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Lecture 25

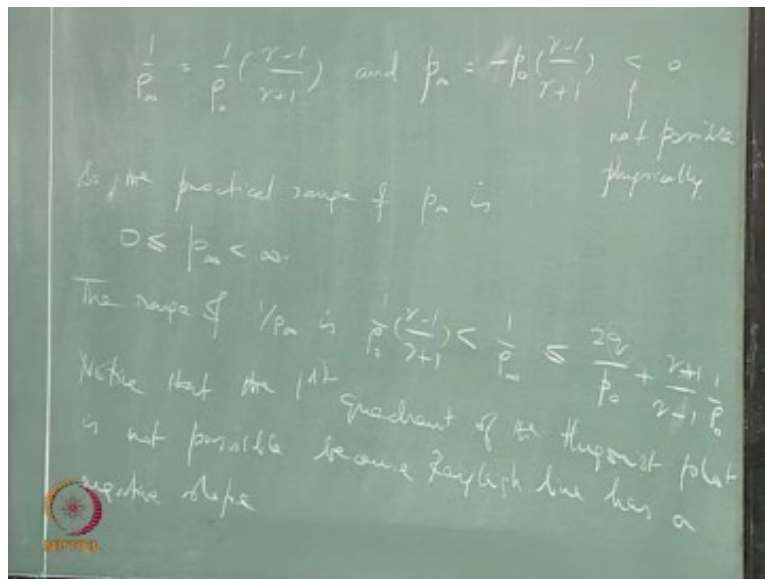
Rankine-Hugoniot Relation 3

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So let us now look at the asymptotes of the hugoniot.

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So we could show that there are some tots is the way we actually try to find out the SM thoughts of this hugoniot curve is to substitute that $1 / \rho \infty$ goes to ∞ and then find out what is the value of $p \infty$ for $1 / \rho \infty$ going to ∞ to get this system taught for you to get this asymptote you now substitute $p \infty$ is equal to ∞ in the hugoniot relation and then try to find out what is the value of $1 / \rho \infty$ right, so can be shown to be that means you show you show us them like you can do this as an exercise and it can be like a typical first question or an exam problem.

$1 / \rho^\infty = 1 / \rho_0$ times $\gamma - 1 / \gamma + 1$ and $p^\infty = - p_0 - p_0 \gamma - 1 / \gamma + 1$ now notice that γ is ratio of specific heats which is greater than one, so let us suppose it is about 1.25 or something like that so this is going to be like point something divided by two points something so $\gamma - 1 / \gamma + 1$ is going to be always less than 1, and so $1 / \rho^\infty$ is always going to be less than $1 / \rho_0$ and therefore you a asymptote is going to be to the left the origin all right and similarly a $\gamma - 1 / \gamma + 1$ again is less than one and it is going to be a fraction of P_0 .

That means it is going to be less than less than the less than the value of value at the origin but with a negative sign right, so it is now going to the other side and obviously that is not possible right so this is less than zero not possible not physically possible right, so the practical range of P^∞ is 0 less than three ∞ less than ∞ it can go upto ∞ that is possible, so what is the range of ρ^∞ that is not going to span between where the hugoniot in intersects ,so that is suppose that is how its intersecting.

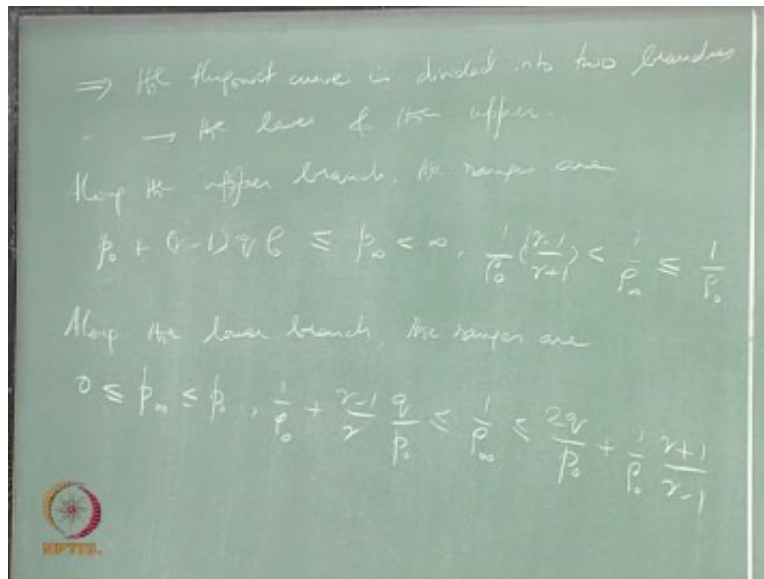
So that goes asymptotically to the lower bound there which is which is impossible but $P^\infty = 0$ is about where you can go down go up 1 or 0 ∞ you cannot go below this because p^∞ cannot become negative right therefore this is like your uppermost value of $1 / \rho^\infty$, so you can try to find that out by plugging in $p^\infty = 0$ in there and kind hugoniot relation right and then try to solve for $1 / \rho^\infty$ and get an expression the lowest value is when p^∞ becomes ∞ , so that is corresponding to this particular value itself.

