Aircraft Structures - 1
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Lecture No -38
Theory of Elasticity - Torsion Problems

Welcome back to aircraft structures one course and this is Professor Anup Ghosh from Aerospace Engineering Department, IIT, Kharagpur. We are at the first lecture of the module 8 in sequence this is the 38th lecture and we will attempt the problem of torsion in theories of elasticity approach.

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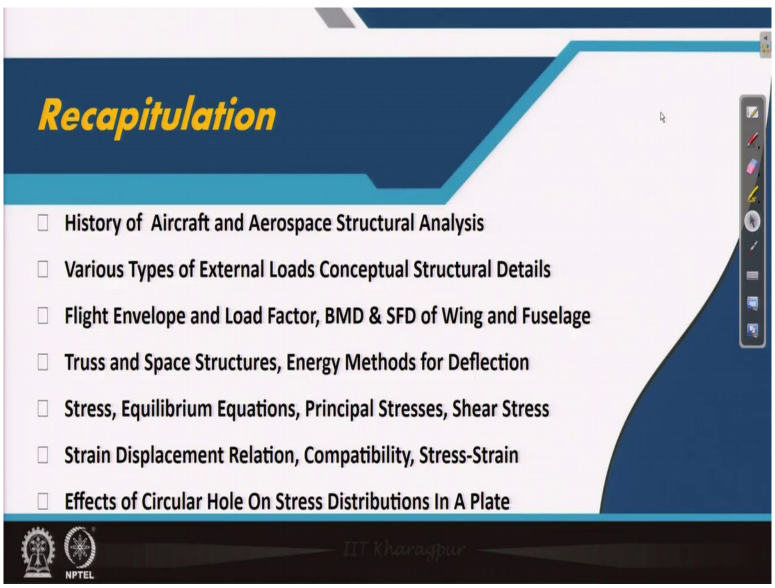

So in that connection first thing what we get is the recapitulation slide in this recapitulation slide we have covered a history of solid mechanics or structural analysis starting from the beam analysis then how before the beam analysis how the theory of elasticity came in how the linear material non-linear materials are described. And then to beam analysis plate analysis vibrations who did what all those things we have we have learned.

And we have seen that the development is still going on the most recent development may be in the area of cell analysis which is very, very important structure while we try to analyze aircraft structure where all the members probably are curved in nature and thin in dimension in the lateral direction so it belongs to the cell group after that what we have learned is that how an aircraft experiences load or how does it withstand load where from the loads come in case of aerodynamic loads.

There are different conditions like rolling pitching or normal aerodynamic lift load or may be gassed or maybe an engine out condition from propulsion side all those loads where and how it comes we have discussed so far. To some extent we have also seen different type of aircraft structures how those are fabricated? How does they look like? How do they look like? And then we have seen different types of loading envelope flight enveloped a new concept is applied for different types of aircraft how the inertia load plays a big role.

And we see that that for civil aircraft where the maximum g force or inertia loads may be limited to up to 2.8 to 3.5 or maybe approximately said as 3 but whereas in case of aerobatic aircrafts or a fighter aircraft it goes to maybe 5 5.5 or 6g so it is it is a huge amount of change of force we need to take care. So depending upon the type of aircraft we need to develop the flight envelope and we need to consider the parts of the structures to design to withstand those inertia loads.

Not only inertia loads it also experiences many other loads as we have discussed we have seen the bending moment shear force diagram of whole aircraft segment and divided in 2 parts in orthogonal direction that is fuselage and wing and then shear force bending moment we have done. We have done stress sorry 3 dimensional truss solved problem of landing gear and then we have gone to the theory of elasticity.

But before we go to the theory of elasticity we have solved problems related to deflection to find out deflection, different energy methods we have learned and in amongst the energy method a very, very important method we have learned that is the Rayleigh ridge method. So with that note we went further for theory of elasticity we developed several equations 15 equations namely may be and more because compatibility is also involved there.

We have seen compatibility in terms of strain we have seen compatibility in terms of stresses compatibility is something which has to be maintained within a structure while we define in mathematical way. In practical world all structures are continuous there is no problem but while we define it mathematically we need to define accordingly and after that solving a few problems initial small problems with inverse and semi inverse approach.

