Aircraft Structures - 1

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Lecture No -40
Torsion of an Elliptical Bar

Welcome back to aircraft structures 1 course this is Professor Anup Ghosh from Aerospace Engineering Department, IIT, Kharagpur. We are in the continuation of 8th week lecture this is the 40th lecture in that equations sequence today we will try to solve problems related to torsion of an elliptical bar. This is very interesting problem this shows the mathematical way of how the warping takes place.

We have talked about warping in the last two lectures I have shown you drawing figures also but in this particular case for rectangular bar I have shown you how the warping takes place we have done experiments and we have seen observed those things we do regularly that type of experiments as a regular lab of of B.Tec students and with that note we will proceed further to solve today's problem.

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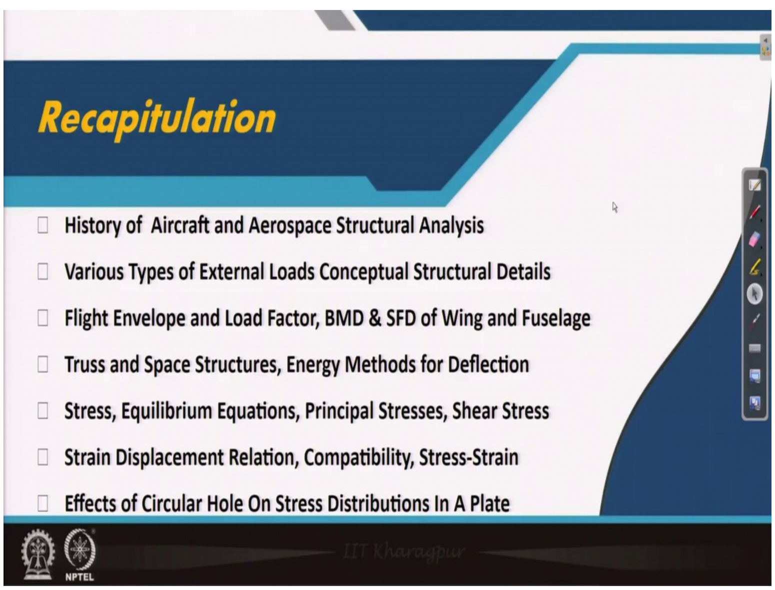

In this before we go into solving the problems what we will do we will go for a recapitulation as it is listed there today I would not discuss much about what we have done. I will try to simply read it history of aircraft and aerospace structural analysis various types of external loads conceptual structural details we have covered we have also come across a flight envelope and different load factors experienced by different types of aircrafts why it is different how it is different all those things we have discussed.

We have drawn bending moment shear force diagram considering typical loading unit load method we have used we have considered those things and solved found out the surface bending moment diagram. Then we have come to the three dimensional structures or the space structures solved a few problem related to landing gear. Then energy methods we have solved different problems not only determinate indeterminate problems also we have solved.

Found out indeterminate reactions redundant forces and then in that method we have come across two unit load method dummy load method and other methods related to Rayleigh rich method. Rayleigh method is really very, very important 1 and we follow those in further derivation or further numerical processes then we have learned the stress elasticity approach elasticity approach of solving problems I have already said many times why theory of elasticity is so important.

In a brief we can say unless we learn the theory of elasticity approach of solving a problem it is difficult to have a insight into the development of stresses and strains inside a body while it is loaded. Unless we have the insight it is difficult to predict catastrophic situations or failures and those criteria. So that is the reason we need to study the theory of elasticity approach and we have covered those things. 1 very, very important problem we have solved in a week long session that is the circular hole stress due to a circular hole on a plate.

While it is loaded uniaxially even in biaxial nature we have seen if the hole is elliptical how it changes. And in the last two lectures what is not listed here we have developed equations to solve torsion of cylindrical bodies it is not considered we will our approach is not considering the section as circular and at the end of this lecture we will come to the section as circular and we will conclude that how it is conforming to the usual solution. With that note we proceed further.

