Aircraft Structures - 1

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Lecture No -30
Formulation of Elasticity Problems

Welcome back to aircraft structures one course this is Professor Anup Ghosh from Aerospace Engineering Department IIT Kharagpur. We are in the sixth week lectures in sequence it is it is the 30 lecture. We will have covered relations between stress strain, strain displacement all these relations we have covered now it is time to solve problem. But theory of elasticity approach of solving problem is not very well appreciated problem for many.

So we need to think a lot to solve those problems and people have spent considerable time to find solution for those problems. Now we are learning probably learning is not much difficult much not much time consuming but it is we are continuing with that. So what we have done so far we have done so far in the elasticity portion with help of the next slide we will discuss.

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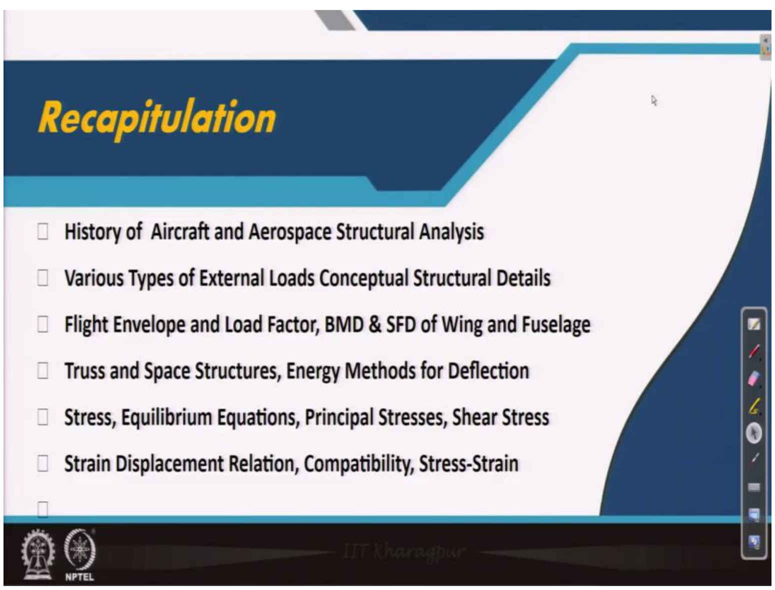

As a recapitulation slide appears it is relating related to the whole or whatever we have covered in our course till that starting with history of aircraft and aerospace structures or solid mechanics. How people started it how Leonardo da Vinci did his the first experiment and then slowly we have come across to many things and to the derivation of cell and other things. Now various types of after that we have discussed about various types of loads experienced by our structure the aircraft structure.

In different condition it experiences different type of load when it is airborne it is not only the loads coming from air and the engine it is it experiences. It also experiences body forces or the inertia forces because of the movement maneuver we say there so man for maneuvering different type of g forces comes and that is popularly known as that how much g it can withstand or the body force it is experiencing.

So with respect to that in correlation to g we have also come across to the flight envelope a flight envelope is the envelope for which we define how much a structure should withstand how much g a structure should withstand and it varies from aircraft to aircraft a type of aircraft definitely for a aerobatic aircraft or a fighter aircraft the g experienced is much more than an agricultural aircraft or a civil aviation aircraft or maybe a glider one.

So keeping in mind those maneuver difficult maneuvers and experience of g a flight envelope is generally prepared and those flight envelope guides us for the design. Now after that bending moment shear force diagram what you have done for different beams probably in your mechanics course for cantilever simply supported many more. We have done those things surface and bending moment diagram for wing and fuselage.

We have considered those separately and we have solved typical problems in association to that we have come across then learn the truss structures. In truss structure we have solved landing gear problems in relation to truss and not only landing gear problem there are many other three dimensional structures available inside the aircraft. So those we may also solve using the approach what we have learned and then we have started discussing the stress strain relations theory of elasticity all those points.

