Introduction to CFD
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Lecture - 57
Basics of Turbulence Modeling (continued)

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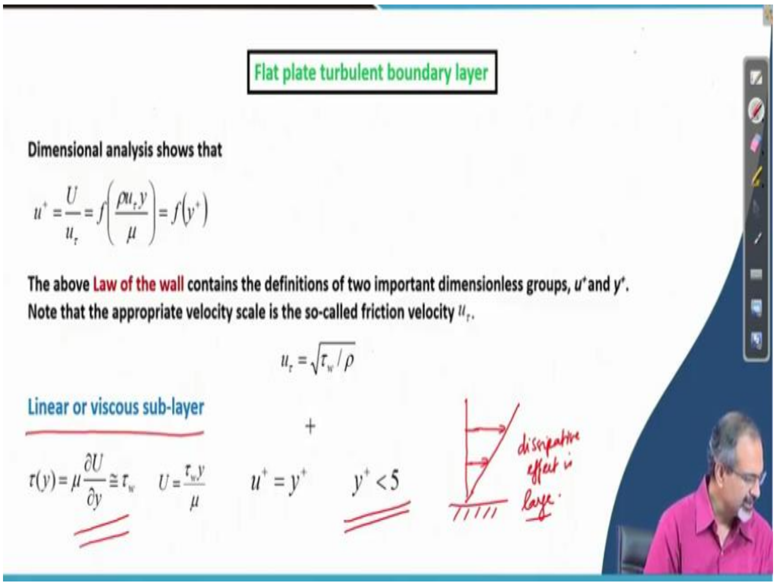

In this lecture, we will begin our discussion with flat plate turbulent boundary layer. So, in the previous lecture, we already had started discussing about it and we had discussed about u plus y plus as the non dimensional velocity and a distance away from the wall and also the definition of the friction velocity. So, we will continue with that part to the different layers that we can define within the terminal boundary layer.

So, the lowest layer which is closest to the wall and which exists up to y plus less than 5 which is a very small distance away from the wall, we get a layer which is purely dominated by viscous effects. So, this is called as the linear or viscous sub layer. Now, in this region the viscous law applies that your shear stress is defined by the viscosity coefficient times the velocity gradient and it is essentially constant.

It is equal to the shear stress at the wall that means it is essentially a linear profile. So, the profile will typically look like this. There is no non linearity in the profile and it is very close to the wall. No turbulent effects or fluctuations are visible at this layer. Why is it? Because you are at such small Reynolds number that the dissipative effect is large. And therefore, the turbulent fluctuations will decay by the time they approach the wall so closely.

So, it essentially boils down to a viscous region. In this region, y plus is equal to u plus. And then you can actually easily show this because a small calculation will reveal that. That is you can express U as; and then you just rearrange it.

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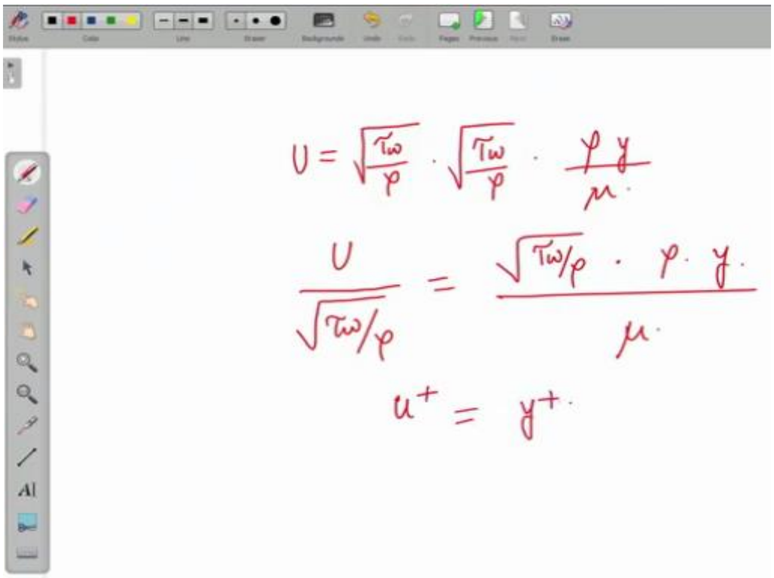

So, what are we creating? We are creating the U tau the friction Reynolds number which helps us to define the u plus and y plus. We show that they are equal. This is how it works in the viscous sub layer. Now, as we move beyond that we find a region typically between 5 and 30. That means y plus varying from 5 to 30 which is usually referred to as the buffer layer.