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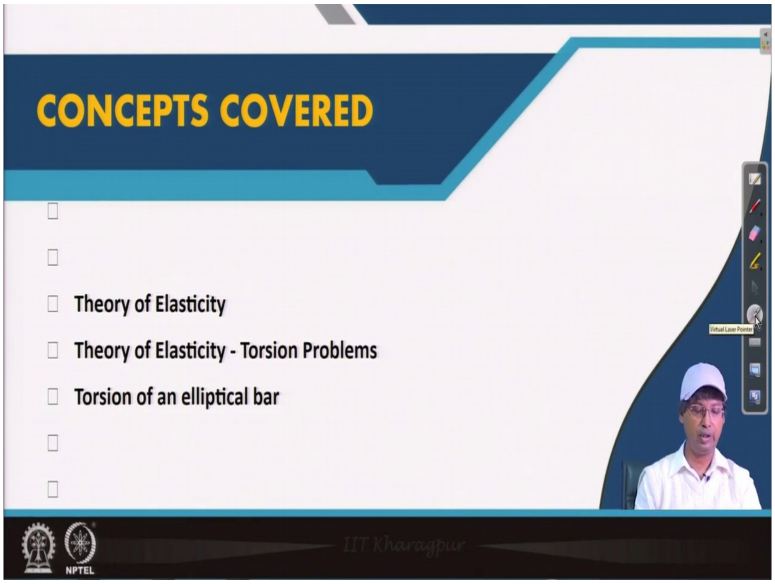

And torsion of an elliptical bar is the problem what we will be solving today.

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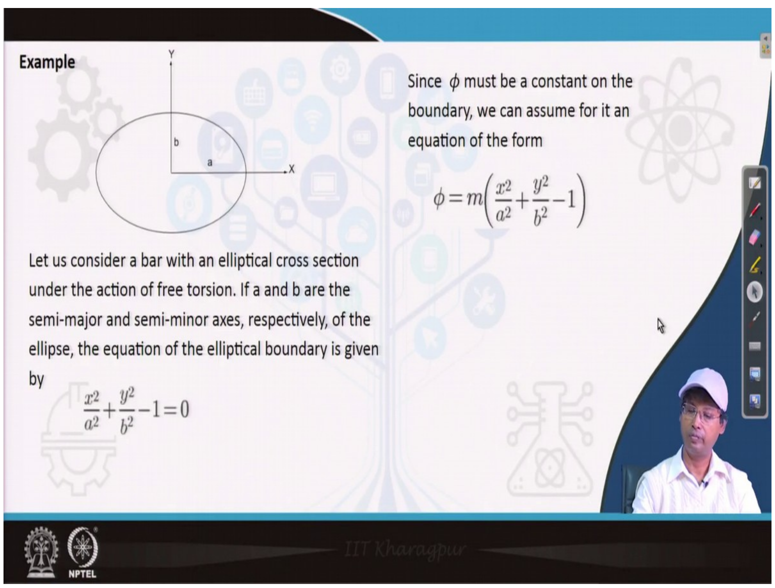

So with that the problem what we will be solving is an elliptical section having a semi major axis and b semi minor axis. Let us consider a bar with an electrical cross section under the action of free torsion again free torsion is there as we have said unless it is free torsion warping is difficult to observe and that is the reason warping has been introduced and has been observed here. If a and b are the semi major and same inner minor axis respectively of the ellipse the equation of the equal elliptical boundary is given by x square by a square + y square by b square - 1 is equals to 0 its quite well learned.

Again since phi must be a constant on the boundary we can assume for it an equation of the form what is equal to similar to the equation of the ellipse. So phi is equals to a constant m multiplied by the x square by a square + y square by b square – 1.

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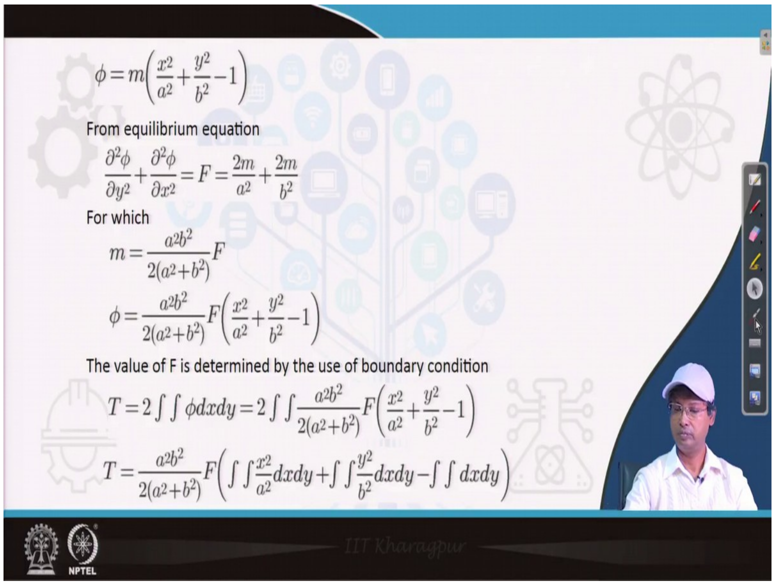