Stress strain in the first week class we have defined stresses we have defined equilibrium equations we have defined transformation of truss. How do we transform for trusses from one plane to the other and during that transformation we found that there is a plane where shear stress is equals to 0 or in a better way we should say there are three planes orthogonal planes where shear stresses are 0. And the normal stresses in those planes are known as the principal stresses.

Those principal stresses follow some certain set of invariant property we have three invariants stress invariance and similarly we have strain invariance also. But strain invariants we have not discussed it is given you as a scope to explore on your own or the principle strain properties all those things I would suggest you explore on your own and I may go into that but after that the most important thing what we have derived is a related to the strain displacement relation.

The strain displacement relation we have derived from the tensor calculus approach or with considering vector and tensors and then we got the complete equations of strain displacement including the non-linear terms. Then in the last class we have considering the only the linear part we have come across to the complete compatibility equations compatibility ability equations are important for unique solutions and that has to be maintained.

So so we we have learned what are the equations and we need to we need to satisfy those conditions for any analysis. Then the stress strain relations that we are probably already introduced to us we have got some relations and with that scenario let us move forward for today's topic.

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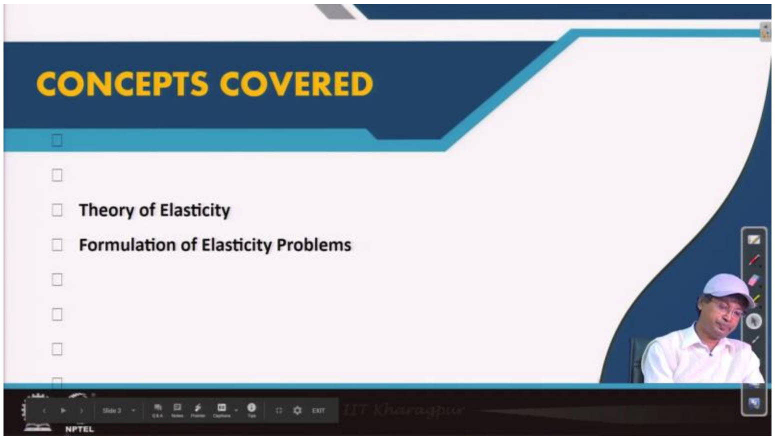

Today's topic or this lectures topic is a formulation of elasticity problem.

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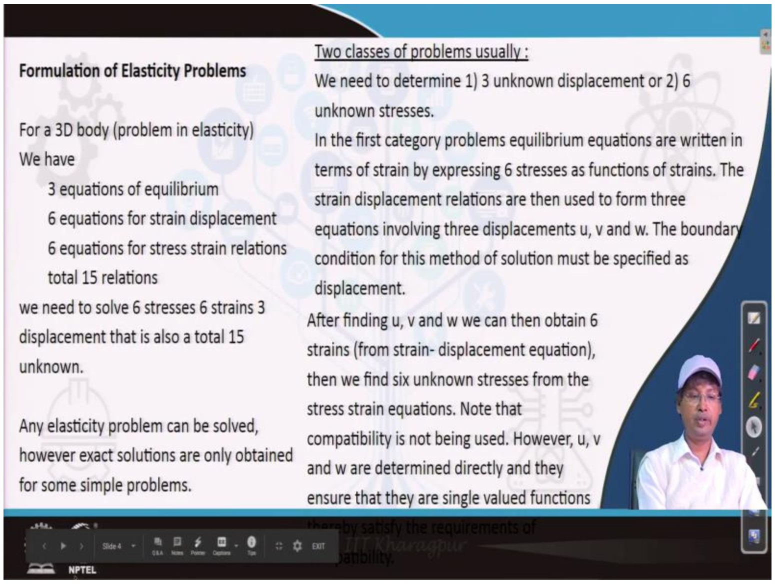

So let us look into it how the elasticity problems are formulated formulation of elasticity problems for a 3D body so we have we are continuing with the formulation of elasticity problem for a 3d body problem in elasticity we have three equations of equilibrium 6 equations for strain displacement 6 equations for stress strain relations total 15 relations we have we need to solve 6 stresses 6 strains 3 displacements sigma epsilon u v w or x y z and that is also total 15.