We have come across to the problem where we have discussed the effect of a circular hole in a plate which is uniaxially loaded. And we have seen that there is a huge stress concentration because of those loads may be observed that is up to the 3 times of the tension applied and that is the reason it the crack opens transfers to the direction of the application of the tensile stress. We have seen how we need to take care of the holes by reinforcement we have seen how a crack propagation takes place and why it is like that with help of ellipse and stress concentration equations.

So after that we will try to solve this week we will start to solve this week problems related to torsion.

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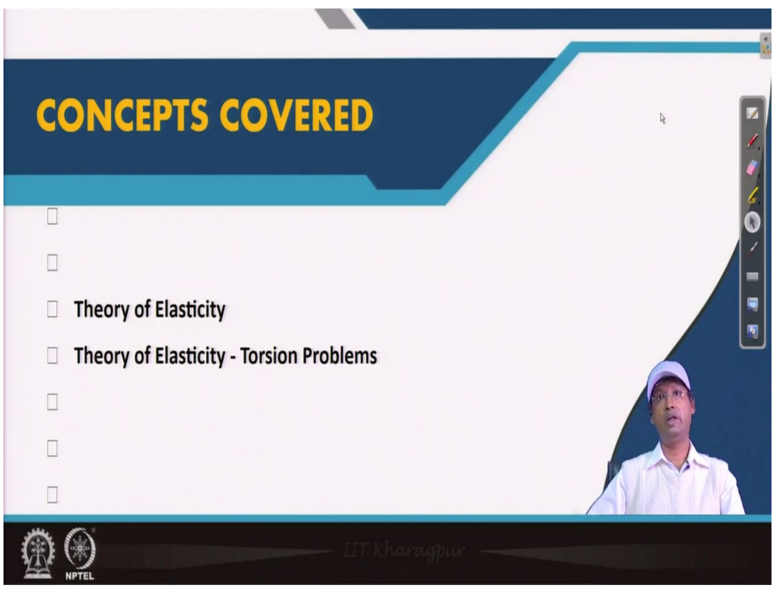

Torsion problems what you have solved in your previous mechanics course probably is little bit different than the way it will be approached in this section. The basic difference is the considerations of warping. What is warping? That is a big question while there is out of plane displacement out of plane means which plane we are talking about. So if we consider in most of the cases what you have seen is that we considered we have considered a cylindrical shaft which is fixed here and we applied some torsion T and what we see say we have observed or assume that if we consider any of one of this plane that plane remains at that plane but it rotates.

But there is no displacement in this direction that is what we said as outer plane displacement so out of plane from the plane of the cross section of the circular member. So if there is some displacement out of plane we call that as a warping. Warping is a phenomena we need to learn because practically if we do if it is a non circular section we observe a warping very predominant way. For example if we consider a rectangular bar what will happen if we apply a torsion?

This will deform like this and this will continue for all sections. I have exaggerated the thing it is difficult to draw all the portions but this outer plane this corner is coming down and the other 2 corners are going up this type of phenomena is known as the warping. So if it is a non circular section we need to take care of that and how do we take care that is first established or developed by Prandtls. And those assumptions the way it has been done that we will learn slowly in very small segments of small lectures. So let us proceed with that note.

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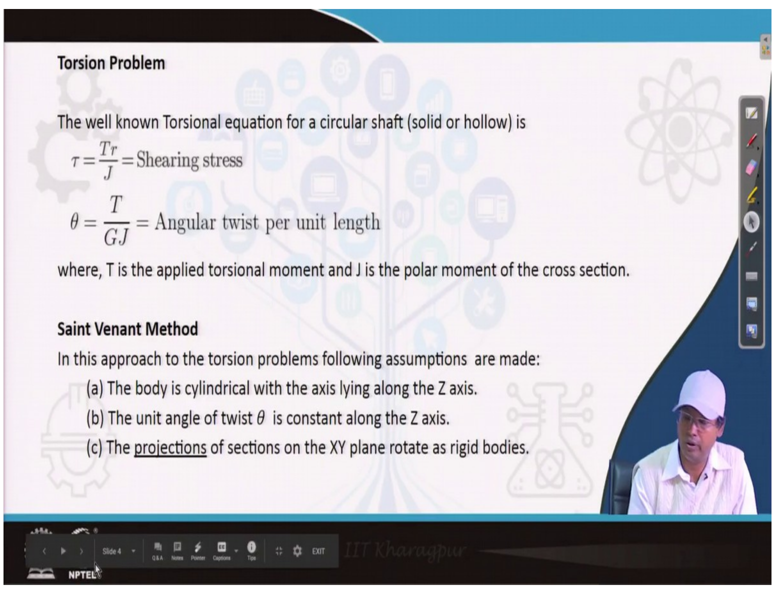

So what do we see the torsion problem the well known torsion null equations for circular shaft solid or hollow is tau shear stress developed is T r by j and the theta per rotation per unit length twist angle of twist or rotation per unit length is where t is the applied torsional movement and j is the polar moment of the cross section. As I said Saint Venant started the analysis and then later

Pandors also worked on it. In this approach to the torsion problems following assumptions are made.

