Indian Institute of Technology Madras

NPTEL
National Programme on Technology Enhanced Learning

COMBUSTION

Lecture 23
Rankin –Hugoniot Relations

Prof. S R Chakravarthy
Dept.of Aerospace Engineering IIT Madras

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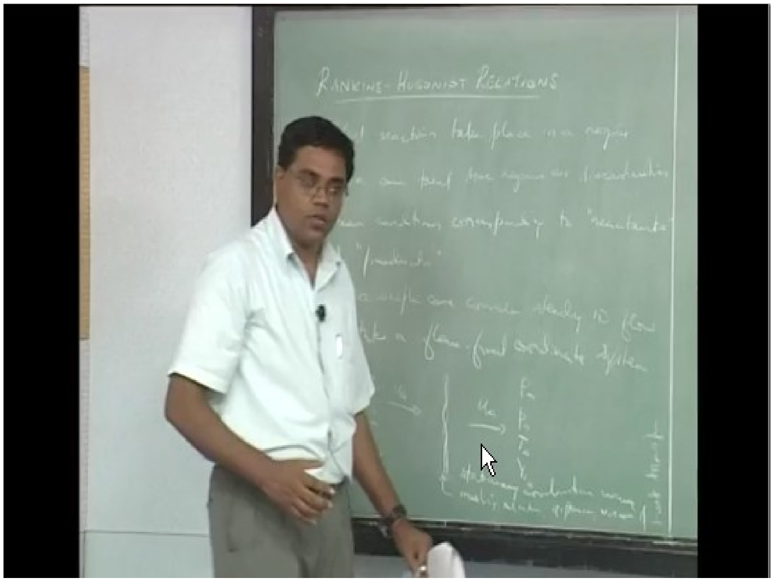

So let us now start doing what is called as Rankin Hugoniot, so you are not talking about the relation that Rankin has with her on you but right but you must have come across Hugoniot when you were doing guest dynamics for example so you do not come across you go to you so what did we do at the time when you if you did they gas dynamics course what you typically do is you are looking for what is called as dump condition across shocks right so you want to now treat shock as a discontinuity and then want to conserve equations across just saying.

So much mass came in on this side and it should be equal to so much mass that side and momentum here equals momentum there and so on we try to do something similar here except that we want to now consider a reaction zone instead of a shock necessarily the reason why we want to do that obviously is we find that combustion is too complicated right so is it possible for us to actually live without combustion right or without having to worry about combustion without having to solve combustion problems.

Is it possible for us to actually deduce some gross features for the flow of reactant gases into a reaction zone by looking at how what comes out of it in relation to what went in right without having to actually look at what happens inside so we want to now treat the region where the reactions are happening like a black box and then simply want to look at what is going in and what is coming out right, and see if we can learn something about so the justification for this is that most reactions happen in a very thin zone.

And so it is possible for us to action you do not necessarily want to say that we are adopting this is like a surface of discontinuity we do not want to really have like a jump condition across the surface but it is jump condition across the region, so it could be a little thick all right but that is okay but you know if it is going to fill up the room then you are not going to be able to do this so obviously we want to have it confined to some region which across which you can apply jump conditions. So we notice that most reactions take place in a region in a region so we can treat these regions as discontinuities between conditions corresponding to what we would callas reactants on one side and products.

On the other side right so as a simple case of course let us consider a 1d steady situation right so consider a steady 1dflow right and take a flame fixed coordinate system that means you are not going to say if you know had a flame that is occurring over here we are simply going to say that whatever is coming in is coming in at a velocity U 0 and what's going out is going at a velocity U ∞ and this has condition conditions ρ 0 V 0 T 0 Y I 0 and this has conditions ρ ∞ from P∞T∞ Y R ∞ right I go I goes from 1 to N for n species right.

So what that means is so how did you draw this boundary how do you now decide that what is outside of this region outside of this region outside of this reaction zone outside.

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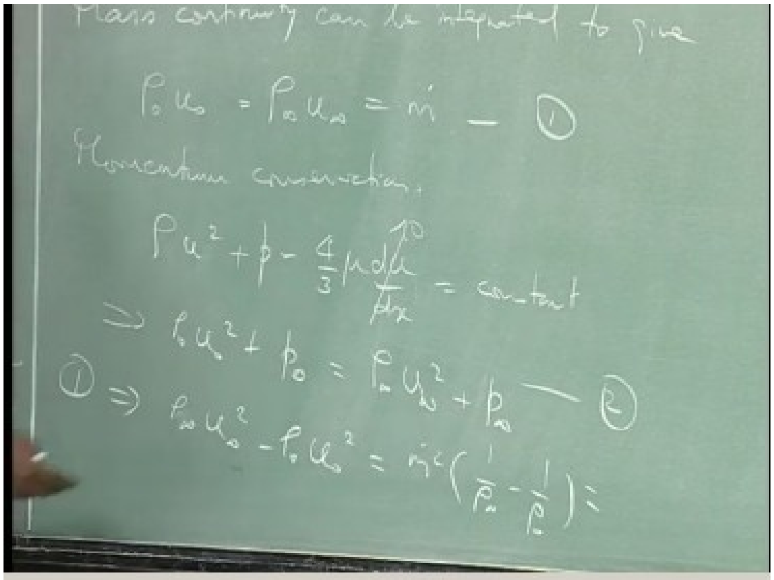

Mass continuity can be integrated to give

$$\rho_0 u_0 = \rho_\infty u_\infty = \dot{m} \quad — ①$$

Momentum conservation,

$$\rho u^2 + p - \frac{4}{3}\mu \overset{0}{\cancel{\frac{du}{dx}}} = \text{constant}$$

$$\Rightarrow \rho_0 u_0^2 + p_0 = \rho_\infty u_\infty^2 + p_\infty \quad — ②$$

$$① \Rightarrow \rho_\infty u_\infty^2 - \rho_0 u_0^2 = \dot{m}^2\left(\frac{1}{\rho_\infty} - \frac{1}{\rho_0}\right) =$$

The reaction zone on either side nobody answer present no gradients in any quantity right is present that means all the variations are happening only here that mean you started out with a uniform ρ 0P 0 T 0and why not all the jumps that are happening right or happening only within this and then you are now ending up with a T ∞ P ∞ ρ ∞ and why I infinities on the outside.

