind we are considering both as right hand Cartesian coordinate system and unless you prepare one template following that it is difficult to show you. So, I would suggest you may do it and experience it.

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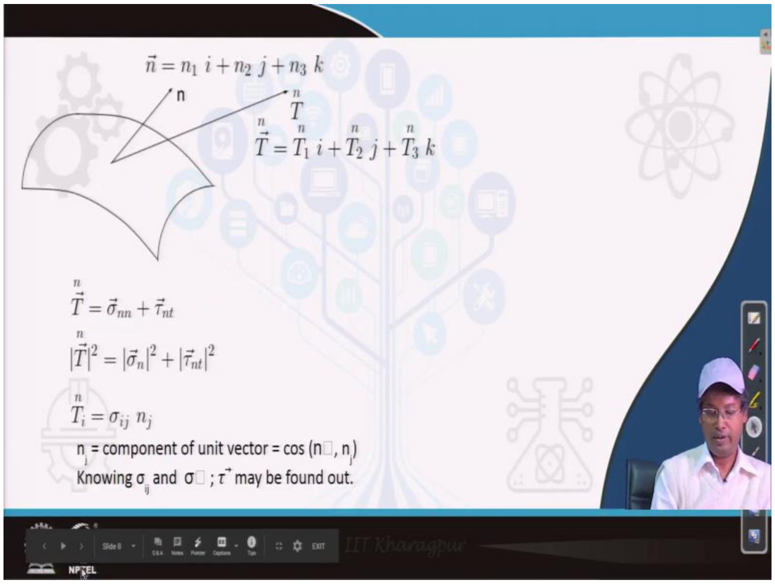

So this is a small topic introduced in between just as a reminder we need this always in different condition we will repeat it again it is something very interesting. It is a T this surface normal is given by this I think I need to go backward yes this surface normal is given by n 1 i into j and n 2 k and the traction force is given by this T 1 T 2 and T 3 n is because it is on the n plane the plane which is denoted by n. And the internal stress system we can have that is equals to sigma n and tau nt which is acting and the amplitude we can easily find out simple from the vectors rule.

Already we have considered and if we expand this which is the equilibrium equation nothing else. This is the normal and tangential component of this the stack traction force and this is the internal force sigma ij which is in balance this is the equilibrium equation we have found out. So, if i know all these components we can easily find out these components and if we can find out these components if one of these are known we can find out the other is once we know the coordinate system nj is the components of unit vector that is cos unit normal this is n with respect to the j.

And this is similarly sigma j and sigma tau may be found out that means here it is talking about this ok. So, um we move forward to the next slide.

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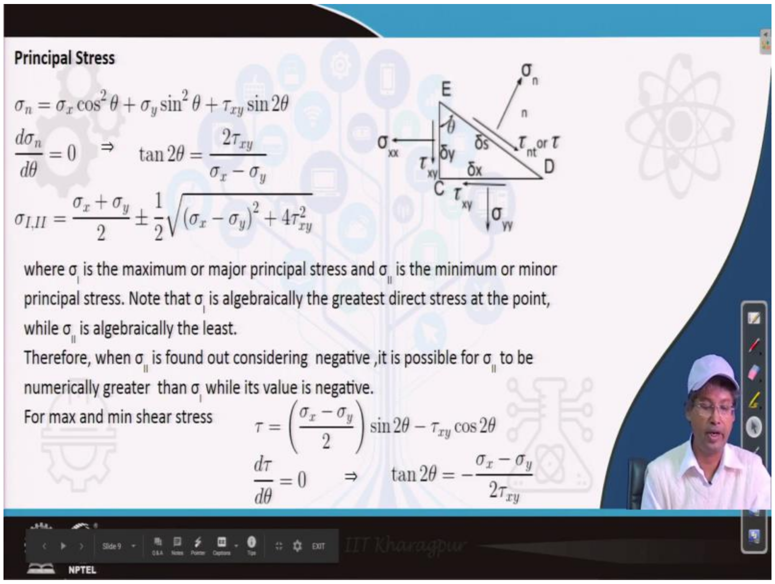

Principle stress: Principle stress is really important point to discuss we will introduce it with respect to two dimension first and we will go forward with advanced mathematical derivation for three dimension we will find out and again we will discuss principle stress in the next class lecture and we will see. So, in this condition, what we do is that we have sigma n considering a plane which is at an angle theta.

And if we want to find out the axis system or the plane along which this is maximum value we need to find the partial derivative of with respect to theta that is what is done here. If we do that derivative if you please note that that leads to this equation we will come to that later 2i cos theta sin theta sigma x minus will come because cos is there this is twice sin theta cos theta and this is tau xy cos two theta two will come definitely in all the cases two are there.