They of the asymptote right so the range of range of $1 / \rho^\infty$ is $1 / \rho_0 \gamma - 1 / \gamma + 1$ which is what we have for does not taught we saw just a little earlier and you are now going to really touch it because it is in as taught and therefore you are not going to really touch $P^\infty = \infty$, so it is you are always less than that and, so $1 / \rho^\infty$ is always greater than this not really greater than or equal to but you can go up to greater than or equal to what is now called to $q / p_0 + \gamma - 1 / \gamma + 1$ I am sorry this is $\gamma + 1 / \gamma - 1$.

$1 / \rho^\infty$ now notice that this is actually dependent on q that means if your q is larger the hugoniot shifts to the right and above and top and therefore the point at that point where it intersects the $P^\infty = 0$ axis shifts further and further outright, so sure enough as you increase your

q this term is going to linearly increase right, so this is this is a range now what we also noticed was a notice that a the first quadrant of the hugoniot plot is a is not possible because a the ray really line has a negative slope so this divides the you go hugoniot into two parts the upper branch and the lower branch.

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So can now begin to see what t those limits are for these each of these branches each of these branches, so the hugoniot new curve is divided into two branches of the lower and the upper right, so along the upper branch along the upper branch the range is the ranges are the ranges are $P \cdot P_0 + \gamma - 1 \cdot q / \rho_0$ not less than or equal to P^∞ less than ∞ that is going from here that is going from here all the way up to ∞ right, so what we are actually locating this is $P_0 + \gamma - 1 \cdot q / \rho_0$ so as q increases as q increases this curve is going to go up.

So the point where this intersection happens is going to go up right so P^∞ then the lowest value of P^∞ will keep moving up as q increases, so this can obviously be obtained by plugging in $1/\rho_0 = 1 / \rho_0$ in your Rankine hugoniot relation right, so each of these expressions can actually be derived by plugging different things in the Rankine any organization okay a question is for what do you need to plug what is something that you have to think about and use your mind.

And you can get these expressions but these are like little problems that you can face in your exams this is like a fertile ground for asking lots of little questions right just to just basically just your algebraic skills not really combustion skills okay then this recipe ∞ is concerned and of course the lowest value of ρ_0 you can do ρ_∞ I am sorry $1 / \rho_\infty$ that you can get is the SM taught and the highest value is $1 / \rho_\infty$ itself that is very easy for you to figure out right.

So again we can write $1 / \rho_0 + \gamma - 1 / \gamma + 1 / \rho_\infty$ less than or equal to $1 / \rho_\infty$ that is a range for the $1 / \rho_\infty$ along the upper branch along the lower branch the ranges are the ranges are again if you are not looking at ρ_∞ it is going to go from here that is the highest value is P_0 and the lowest value is this right which is zero therefore.

So you it is going to go from 0 less than or equal to P_∞ less than or equal to P_0 okay and the you not look at the lower branch it starts from here, so the lowest value of $1 / \rho_\infty$ is corresponding to this value which is which is obtained by plugging $\rho_\infty = P_0$ right, so if you now plug $\rho_\infty = P_0$ in the ranking you go near relation you should, now get an expression that we should not be able to see readily depends on q linearly, so $1 / \rho_\infty + \gamma - 1 / \gamma q / P_0$ less than or equal to $1 / \rho_\infty$ less than or equal to that is the other end where the ρ_∞ becomes 0.

This is something that we have already figured out that is just value right so that is $2q$ divided by $\rho_0 + 1 / \rho_\infty + \gamma - 1 / \gamma - 1$ good.

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Rankine-Hugoniot Relations

- Asymptotes of the Hugoniot equation can be shown to be

$$\frac{1}{\rho_\infty} = \frac{1}{\rho_0} \frac{\gamma - 1}{\gamma + 1} \text{ and } p_\infty = -\rho_0 \frac{\gamma - 1}{\gamma + 1} \quad (295)$$


Range of p_∞ and $\frac{1}{\rho_\infty}$

$$0 \leq p_\infty < \infty \quad (296)$$

$$\frac{1}{\rho_0} \frac{\gamma - 1}{\gamma + 1} < \frac{1}{\rho_\infty} \leq \frac{2q}{\rho_0} + \frac{\gamma + 1}{\gamma - 1} \frac{1}{\rho_0} \quad (297)$$

- Range of upper branch of Hugoniot

$$\rho_0 + (\gamma - 1) q \rho_0 \leq \rho_\infty < \infty \text{ and } \frac{1}{\rho_0} \frac{\gamma - 1}{\gamma + 1} < \frac{1}{\rho_\infty} \leq \frac{1}{\rho_0} \quad (298)$$

