Aircraft Structures - 1
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Lecture No -28
Introduction of Strain

Welcome back to aircraft structures 1 course this is professor Anup Ghosh from aerospace Engineering, IIT Kharagpur. We are at the beginning of the module 6 or the 6th week in sequence the lecture number is 28. We have covered in last week lectures last module lectures different way the stress and in this week we will cover strain as well as we will solve a few problems to give you some insight about the analysis.

So we will discuss more on strain this is this is very important lecture. The lecture may be may outcome to a very small lecture but it is very important I would suggest if you are in the process of learning or in advance studies better you remember this particular lecture it will help, you definitely it will help you. With that note let us move forward and as we move forward the first slide comes with the recapitulation.

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It is always I feel to better to recapitulate that helps to memorize things gives us the reminder what you have learned all already. And as a whole at the end of the course you will see if you look at this and attend this few seconds you will find that you can visualize the total course very well and that helps that really helps not a big deal but really helps. I have learned this from a very renowned profession. I am lucky to learn things from him.

Anyway so we have learned history of aircraft or aviation to some extent and we have learned how physicist has developed the subject of mechanics solid mechanics and from there how the structural analysis has come up. In mathematical approach was the first approach that we have already started we have started in last week as well as in the last to last week while we have discussed about complementary strain and energy strain energy and total potential energy everything we started.

And those are the basic things where from the total subject has evolved to do to the date what we have learned. Various types of external load is the next topic we have covered for experienced by aircraft and how those loads vary in different condition while it is in runway or is its airborne or maybe during landing. And we have also learned the flight envelope load factors. We have we have also learned shear stress bending moments experienced by an aircraft.

We have learned 3 dimensional structures like trusses and in the last week that is just in the last week what we have learned is that about stress. How the stresses are we have come across different types of stresses components of stresses for isotropic stress we have come across 6 components finally. Considering the complementary shear stresses to be equal initially it was 9 components.

And then we have come across of different orientations transformations from transformation we have seen that we can find a coordinate system or a set of set of stress sigma 1 sigma 2 sigma 3 which are normal stresses were known as principal stresses where in those particular planes where those are acting there is no shear stress. And from there we have come across about shear stress also at any plane.

And we have come across good examples like octahedral stress and also we have come across about the stress ellipsoid. So those things you have learned we have learned in the last week with that note we will move forward to the definition of strain. And this week we will be covering mainly the strain this is what already we have seen.

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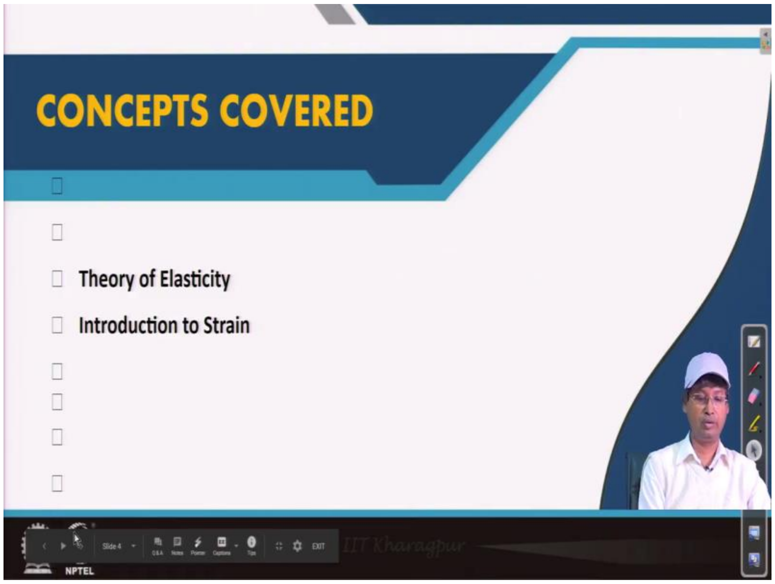

So what we are going to do is the strain introduction to strain we will be doing and I will try to introduce the strain with as much detail as possible in different books it has been introduced in different way. I will try to follow a general approach with a concept of vectors and tensors. Let us see how do we understand that.