So phi as we have defined in the last slide from equilibrium equation if you substitute this what do we have del two phi del y 2 + del 2 phi del x 2 is equals to constant as we have seen and that constant we get the value of m its simple rearrangement we have done. And if we substitute back to that the phi takes the form as it is said a square + b square by twice a square + b square multiplied by F and the remaining portion is part of the ellipse equation.

The value of S if is determined by the use of boundary condition. So that boundary condition whatever we have earlier stated we have not derived this but it can be derived from the boundary condition surface forces boundary condition easily. So if we put the value of phi here and we carry out write the equations in a segmented manner what do we have we have x square by s square d x d y square by b square dx dy and this is - dx dy.

So if we this terms are quite familiar is not it, if you try to remember we will find that this you have already solved.

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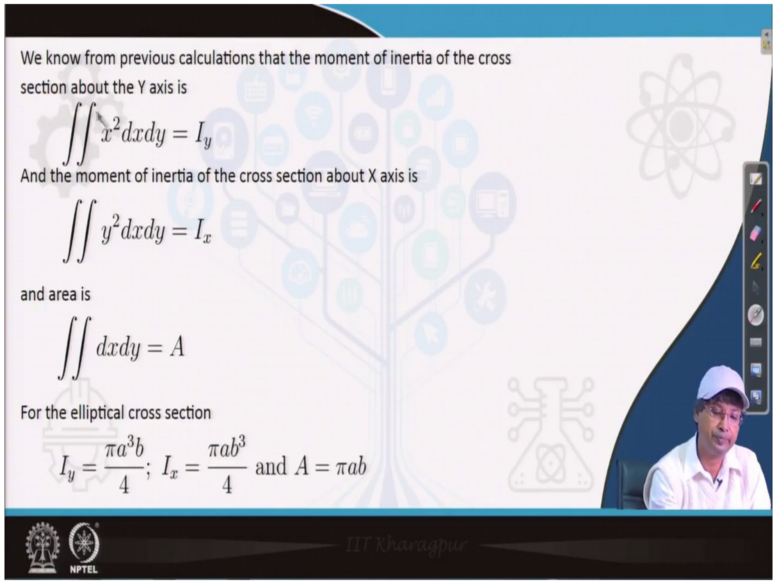

As area moment of inertia so we know from the previous calculations that the moment of inertia of the cross section about y axis is x square by del dx dy that is I y, similarly this is y square dx dy is I x integration definitely is there and integration dx dy is equals to the area. So now if we find the integration of these things so I x is equals to pi a cube b by 4 I y is equals to pi a cube b by 4 I x is equals to pi a b cube by 4 and a is by ab as we know.

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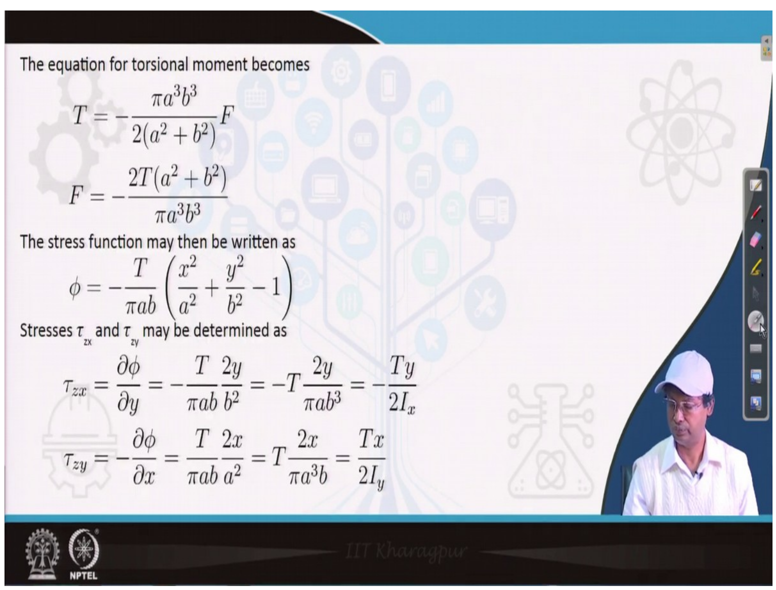