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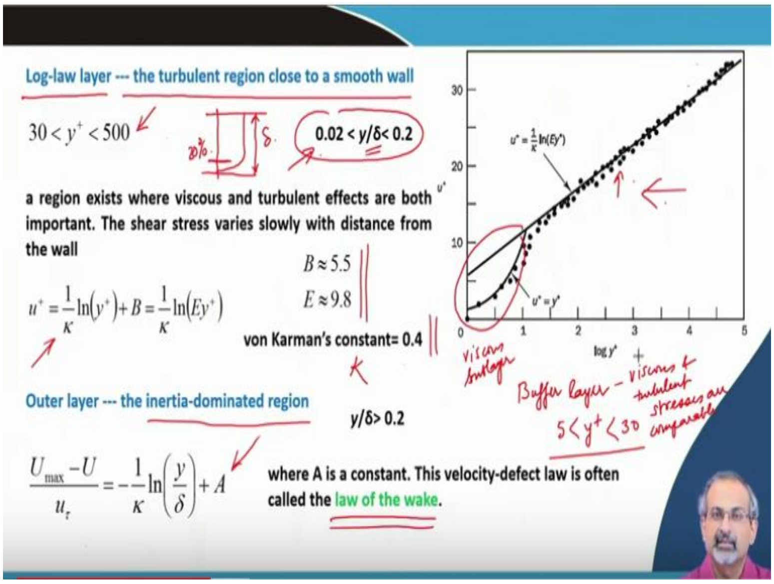

And in that buffer layer, we have the viscous and the turbulent stresses which are of comparable magnitude. So, we just take note that there is a buffer layer which is typically lying between y plus 5 and 30 where viscous and turbulent stresses are comparable. And then beyond that we have a so called log law layer which exists between y plus 30 and 500 in terms of the turbulent boundary layer thickness.

This is typically between 0.02 to 0.2 of y by delta ratio. That means if you have a turbulent boundary layer like this and this is the total thickness then you now know that the log law layer spreads up to just 20% of it. It starts from as small as 2% and spreads up to 20%. So, in this region, of course, viscous and turbulent effects are both important but viscous effects become weaker and turbulence takes over gradually.

And the functional dependence between u plus and y plus can be shown to be given by this equation, where the values of the constants are given here for smooth walls. And you have the von Karman's constant Kappa given by 0.4 or 0.41. Beyond that you have an outer layer which is essentially inertia dominated region and therefore, you do not any longer find very active participation of viscous stresses anymore.

So, that region is given by this equation, which is often called as the law of the wake. So, if you plot this variation with a u plus versus log y plot or log y plus versus u plus plot then it looks more like this. So, you can imagine that this part is essentially the viscous sub layer. And then you can find the next region up to 500 which means, it will be up to a log y plus value of the order of this range which is essentially linear because you are plotting it on a log scale along the x axis.

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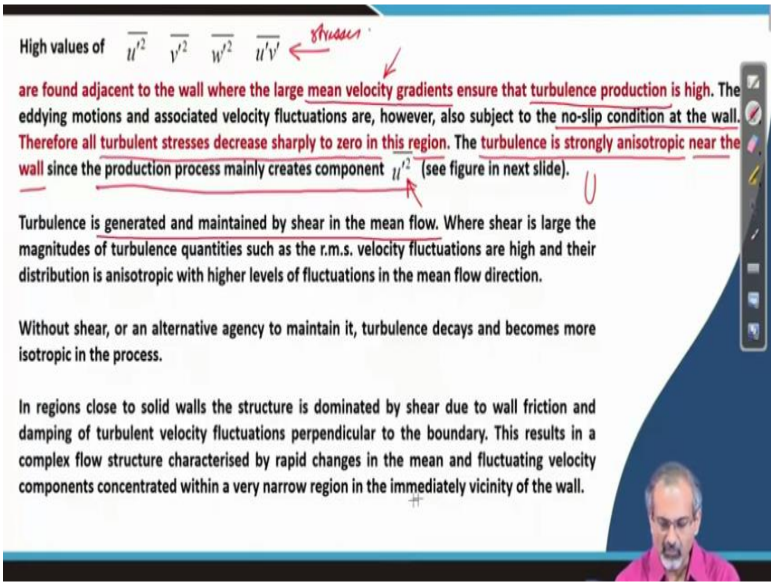

