Aircraft Structures - 1
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Lecture No -33
Equilibrium Equation in Polar Coordinate System

Welcome back to aircraft structures 1 course this is Professor Anup Ghosh from Aerospace Engineering, IIT, Kharagpur. We are at the beginning of the module 7 or the 7th week lectures we will be discussing equilibrium equation in polar coordinate system because the type of problem what we will attempt this week is requires the polar coordinate system. Polar coordinate system I want to mean not in general the coordinate system will be our highlight of discussion.

Our main aim will be to discuss the discuss and develop the equilibrium equations compatibility equations and other theory of elasticity equations what we have already learned with respect to Cartesian coordinate system. So those Cartesian coordinate system expressions we need to convert in polar coordinate system and in this particular case it is not exactly polar coordinate system what we will be using we will be using a similar to a cylindrical coordinate system.

So the difference between polar coordinate system and cylindrical coordinate system may be imagined something like that the polar coordinate system describes with angle and radius a position of a point in a globe or in a circular manner or in a spherical manner whereas in case of cylindrical system what we can imagine that it is it is a cylindrical body which is being described by the system.

The other way in mathematical sense what do we say that it has a an axis of symmetry. So for things which are having axis of symmetry generally we consider the cylindrical coordinate system and accordingly we go forward. So we will come to the equilibrium equation derivation in this small lecture step by step we will learn how it is done.

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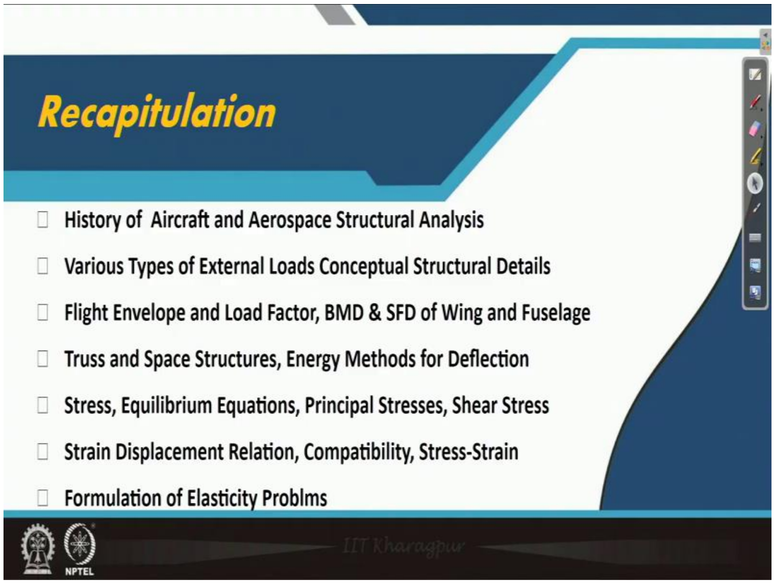

And before that as usual we let us come back to the slides of recapitulation it is important slide in the sense what we have covered so far in the lecture series or lectures whatever we have um so far described. So history of aircraft and aerospace structures analysis is the first topic what we have discussed. In that we have learned that famous physicist have contributed a lot while we discussed about the structural analysis they have discussed how what is the material a material behaves how a material behaves.

How many constants do we need to describe a material behavior? There was so many confusions initially and then finally there was a consensus that for isotropic material again if we talk isotropic, what is isotropic? That is also a matter of question so they defined discussed isotropic they defined that E and u are the 2 constants which are sufficient to describe linear elastic material again.

I use the word linear, linear elastic so they also have discussed linear elasticity non-linear elasticity all those things and slowly they have developed they have proposed analysis theories not only static analysis theories they have proposed discussed analysis theories related to dynamics also or say the time dependent load. And time dependent load how structures behave and then and then we got defined that what is structures.

How anything bears a load and then we have come across that may be there are some specific structures which are predominantly used like plates and cells and we have also seen that the cell structures development have taken place in the last few years back. The basic fundamental developments have been done by scientist physicist researchers in maybe in last 50 years also. So with that we have looked into the history of aircraft also development of aircraft starting from the Kitty Hawk by Wright brothers.