So with that note we come to a simple solution for T and we write that F may be written as – 2T x square + v square by pi a cube b cube. So now we have 1 more expression for the a earlier we had an expression with respect to theta. Now from the boundary conditions implementation what we have the expression of F the constant as in terms of T. So easily we can correlate those two so but it is not done here it may be done later.

So the stress function may be then written as - T by pi a b x square + by a square + y square by b square - 1 is equals to this. Now once we have the stress function in terms of the applied torsion. So we can easily find out the shear stress shear stress experienced by a particular section. So as I mentioned z along x z along y del phi del y and we simply take derivative of this you can easily do I think and what do we get is - T y by 2 I . Similarly for the other direction in the y direction in the z plane the shear stress is T x by 2I y.

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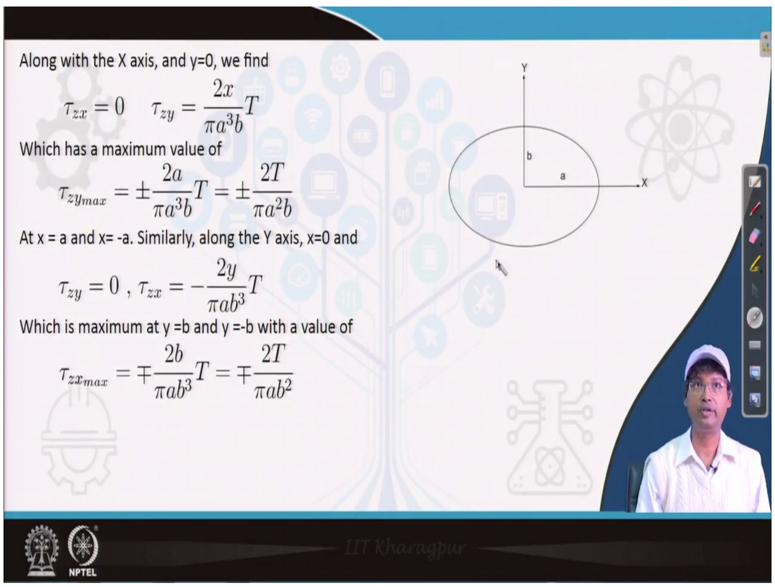

So to have some discussion with respect to the shear stresses as we have seen. So if we look at the expression along with the along the x axis and y equals to 0 we find that that means if we follow this axis what do we have since y is equals to 0 the y expression if you look into the previous slide you will find that tau zx is equals to 0 and tau zy is having some value. So if we try to draw it here we have only value may be increasing linearly why because it is a function of x.

And this value is the maximum value what do we have if we substitute x is equals to a we have this value is 2T by pi a square b and we do not have the other component zx is missing in this direction there is no shear stress shear stress is acting only in this direction and the maximum value is plus minus this because 1 may be this direction the other may be in the other direction that is the reason it is said plus minus this.

Similarly at x equals to a at x equals to - a up to this is this one. So similarly along the y axis the x is equals to 0, if we follow this line x is equals to 0 what do we have again? We have that this is equals to minus so if it is equals to minus we can have and this value is the maximum value that is 2T by pi a b square ok so that is what at the plus minus even here also we will have so with that note we observe that for an elliptical section at the boundary the stresses are not same.

It is different and it is not that every point there are tau xz x as well as tau yz in between any point if you consider whatever may be the point we have values tau zx as well as tau zy I think I have written wrong zx and tau zy. So with that note and with the distribution of shear stress this is very, very important understanding in my opinion we will proceed further for this type of solution what else we have to see to have some insight.

So unless a repeat unless it is on this line there are shear stress components both tau zx as well as tau zy and with that note we proceed further to solve or observe other properties.