We have had a look at the different regions within the turbulent boundary layer existing on a flat plate and within these regions we have certain properties which we need to discuss about. So, we find high values of these parameters which are found adjacent to the wall where the large mean velocity gradients ensure that turbulence production is very high.

And these are essentially stresses which are developing because of the fluctuations. Now, the eddying motions and the associated velocity fluctuations are also subject to the no slip condition at the wall. So, as we said that as they approach the wall, they also have to rapidly decay down. So, turbulent stresses decrease sharply to 0 in the region as they approach the wall.

So, on one hand, because the mean velocity gradients are going to be high, why the gradients are very high? Because, you are seeing that the mean velocity changes from the free stream value just at the edge of the boundary layer to a value of 0 as you approach the wall within a very small thickness. That is the boundary layer thickness itself, which will give you a very large mean velocity gradient.

And as you know that means heavy shear and if there is heavy shear, there is heavy turbulence production, if there is heavy turbulence production, then the stresses are also going to be large. It is all known to us. So, however, when you approach the wall again these fluctuations have to decay. So, on one hand, they become very sharp as you get into this region. And again they have to go to 0 after they grow rapidly.

They again have to become 0 as they approach the wall. So, as a consequence, the turbulence becomes strongly anisotropic near the wall, because it has a strong directionality. And it is seen that if you are talking about the U component of velocity existing near the wall, then it will show that the main contribution will influence the u dash square quantity. So, we already said it is generated and sustained by shear. And shear has to be maintained in order to keep turbulence going.

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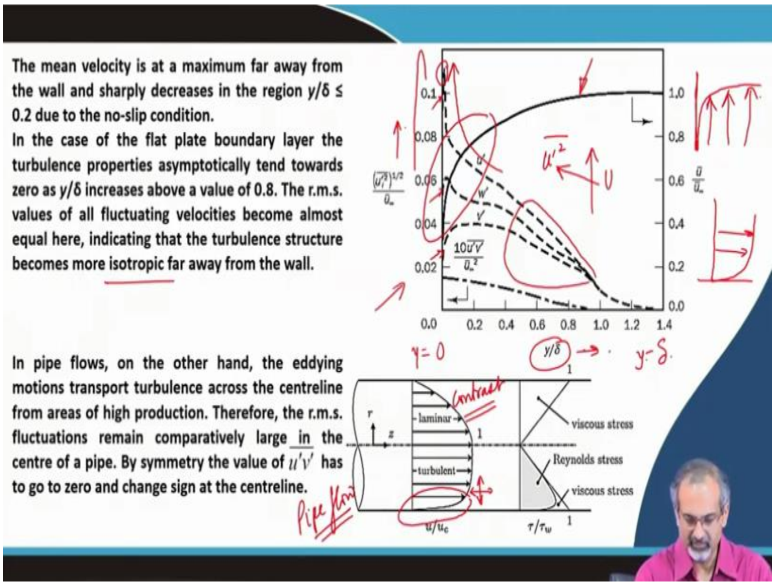

Now, this is typically a plot which indicates the variation of the stresses near to the wall. So, most importantly we see the velocity profile over here. So, most often how we plot it is like this. You might have seen plots of boundary layer being put like this. But here, notice that it is the other way around. That means the plot is made like this. So, it is like turned around. So, you have to interpret it that way.

So, velocity profile here as shown in this figure is like this, which means this is the edge of the boundary layer and this is y = 0. It is like that. So, along this direction you have a y by delta and the other direction you are plotting the different turbulence stresses, which are scaled with respect to the free stream velocity. Now, as you can see that in the previous slide we had talked about this quantity being more strongly influenced.