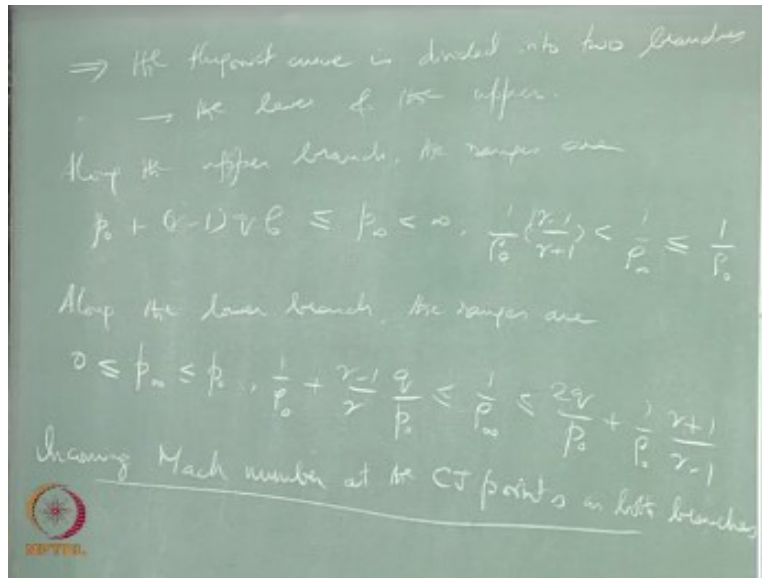


$$0 \leq p_\infty \leq p_0 \text{ and } \frac{1}{\rho_0} + \frac{\gamma - 1}{\gamma} \frac{q}{\rho_0} \leq \frac{1}{\rho_\infty} \leq \frac{2q}{\rho_0} + \frac{\gamma + 1}{\gamma - 1} \frac{1}{\rho_0} \quad (299)$$

So these are these are just algebraic expressions that you can get for the different limits for the upper branch the lower branch separately okay, now as we noticed we can clearly see from here that the upper branch corresponds to a compression wave that actually DC the rates of flow and therefore we call it a detonation wave the lower branch is a expansion wave which accelerates the flow and so we call it a deflagration wave okay these are things that we noticed the next thing that we want to point out was a we wanted to actually.

Find out what is the speed at which the detonation waves travel and what is the speed at which the deflagration waves travel, so the way we wanted to actually look at the disk what is the speed of the lowest the slowest in detonation wave and what is the speed of the fastest deflagration wave and obviously we can see that they are sort of poles apart okay, so that is the slowest a detonation wave is still going to be much faster than the fastest deflagration wave so let us see what they what they are tell us okay.

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And these are corresponding to the Chapman-Jouguet detonation and the deflagration waves so what we want to actually look for is what is the incoming Mach number incoming Mach number at the CJ points at the CJ points on both branches alright, so this is what we should be looking at now so how do you locate the two CJ points right, obviously you look at any point any point that is like a solution of both intersection of the Rayleigh line and the Hugoniot is by solving them together right.

So the Rayleigh line has its p_∞ and ρ_∞ showing up and the Hugoniot curve has its $1/p_\infty$ and $1/\rho_\infty$ showing up, so that is to say that if you now solve for p_∞ and $1/\rho_\infty$ together with these two equations you now should get a pair of p_∞ and $1/\rho_\infty$ that corresponds to coordinates that satisfy both the equations and that means that that is a point where both of them intersect right but in addition to that are the CJ points we also have to match the slopes because the Rayleigh line.

Is tangential to the Hugoniot curve right, so let us first try to get the end states at the CJ points in states as in the ∞ conditions right, so at the CJ points the slopes of the Rayleigh line and the ∞ curve match, so that means we had a first of all find out what is the slope at any point on the Hugoniot curve and then match it to the slope of the Rayleigh line so that the slope of

there you go into your curve is $DP \propto D / D 1 / \rho^\infty$ this is something that you can actually obtained by simply differentiating.

The you go hugoniot curve with respect to $1 / \rho^\infty$ right, so wherever you are getting a p^∞ you try to now differentiate that with respect to $1 / \rho^\infty$ right, so then you get a $p^\infty - P_0$ times sorry $- 2 \gamma / \gamma - 1$ times $p^\infty / \gamma / \gamma - 1$, $1 / \rho^\infty - 1 / \rho^\infty + 10$ would ρ^∞ that is the slope expression that you get for any point on the hugoniot curve this is actually the slope.