Which is it is no uniform on the other side so all the variations are happening there what that means is this is a this is now a essentially a stationary combustion V stationary combustion wave involving not only reactions but also all the heat transfer viscous effects diffusion everything right because all the all the diffusion he turns from viscous effects all of them based depend on ingredients right and then it says all the gradients are contained in this so diffusion viscous and heat transfer.

So maybe I should just write here that is okay so that is what is going on within this very small region is it okay can we can we do this do you have a feel for what is going on if we if you were to deal with geodynamic shocks and treat that as a condition tenacity we will find that the shocks are only about a few mean free path stick okay, and that is very thin strictly speaking but few is less than 10 mean free path stick well we are here I is not going to be so bad or so thin right so if you now think about like a flame like in a Bunsen burner you are expecting.

It to be less than a millimeter thick or around that okay, some conditions it could be around about a millimeter or so that is still quite small quite thin right that is for a flame is it possible for us to now think about like a flame that happens along with a shockwave right like reactions happening along with the shock wave or in other words.

Whatever shocks that you have come across were because they were actually propagating in non reacting unit mixtures what happens if you now were thinking about shops propagating in reacting mixtures what will happen will you have reactions that happen along with it right so is it possible for us to deduce those then will we have what kind of a flame but in general what you are basically thinking is whatever we are thinking about its recently tenant okay, we can think about this is say a little black box.

That is quite thin and on either side and all the greens are in there and on either side you have uniform flow of reactants on one side and products on the other side okay now if I had this picture this is obviously a flame fixed coordinate system if they want to have like a lap fixed coordinate system what do you think will happen so you now have this you not coming in here and trying to make this stop right that is because we are we are traveling with the shock so if you had a still set of reactants and the shock actually tries to go.

I should not say shock strictly speaking could be a flame just a reaction zone right so this travels like this that means the reaction zone is eating into the fresh reactants maybe still fresh still reactants right so this is strictly speaking supposed to go like this but I now find out mark my words now I now find out at what speed this goes and then try to oppose that with an equal amount of velocity for the reactants to stabilize this just to stay there okay, and then it did not stay otherwise I allow to travel with it that means I should know what velocity.

This goes so that I can travel along with that to me make it stationary with respect to me so there is a little problem and in terms of like having to know what this velocity is or shall we get it with this set of equations that is not true what we are really hoping for is if I were to give these I can get these but in this problem I do not even know this strictly speaking keep that in mind okay that is going to come back and haunt us a little later so our goal is to obtain product conditions given reactant conditions right.

So question is our or all of that noon okay specifically you not like I have this question with me and I have to seek an answer as we go along but let us, suppose at the moment I know what does you notice and see what happens, so how are you going to obtain this these product conditions obviously you have these conservation equations that you want to integrate across the shock or the or the reaction zone. So mask continuity or mass conservation can be integrated right across this discontinuity to give you can integrate this.

And then find out that all it matters is we have to substitute the end conditions starting from one side to another side if you now travel X this way for example then you get ρ not equal or not you not equal to or not ρ∞ u infinity you are so familiar with this that you do not even question how this into how it how we did the integration and goddess right so I am just going to take liberties and say this is how this is how I got it okay but then if so this is actually the incoming mass flux and we are now saying that is outgoing mass flux and they are equal.

So if they are equal then can I now say both of them are equal to some common m dot that is like a mass lock in a system so the mass flux kind of tells you the size of the or the strength of the system okay higher the mass flux stronger is your propagation faster you is your propagation. So let us call this equation one today, so momentum conservation so we have grown our units quiet plus well actually strictly speaking we should we should be able to integrate and say $\rho u^{2} + P -$ 4 / 3 to a constant but then we also notice that we want to integrate this.

So across this region you do not have this term because you do not have any derivatives when you are now trying to apply this to the left hand side or right hand side, so this can be written as $u0^{2} + P0$ equals ρ∞ right, so let is call this two this is also something that you are familiar with but I should I should take a little bit credit to say that we did not get this dude this is true not just for in viscid flows there is three even for viscous flows okay. So we could actually keep this and try to evaluate it outside the discontinuity and find that Du/ DX = zero.

So it is like you can now viscosity alright but so long as you do not have velocity gradients you do not have a viscous effect right so the viscous effects are negligible on either side of the discontinuity and therefore you get it to behave like an invested flow that's how in the Cites emissions typically work right. Now we want to know how do you want to do this keep in mind what we want to do is our goal is to obtain product conditions given reactant conditions all right and the way we want to do this is we want to deal with it in some sort of a thermodynamic fashion that is if I were to give you two thermodynamic properties that are representing the state of the system.

When they were reactants is it possible for me to actually find out corresponding thermodynamic properties for the state of the system and they have become products right. So it is sort of like a global approach and we are we are putting a black box around you are only worried about what happens between when we started out and when we ended we are not really worried about how this change happened right that is very symbolic or symptomatic of their thermodynamics therefore I should be interested in describing.

The state the initial state of the system thermodynamically and correspondingly look for obtaining the corresponding thermodynamic quantities for food is described the final state that is the products well I started with mass conservation I had ρ not and you not ρ is certainly a thermodynamic quantity but you is not that is a flow quantity right and similarly I can say ρ 0 u 0 square +V 0 ρ and PR but you not is not that is a flow quantity, so is it possible for me to somehow camouflage the flow and then deal try to deal with only the thermodynamic quantities the answer is so long as I am going to use m dot okay and treat it as a given and keep in mind this is related to u 0 right and then we cannot quite treat it as a given okay we do not know exactly.

So there is a little problem but so long as I am going to couch you not or you ∞ either of them as a matter of fact into m dot and it is so nicely it is so nice that both of them can actually get hidden and aimed out okay this equation will permit us to do that so I am going to use m dot whenever I try to get a velocity right and then I am going to write like you not as m dot divided by ρ 0 u and u ∞ as m dot divided by ρ ∞ so I try to do that then.