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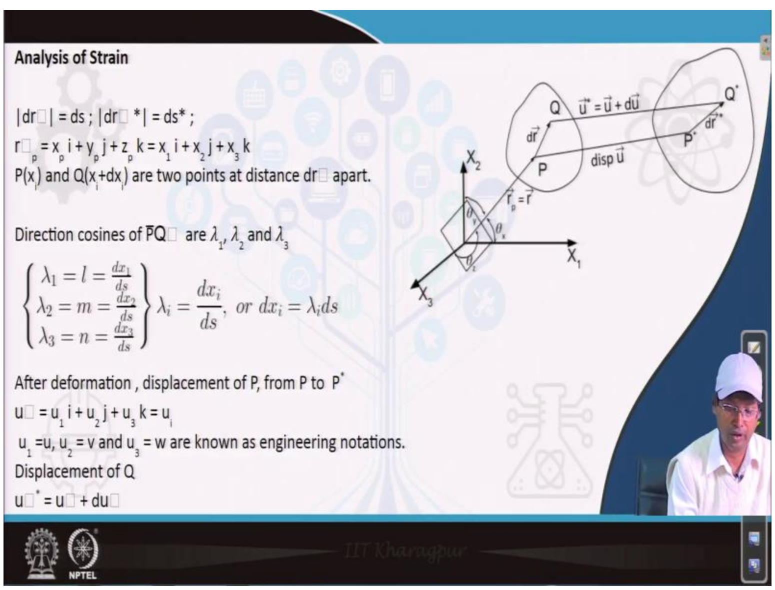

So in this it is again some mistake with the interpretation of slide these are all vectors. So with that note let us start our discussion. This figure is really interesting figure what we are trying to see is that a vector is there as r the tip of that vector is p and an incremental length of that p vector is q which is given by dr. And we see that we are considering that that amplitude dr is equals to ds. Now we also define dr star please consider that thing for misrepresentation in digital media.

Similarly we are considering that after deformation, deformation means it is under stress from external system and because of the stress that r vector has come here and that p has changed position to p and q has p star and the q of r has changed position to q star. And please look at that there is twisting and all other possible change of lengths as far as possible is represented in this figure the overall this figure is changed it is twisted it is length is increased.

All those things all possible ways we have put here and from general consideration since you are introduced with the components of stresses we can easily imagine that that many components of strains are also there. And we define that this r vector as the r p as x p i x p j x p k or x 1 x 2 x 3 as i j k using the unit vector now as it is said there I mentioned that delta r is a is an increment from here and that is the point q x + dx i is the are the 2 points to a distance of dr apart.

And that dr apart these 2 points are changing to these 2 points dr changes to dr star and because of that the displacement involved is here is u and here is u + delta u let us try to see what happens this is a general discussion on the direction cosines involved and we will be using the direction coefficients that is the reason direction cosines of p q are lambda 1 lambda 2 lambda 3 sometimes it is given by l m n sometimes if it is unit n 1 into n 3 all those way it is given and thus the reason it is given as dr is ds the x y z as it is can be said x 1 x 2 x x 1 x 2 x 3 here.

And similar way we define those in a general way we define this way dx i is equals to lambda ds after deformation the displacement of p from p to p star this we have said and u is u i u y where you are 1. Again it is some similar terms are introduced here though we will be using mostly the tonsorial way. But these terms are introduced these these or these are introduced to give you the correlation with the already experienced or learned system.

We are bringing back all those things again and again. So in that way 1 2 3 represents u v and w are known as engineering notations and displacement of q is as we mentioned u + u star is equals to u + delta u. So with this understanding of the system and positions and vectors direction cosines we move forward to the next slide.