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The slope of the Hugoniot curve

$$\frac{dp^\infty}{d(1/\rho^\infty)} = \frac{(p^\infty - P_0) - \left(\frac{2\gamma}{\gamma-1}\right) p^\infty}{\frac{2\gamma}{\gamma-1} \frac{1}{\rho^\infty} - \left(\frac{1}{\rho_0} + \frac{1}{\rho^\infty}\right)}$$

Slope of the Rayleigh line is $\frac{p^\infty - P_0}{1/\rho^\infty - 1/\rho_0}$

Equating the two, $p^\infty = \frac{P_0 \rho_0}{\rho_0 \left[(\gamma+1) \frac{1}{\rho_0} - \frac{1}{\rho^\infty} \right]}$

or, $\frac{1}{\rho^\infty} = \frac{1}{\rho_0} \left[\frac{\gamma P_0}{P_0 \left[(\gamma+1) \frac{1}{\rho_0} - \frac{1}{\rho^\infty} \right]} \right]$

At any point okay we are still not saying that this is that the at the CJ points at the CJ points it turns out that the slope should now be equal to the real a slope right, so slope of the rally slope line what the slow is really line is a straight line okay and, so long as we do not want to use the m dot squared expression that is like a no-no yeah because we want to deal with only the thermodynamic properties so for a straight line it is just $P^\infty - P_0 / \rho^\infty - P_0 / \rho_0$ or that is it right so $P^\infty - P_0 / 1 / \rho^\infty$.

So equating the two equating the two we can we can we can get so we can get like a for example $p^\infty = P_0 / \rho^\infty$ times $\gamma + 1 / \gamma - \gamma$, so here what we have done is we have eliminated or we have

now written expression for P_∞ in terms of the other three p_0 , ρ_0 and ρ_∞ , okay in fact we always should be looking for ρ_∞ to show up as $1/\rho_\infty$ so sure enough this is $1/\rho_\infty$ here this is $1/\rho_\infty$ here and we are also getting it as ρ_∞ everywhere right here as well as here okay, so essentially that means we should now be in a position to eliminate ρ_∞ in favor of P_∞ or rather $1/\rho_\infty$.

Or if you want to do it the other way you can now try to get an expression for $1/\rho_\infty$ in terms of P_∞ , p_0 and $1/\rho_\infty$ okay so or $1/\rho_\infty = \frac{1}{\rho_\infty} \times \gamma \frac{P_\infty}{P_0} \times \frac{1}{1+\gamma}$ rather you used to writing $\gamma + 1 = \frac{\rho_\infty}{p_0} - 1$ okay, so you can use either of these expressions.

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Rankine-Hugoniot Relations

- At the CJ points, slopes of the Rayleigh and Hugoniot curves match
- Slope at any point on the Hugoniot curve

$$\frac{dP_\infty}{d(1/\rho_\infty)} = \frac{(\rho_\infty - \rho_0) - \frac{2\gamma}{\gamma-1}\rho_\infty}{\frac{2\gamma}{\gamma-1}\frac{1}{\rho_\infty} - \left(\frac{1}{\rho_0} + \frac{1}{\rho_\infty}\right)} \quad (300)$$
- Slope at any point on the Rayleigh line

$$\text{Slope} = \frac{\rho_\infty - \rho_0}{\frac{1}{\rho_\infty} - \frac{1}{\rho_0}} \quad (301)$$
- Equating the two slopes

$$\rho_\infty|_{CJ} = \frac{\rho_0 \rho_0}{\rho_\infty \left[(\gamma + 1) \frac{\rho_0}{\rho_\infty} - \gamma \right]} \quad (302)$$
- $$\frac{1}{\rho_\infty}|_{CJ} = \frac{1}{\rho_0} \left[\frac{\gamma \rho_\infty}{\rho_0 \left[(\gamma + 1) \frac{\rho_\infty}{\rho_0} - 1 \right]} \right] \quad (303)$$



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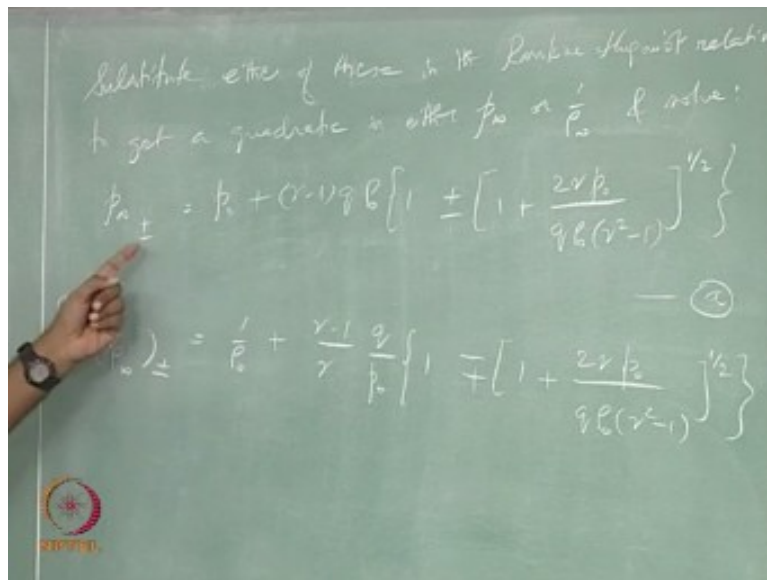
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So these are both at CJ points both is both these expressions are valid at CJ points okay does not really solve much we are not really got the coordinates yet because here ρ_∞ depends on ρ_∞ , so what does it meant by saying getting the coordinates, we should now be able to write P_∞ only as a function of P_0 , ρ_0 and Q okay and $1/\rho_\infty$ we should be able to write in terms of P_0 , $1/\rho_\infty$ and Q but this is the mixed thing we just can eliminate because we have a or we can write explicit expressions for P_0 at the scene ρ_∞ at the CJ point and $1/\rho_\infty$ at the CJ point.