And then we have come across more than 100 years almost 120 little less than 120 years and we have crossed huge steps starting from monocoque to semi monocoque structures and from structures point of view from isotropic material used to the orthotropic material use. From material use of metal to plastic like that we have learned the other developments mainly we have put our stress in terms of structural analysis.

Then various types of external loads conceptual structural details we have learned. So because and structure a structure is supposed to withstand loads. So where from loads come into that structure or on an aircraft. So that is important so we have discussed various situations various flight regimes maneuvers during which portion gets stressed more which portion is designed in a overall manner we have discussed that.

We have seen that there are specific groups who takes care of all these design part it is not that somebody is sitting on a desk and designing an whole aircraft. There are experts who finds out the type of loads and they estimates those there is a an agency known as airworthiness agency in every country almost it is there. So they looks at the critical conditions from where the maximum load is encountered by an aircraft.

So according to that some schedule procedures have already been laid out and people follow that engineers designers follow that and those various types of loads we have discussed to some extent in overall manner. We have seen how the structures are fabricated from thin wall members from forming using the process of forming how thin wall thin plates thin sheets are bend according to the required section type.

And then those are used how those rivets are done all those things we have just discussed articulated portions of wing fuselage we have seen. We have not analyzed those parts are may be done in detail in some other course. Then other things like flight envelope we have seen load factors we have seen many, many more things shear force bending moment diagrams all those things on fuselage and wing we have seen.

We have seen energy methods for deflection we have seen approximate methods we have seen studied theory of elasticity Cartesian coordinate approach we have solved problems we have seen that there are 15 unknowns we need 15 equations considering equilibrium stress strain and strain displacements. And then what we have come across today is that formulation of a problem which requires some other coordinate system. We have solved problems also in last lecture and previous to that.

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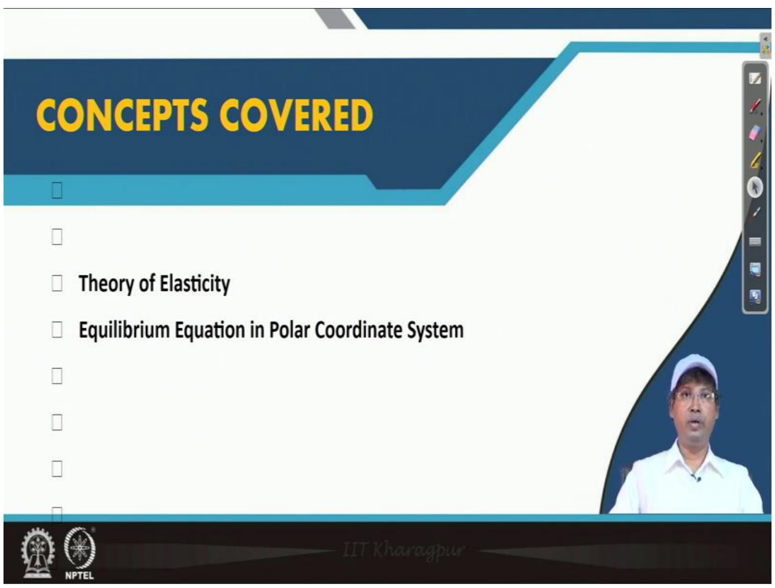

And today we will go into the formulation or say derivation of the equilibrium equation in polar coordinate system. So let us start with that.

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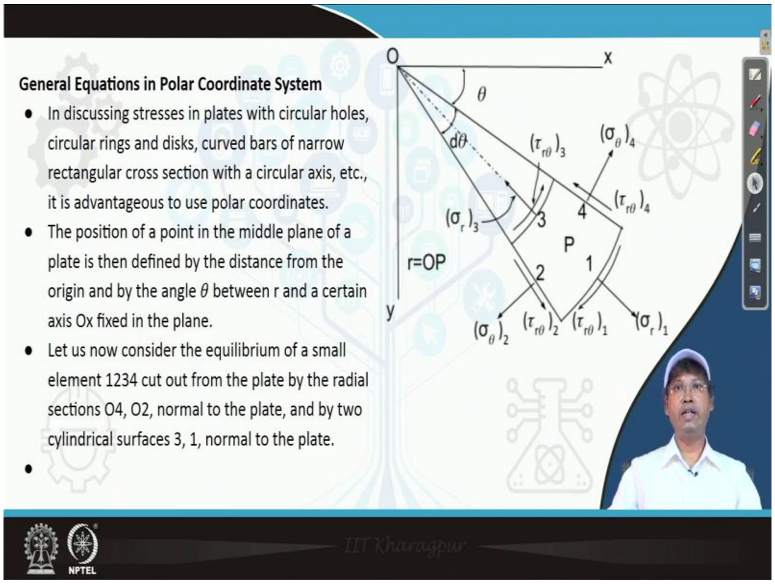