So if we make that thing is equals to 0 we get an angle tan 2 theta is equals to 2 twice tau xy by sigma x minus sigma y I would suggest you carry out this that simple mathematics is not the topic of discussion here. Here the topic of discussion is that see we with that value substituting that we can have two values of sigma that is sigma 1 and sigma 2 this derivation is available in all books I would suggest you refer that.

We can have the two values two roots of sigma n which with replacement of theta as sigma one and sigma two this is the convention it is given sometimes sigma numeric one numeric two not that roman 1 roman 2 which is sigma x plus sigma y by 2 plus minus half root over sigma x minus sigma y square plus 4 tau x y square. Now question is that this is these are the two normal stresses where we have tau equals to 0 that is the reason i said if you derive this we get this equation.

And this if we put this equation is 0 that is nothing but the tau is equals to 0. So, we have that plane where tau is equals to 0. So, in the other sense in a in it is a very common practice to say that principle stress is the stress or the maximum stress at a plane where tau or the shear stress is equals to 0 and those are the maximum values with possible combination of orientation of axis system different way orientation of plane where we are trying to find out normal stress where sigma 1 is the maximum or major principle stress and sigma 2 is the minimum or minor principle stress.

So this major and minor terms are used popularly. Note that sigma 1 is algebraically the greatest direct stress at the at that point since it is having a plus sign while sigma 2 is algebraically the least. Therefore when sigma 2 is found out considering negative sign it is possible that sigma 2 to be numerically greater than 1 while it is sigma 1 while its value is negative. So, this is a good point to note that the compressive stress may become larger in numeric value in some condition.

So, it is not that since it is having a plus the material the maximum stress is experienced by while we are considering plus value only. Plus value is only showing that the material is in tension but while it is negative sign it shows that the material is in compression. So, in this point for the maximum shear stress if we talk about the tau we have in similar way we can have that it is the angle tan 2 theta again we can find out but it is inverse and minus sign is there with respect to this.

So please note that this is a different angle so we have a different angle in this context it is better to remember the Mohr cycle sorry Mohr circle of stresses where we put in tau and sigma axis.

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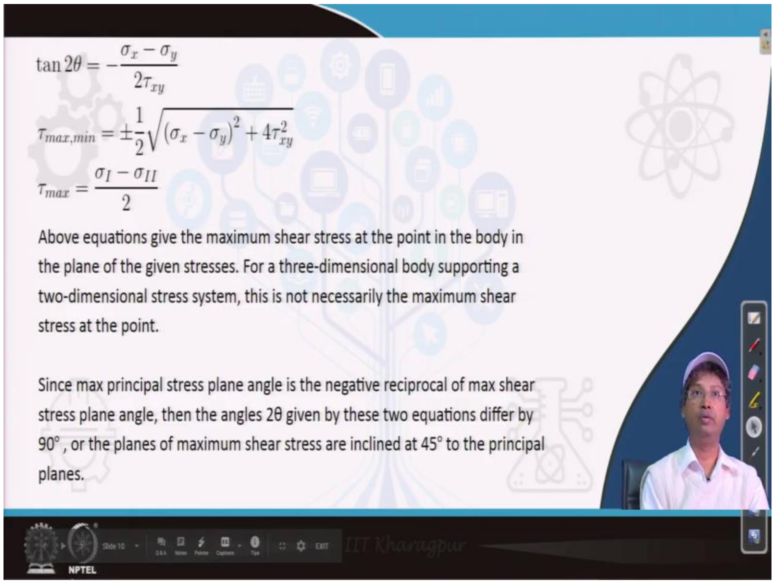

The values of at different orientation of plane what are the normal and shear stresses are and if you look at those values that in general case if it is the Mohr circle this is sigma this is tau. We will find that this is the sigma 1 and sigma 2 or may be the other way. I think better to show it in the other way otherwise confusion will create. This is 2 this is 1 you please note that in this condition at these two points tau is always 0 but while tau is having maximum value sigma is not equals to 0.

So that does not mean that the normal stress is not there is no normal stress on that particular orientation. So, keeping in mind that thing we would like to discuss one interesting problem. So, before we go to that interesting problem which I will discuss partially and i would like to ask you to explore on yourself tan 2 theta, we have now put for the maximum value. These values if we put in the equations previously said in the last one we get that the tau max value is nothing but sigma 1 minus sigma 2 by 2.