In terms of the others which will still contain the other term p^∞ will contain $1 / \rho^\infty$ term and $1 / \rho^\infty$ will contain p^∞ okay, so there are there are these other things so we still have not solve the solve for the coordinates which is which is what we are now going to do next.

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So substitute a three of these right in the Hugoniot should strictly speaking say Rankine Hugoniot to give Rankine's view relation, so that what happens is if you now right p^∞ in terms of $1 / \rho^\infty$ in the Rankine Hugoniot have p^∞ at all in that expression you will now have an expression that is only on $1 / \rho^\infty$ which you which means you can now write what is one of the ρ^∞ in terms of everything else right.

So too so but unfortunately you are now going to get a quadratic okay in either of these well but it is actually fortunate if you physically fortunate mathematically a bit unfortunate you are dealing with quadratic equation, but it is physically fortunate because this says only CJ matching the slope simply means that we are just working at any CJ point it could be the upper CJ point or the lower CJ point either of these CJ points the rail a line is tangential Hugoniot it is done distinguishing between those two right.

And we are therefore looking for two solutions to p_∞ and P_∞ at the C-J points one of which will correspond to the upper C-J the other one corresponding to the lower C-J point so here so substitute either of these expressions in the in the in the ranking you go near relation to get a quadratic in either p_∞ or $1/\rho_\infty$ and solve the quadratic so if you now solve the quadratic you get $(p_\infty + \text{or } - = P_0 + \gamma - 1 q \rho_0 \text{ times } 1 + \text{or } - 1 + 2 \gamma p_0 / \rho_0 \gamma^2 - \rho_1)^{1/2}$ okay.

Now let us call this small a the expression for $P_\infty + \text{or } - 1$ and $1/\rho_\infty + \text{or } - = 1/\rho_0 + \gamma - 1/\gamma q/p_0 \text{ times } 1 - \text{or } + 1 + 2 \gamma p_0 / q \rho_0 \gamma^2 - 1$ so that is kind of like the same as what we had before her to the $1/2$ right okay so here what happens is when you now say + or - this + goes with the + here this - goes to the - here right but here this + or - is such that this + goes with the - here and this - goes to the + here right so very safe see basically say + or 1 you simply mean upper C-J or lower C-J.


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Rankine-Hugoniot Relations

- Use either of the above equations in the Rankine-Hugoniot equation to get a quadratic in p_∞ or $\frac{1}{\rho_\infty}$ which can be solved to get

$$p_{\infty \pm} = p_0 + (\gamma - 1) q \rho_0 \left[1 \pm \left[1 + \frac{2\gamma p_0}{q \rho_0 (\gamma^2 - 1)} \right]^{0.5} \right] \quad (304)$$

$$\frac{1}{\rho_{\infty \pm}} = \frac{1}{\rho_0} + \left(\frac{\gamma - 1}{\gamma} \right) \frac{q}{p_0} \left[1 \mp \left[1 + \frac{2\gamma p_0}{q \rho_0 (\gamma^2 - 1)} \right]^{0.5} \right] \quad (305)$$



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That means we are expecting that for the upper C-J your P_∞ should always be greater than P_0 therefore $P_\infty +$ will correspond to $p_0 +$ this okay lower C-J is going to be having a p less than P_0 therefore $P_\infty -$ should be equal to $P_0 +$ this - with away the negative sign that means you are going to have a less than and similarly here $1/\rho_\infty$ you are not going to have or 1

$1/\rho^\infty$ that is for the free for the upper C-J you need to have a $1/\rho^\infty$ that is less right therefore you need to have a negative sign / there and for the deflagration C-J point the low CJ point $1/\rho^\infty$ is greater therefore you need to have a +, so the lower sign will always correspond to a LCJ upper sign corresponds to you UCJ right.