The body is cylindrical do not think that cylindrical means a circular cross section the body is cylindrical with the axis lying along the z axis, axis system will see in the next page we have drawing for displacement equations we will see. The unit angle of twist theta is constant along z axis. Here comes the most important assumption the projections of the cross of sections of x y plane rotates as rigid bodies.

Earlier we did not use this term projections we said that the sections on the xy plane rotates as rigid bodies. So that is the difference here we say the projections of this so with that considerations we will go. So if a particular section is projected on the xy plane that rotates as a rigid body not that section completely. So there is out of plane deformation and we need to take care of that.

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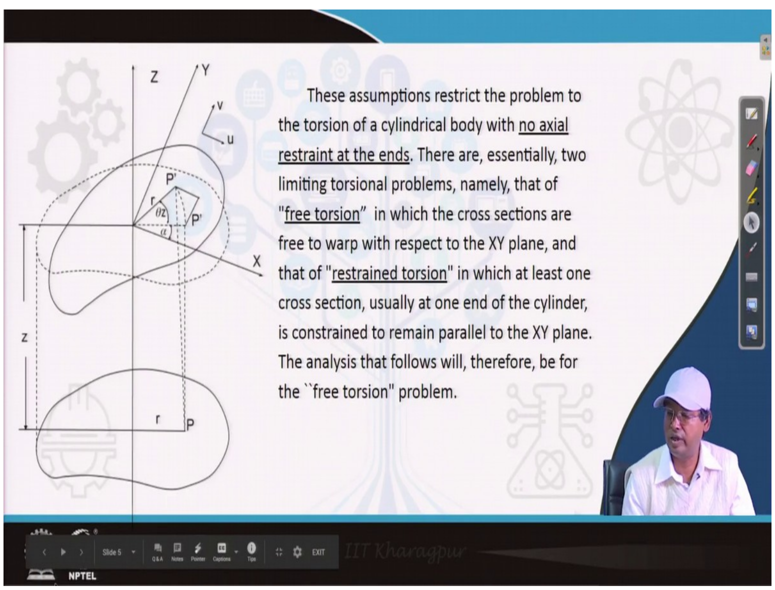

So let us try to define the displacement first with respect to torsion. In this section it is the description of the problem first this assumption restricts the problem to the torsion of a cylindrical body with no axial restraint at the ends. Why it says like that let us read and will come back again. There are essentially 2 limiting torsional problems namely that of free torsion in which the cross sections are free to work, please note that in case of free torsion sections are free to warp or free to deform out of plane with respect to the xy plane.

And that is restrained torsion and that of restraints torsion in which at least one cross section usually at one end of the cylinder is constrained to remain parallel to the xy plane. So here what we are considering that no plane is restrained even the case we discussed for the circular the figure I have drawn in the last slide it is not like that no part was even this plane as well as also this plane no plane is restricted to deform out of the plane or the xy plane here with respect to the xy it is quite clear.

The analysis that follows will therefore be the free torsion problem. So what we will consider in this particular case is for the free torsion. So that for the free torsion what how are we going to define the displacement and how do we get the first starting equation with that small note we will end today's lecture. So let us try to go forward.