So in that what we will do the general equations in polar coordinate system we need to find out the first equations what we will attempt today is the equilibrium equation. Before we go into the equilibrium equation it is to better to describe what is there on the right hand figure this figure what do we consider and how do we do that is described here also in a very concise manner. What we are considering that again as I told you we are considering asymmetric portion.

Say there is some structure which is asymmetric about the z axis, z axis is not shown which is from this point towards me or on the other side. So if it is if we follow the right hand coordinate system it is going from me to the board that means it is coming this way and it is going the other way to the board so it is something like this, x is coming to y and z is coming down. So anyway that part the axis of the symmetry we would not discuss here.

But we will discuss here that we are considering a small element given by this 1 2 3 4 and named as P, P is the center point of that element and what do we see that there are 2 the radius of point P is as it is said OP is equals to r up to this point it is r and we see that there are 2 red other radial components which are d theta by 2 apart from the theta or on the other sense we can say that from this point it is d theta apart 2 radial planes which are perpendicular to xy plane is cutting the element.

And 2 cylindrical plane say from this point to this point these points to this point are cutting this element. So this element is having 2 straight edges 2 curved edges given as name 1 2 3 4 and this element is theta at theta degree angle with respect to the x. Now we have we need to find out the equilibrium of that particular element to find out the equilibrium of that particular element we have given 3 components of stresses here.

Those components are sigma r sigma theta and tau r theta ok. So it is to better to define what is r what is sigma r and what is sigma theta. Sigma r is for this part particular element or in this coordinate system we are describing the stress experienced in the radial direction so that is the reason r is given here and theta, theta is perpendicular to any particular radial line or plane ok this is perpendicular to this plane given a notation sigma theta.

So perpendicular to this plane acting outward as tension considering positive is given as sigma theta and as usual there are complementary shear stresses obscure stresses come always in pair. So those are given here tau r theta tau r theta tau r theta tau r theta and with respect to the plane the name subscripts and brackets are introduced as 1 2 3 4. And these are also given 1 2 3 4 considering the plane where it is acting.

So let us see what is written here and read almost the same thing is described here in a very concise manner. Let us see in discussion stresses in plates with circular holes circular rings and discs card bars of narrow rectangular cross section with a circular axis etcetera it is advantageous to use polar coordinate. So why do we need polar coordinate that is what is said it is advantageous to discuss it in polar coordinate while there is a problem of a circular hole in a plate.

Circular rings or discs curved bars or narrow rectangular cross sections bars of narrow rectangular cross sections with a circular axis. In those cases if we consider if we discuss with this type of coordinate system it becomes helpful. The position of the point in the middle plane of a plate is then defined by the distance from the origin and by the angle theta between r and a certain axis OX fixed in the plane.

Let us now consider the equilibrium of the small element 1 2 3 4 cut out from the plate by the radial section O 4 O 4 O 2 O 2 it is something like a plane acting on this normal to the plate that is the reason it is saying normal to the plate it is difficult to; so it is acting like this and by 2 cylindrical surfaces 3 1 normal to the plate.