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Substitute either of these in the Rankine-Hugoniot relation to get a quadratic in either p_2 or $\frac{1}{\rho_2}$ & solve.

$$p_{2\pm} = p_1 + (1-\gamma) \frac{p_1}{\gamma} \left[1 \pm \left[1 + \frac{2\gamma p_1}{\gamma P(\gamma-1)} \right]^{1/2} \right]$$

and

$$\left(\frac{1}{\rho_2}\right)_{\pm} = \frac{1}{\rho_1} + \frac{\gamma-1}{\gamma} \frac{p_1}{p_1} \left[1 \mp \left[1 + \frac{2\gamma p_1}{\gamma P(\gamma-1)} \right]^{1/2} \right]$$

Upper sign \rightarrow UCJ
 Lower sign \rightarrow LCJ

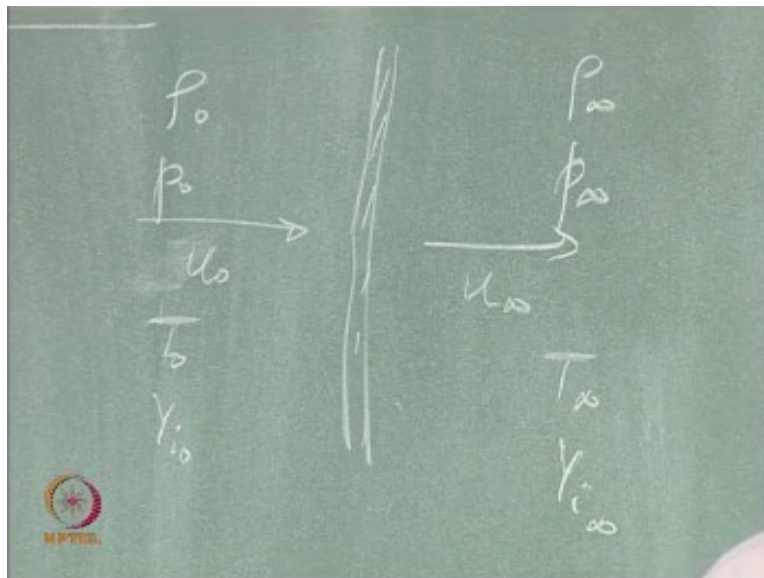
So upper sign corresponds to UCJ lower sign corresponds to LCJ so we now got the coordinates great so of course we need to call this b right, good I mean the in fact getting this by this particular thing now substitute those two expressions in the you go Hugoniot curve expression the Rankine Hugoniot and get the quadratic and solve the quadratic to get these this is actually done for each of those separately right, you will straight a little but it is going to take like some amount of time doing all this stuff okay.

Well it could be a bit boring but you wait through the mathematics to see how the terms basically get grouped the way they are okay, and then it is going to be pretty instructive for us because what we are going to do next. Now what we want to do is what do you want to do, what we want to do we want to now look at what is incoming Mach number, why do you want to look at the

incoming Mach number because we want to know how fast the CJ detonation wave and the CJ deflagration wave are going to travel right.

Now keep in mind how fast as the wave going to travel is an information that is contained in U_0 okay, because in a way fixed coordinate system you are going to have the reactants travel to the wave at that speed and that information is now in our scheme of things embedded in \dot{M} because \dot{M} is the one that is containing $\rho_0 U_0$ as well as ρ_∞, U_∞ so it serves purposes of carrying the flow information on either side of the wave right, and since we are actually looking at the incoming Mach number we should be focusing on U_0 that means we are looking at \dot{M} through which we will try to look for $\rho_0 U_0$.

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And \dot{M} does not show up in the Hugoniot curve it is showing up only in the Rayleigh line right so we should now go back to the Rayleigh line and say it is sort of like saying look at whatever what we have done we have substituted the slope matched information into the Hugoniot curve to get this but can I now look at how the Rayleigh line is going to be like if I plug these coordinate points in the Rayleigh line and then get an \dot{M} information through that you see what I am saying okay.

So we have really not used in Rayleigh line information except to note that that is the slope, but that is just a thermodynamic that is just the slope written in terms of thermodynamic variables we have not brought in the fact that this is equal to $-\dot{M}^2$ right, so that is what we want to do now.

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The slope of the Hugoniot curve is

$$\frac{dp_0}{d(V/V_0)} = \frac{(p_0 - p_0) - \left(\frac{2\gamma}{\gamma-1}\right)p_0}{\frac{2\gamma}{\gamma-1} \frac{1}{p_0} - \left(\frac{1}{p_0} + \frac{1}{p_0}\right)}$$