Because, the stream wise velocity is along this direction that is U. And therefore, it strongly enhances this quantity while the others are smaller. Again as you get into the turbulent boundary layer, you see that all these traces are growing because the mean shear is increasing in the flow. And then where does it peak, where the shear is highest as you can understand shear scales with the velocity gradient.

So, the gradient is most rapid as you approach the wall and that is where these values are actually becoming much larger they are peaking over here. But then though it is not very clear from the figure, you can probably understand that once it reaches the peak, it tends to again drop. It is just around visible here at the corner that extending to drop. These are also tending to drop and that is happening because they are approaching the wall.

So, this is typically how the stresses arrange themselves near the wall. Again remember that as you approach the boundary layer edge, the turbulence becomes weaker, because there is hardly any shear to sustain it and turbulence also becomes broadly isotropic. Now, if you were to look at pipe flow, you will also see similar features in some manner. Again, an interesting point to see is the contrast between how a laminar velocity profile will look and how a turbulent velocity profile will look.

As you can see, the turbulence velocity profile is much fuller close to the wall. Why is it possible? It is possible through the fluctuations in the velocity components which enable mixing and therefore, percolation of momentum much closer to the wall. But then that means much larger velocity gradients close to the wall. And therefore, there could be much augmented stresses, which have to be encountered when keeping a turbulent flow alive.

Therefore, it takes more power to drive a turbulent flow. But turbulent flow is very robust, it is less susceptible to flow separation for example. So, when you have flow past a car or a bus or an aircraft, on one hand, you spend more fuel to keep that flow going that means to make the vehicle propel through that region, but then at the same time, the flow will not very easily separate from that surface and therefore, produce a large pressure related drag.

In the pipe flow, a very important aspect is that you can see that the Reynolds stresses are becoming larger as you get closer to the wall. They are peaking here in this region and then again rapidly falling to 0 as you approach the wall. Viscous stresses on the other hand tend to remain constant. And as you approach the wall viscous stresses become more active. Again in the pipe flow about the symmetry axis, there will be certain stresses which will change sign. For example, the u dash v dash over bar will change sign at the centerline.

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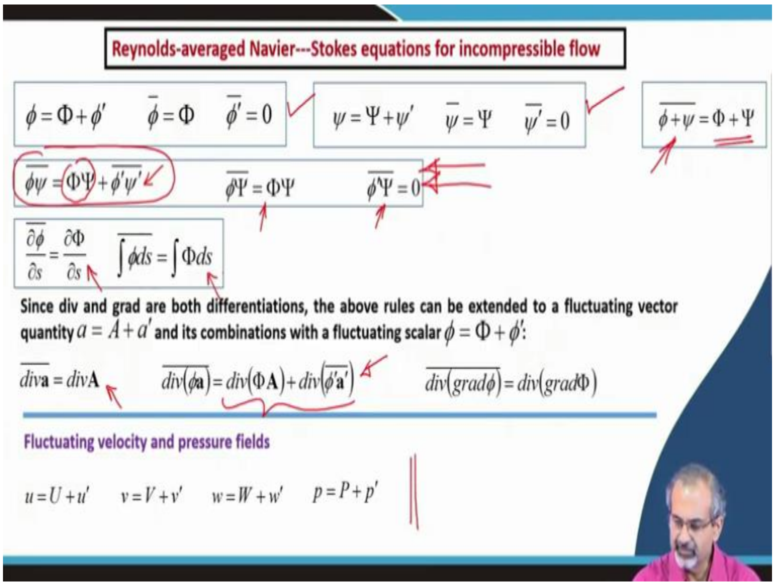

We come back to the Reynolds averaging concept which we had discussed in one of our previous lectures and we try to now apply it to Navier Stokes equations for incompressible flow. So, we broadly review the idea behind Reynolds averaging. So, if we take two different flow properties, phi and psi, these are already known to us. And then if you combine phi and psi and take Reynolds averaging the mean values the sum of the mean values would be available.