So 1 implies do not row ∞ u ∞ squared ρ 0 u 0 squared s you can write this as m dot squared times 1over ρ ∞ _ 1 over ρ n and that would be equal to so what am I doing I took this ρ0 u 0 squared to the right hand side so I had this difference so I have to take this P ∞ to the left hand side and I have a P 0 - P infinity right.

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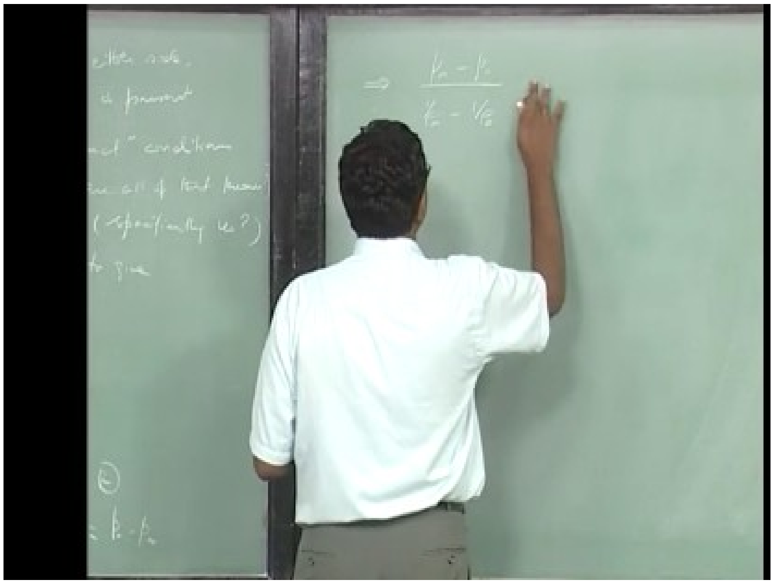

So from this I can write P ∞ _ P 0 divided by 1 over ρ ∞ _ 1 over ρ 0 =_ m dot square okay.

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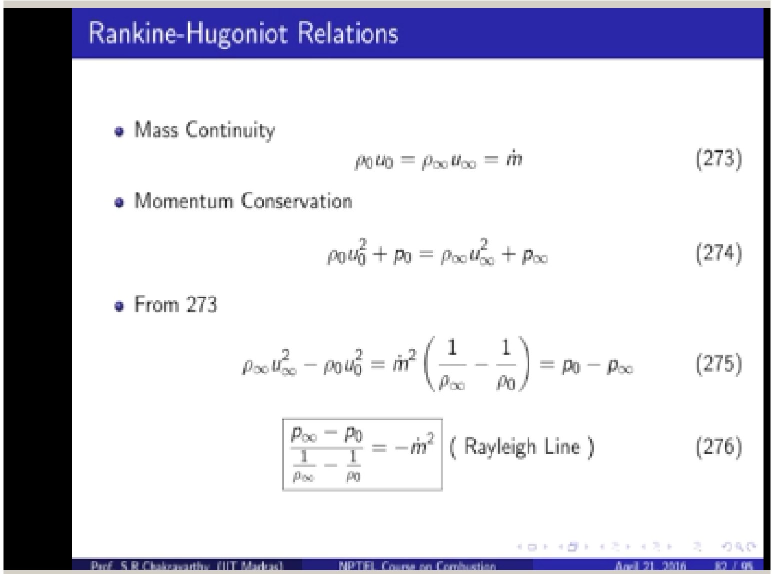

Rankine-Hugoniot Relations

- Mass Continuity

$$\rho_0 u_0 = \rho_\infty u_\infty = \dot{m} \tag{273}$$

- Momentum Conservation

$$\rho_0 u_0^2 + p_0 = \rho_\infty u_\infty^2 + p_\infty \tag{274}$$

- From 273

$$\rho_\infty u_\infty^2 - \rho_0 u_0^2 = \dot{m}^2 \left(\frac{1}{\rho_\infty} - \frac{1}{\rho_0} \right) = p_0 - p_\infty \tag{275}$$

$$\boxed{\frac{p_\infty - p_0}{\frac{1}{\rho_\infty} - \frac{1}{\rho_0}} = -\dot{m}^2} \text{ (Rayleigh Line)} \tag{276}$$

So very interesting that this analysis is amazing and in my in my opinion okay, what you are looking for is to go back and look at the goal again we are looking for product conditions given the you know given the reactant conditions and we just decided that we are by conditions we will actually talk about thermodynamic quantities right and then we just went through these and then found that ϱ and P are the thermodynamic quantities so if I were to be able to give $\varrho_0$and P0 I should be able to get P ∞and ϱ ∞ this is what I am trying to look for all right and in trying to get this equation I have already used up the two equations okay the demos conservation.

And momentum conservation what am I getting out of this so if I were to plot and this is the beginning of some lot of fun that you are going to have in the coming days right they find out if I am not going to plot P ∞ on the y-axis and 1 over ϱ∞ on the x-axis now keep in mind one over ϱ means specific volume so this is like a PV plane right and if I were to now locate numerically the value of 1 over ϱ_0 and let us say P not all right that is to say I am starting with a modest pressure for the reactants yeah and a modest of 1 over ϱ_0 t that means like a fairly high density okay that is what I am starting out with.

And this is my initial condition I am trying to locate for this initial condition where the final conditions should be and this equation is basically telling me that I can look for my final condition anywhere along a straight line okay which connects this point to the final condition with a slope equal to what minus m dot square right now m dot is mass flux there cannot be negative strictly speaking okay it can be negative only if I if I if I had the flow going back backwards like this so I am not really thinking about the reactants flowing out of the reaction zone I am looking at the reactions always reactants always feeding into the reaction zone.

So I am thinking m_0 is positive that is okay but m dot squared should obviously be positive so that is not even worry about it right and therefore negative m dot squared is going to be negative so the slope of this line is going to be negative so I am going to be looking for a line that always is having a negative slope you know that never happens you want to draw a line that passes through a point and always goes a little bit of a away from that point okay.

So we now push the point to wherever the line goes and then say this is the line along which I am expecting the final solution to lie right any point along this line is where the final solution should lie we have made remarkable progress in the last few minutes what did we do we now said given the initial conditions described thermodynamically by two quantities right we have now zoomed in on the possible solutions to lie along only one line as opposed to a huge plane right.