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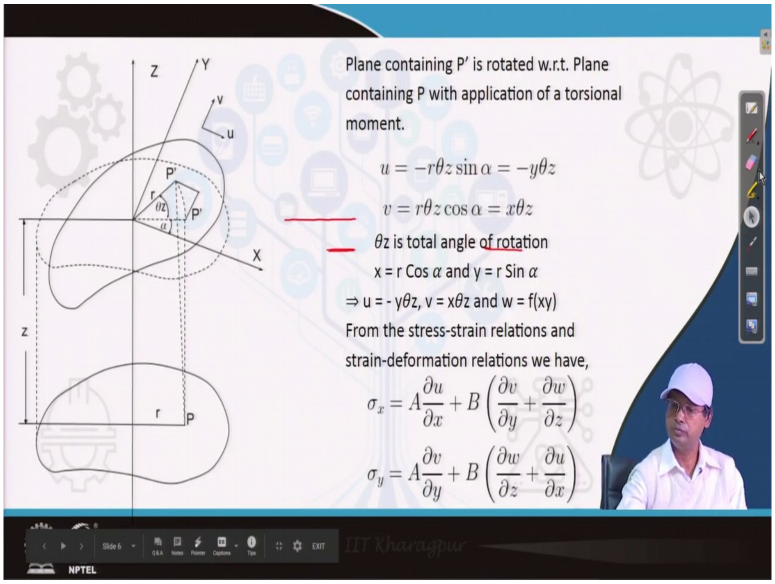

Here we have it so initially this section this is the section of the cylindrical body bottom section this dotted one is the top section and then there is a torsional force applied on this body and we have seen that theta is the rotation per unit length. So, the total rotation here is theta multiplied by z because from this point to this point or the segment of the cylinder cylindrical body we are considering is the off length z. Now with that considerations; considering xy is like this similarly xy is also here at this plane we are considering the displacement at this particular point.

We define that a point p which is r its distance apart from the axis or the z axis is rotated to p prime here both are p prime because initially this was the p prime point and then after rotation after application of the torsion it has rotated to this point. Now with this concept what uh how do we get the displacement and other equations let us try to see. Plane containing p prime is rotated with respect to the plane containing p with application of a torsional moment.

Now u is equals to r theta z sine alpha and v is equals to r theta z cos alpha this is not derived here why the minus is coming why it is not like that I would suggest you to do this as homework because if we go for the geometric way drawing the components and other things it becomes difficult to understand. So we need to find u v as we have discussed that finding u and v in geometric way is much more difficult so following trigonometric approach probably will help.

So, what we can assume that as it is the drawing what is prevailing here we will draw the same thing same phenomena in a different way. So let us this is the xy plane and with r I think I need to revise it, say this is the point represented as x 1 y 1 and after and this is the angle alpha and after rotation it goes somewhere here which is x 2 y 2. Now this is theta z say if we simply consider u, u is equals to r cos alpha - r cos theta z + alpha.

So if you simply do it you will get this relation and the similarly v also you can find out so with that I think we can proceed you can solve this easily. So what do we have since we have the other relations like x is equals to r cos alpha and y is equals to r sin alpha u becomes minus of y theta z and v becomes x theta z, z is not subscript please be careful z is never subscript. So that is what is written here repeatedly.

And we define one more important function w function of xy this function is the difference with respect to our previous things from the stress strain relation this is known as the warping function from the stress strain relations and strain displacement deformation relations we get this, this will be repeated in next slide also we will use it sigma x sigma y and all those will let us see how do we come to the equilibrium equation.

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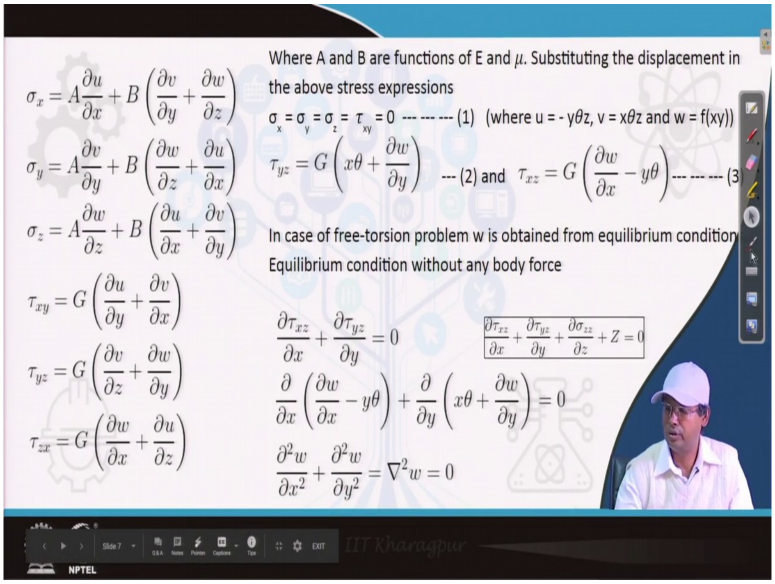

So with this note what we see is that we have all the 6 components of stresses here and if we substitute these values what we have got the displacement because of the torsion. If we substitute all those values here we finally get that sigma x sigma y sigma z and tau x y is equals to 0 you can easily substitute this and check have checked several times please note that this A and B are written in a generic form which are function of alpha E and mu.