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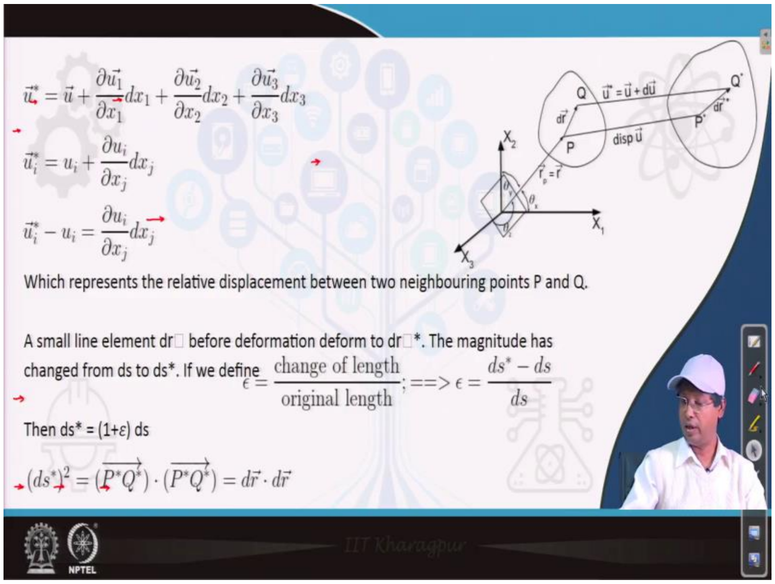

All vectors are now ownerless so we need to erase those vectors are erased. Now what we have is that this is the increment u star yes it has gone back that is the problem so u star as we have seen that is getting increment with respect to the coordinate system or that is the way this is the gradient in that particular direction of x 1 and that is why the dx change is given as del u 1 by del x 1 del u 2 by del x 2 by dx 2 in the y direction x direction y direction and z direction in 3 orthogonal direction we are get giving the increment.

And that is consisting of the du portion and then du gets redefined in this term this way as the separate increments of that particular direction or 3 orthogonal Cartesian directions. And accordingly what we have is that u star i - u i is equals to this. So with this what we have which represents the relative displacement between the 2 neighboring point p and q. A small line element dr before deformation deform to dr star this we are repeating.

Then the magnitude has changed from ds to ds star and from the usual definition of strain this is has come close please excuse me if we define epsilon is equals to change of length by original length that gives us ds star - ds divided by ds and a simple algebraic + 1 if we do in both sides we get this relation which gives that that ds star or the changed length this length is equals to 1 + epsilon multiplied by the ds the original length.

Now ds star amplitude if you talking about is nothing but the dot product of these 2 p q and dr star and dr star so what we can do is let us see how can we define that p q.

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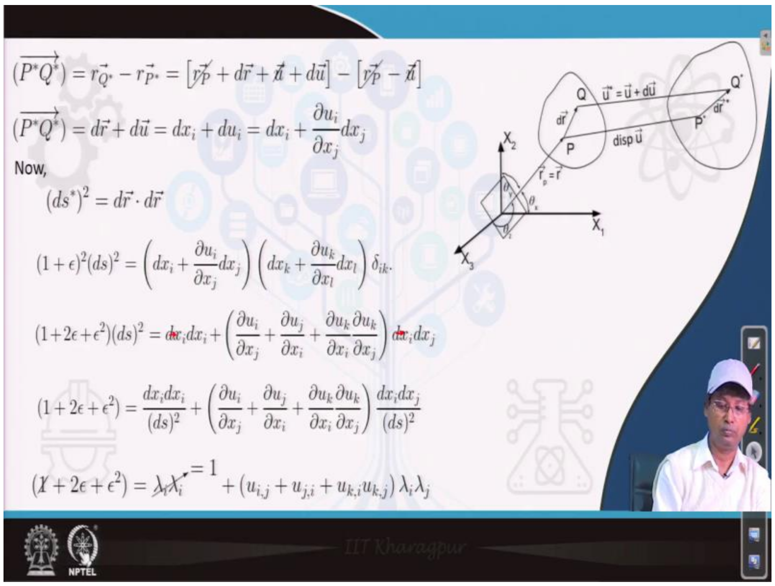

So this is star component this is star component that is the reason we will come there so p q star what we have found is that r q a vector I have not drawn from here to here and a vector r p star that is from here to here that is the difference will give me that p s star q star vector so that is written here and it is exponent as r p + dr and u + delta u and this is the rq what we see and this is the rp what we see and if this the common terms cancels out.

And we have that this is equals to dr - du and actually while we substitute the values of this what we have is that this is equals to dot product of these 2 or in the other way if we see that this is this is equals to P star Q star dot P star Q star dot so this we have as this expression and those expressions are substituted here and the previous slide what we have already found out that is put here.