Until now we could have actually expected the solution to lie anywhere in this plane but now it is all confined to just one line that is amazing right we have reduced a plane to but a possible set of points in a plane to a possible set of points in a line that nice but then there are infinite number of points in a line just as well as there are infinite number of points in the plane so what is V it is all infinite number of possibilities right so how do I now pick exactly that solution that that is going to be lying on this line I need at least some more information I should now be looking for this line to intersect with some of the line maybe or there must be some way by which I can choose a particular point along this line.

So we have exhausted mass conservation and momentum conservation together to now get a line we now are looking for a energy conservation to help us with trying to locate a point along this line right so this like a very step-by-step thing from a plane to a line from a line to a point is what year you are going so this line is what is called the Rayleigh line you and the slope of the Rayleigh line is essentially MDOT squared I am not saying negative end up end up MDOT squared because I have already taken the negative sign an account and drawing the line that is inclined from left top to right bottom okay so the slope of the line contains so what does MDOT signify for us it's actually the flow the flow information is hidden in MDOT.

And the slope of the line indicates how fast it is right if the if the line were very shallow that means its MDOT is quite low if the line we are steep MDOT is high all right so that is going to tell us how fast the flow is or how slow the flow is Internet's going to tell us how fast the way were propagating to still reactants or slow okay but there is something that we do not know okay so keep that in mind, so now you look at the energy equation to get initiative the energy equation the energy equation reduces to H_0+1/2 u 0 squared equals H ∞ +1/2 u infinity squared let us call this 3 now of course you know have to start thinking about what these hitch knots.

And hitch H ∞are so notice note that note that P 0 equals ϱ_0 R_0 T_0 t P ∞=ϱ,∞ or ∞ these are specific gas constants and these are going to be specific to the reactant mixture for or not and the product mixture for R∞ right so which means we should know what is the molecular weight of the mean molecular weight of the reactants and mean molecular weights of the products which means we need to know the composition of the reactants and composition of the products which means we know why I why I knots but we have to actually get why I ∞ right so how do you do that we have to integrate the species conservation equation they try to try to do that so trying to relate so as well as well let us just I do not do that immediately.

So before we do that let us just say no again also H_0 = σ I= 1 to n why I 0 H I 0 where Hi-Δ HF I superscript 0 plus CP i-- t not assuming calorically perfect gas if not you have to write integral T superscript not 2t CPI DT okay T superscript not - t subscript not K CPIDT is what you should write and then similarly H∞ is σi=1to n why I ∞ H I ∞ H I ∞y =Δ hf i superscript not plus CP i-- t ∞ okay so in on the cases we are assuming calorically perfect gas here to simplify things for us ourselves here you can see that the fate of H∞ not only lies 14 knowing T ∞ given T_0.

But also in knowing YI ∞ is given YI $_0$ okay. And the reason why I was talking about YI∞ is in relation to YI_0 was to get the mean molecular weights for the reactants and the products and that is because we wanted to relate the t_0 to p_0 and ρ_0 that is what we are comfortable with okay, we are comfortable with p_0 ρ_0 or P ∞ and ρ∞ we wanted to write T_0 and T ∞ in terms of the pressures and densities rather than keep them as temperatures for which we wanted to use R ∞ and R_0 and that involves molecular weights and involves YI and so on that is what we were talking about but we also find that HI is require Y I for performing your H_0 and H ∞.

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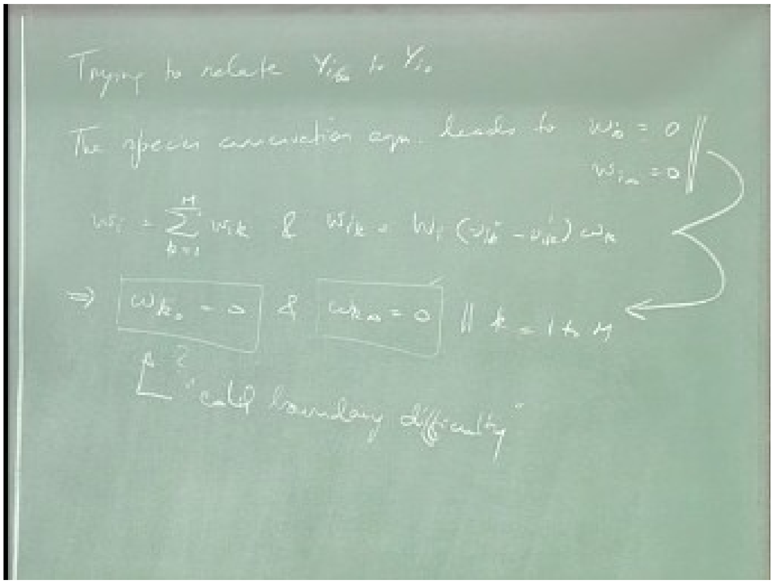

Trying to relate $Y_{i\infty}$ to Y_{i0}

The species conservation eqn. leads to $w_{i0} = 0$ //

$w_{i\infty} = 0$ //

$w_i = \sum_{k=1}^{M} w_{ik}$ & $w_{ik} = W_i(\nu''_{ik} - \nu'_{ik})\omega_k$

$\Rightarrow \boxed{\omega_{k0} = 0}$ & $\boxed{\omega_{k\infty} = 0}$ // $k = 1$ to M

?

"cold boundary difficulty"

So we will try to relate trying to relate yI ∞ to YI_0 right, how do you do this? So let us now try to try to integrate the species conservation equation across the discontinuity right so if you now try to equate the species conservation then the species conservation the species conservation equation leads to notice what we had, we had we had for the species conservation the convection term and the diffusion term and the reaction term okay, the convection of the diffusion terms involve derivatives. So they do not really exist here and what really it boils down to is to say that simply your W I_0 = 0 and w I ω = 0 right.

Now you could recall that you can say W I is ω K X = 1 to M W IK therefore or yeah Wik and wik = WI μ I k'' - μ IK' ω k so what this means is if you not trying to take these two together right then this simply means what you are saying is ω K $_0$ = 0 and ω K ∞ = 0, this is what it is going to of course you can say that for both of these you can say K = 1 to M for each reaction right.