The most important and distinguishing fact comes in this box where you take a product and then what comes out of it is a product of the means and then a time average of the fluctuations. This is a very important fact, which will be seen how they affect the equations, when we do a Reynolds averaging of the equations. In this what we are doing is we have phi times the mean of psi which gives you a mean of phi times the mean of psi.

And then here it is the fluctuating part of phi multiplied by the mean of psi which will give you a zero. When it applies to differentiation or integration the rules similarly apply. So, what you see is primarily the mean quantities are essentially making it to the final forms. So, divergence or gradient these are also differentiation operations. So, things work very similarly.

And then these rules that we have discussed above can be extended to a fluctuating vector quantity a, and its combinations with a fluctuating scalar phi. So, we were previously discussing about primarily scalars combinations of scalars, but here it could be a combination between a vector and a scalar also. And vectors are the ones which we are more often handling in conservation equations.

So, these are the rules of the game. And as you can see that this looks very similar to what we saw in this box. And that is going to be a very important fact for us, when we look at Navier Stokes equations. So, just like the way we look at these scalar or vector variables being decomposed using Reynolds decomposition, we can do it for our velocity components and pressure as well.

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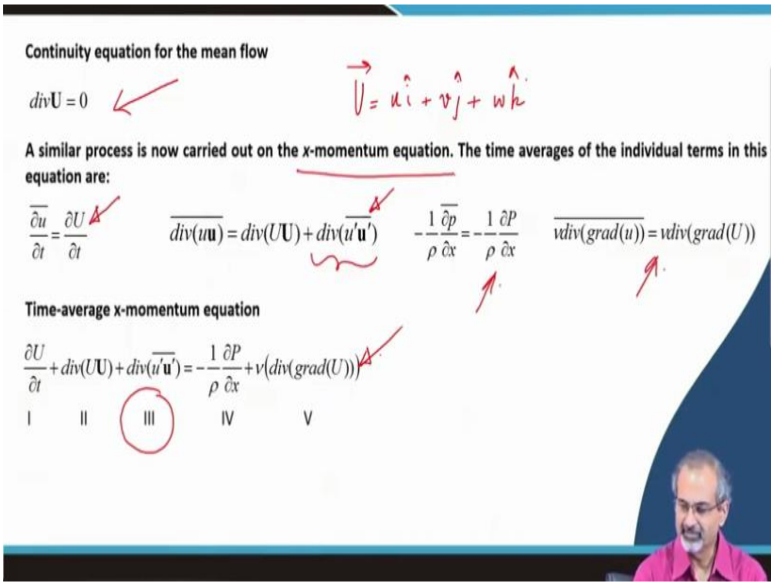

So, if you apply these rules to the continuity equation for the mean flow, you will find that this is how it shows. Here U is of course written in a vector form. So, in general U is a combination of different components. And putting these things together, if you now look at the x momentum equation, the time average of the individual terms in the equation will be like this.

This is the time average of the unsteady part. This is divergence of the convective term, the time averaging of that. And as we said that this is the additional term which is being produced the pressure gradient term and the diffusion term. So, the time average momentum equation x momentum equation shows up like this and as you can see that the third term is the new entry.

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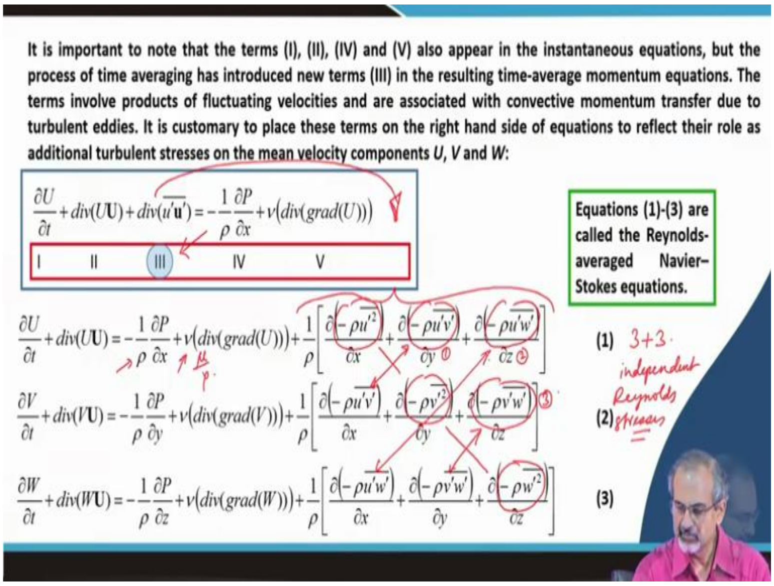