And what we see is that I have skipped some steps in this because that is a completely tensorial notation of multiplying 2 incremental vectors. We need to already you see some are introduced Kronecker delta because of the dot product is already introduced. And because of the introduction of that dummy variable k l all these things there is a complicated tonsorial calculus is involved. Thus the reason I have jumped those few steps.

And why if we see we get if we carry out this we get these equations and the equation becomes that del x dx i multiplied by dx i or square of it we generally do not write a square in that sense so the other components becomes del u del u i del x j del u j del x I del u k del x I del u k del x j these 2 are repeated. So if these 2 are repeated what happens I will have summation over that but before that let us see how does theses get modified for with respect to the left hand side.

So now ds can easily be divided on the right hand side and we can have that this is equals to lambda i into lambda i and this is also lambda i and lambda j this is x i and x j that is there is an i j and this is x i and multiplication of this or something like cos square theta + cos square alpha + cos square beta + cos square gamma is equals to 1 so following that for 3 dimensional case we have direction equations summation of those equals to 0 this and this cancels out.

The remaining term is twice e + epsilon 2 twice epsilon + epsilon square is equals to this term so finally what we see is that we considered a vector r and that vector is changed position to here as the final r vector with p star. And associated increment of that particular vector dr is also changed to dr star and that thing we have found out that what is the strain this is the strain from here ds star square we have found out.

And ds star square is nothing but this p q dot product of these 2 and those 2 vectors we have found out dot product using the tensor calculus and we come across some relation with respect to the strain and displacement variables involving u y u j and u k sorry u i j where i j varying from 1 to 3 so with that note we move forward to the next slide.

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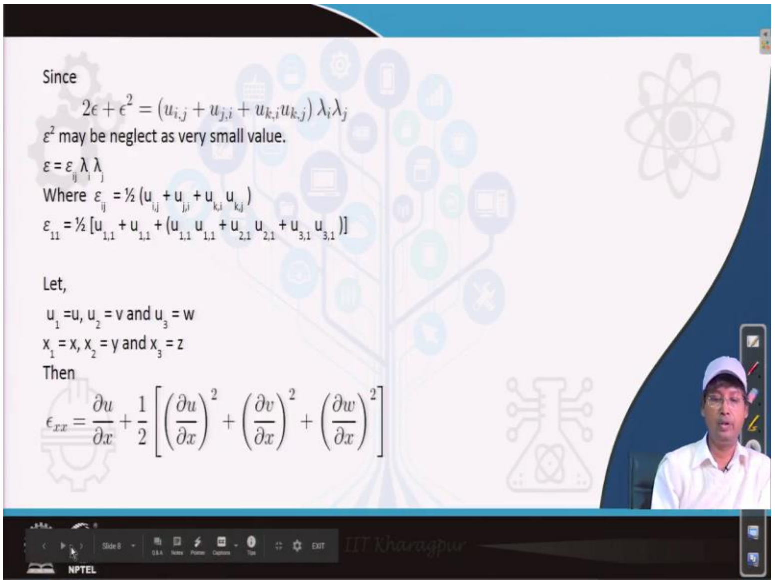

Yes this is what we have seen in our last slide it is written in a very clean way. Now epsilon square may be neglected as a very small value please consider this epsilon and this epsilon as same. These are 2 font problem only so if this is neglected because very small terms if made square that contributes a very, very small way that is the reason that is neglected. And if we express that epsilon is equals to epsilon i j lambda i lambda j.

What we can see that this epsilon i j is equals to half of u i, j that means the derivative of u i with respect to j derivative of u j with respect to i and derivative of u k with respect to i multiplied by this and summation T understand this tonsorial notation one example is carried out. In this example i and j is considered as 1, 1. So what happens half remains outside as half this is 1, 1 that means del u 1 del u del i or del x i this is said that del u again same quantity because both are same.

And they are here we have summation of those these are since it is 1, 1 all are coming as square but it is not always the same way we can elaborate it, we can experience it. So I would suggest you may consider that as a homework. If we elaborate this as a tensorial notation putting i and j 1 2 3 and k is definitely sum over i j and since it is repeated we need to consider those as sum of those 2 terms.