So now we are ready with our status to solve it how to solve that is what we will cover. Now an elasticity problem can be solved however exact solutions are only obtained for some simple problems here lies the key people have tried a lot to solve all the problems using this mathematical approach. And it is really difficult to solve using this mathematical approach all the problems there are a few problems that people have tried and solved and those we will learn.

And there are two basically ways of solution one is solving the three displacement first and then solving the stresses the other is the reverse way. But those we will learn but this actually this as the statements stated here that actually initiates the process of approximate solution and modern day computational solution process. So involving finite element method not only finite element method finite element method is the probably the first method that is why people say always finite element method.

There are many other methods similar methods like boundary element methods and so on. So those approximate methods are invented and using these conditions satisfied these conditions satisfied for a smaller domain they go for the larger domain analysis and using computers and modeling of those solids in a even if it is complicated. We can solve those problems find out solutions for that and we cannot solve find out solutions for all problems following the elasticity approach.

But the question may come why then do we need to learn if it is not able to solve because the approximate methods what just now I said most popular is find element method nowadays probably people are more referring more with the commercial names those things but fundamental basis of those are also this theory of elasticity from here using some functional analysis approach or some approximate method approach like the Rayleigh method or some other functional analysis approach we get the basic equations for the smallest unit.

And then we assemble those units and get the approximate solutions. So how good we approximate the displacement behavior from the theory of elasticity point of view that gives us how accurate we are in finding the solution. Anyway let us move forward 2 classes of problems usually we need to determine 3 unknown displacements or 6 unknown stresses whatever we do we can do the other using the relations whatever we have in our hand.

We have 15 relations is not it. In the first category this category problems in the first category problems equilibrium equations are written in terms of strain by expressing 6 stresses as function of strains. Equilibrium equations are written in terms of strain. The strain displacement relations are then used to form three equations involving 3 displacements u, v and w. The boundary conditions for this method of solution must be specified as displacement.

So in this approach this is the way we solve after finding u, v and w we can then obtain 6 strains from strain displacement equations and we find 6 unknown stresses from the stress strain equations. Note that compatibility is not being used however u, v and w are determined directly and they ensure that they are singular valued functions. So with this note um as I have already said unless uvw are having single valued function we cannot do that is the reason in many cases after going through the elasticity course many people say that what is the need of compatibility equation.

Because the type of problem we solve that those are already solved by famous physicist and scientist and they have already satisfied the conditions of compatibility. We generally many times skip that part we do not show that it is satisfying the compatibility condition but that does not mean that it is not required to be satisfied. So please keep it in mind compatibility has to be satisfied so with that note we move forward to have some more discussion on that solution process.

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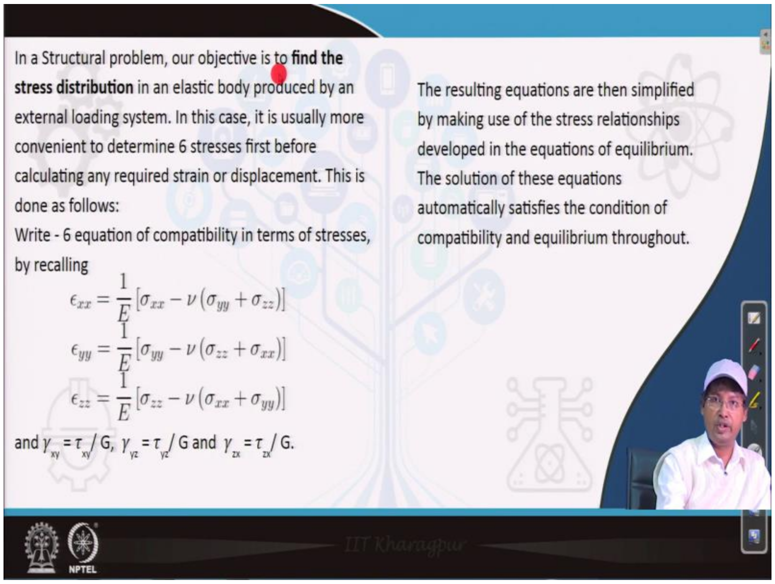