Above equation give the maximum shear stress at the point in the body in the plane of the given stress given stresses. For a three dimensional body supporting a two dimensional stress system this is not necessarily the maximum shear stress at that point so that is the reason we are repeatedly saying it is a two dimensional case we are discussing. Since maximum principle stress plane angle is the negative reciprocal of the maximum shear stress angle then the angle 2 theta given by these two equations differ by 90 degree.

So or the planes of maximum shear stress are inclined at 45 degree to the principal stress. In this case I would like to give you a good problem that is what I thought. If we consider one element and if it is under this stress condition there is no shear stress acting and if we imagine one element inside which is having 45 degree angle I would like to suggest you to find out what is the stress condition stress status in this element.

And this is very interesting problem with correlation to a problem while we twist a chalk and it fails, it fails in a very particular orientation that may be discussed in some other way in some other opportunity but this is very interesting problem you can solve it and observe it. So, let us move forward with our next topic.

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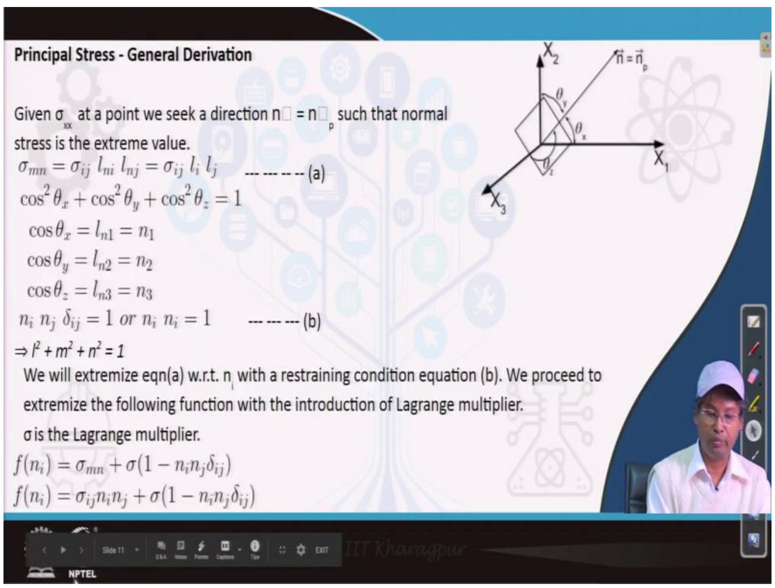

Next topic is really is an advanced topic i would will try to try to cover as much as I can it is as much as I can, it is derivation general derivation of a principal stress we have derived in two dimension but in three dimension it is difficult to derive. So, with respect to the tensor calculus and LaGrange multiplier concept we will try to derive. Given sigma xx at a point we seek a direction n equals to n p as it given.

Such that normal stress is the extremum value that is the problem in mathematics point of view if we talk about we want to extremely find out the extremum or the extreme value or the maximum minimum value of the normal stresses in a orientation which gives the maximum. So, if we consider that sigma m n is a general expression of stress and if we consider only the normal stress of that component that becomes in boils down to slowly from m n reduces to only n.

And then it is once direction and we get that sigma ij and direction cosines if we multiply it becomes this way. So, the normal stress at any plane which is l and l ni l nj direction cosine is having n p say is denoted by distinction. Now this is the usual way of telling that this property to satisfy. And this property to satisfy is direction cosine property is to satisfy and that same thing is said again into different way.

And this is also l square m square this is nothing but this, these are also same thing so it is stated in a different way to give you understanding for the thing which we are going to do. Now we are bringing one concept of Lagrange multiplier we will extremize the equation a the normal stress to be extreme extremized with respect to i j but what is the constraint with respect to n j as I said with a restraining condition equation b that means it should maintain the we are transferring the coordinate system.

But that coordinate system should follow the Cartesian called coordinate system rule that is the Cartesian coordinate system rule this is the Cartesian coordinate system rule that is the reason is saying that restraining condition equation b it has to follow that. We proceed to extremize the following function with the introduction of Lagrange multiplier. We are going to extremize this function where the normal stress is given by this we are considering that the sigma is some value the normal value of it and it is following the same rule of Cartesian coordinate system as it is given here that is substituted back with respect to n 1 n 2 here. And we are supposed to take the derivative of it ok.