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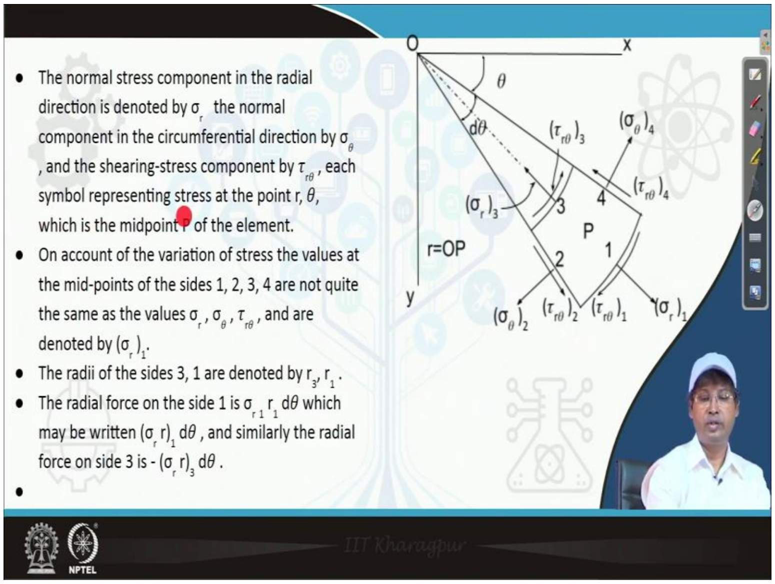

So it is the same figure we need to refer this figure for repeated leaf or derivation so the normal stress components in the radial direction is denoted by sigma r the normal component in the circumferential direction is given by sigma theta and the shearing stress component tau r theta. Each symbol representing stresses at that point r and theta this is the point r theta which is the point P of the element point P of the element.

On account of variation of stress the values at midpoint of sides 1 2 3 4 are not quite the same as the value of sigma or sigma theta r theta and are denoted by sigma r 1 r 2 r 3 like that. So it is said that since there is we are considering that there is a variation. So if that is the reason these are components are given some other subscripts. The radii of the sides 3 and 3 1 are denoted by r 3 and r 1, the radial forces on the side 1 is sigma r 1 r 1 d theta sigma r 1 d theta is this area d theta multiplied by this we are considering unit width in the z direction that is the reason it is not coming.

So this length is r 1 d theta multiplied by sigma r 1 so that is the force considering unit width unit depth z direction it is the force may be written as sigma r r 1 d theta and similar to the radial. Similarly the radial force on side 3 is minus is given because it is acting in the opposite direction that can also be got considering this and we get this equation. So these 2 components of forces we have. Now let us see the other components.

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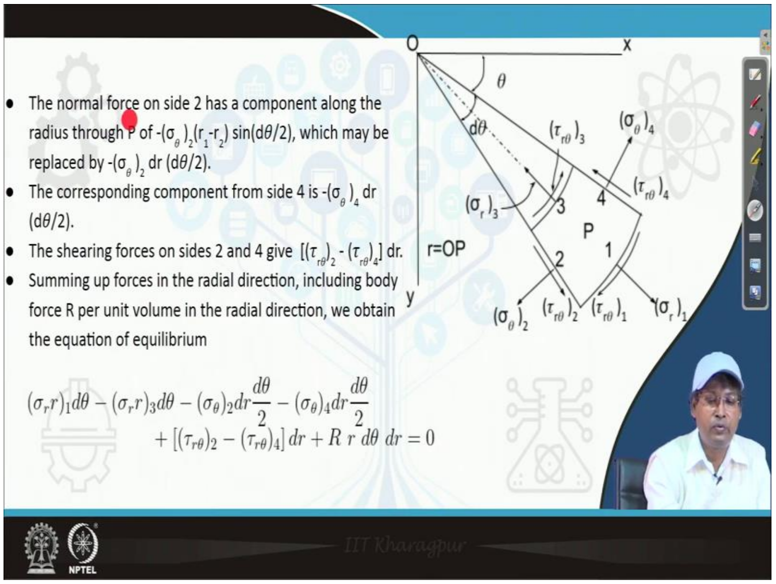

The normal force on side 2 has a component along the radius through P of sigma theta 2 r 1 r 1 minus r 2 sine d theta by 2. So if this is d theta by 2 it is very, very thin line. So this component is as it is given here as sigma theta 2 r 1 - r 2 sine d theta by 2. So it is acting in this direction that is there is in the minus is coming here and sin d theta by 2 is very small angle that is the reason we can directly consider that it is equals to d theta by 2 that is what is said.