In a structural problem our objective is to find the stress distribution in an elastic body produced by an external loading system. In this case it is usually more convenient to determine 6 stresses fast before calculating any required strain or displacement. This is done as follows. So write 6 equations of compatibility in terms of stresses. So in the second approach to find the stresses first we need to write the compatibility in terms of stresses that is what we will try to do try to see.

And to do that what we do is simply recall first the equations of stress and strain 6 equations we have the resulting equations are equations are then simplified by making use of the strains relationships developed in the equation of equilibrium the solution of this equations automatically satisfy the condition of compatibility and equilibrium throughout. So let us see how do we do compatibility in terms of stresses.

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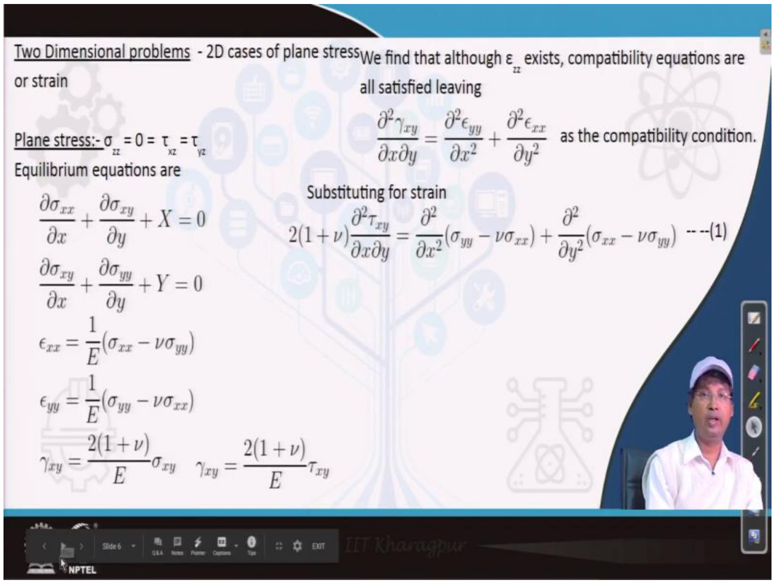

Again to make it simpler to understand it properly we have reduced our domain we have come to the two dimensional problem two dimensional case of a plane stress or strain plane stress or strain we will be solving. So in this first to consider plane stress in today and strain will be discussed in the next lecture. So in case of plane stress as we have already said that in the z direction all the stress components are 0 that is sigma zz equals to sin 0 and tau xz tau yz are also equals to 0.

If we put this condition to equilibrium equation it reduces to this xy are the surface forces stress and relation also reduces to this and compatibility. This is also with substitution of the other relations we can have the stress strain directly with respect to the g is something like this and then we find that although if such z exists compatibility equations are satisfied leaving this equation only.

So if we substitute all these values and the compatibility equations even though the there is a value of f z we can we come across only to this equations to satisfy that is del 2 gamma xy del x del y is del two epsa y del x 2 del two epsilon x del y 2. And then substituting for strain here if we substitute all these strain components but we have a relation it is again I have skipped that part I did not find it simply copying pasting the equations and to show you.

I would suggest you simply substitute these equations and get get the equations here substituting this here and get this equation here. And then if we move forward with this equation what we have we need to use the remaining one is not it.