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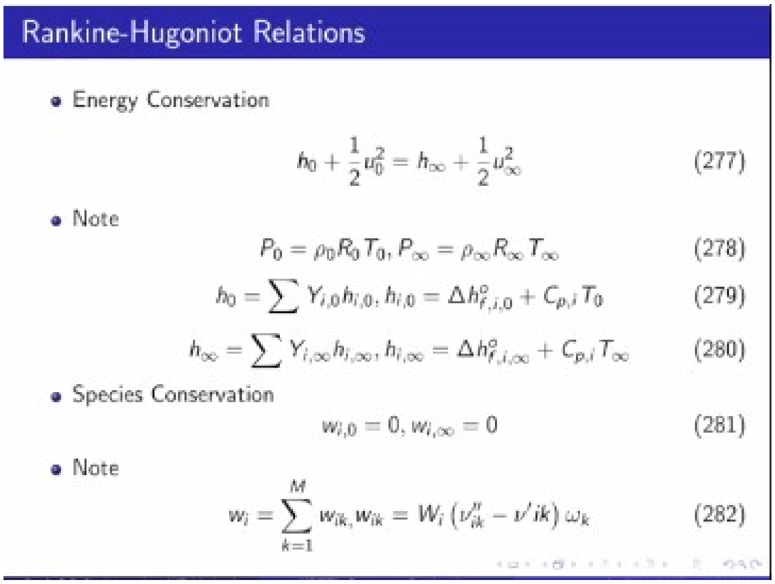

Rankine-Hugoniot Relations

- Energy Conservation

$$h_0 + \frac{1}{2}u_0^2 = h_\infty + \frac{1}{2}u_\infty^2 \tag{277}$$

- Note

$$P_0 = \rho_0 R_0 T_0, P_\infty = \rho_\infty R_\infty T_\infty \tag{278}$$

$$h_0 = \sum Y_{i,0} h_{i,0}, h_{i,0} = \Delta h^o_{f,i,0} + C_{p,i} T_0 \tag{279}$$

$$h_\infty = \sum Y_{i,\infty} h_{i,\infty}, h_{i,\infty} = \Delta h^o_{f,i,\infty} + C_{p,i} T_\infty \tag{280}$$

- Species Conservation

$$w_{i,0} = 0, w_{i,\infty} = 0 \tag{281}$$

- Note

$$w_i = \sum_{k=1}^{M} w_{ik}, w_{ik} = W_i \left(\nu''_{ik} - \nu' ik\right) \omega_k \tag{282}$$

This is for a multi step reaction nowhere we have a little problem which is this is alright this is more or less alright because your reactions at the end of the reaction zone are going to come to completion that is how you actually figure out that the reaction zone is over right and how did they come to completion because you actually consumed the reactants in those reactions, so if you now look at reactant profiles they would actually go to zero or at least one of the reactants will go to zero.

And if the reactant concentrations were present in these terms right through the law of mass action you know that the reactant one of the reactant concentrations goes to zero that means if it is if it is come if either of the reactants that are happening in reaction goes to zero concentration then the reaction is complete is what you are basically thinking about.

But here as far as this is concerned ω K0 the question is it zero, on other words are you having reaction rates equal to zero here I thought so because I thought all these this is the region that contains all the reactions right this is the region that is not supposed to contain any reactions is it right we have all the reactant concentrations yeah and then we also have a temperature T_0 at which they can react.

So strictly speaking there is nothing that is really stopping reactants at like let us say a reactants temperature or say room temperature there is nothing that is stopping them from reacting it is just that they have to circumvent the activation energy and it's take a longtime, so the reaction rate so very low but it is not necessarily zero. So this is going to be some very low number maybe10 to the -14 but it is not zero right.

So while we are now stinking that you do not have any gradients over here that is because we are not really taking into account the heat conduction that is happening because of chemical reaction so there is like a Mile gradients possibly but you are disregarding that saying that there is like a line that we are drawing and saying no gradients on the other side simply we similarly we are also drawing a line saying no reactions there but that that is not going to numerically work out.

So this is this is what is termed as in most combustion problems this is what is called as the cold boundary difficulty in other words if you want to supply an upstream boundary condition on the cold reactants in most combustion wave problems you will find that it is not really strictly true to say that it is not reacting at all right.

But then we suppose and this is this is primarily because of the way the Arrhenius law works the Arrhenius law is essentially an exponential dependence on temperature for the reaction rate constant and that that is not really going to zero identically as you go to room temperatures of the reactant temperatures it still exists at a finite nonzero infinitesimal maybe value.

So we circumvent this in instance in some sort of an ad hoc manner okay we do not we do not do this perfect way of integrating things across the jump and all that stuff for the species conservation we could do this for the mass conservation we could do this for the momentum conservation we could do this for the energy conservation but the species conservation is not going to actually lead to this the reason is this all these are actually rate equations okay.

So this is actually a mass flow rate this is the momentum flux rate and this is a energy rate and similarly this is a species flow rate is what the species conservation equations is all about, but what we are primarily looking for is only the composition we do not we are not really worried about the rate of change of composition or the rate of flux of composition and so on. So we could actually fall back to thermodynamics instead of looking at rate equations like species conservation equation in this case.

So typically we sort of been canalize in this do not worry about this and then resort to thermodynamics and in thermodynamics we have seen how the product concentrations actually can be worked out given the reactant concentrations but the caveat there is we are assuming equilibrium right. Whereas in all these rate processes that we are doing we are not necessarily assuming equilibrium that means we are allowing for changes to happen and it manifests in saying the reactants themselves are undergoing change even far upstream of where you thought with the reaction zone right.

So strictly speaking it says no equilibrium, so we wink our eyes on that we just say do not worry about it we were assume equilibrium we can try to get her wire infinities from an equilibrium condition and recall what we did when we when we did that we started out with atom conservation on either side that was like balancing the chemical reactions four additional equations that we needed for extra unknowns on product, products we assumed hypothetical partial formation equilibria and so on so he had ways by which you could you enumerate the wire infinities assuming equilibria so that is what we do.