So, we have indicated the third term here separately and we would like to expand it now. And again apply this concept to the y and z momentum equations as well. So, it is usually practiced this way that this additional stress term is taken to the right hand side. So, at source though it has a convective connection, but it actually shows up as an additional stress. And therefore, we would like to club it with the viscous stresses on the right hand side as a source term.

And therefore, that term when we take it to the right hand side and we express it, we usually put it this way. We generally get density associated with it so that we can keep parity with the remaining terms as you can see that all these terms are actually divided by the density. And therefore, comes these respective terms which we call as the Reynolds stresses.

Now, incidentally you will find that there are essentially six independent Reynolds stresses which are produced as a consequence. So, three of them are normal stresses. So, these are the three normal stresses while the rest are shear stresses. So, this is one this is another third one would be this. Now, of course, this and this are identical. So, are these and these and therefore, they are not independent. So, in all you have 3 + 3 independent Reynolds stresses.

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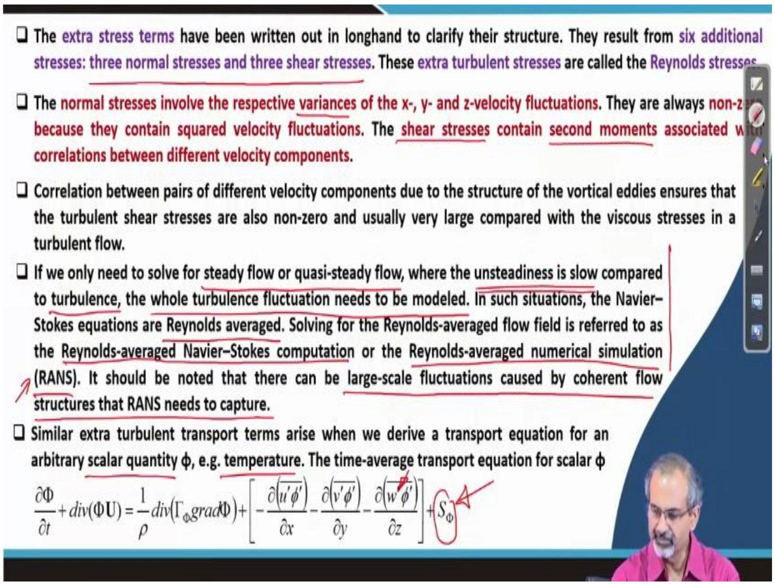

So, we already talked about these points, which have been highlighted here. So, the normal stresses essentially are the respective variances along x y z directions. And then the shear stresses are the second moments associated with correlations between different velocity components. So, correlations between pair of different velocity components are nonzero because of the nature of fluctuations in a turbulent flow field.

So, if you have a u dash produced somewhere that may induce a v dash as a consequence. Because there are also mass conservation issues which will drive such fluctuations. And therefore, often there are strong correlations between them. That is why both the shear stresses as well as normal stresses may be produced in a turbulent flow field. Now, if we need to solve for steady flow or quasi steady flow where unsteadiness is slow with respect to turbulence, then the whole turbulent fluctuation needs to be modeled.

In such situations, Navier Stokes equations are Reynolds averaged. So, that is basically the idea behind why we are pursuing Reynolds average at this point to be applied to Navier Stokes equations. So, solving for the Reynolds average flow field is referred to as the Reynolds averaged Navier Stokes computations or Reynolds averaged numerical simulation often written as RANS.

Also note that large scale fluctuations caused by coherent flow structures need to be captured through the RANS approach. So, this is the basis on which we are taking forward this Reynolds averaging to be applied to Navier Stokes equations and therefore, coming up with a possible way of tackling turbulence. Now, in turbulent flow fields we may also be dealing with scalar transport, let us say we are talking about transport of temperatures.