Let u 1 equals to u, u 2 equals to v and u 3 equals to w then same expression becomes this expression. So it is in index notation 1 2 3 it is in Cartesian notation or x y z notation u v w are the displacement fields and derivatives are taken with respect to x and x x is defined this way. Now here it is better to introduce that probably you were introduced with the term that epsa x x s is equals to del u del x where from this term comes this is what we get because of consideration of that particular generic case of strain derivation.

We considered any type of movement and we have in that way we have considered some higher order terms or non-linear terms. So this part of strain is generally known as the non-linear strain and this is the known as the linear strain.

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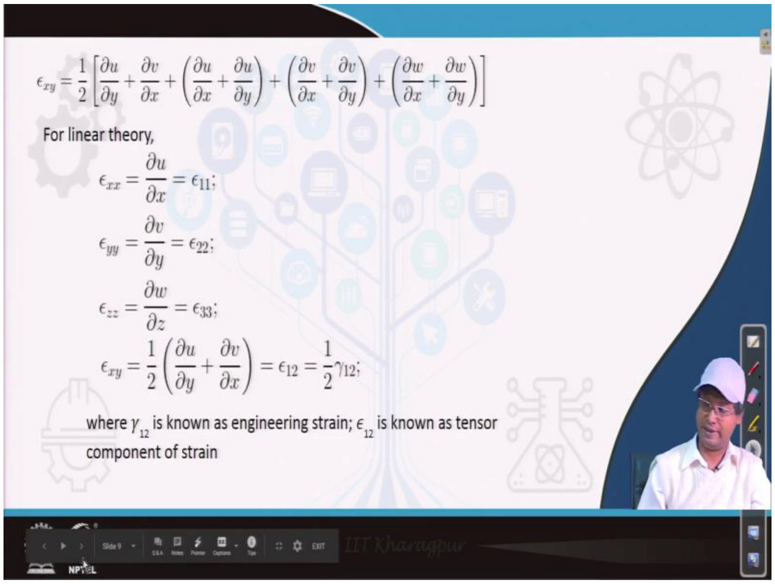

So in a general way if we write it down what it looks like let us say if it is some shear strain to be denoted so like that epsa x x considering 1 2 also you can find out and as I said it is does not is a square it is sometimes this way written and in general epsa x y is something like half of del u del y + del v del x that is what we are familiar with but this terms are coming here as the non-linear terms. For linear theory we generally do not consider those terms and it becomes something like epsa x x equals to del u del epsa y y is del v del y it says that del w del del z and x y is equals to half of del u del y + del v del x and similarly we can write the other strain components also.

Where there is a good relation between these 2 epsa and it is known as a gamma 1 2 is known as the engineering strain and epsilon 2 is known as tensor component of strain. So please keep it in mind generally these 2 convention is followed either epsilon or gamma is used and with this notation we will again yes please stop please recording it is multiplication yes to stop **(FL: 26:05)** last slide what I have written yes here it is written square is considered there it is written yes good pointed out good pointing out.

So better to repeat this slide **(FL: 26:27 to 27:44)** so with the generic derivation of strain we have already seen the epsilon x x considering from the index or tensorial notation to the Cartesian notation how does it changes to epsilon xx in case of epsilon xy or the shear strain similar way we can derive from the index notation what is the expression comes it comes as half of del u del y + del v del x and this is not plus please note that it is a typographical mistake this is multiplication this is also multiplication and this is also multiplication do not think that it is deleted.

This plus signs have come by mistake but actually if you observe the previous expression you can is also easily get but for correction purpose it has been corrected here. And what we do we move forward considering this particular portion or the non-linear portion as it is said to be discarded for linear theory and most of the derivations most of the work whatever we do we follow the linear theory.

And probably our world is running following the linear theory whatever engineering design in general we do we consider linear theories. Nonlinear theories are still in research state implementation probably is there in some cases but it is not may be very high precision strain wherever it is required or the material which is a which shows too much deformations along with non-linearity in its deflection in those cases we need to consider.