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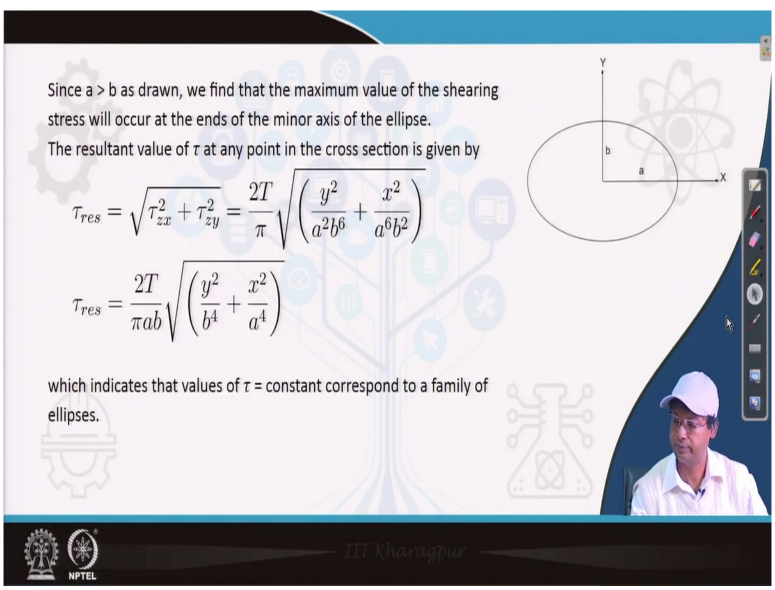

Since a is greater than b as drawn we find that the maximum value of shearing stress will occur at the ends of the minor axis of the ellipse. So from the expressions easily you can say one is what is the expression one is 2T pi a square by b the other is 2T pi a b square since a is larger this value is more that is what is said. So the amplitude here will be more whereas amplitude here will be less.

The drawing what I drawn last time probably showing the same amplitude but it is not the same amplitude what we have is that to notice that we find that the maximum value of shearing stress will occur at the ends of the minor axis of the ellipse. So the maximum value will occur at this place or at this place these point this point. The resultant value of tau at any point in the cross section is given by it is as usual it is very easy formality formula to observe and we have the resultant value as given here as 2T by pi a b over square root of y square by b to the power 4 + x square by a to the power 4.

Which indicates that the value of tau is constant and correspond to a family of ellipse so family of ellipse means where this the a by a b ratio is maintained and there is a relations which satisfies this equation in those ellipse it is constant all resultant shearing stress is constant. So with that node we proceed further.

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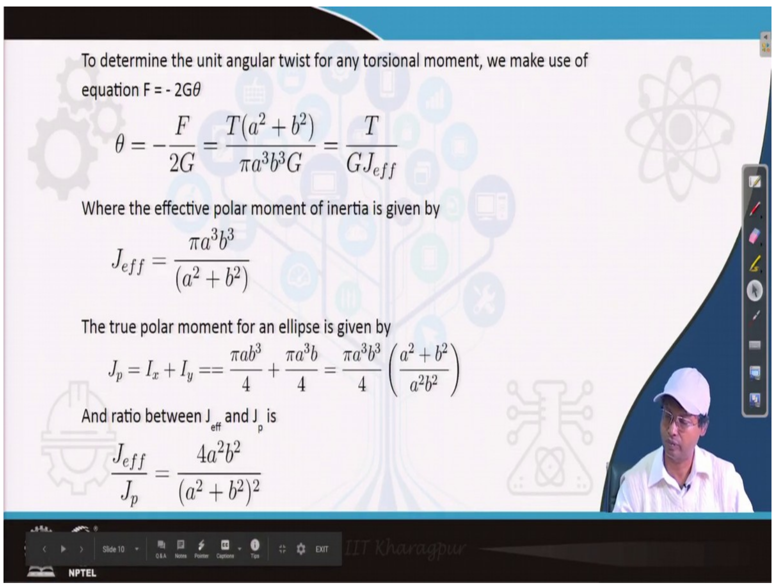

To determine the unit angular twist of any torsional moment we make use of the equations as we have done earlier F equals to - 2 g theta and then if we put that what we have as I mentioned in the previous slide that there we got expression with F with respect to T and put it to find out phi here we are putting the same value and getting the equations for theta, theta is equals to T by G J effective where from the J effective is coming?