So, we were some time back talking about exchange of energy through turbulent fluctuations. So, that could be heat and therefore, there will be temperature differences in the region. And then you would have a transport equation associated with such a quantity a scalar quantity like phi which will look like this. You can have additional source terms like this. But importantly, there would be correlations of this kind between the scalar fluctuations and the velocity fluctuations.

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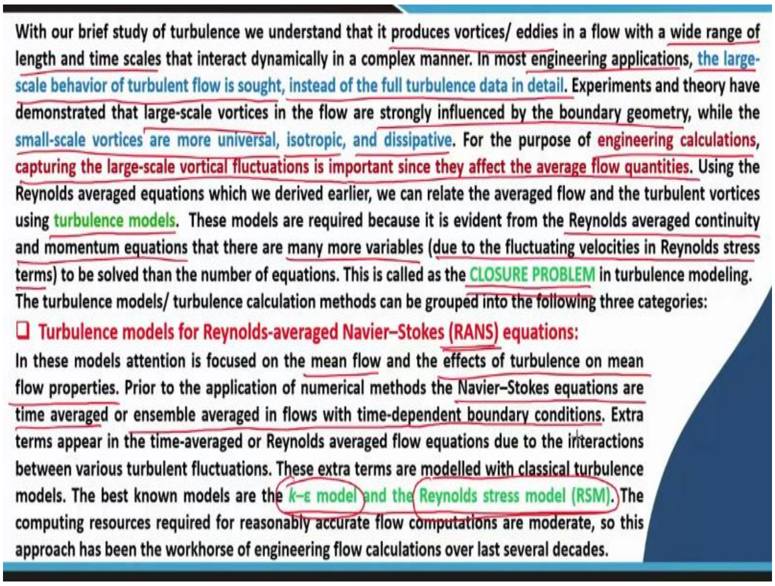

So, till now, we have discussed a little bit of basics of turbulence already and we have understood certain behaviors of turbulent flow fields from which we can at least conclude that any turbulent flow field produces an array of vortices of varying sizes and they have wide range of length and timescales. And in engineering applications, our intention is always to come up with the large scale behavior of turbulent flow instead of looking at full turbulence data in detail.

That means instead of looking at the whole range of vortex structures, their fluctuation data etc, which will be an enormous exercise to do and computationally very challenging. We try to extract the broad features of the flow. That is how it is relevant in engineering applications. And experiments and theory demonstrate that large scale vortices in the flow are strongly influenced by boundary geometry, the nature of the geometry of the problem.

While the small scale vortices are more universal isotropic and dissipative and they are not essentially influenced by the geometry of the problem. And in engineering calculations, the purpose is always to capture the large scale vertical fluctuations and try to come up with average effects of those large scale structures. Now, in order to do that we seek turbulence models.

That is because the exercise that we did just one or two slides back that Reynolds averaging brought out one fact that due to the presence of many of these fluctuation velocity correlations, which figured as the Reynolds stresses, there are many more variables to solve, then the available conservation equations that we have. So, this is because of the correlation terms. And therefore, we need some way out of it.

And this is essentially what is meant by the so called closure problem in turbulence modeling. So, we need to have a supporting number of variables and equations, which equate each other at this point it is not there after the Reynolds averaging exercise. So, we will seek turbulence models of suitable nature in doing that. And at this point we have already discussed briefly about the RANS approach.

Before we finish this lecture, just a minute we discuss about the two main categories of RANS based models one is the so called k epsilon model and the other category is the Reynolds stress models. And in these models, attention is focused on the main flow and the effects of turbulence on main flow properties. So, here the Navier Stokes equations are time averaged or ensemble averaged in flows with time dependent boundary conditions.

And extra terms that appear in the time averaged or Reynolds averaged flow equations due to the interactions are essentially taken care of with some model equations. For example, in the k epsilon model, they are all accounted through an equation for turbulent kinetic energy k and one equation for the turbulent kinetic energy dissipation epsilon. And these models are not very expensive to run.

And therefore, they have remained the main stay for engineering turbulent calculations. So, with this we complete this lecture we will discuss further in the next lecture on other turbulence models. Thank you.