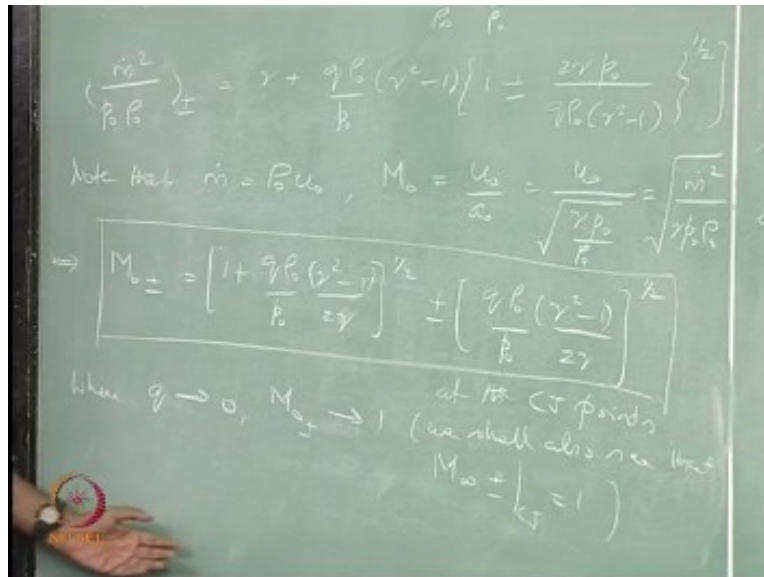
Slope of the Rayleigh line is $\frac{p_0 - p_0}{\frac{1}{p_0} - \frac{1}{p_0}}$

Equating the two, $p_0 = \frac{p_0 p_0}{p_0 \left[\frac{2\gamma}{\gamma-1} \frac{1}{p_0} - 2 \right]}$

or, $\frac{1}{p_0} = \frac{1}{p_0} \left[\frac{2\gamma}{\gamma-1} \frac{1}{p_0} - 2 \right]$

So we want to now notice that the slope that we had used for the Rayleigh line is actually equal to $-\dot{M}^2$ and then get to the flow information.

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So using a and b in the in okay, so simply just go back and write what it is $p^\infty - p_0 / 1/\rho \propto -1/\dot{\rho} = \dot{M}^2$ right, we can now get our $\dot{M}^2/p_0\rho_0 \pm = \gamma + q p_0/p_0 (\gamma^2-1)[1 \pm 2\gamma p_0 /q p_0(\gamma^2-1)^{1/2}]$ okay, note that $\dot{M} = \rho_0 U_0$ and the Mach number M_0 is going to be written as U_0/a_0 which is nothing but $U_0/\sqrt{\gamma P_0/\rho_0}$ if you now notice that \dot{M} is nothing but $P_0 U_0$ so now you write U_0 as \dot{M}/ρ_0 okay, so you now plug that over here you will now get your this is nothing but $\sqrt{\dot{M}^2/\gamma P_0 \rho_0}$.

So we had $\dot{M}^2/P_0\rho_0$ alright okay, and then we now are looking for $\sqrt{\dot{M}^2/\gamma P_0 \rho_0}$ is your Mach number or M_0^2 is now going to be $\dot{M}^2/\gamma P_0 \rho_0$ on that I mean this is something that is basic it is nothing particular about whatever it is like just putting these things together right, so from here we should now be able to write $M_0 \pm$ as $[1 + q p_0/p_0 \gamma^2 - 1/2\gamma]^{1/2} \pm [q p_0/p_0 2\gamma]^{1/2}$ so it is sort of like falling into some sort of a pattern there and this is what we were looking for so because it is going to say something to us so at the CJ points okay.

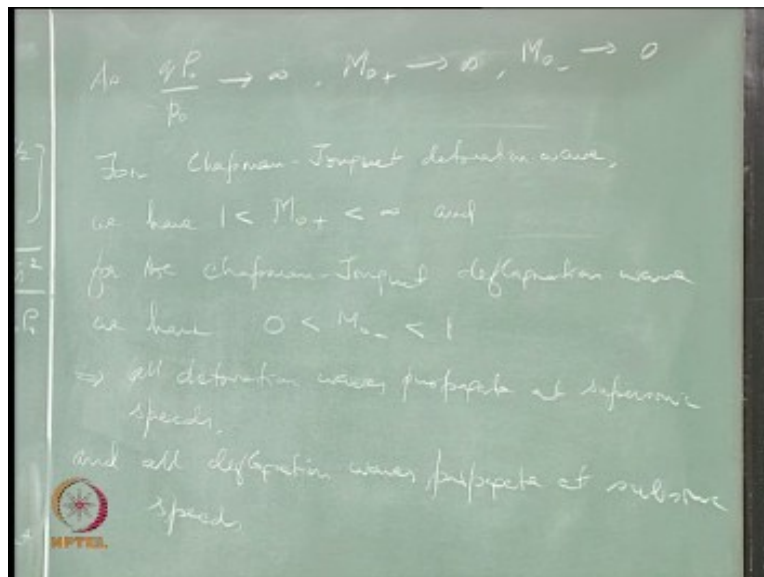
So what we can see from here is when q goes to 0 right, so when q goes to 0 this goes to 0 right, so you just get $M_0 \pm = 1$ all right, so and q goes to 0 we get $M_0 \pm = 1$ I can say goes to 1 right, and we would also see that $M^\infty \pm = 1$ at CJ points equal to 1 well that is something that we will do next, okay. So what is that simply means is nothing is happening when you do not

have any heat release it is as if like you had a wave that is going at the sonic speed okay, and not really changing anything about your gases, so gases are basically non reactive gases that means you do not you are not having any heat release and that is as if like you had gases go at sonic speed and nothing happened to it okay, that is not a very interesting situation for us.