We are in it is quite known that theta is equals to 2 by GJ J is the polar moment of inertia of the section. Now but in this elliptical case what we observe that J effective or from this equation the J effective is having some different value that is pi a cube b cube by divided by a square + b square and whereas if we find out the usual way of a polar moment of inertia of that particular section of ellipse as we have described earlier with semi major axis as a and say minor axis as b we get the expression something like this is written in a special way to cancel out a few terms.

So if you take a ratio of this and this what do we have is equals to 4 square b square divided by a square + b square whole square and let us try to observe what happens if we assume J is J p or e is equals to J effective.

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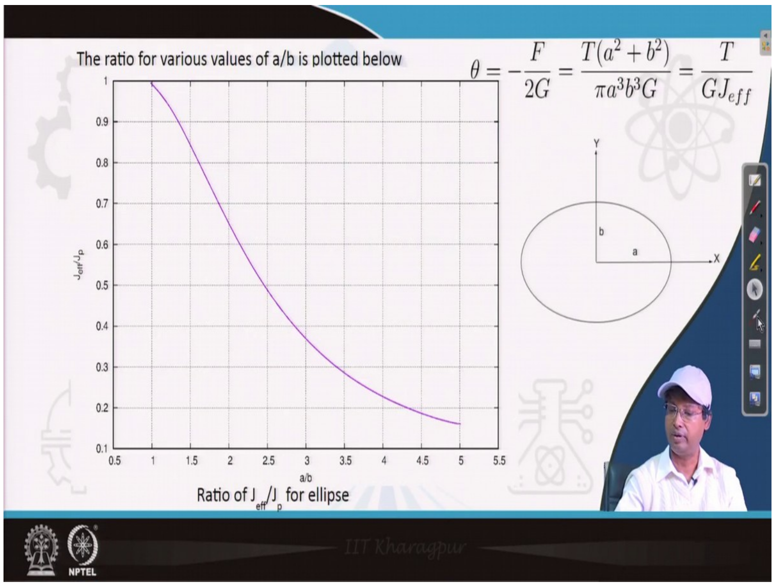

To study that if with respect to the variation of a and b we have prepared a plot it is available in almost all good books. So with that what is that plot we have plotted the J effective by J p in the y axis and in the x axis we have plotted the ellipse semi major and say minor axis ratio that is a by b as it is pointed out here. So what do we see if it is 1 that means it is a circular section in that particular case there is no change of this value the ratio that ratio is equals to 1 but as it becomes the ellipse there is a considerable change of this value.

At some point while the a by b is equals to 5 or it is very, very slender or thin ellipse if we talk about in that particular case it is the value is about 0.2 here about say about say 4.25 or somewhere it is about 0.2 and that is a considerable change in value of J p and definitely there will be a constant they were sorry considerable change in the tau as well as theta. But a simple observation you can see that what is happening the J effective is increasing that is the reason we get the value in fraction.

So as it becomes more elliptic from the circular the J effective value is increasing and from there the theta value is decreasing. If we look at that manner what is happening if the torsion applied torsion remains same the angle of twist reduces right. So with that note we proceed further.

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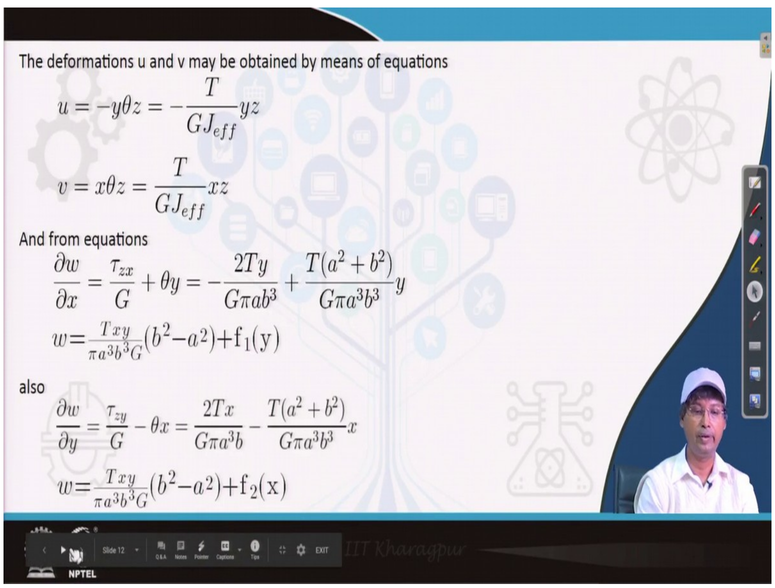