And this change of length that radius is considered as dr so this becomes minus sigma theta 2 dr d theta by 2. Similarly from this also we will have 1 more component. If I draw it will become little bit clumsy so we can easily imagine that 1 more component will come say that is also coming in this direction. So similar way that 1 is this is also d theta by 2 and accordingly we get this value. Now the shearing forces on side 2 and 4 gives side 2 and 4 this is side 2 and this is side 4 considering 2 this side positive this minus this dr is the total force considering again z unit depth.

Summing up forces in the radial direction including body force R per unit volume in the radial direction we obtain that this is coming here as sigma r 1 sigma r not this 1 this is described in the previous slide this component this component is this component this is this component as it is given this is this component as it is given and this cr portion here whatever is given and this is the body force. So we have in the radial direction this is the equation and we see how this equation changes.

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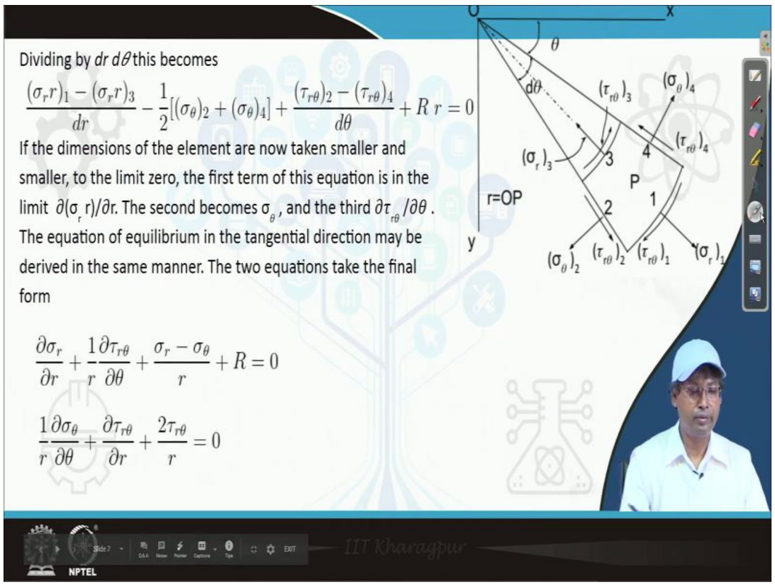

So in this what we have we divide the previous equation with dr and d theta and what do we have is that sigma r r 1 - sigma r 3 by dr half sigma theta 2 + sigma theta 4 tau r theta 2 - tau r theta 4 by d theta are smaller equals to 0. So if the dimensions of the elements are now taken smaller and smaller to the limit 0 this is nothing but considering the limit. The first term of the equation is in the limit of del sigma r r dr this gives del sigma r dr.

The second becomes sigma theta it is average of 2 sides and the third this also becomes del tau r theta del theta now if it is a multiplication of 2 terms if we expand this using del du v del x that multiplication of 2 variables derivation concept if you use the equation of the equilibrium in the tangential direction may be derived in the what do we get is this equation sorry this is for this statement.

So what do we get is this equations from this component we get this as well as we get this component sigma r by r and the remaining is whatever we have this is this component r is getting divided this is this component. So all the components after getting it in the radial direction we have this equilibrium equation and the process what is shown already considering those process if we consider in this direction the equilibrium and in that direction equilibrium whatever we can get is the equation the equation of equilibrium in the tangential direction may be derived in the same manner the 2 equations take the final form as 1 by r del sigma theta d theta del theta + del tau r theta del r + 2 tau r theta by r equals to 0.

So we get the equilibrium equations considering the body forces in case of polar coordinate system or particular case the cylindrical application or cylindrical coordinate system because we are not considering the z considering as a symmetric portion of that.