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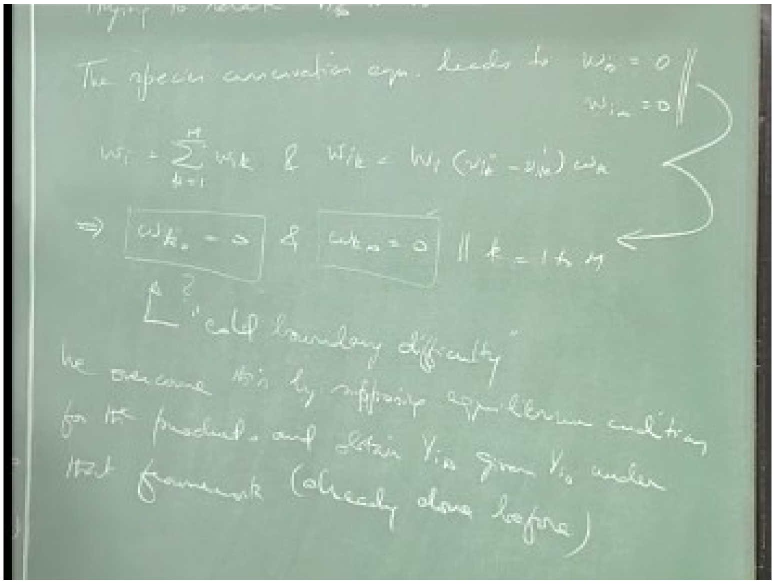

The species conservation eqn. leads to $w_{i0} = 0$ //
$w_{i\infty} = 0$ //

$w_i = \sum_{k=1}^{M} w_{ik}$ & $w_{ik} = W_i(\nu''_{ik} - \nu'_{ik})\omega_k$

$\Rightarrow \boxed{\omega_{k0} = 0}$ & $\boxed{\omega_{k\infty} = 0}$ // $k = 1$ to M

? "cold boundary difficulty"

We overcome this by supposing equilibrium condition for the products and obtain $Y_{i\infty}$ given Y_{i0} under that framework (already done before)

So we overcome this by supposing equilibrium condition for the products and obtain Yi infinity given Y0 under that framework okay, already done before okay. So let us suppose that we let us not make a big fuss about this we know how to do this we can proceed on dealing with the energy equation so we have the energy equation integrated across the jump and we call this 3 and of course there are as well we have $U0^2$ and we do not like to see U0 we want to replace it with $\dot{M}$ right.

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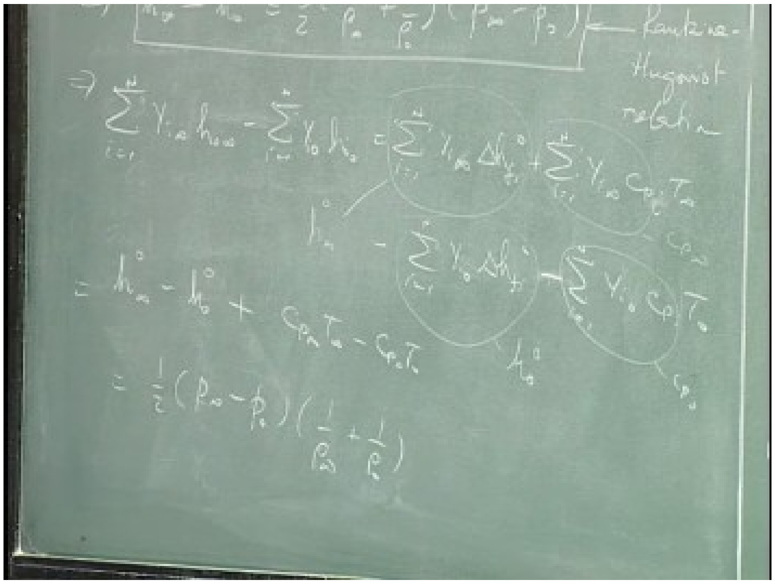

So we use 1 in 3 which means $h\infty-h0=-1/2\dot{M}^2\ 1/\rho\infty^2-1/\rho 0^2$ because we can we can write $U0^2$ as $\dot{M}^2/\rho 0^2$ and then you can also use you can you now notice that you have $1/\rho\infty^2\ -1/\rho 0^2$ can be written as $1/\rho\infty+1/\rho 0$. $1/\rho 0$ and then you can find out if now you say this taken over there $-\dot{M}^2.1/\rho\infty-\ 1/\rho 0$ is $P\infty-P0$ so you could try to use that so $h\infty-h0=1/2\ (1/\rho\infty+1/\rho 0)P\infty-P0$ this is good because this completely gets to get rid of $\dot{M}$ right.

Previously we had this Rayleigh line that was that could actually change its slope depending upon the $\dot{M}$ but now we have a truly thermodynamic relationship no flow information at all that is great because this is actually related to thermodynamics in some sense okay, if you start out with certain energy content in your flow in terms of a mixture of heat of formation and sensible enthalpy right, you will get your reactants, your products to now have so much enthalpy which will have a another combination of heat of formation and sensible enthalpy right.

That is essentially what does means, so we can we can now furthers ay well let us now write this as in fact this is this before we do that let us now in fact this is what is called as the Rankine Hugoniot relation let us just try to do this a little bit more next couple of steps is going to be a bit clumsy but let us just bear with it and see what happens, so if you now write your $h\infty$ as this and h0 as Yi, Y0 hi0 and then you have to write this as ∑ i= 1 to n, Yi∞ Δhfi0 0+∑= 1 to n Yi∞CP∞T∞ R should be CPi - ∑I = 1 to n Y0 Δhi superscript not +R would negative again.

∑i=1 to n Y0 CPi T0 now let us call what happens, now so let us now call this as our h∞0 and let us call this is h00 why would you call this h∞ not h00 not because they are actually standard heats of formation of species I weighted by the product composition here this is weighted by the reactant composition, so this superscript not in both the cases actually indicates standard heats standard conditions and the ∞ and ∞ not basically means you are waiting with respect to product composition and reactant composition respectively.