Now to find out the displacement so to find out displacement the deformation u and v may be obtained by means of the equations already you have we have found out. Now it is time to replace only the value of theta T by G J effective is put there here also it is put it is integrated. Once we integrate it again del w del x is brought back in relation to the standard way what we have seen in the equations and then what we get that w is equals to f 1 of y because it is partial derivative of x and similarly if we go for the w from this equation we have a function of x since it is derivative with respect to y.

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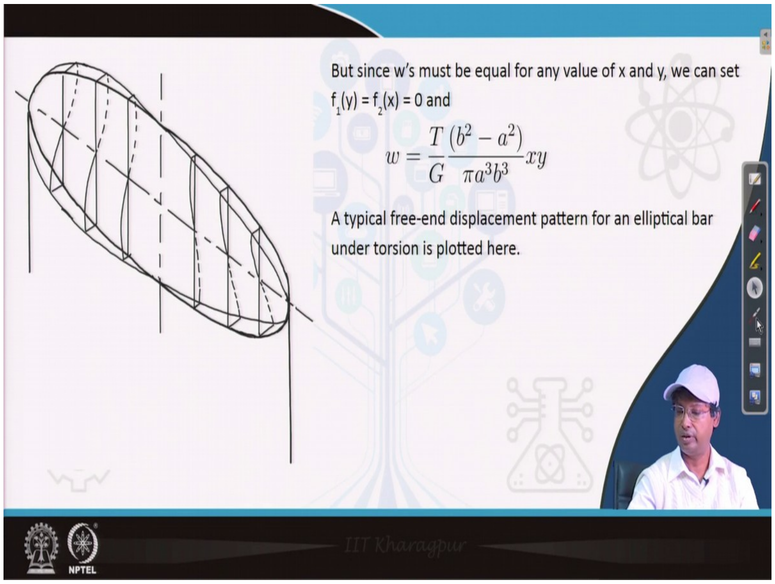

So now it is interesting to note that in both the cases it is expressing w and to have a reasonable solution from mathematical point of view w must be equal for both values of x and y we can state set that f 1 y and f 2 x are equals to 0 but w is a unique number it is not having a different value at the; for different position of xy. So for any value of xy since this w both the functions are certain to lead to a same value the values of f 1 and f 2 f 1 y and f 2 x are equals to 0 and we get that w which is along this axis.

So if I consider better to have some conforming relation. If this is x if this is x this is y this is z. So this is the xy plane and w is in this direction. Now 1 more thing you should notice here it sometimes it is confusing I have tried to draw it as clearly as I can it is digital drawing becomes difficult. So let me give you some try to explain the figure as far as I can. So the initial state of the section is given by this line am I right? Yes I am.

This is the section which is not stressed or which is not experiencing any torsion. But as soon as there is a torsion applied what is happening we observe that there is a out of plane displacement and this point goes inside corresponding this point comes out. Similarly this point goes inside this point comes out. So as I showed you in case of a rectangular cross section the some opposite corners go up and the opposite corners go down.

In a similar manner if we see that this side this side is going down this side is also going down ok its going in this direction its going in this direction whereas this side is coming out this side is coming out. So this is a nice phenomena unless you do experiment unless you observe the section it is difficult to imagine difficult to probably experience using this formula but it happens we have seen to happen its exciting phenomena.

So we if you carry out experiment elliptical section is difficult to find out that is the reason we always prefer to carry out experiment with rectangular section. If you do rectangular section experiment I would suggest that you do draw parallel lines on the section before you give it for torsion and then at the end of the of the experiment while the specimen has failed you please observe that it will take the shape as we have described as we have got in case of elliptical section. So with this note of warping is very, very nice function phenomena we proceed to the next slide.