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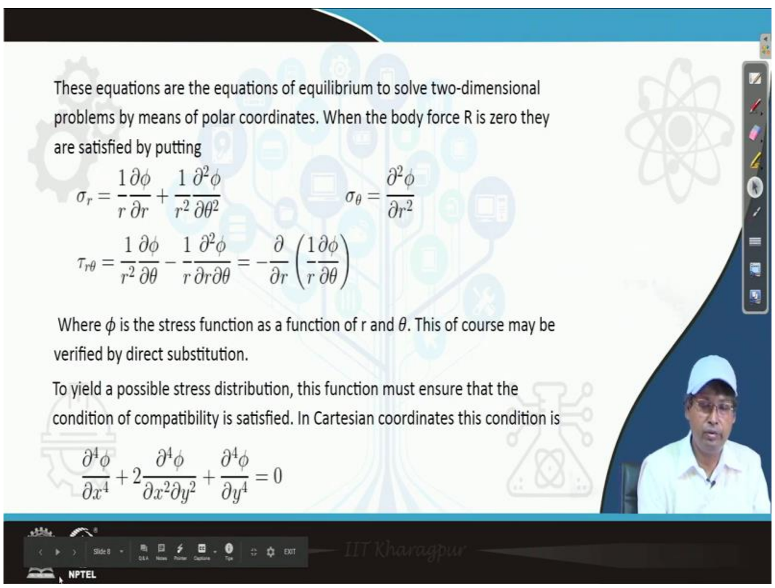

So with that we proceed further the equations are the equations of equilibrium to solve 2 dimensional problems by means of polar coordinates when the body force R is 0 they are satisfied by putting this expression with respect to phi the stress function, the stress function already you have. You are introduced to so this how do we get this expression we will prove in the next lecture but let us consider that if we transfer the stress function expression we get this stress function expressions.

And if we put this stress function expression in the previous equilibrium equation we those are satisfied so with respect to that we define that sigma r is equals to 1 by r del phi del r + 1 by r square delta phi del theta 2 sigma theta is del 2 phi del r 2 and tau r theta is 1 by r square del phi del theta - 1 by r del 2 phi del r del theta equals to minus of del del r of 1 by r del phi del theta this is the popular way generally written where phi is the stress function as a function of r and theta.

This of course may be verified by direct substitution to yield a possible stress distribution this function must ensure that the condition of compatibility is satisfied. The Cartesian coordinates; in Cartesian coordinate this condition is as we have already seen grad 4 phi is equals to 0 or by harmonic equation is this. So this is our next task in the lecture forthcoming lecture we will come across about finding out the equivalent expression in polar coordinate for the compatibility equation.

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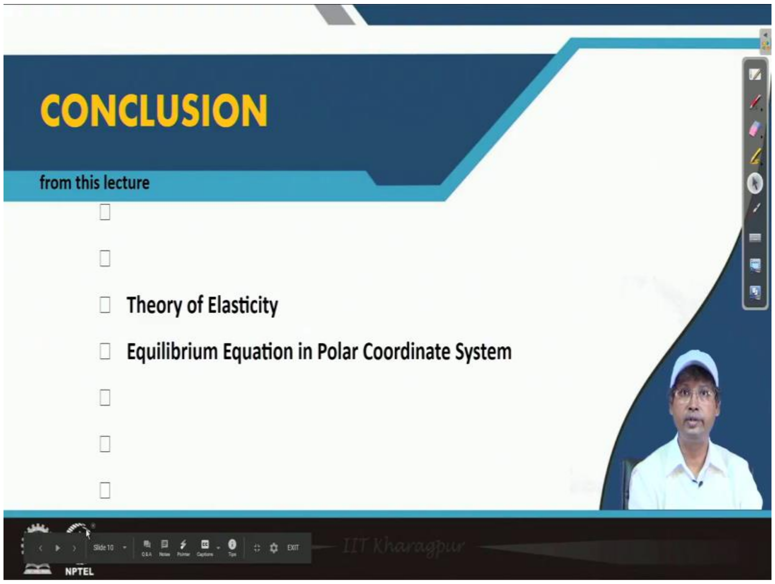

So with this note we come to the almost end of that lecture. And what we have learned in this slide is equilibrium equations in polar coordinate system we have derived we have derived the polar coordinate system equations for a axis symmetric case. And we will further find out the solution or compatibility condition with this topic we come to the conclusion concluding slide today and thank you for attending this lecture we will come back again with the compatibility equation, thank you.