E and mu is the sorry is expressed in terms of E and mu then tau yz and tau xz are the 2 remaining stress components and if we substitute these values here what do we get we get this is a simple substitution in this we get that tau yz equals to z x theta plus del w del y and tau tau yz is this tau x z is z del w del x - y theta. So 1, 2, 3 are the 3 equations. In case of free torsion problem w is obtained from equilibrium conditions equilibrium condition without any, body force which is the third equilibrium condition we need to bring because of because all other terms are 0 only this equation will lead to equation what we is what is useful.

So simple substitution of that here what do we get is that we get a function in terms of w and which may be stated as that grad square w is equals to 0. So for a torsion problem to solve our equilibrium equation is grad square w or the working function is equals to 0. So with that note let us stop here for the development of torsional problem. We will come back in the next lecture with a few more concept and we will go through it.

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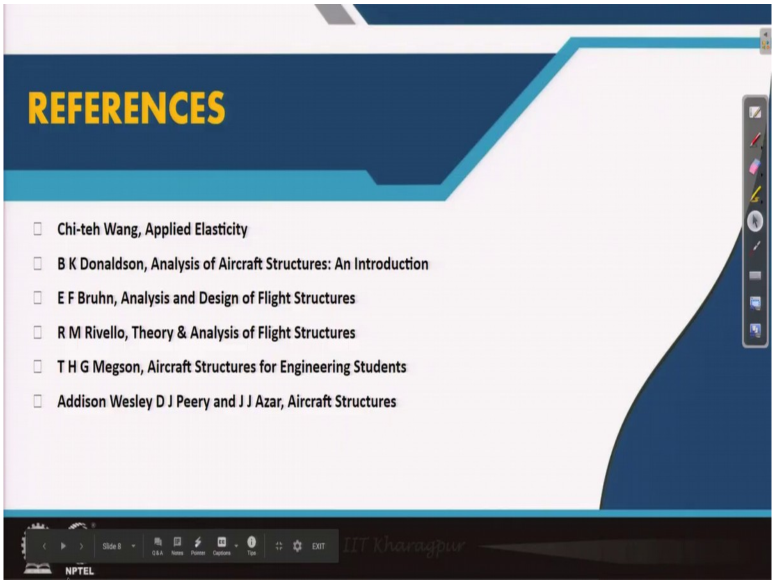

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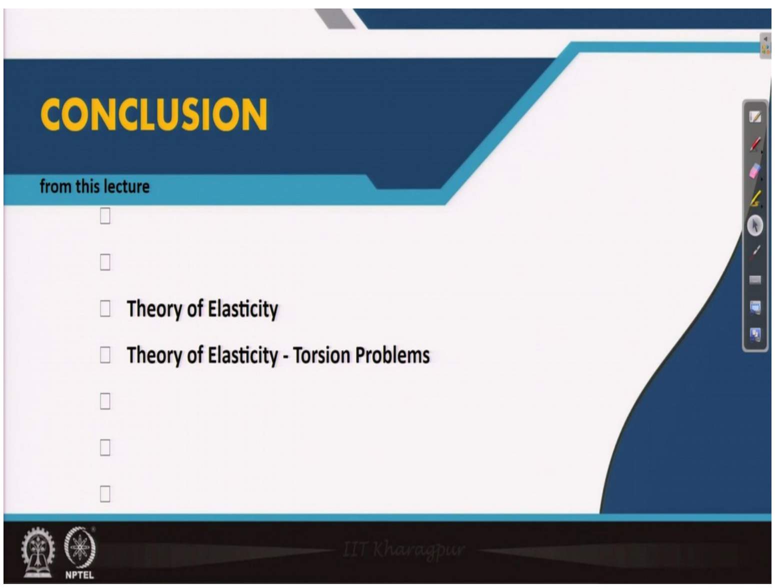

So we come to the standard references what we need to follow and what we have learned or initiated is the torsion problem as the theory of elasticity approach. And at the end I thank you for attending this lecture and we will come back with some more lectures soon, thank you.