So this is equal to h∞0-h00+Cp∞∞-Cp0T0 why would you say CP∞ because T∞ can be pulled out of the summation so ∑Y∞ Cpi is CP∞ right, here you are going to get so this is CP∞ and this is CP0 so it is essentially the compositions that are designing these terms rather than the heats of formation and heats of formation or the temperature there so this is what this what you started out with the left hand side the right hand side can be written now so this is equal to ½(P∞-P0)(1/ρ∞+1/ρ0) right okay.

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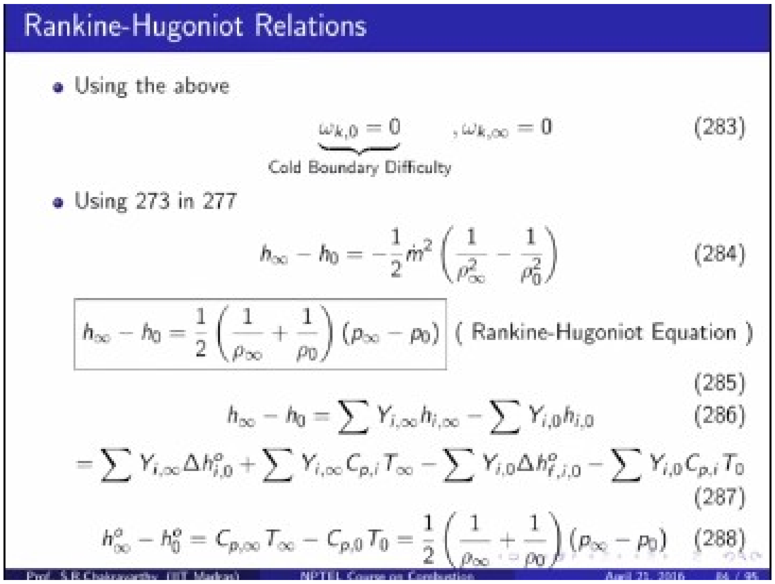

Rankine-Hugoniot Relations

- Using the above

$$\underbrace{\omega_{k,0} = 0}_{\text{Cold Boundary Difficulty}} \quad , \omega_{k,\infty} = 0 \qquad (283)$$

- Using 273 in 277

$$h_\infty - h_0 = -\frac{1}{2}\dot{m}^2\left(\frac{1}{\rho_\infty^2} - \frac{1}{\rho_0^2}\right) \qquad (284)$$

$$\boxed{h_\infty - h_0 = \frac{1}{2}\left(\frac{1}{\rho_\infty} + \frac{1}{\rho_0}\right)(p_\infty - p_0)} \text{ (Rankine-Hugoniot Equation)} \qquad (285)$$

$$h_\infty - h_0 = \sum Y_{i,\infty} h_{i,\infty} - \sum Y_{i,0} h_{i,0} \qquad (286)$$

$$= \sum Y_{i,\infty}\Delta h^o_{i,0} + \sum Y_{i,\infty} C_{p,i} T_\infty - \sum Y_{i,0}\Delta h^o_{f,i,0} - \sum Y_{i,0} C_{p,i} T_0 \qquad (287)$$

$$h^o_\infty - h^o_0 = C_{p,\infty} T_\infty - C_{p,0} T_0 = \frac{1}{2}\left(\frac{1}{\rho_\infty} + \frac{1}{\rho_0}\right)(p_\infty - p_0) \qquad (288)$$

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Now I told you we do not we do not like T's temperatures you want to write this thing right things in terms of pressures and densities so we use our equations of state applied to the end conditions and if you do that and you know in one more step we will hit a point that is ready for us to draw a curve that will intersect with the Rayleigh line and try to get us a solution.

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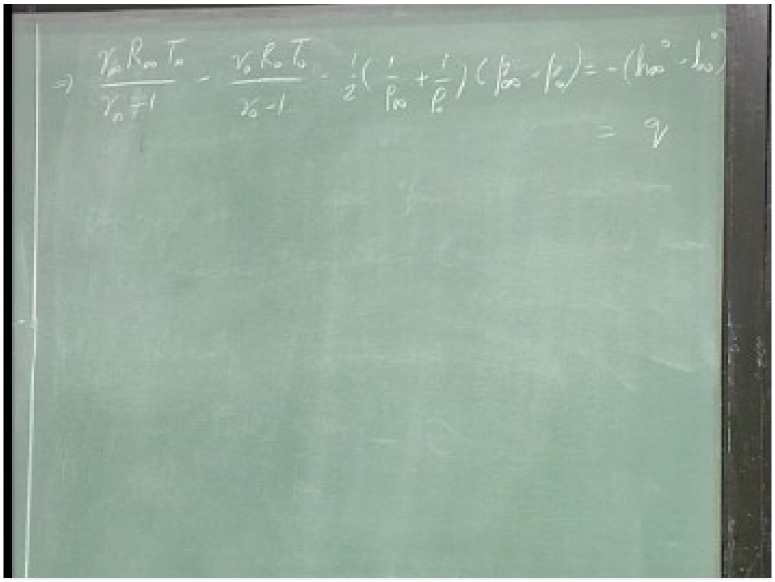

So γ∞ R∞T∞/γ∞-1 so CP is being written as γ or γ-1 r basically –γ0 R0, T0/ γ0-1 -1/2(1/ρ∞+1/ρ0)(P∞-P0) okay, so you try to take this to the right hand side to the left hand side and this term you will now take to the other side with a negative sign so we now call this equal to -h∞0-h00 and we want to call this q right, so the negative of the differences in the standard heats of formation weighted by the product composition.