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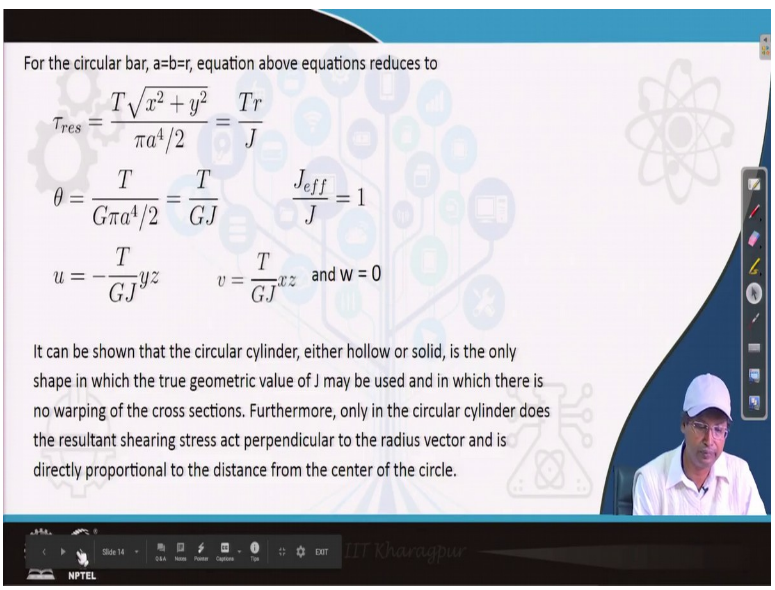

For the circular bar where a is equals to b is equals to r so we come back to the simplified solution considering a and b are equals and is equals to r and we will try to see observe that the derivation whatever we have done that also holds for a circular section that is what is done a is equals to b is equals to r is put here and the tau resultant shear stress if we put those values is equals to t r by J and the theta is T by G J and J effective is also equals to J and it is equals to 1.

And we have u is equals to - of T by G J yz and v is equals to T by G J xz w is equals to 0. It can be shown that the circular cylinder either hollow or solid is the only shape in which the true geometric value of J may be used and in which there is no warping of cross section furthermore only in circular cylinder. Does the resultant shearing stress act perpendicular to the radius vector and is directly proportional to the distance from the center of the circle.

So the resultant as we have seen that there are two components always tau zx and tau zy unless it is lying on the axis. So definitely it is the resultant is not going to be perpendicular to the radius so that is what is concluded in this portion and we observe that particular phenomena.

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So with that note we come to the end of the torsion problem of elliptical section will solve a few more if time permits. So these are the standard reference what we see and with this reference.

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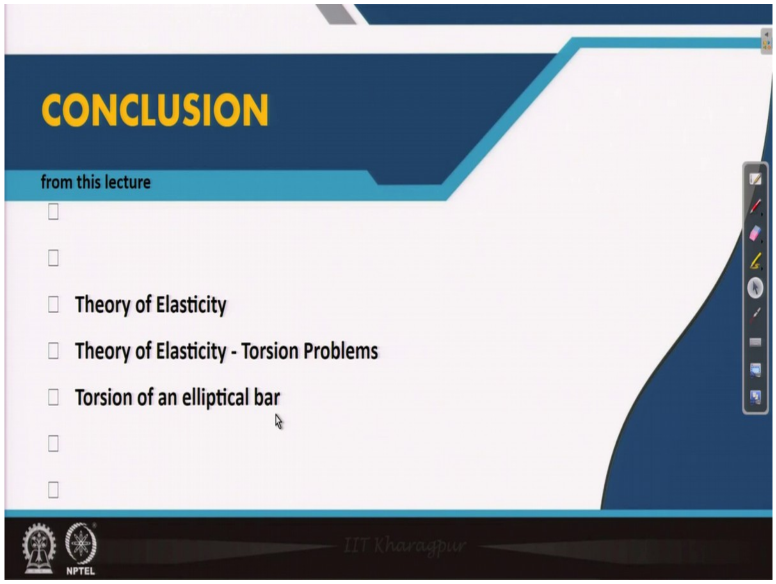

We go to the next slide what we have learned today learned that torsion of elliptical section and not only elliptical section we have got some insight towards the warping phenomena. And we have seen solutions conforming to the circular section also. So with that note I thank you for attending the lecture we will see you later, thank you.