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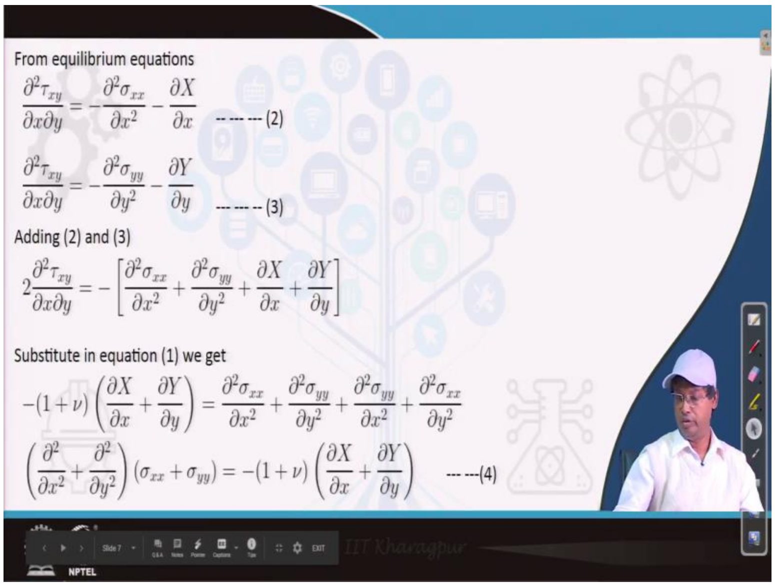

So the equilibrium equation the remaining equilibrium equations 2 and 3 what we have that we can see and that is taken a derivative.

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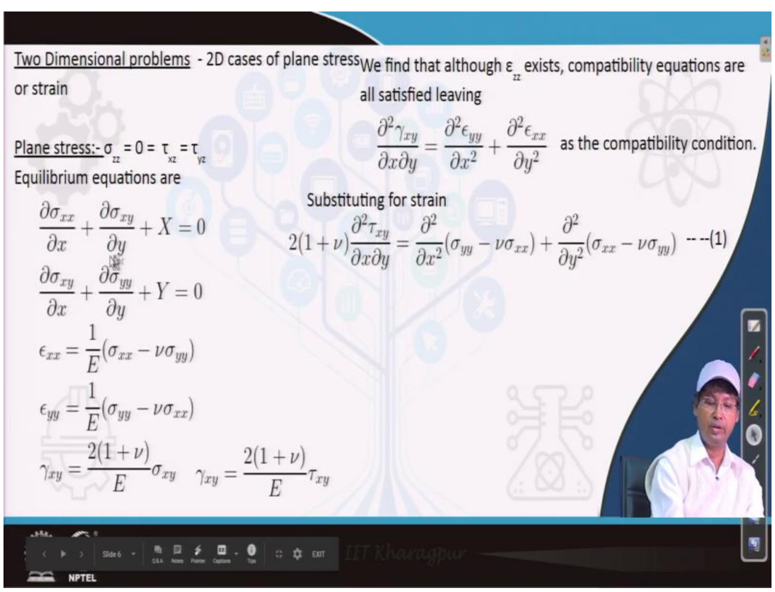

This is substituted this notation is only changed here it is sigma there it is tau is used please keep it in mind that no not a new equation here and that one more derivation is considered and simply rearrangement is done and then those values are substituted in the equation one after adding these two. So what we do we add first these two equations and we get the equations at tau xy we with respect to the boundary conditions the x and y.

And then if we substitute these two in equation one we have a relation which is this that is del 2 del x 2 + del 2 del y 2 sigma xx + sigma yy is equals to minus of 1 + nu del x del y del x sorry del capital X del x del capital Y del y so this is a compatibility in the in terms of stresses. So, with this note which is the second approach to find out the stresses. We conclude we will also find out in case of plane strain how these equations are modified and then we will try to solve a few problems in the next lecture.

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So the reference slide it is as usual it is a combination of the books I am sorry I cannot pinpoint for a lecture which book is followed. So formulation of elasticity problem is discussed in this lecture to some extent and probably you have come across you have learned the process followed to solve a elasticity problem and with that note I thank you for attending this lecture and we will meet again in the next class to solve a few problems, thank you.