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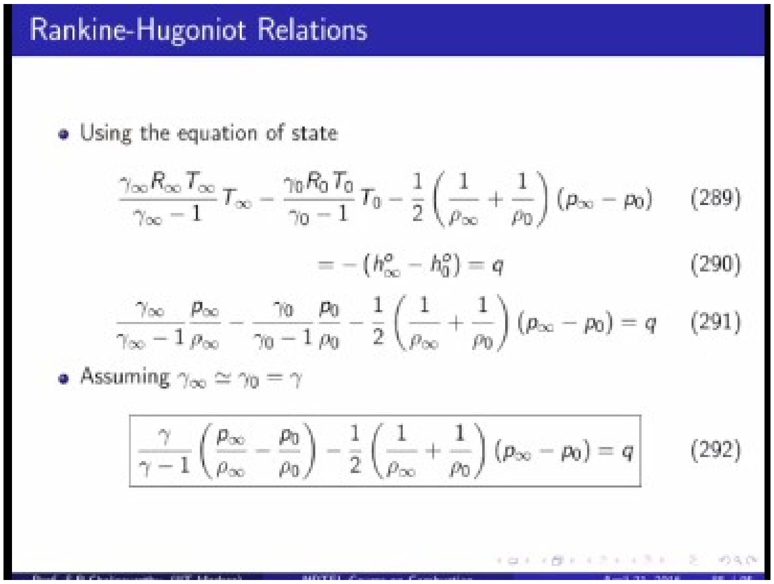

Rankine-Hugoniot Relations

- Using the equation of state

$$\frac{\gamma_\infty R_\infty T_\infty}{\gamma_\infty - 1} T_\infty - \frac{\gamma_0 R_0 T_0}{\gamma_0 - 1} T_0 - \frac{1}{2}\left(\frac{1}{\rho_\infty} + \frac{1}{\rho_0}\right)(p_\infty - p_0) \qquad (289)$$

$$= -\left(h^o_\infty - h^o_0\right) = q \qquad (290)$$

$$\frac{\gamma_\infty}{\gamma_\infty - 1}\frac{p_\infty}{\rho_\infty} - \frac{\gamma_0}{\gamma_0 - 1}\frac{p_0}{\rho_0} - \frac{1}{2}\left(\frac{1}{\rho_\infty} + \frac{1}{\rho_0}\right)(p_\infty - p_0) = q \qquad (291)$$

- Assuming $\gamma_\infty \simeq \gamma_0 = \gamma$

$$\boxed{\frac{\gamma}{\gamma - 1}\left(\frac{p_\infty}{\rho_\infty} - \frac{p_0}{\rho_0}\right) - \frac{1}{2}\left(\frac{1}{\rho_\infty} + \frac{1}{\rho_0}\right)(p_\infty - p_0) = q} \qquad (292)$$

And the standard heats of formation weighted by the reactant composition is essentially what we are saying as the heat released new to chemical reactions in this right so this is exactly what we talked about previously as well. So there is the heats of formation the difference between those is the one that is actually giving rise to the chemical heat release so we will we will just write the next step and stop.

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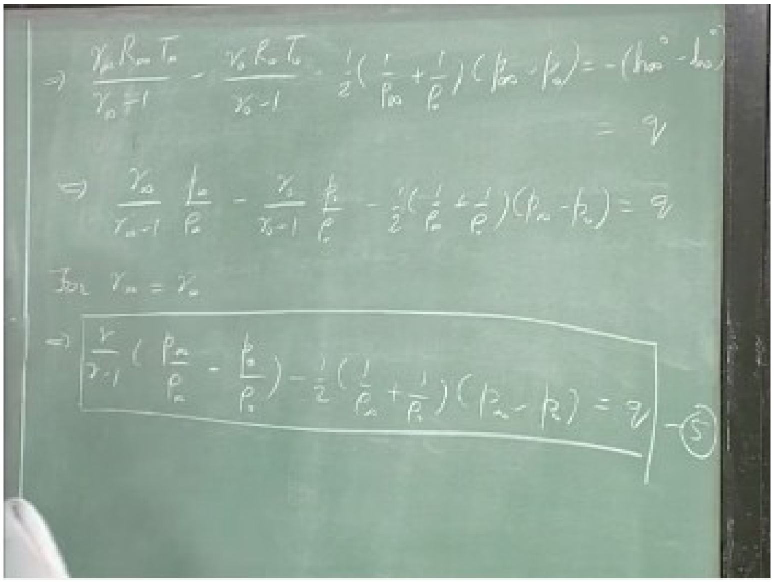

$$\Rightarrow \frac{\gamma_\infty R_\infty T_\infty}{\gamma_\infty - 1} - \frac{\gamma_0 R_0 T_0}{\gamma_0 - 1} - \frac{1}{2}\left(\frac{1}{\rho_\infty}+\frac{1}{\rho_0}\right)(p_\infty - p_0) = -(h_\infty^0 - h_0^0)$$

$$= q$$

$$\Rightarrow \frac{\gamma_\infty}{\gamma_\infty - 1}\frac{p_\infty}{\rho_\infty} - \frac{\gamma_0}{\gamma_0 - 1}\frac{p_0}{\rho_0} - \frac{1}{2}\left(\frac{1}{\rho_\infty}+\frac{1}{\rho_0}\right)(p_\infty - p_0) = q$$

For $\gamma_\infty = \gamma_0$

$$\Rightarrow \boxed{\frac{\gamma}{\gamma - 1}\left(\frac{p_\infty}{\rho_\infty} - \frac{p_0}{\rho_0}\right) - \frac{1}{2}\left(\frac{1}{\rho_\infty}+\frac{1}{\rho_0}\right)(p_\infty - p_0) = q} - ⑤$$

For the day so if you now write your R∞, T∞ as P∞/ρ∞ because that is what we are looking for so P∞/ρ∞-γ0/γ0-1 P0/ρ0- of course keep this equal to q, this is now clean because it is now coming back to within pressure and density you are not really having any temperatures anymore now for just simplify if γ∞=γ0 this is not very bad because γ are typically about 1.31, 41.2 whatever, so we get γ/γ-1 and P∞- sorry you divided by ρ∞-P0/ρ0-1/2 (1/ρ∞+1/ρ0) P∞-P0=q we will just start from exactly this point when we meet again this is the equation 5.

And what I want you to think about is how is this curve going to look like in this plane that means given your P0 and 1/ρ0 look at this curve to find out how in this plane of P∞ ρ and 1/ρ∞ this curve going is going to look like, have a good weekend.

Production and Post Production

M V Ramchandran
G Ramesh
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K R Vijaya

Administrative Assistant
K S Janakrishman

Principal Project Officer
Usha Nagarajan

Video Producers
K R Ravindranath
Kannan Krishnamurthy

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