But those are rare cases we really do not need to consider 1 very specific example may be the inflatable structure people are using but anyway those things are very, very high topic of discussion better not to bring here at this stage. So anyway what we see is that with linear theory it reduces to very easy expressions epsa x x is del u del x if so y by del v del y epsilon z del w del z and epsilon x y is equals to half of del u del y + del v del x.

Similarly the y z you can do and z x also you can do is corresponding u v and w are all these variables will change. And corresponding one more notation is generally followed that is half of gamma 12 where gamma 12 is known as the engineering strand and epsilon 12 is known as the tensor component of strain.

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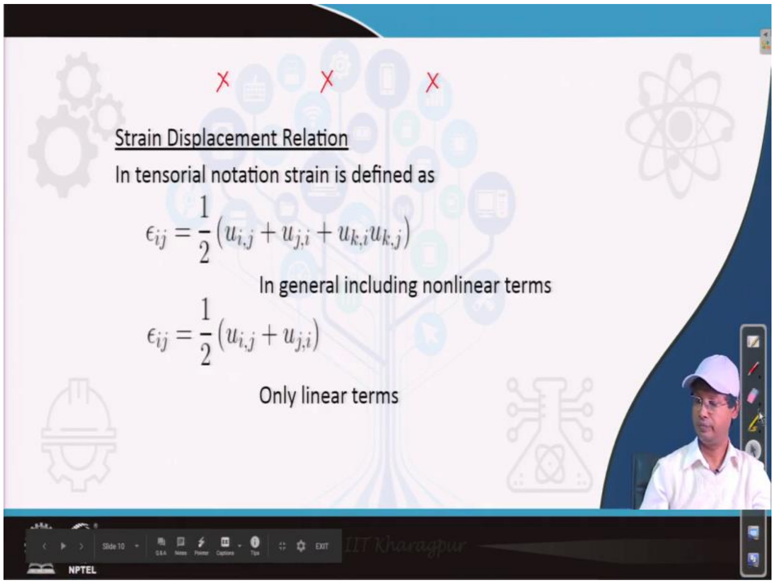

So we move forward, so we come across to the last slide it is better to remember this slide this slide is very, very useful wherever whatever we do. You may like to remember along with the non-linear terms or you may like to remember without the non-linear terms but it is better to remember this if you remember all this up to this it there is not a big deal to remember the remaining portion.

So this is what the strain is defined including the non-linear portion and it is only with the linear terms. So with that note let us conclude today's lecture.

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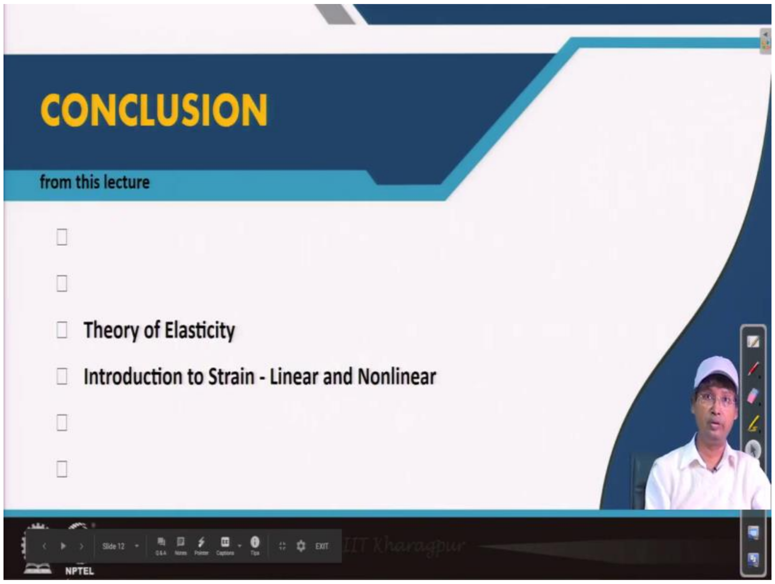

Reference slides are same and we come back to what we have learnt slide and that learnt slide we have learned linear and learned non-linear strain derivation with respect to the displacement, strain displacement relation is learned now we will go further with different compatibility equation of strain and will solve problems in our future lecture. So with that I thank you for attending this class. This lecture will meet again in the next lecture to start or to talk more about strain and problems, thank you.