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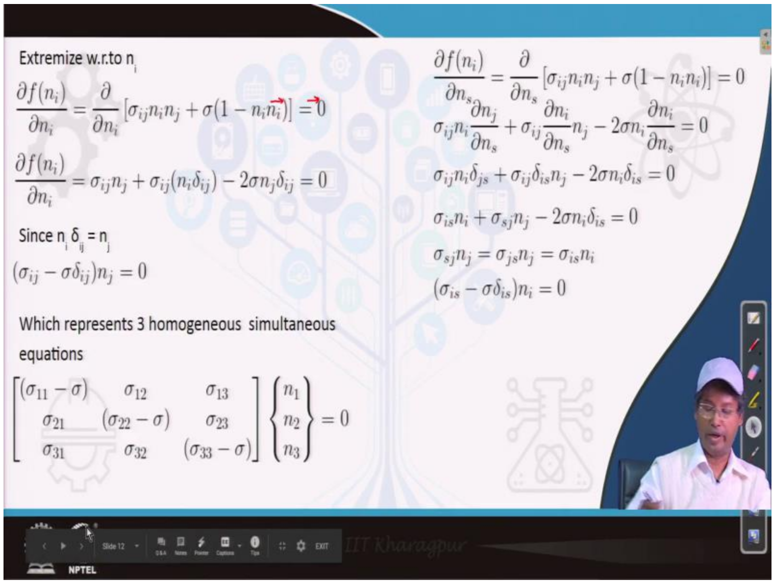

Derivative of it with respect to; what? With respect to the direction right the normal stress we need to find out the extreme value with respect to the direction we need to take the derivative and that derivative is considered here. So, partial derivative of if n i is considered here it is written the same thing once again only this sign is partial derivative sign is put here. Now on the left hand side is something what is done with skip of this steps.

This derivation we are considering with respect to some arbitrary direction s first and if we go on the derivation of s first in this way in between we get this similar to this one equation. So, this is this equation we get n s n j n s is there the first term this it will lead to those two terms these are the two terms we are getting. And then here sigma 1 this is constant we would not get get that this is actually a square term there is the reason 2 sigma ni del ni del n s we are getting.

Now this will lead this derivative will lead to the Kronecker delta this chronic delta I think I have not discussed earlier kronecker delta is better to discuss now I should have discussed earlier. So, if you find earlier you please refer to that Kronecker delta is the delta while both the subscripts are having same value it is equals to 1 otherwise it is equals to 0. So, if I say delta i j equals to one while i is equals to j is equals to 0 while i is not equals to j that is what the Kronecker delta is.

So this derivative actually leads to this Kronecker delta and similarly other chronic delta also we find. Now with this what we have this leads to this this leads to this I think you can easily say that if we if we put this property we get this value this remains in this sense and a good observation and a little knowledge of tensor calculus if you put you will find that this is equals. This step as I said is the in between step this one and if we follow this step this actually leads to this value i s is substituted here with the j, i j and this value we get.

And this is actually the condition of principal stress this sigma is the principal stress value which represents in three homogeneous simultaneous equations if we expand ij to this value and we can find the values of sigma for a nontrivial solution of this part. With this concept I would like to conclude the principal stress will come back again with principle stress concept we will discuss and will go further with different values of principle stress.

As I mentioned important things on the plane where principal stress acts the shear stress is equals to 0. Principle stresses we need to find out in design cases because that may govern to failure of the system, it may be compression it may be tension and the maximum shear stress also we need to find because sometimes the failure leads from shear stresses.

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So, with this note we move forward with the standard books whatever I have covered it is available in these books only.

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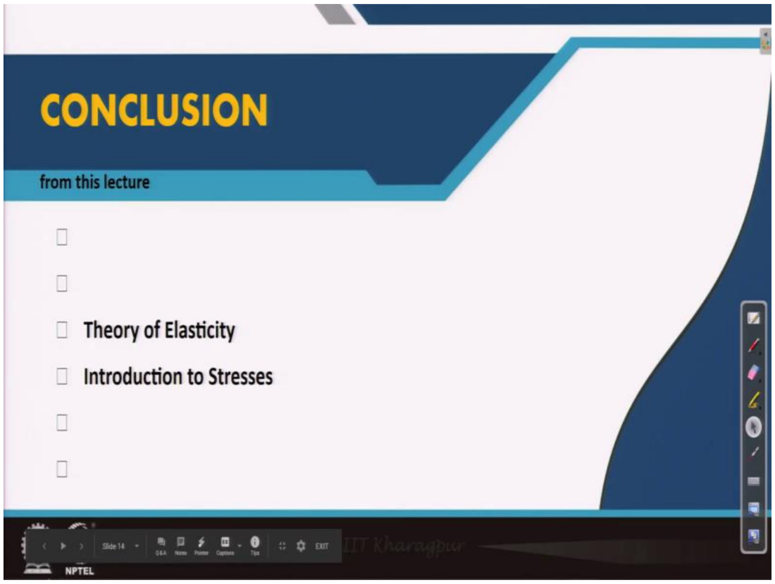

And we conclude the today's lecture thank you for attending and we will move forward for our next lecture, thank you.