But let us now look at the other possibility what we are really interested in is when you have non zero q , now how would you actually look for a non zero q in the Rayleigh line as I said when you now have a q greater than 0 as q increases you now have the Rayleigh line push up like that away from the origin of the Hugoniot right, and as you now go up to q tending to ∞ let us say you now had q going to ∞ right, this curve is going to actually go and hit the Rayleigh line the CJ point is go get pushed to ∞ and this CJ point is going to get pushed ∞ here all right.

And what you are going to now see is either your $1/p^\infty$ is going to go to ∞ or your $1/\rho^\infty$ is going to go to ∞ the P_0 is going to go to 0 over here your one over ρ your p^∞ is going to go to 0 here and you $1/\rho^\infty$ is going to go to the asymptotic this is how it is going to push right, so if you now look at the limit then.

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So as we strictly speaking do not have to look at q going to ∞ itself what really matters for us is in this expression $q \rho_0/p_0$ goes to ∞ then if you now use the positive sign for the UJ that is a detonation wave then both these terms add up together towards each other and then the M_0^+ goes to ∞ right, so M_0^+ goes to ∞ but M_0^- goes to 0 these two terms essentially subtract from each other of course one is like a small number when compared to these infinities that we are talking about so M_0^- goes to 0 right.

So what this means is what you then saying is then the for the Chapman Jouguet detonation wave detonation wave keep in mind when $q=0$ M_0^\pm both of them are going to be equal to 1 that is a lower limit as far as the heat is concerned right, so for the Chapman Jouguet detonation wave we have M_0^+ go from 1 to ∞ right, and for the Chapman Jouguet deflagration wave we have $0 < M_0 < 1$ you see so these are the limits that we are talking about therefore this is how it is going to expand.

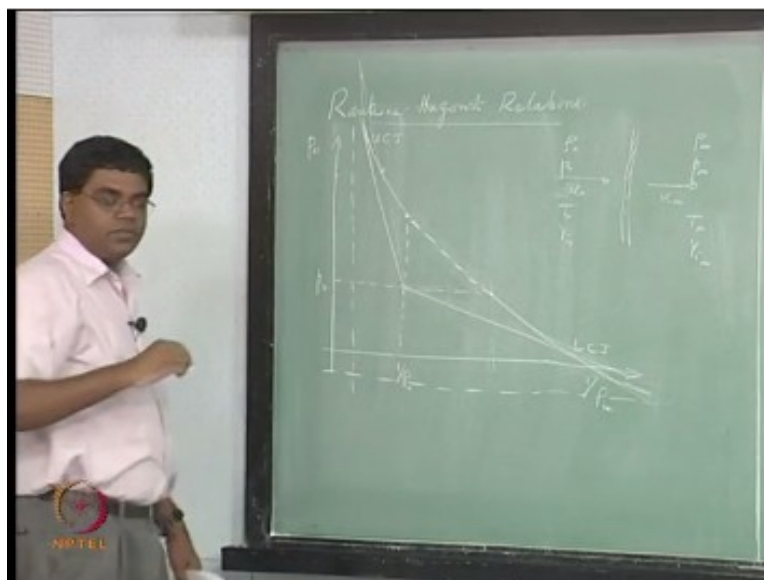
And what that means is the M_0 is not really going to overlap between these two okay, so this is going to go only up to 1 that is going to go only above 1 okay, so that means a detonation wave travels it propagates at supersonic speeds and a deflagration wave propagates at subsonic speeds. Well let us not just jump to that conclusion yet there is just one more step that we have already talked about the reason why and then I want to bring that into generalize this quite well.

The reason why we were actually looking at M_0^\pm at CJ points is keep in mind the CJ detonation is the slowest of all the detonation waves and the CJ deflagration is the slowest of all deflagration waves, right. So if you now have a CJ wave that is supersonic right that is a CJ detonation wave that is supersonic all other detonations are going to be supersonic, because this is the slowest and if a CJ deflagration wave is going to be subsonic then all deflagration are going to be subsonic, right.

So the this implies that right, all detonation waves propagate at supersonic speeds and all deflagration waves propagate at subsonic speeds right, so that is that is exactly what we were expecting or thinking will happen we also thought about a few things which is what happens downstream over here so if this wave is going to travel at supersonic speeds for detonation waves

and compresses the gas the reactants as they now become products they are compressed they also decelerate. So the question is or do they decelerate down to what okay, so the answer there is they could decelerate down all the way down to subsonic conditions all they could get decelerated to supersonic conditions that are still slower than the incoming supersonic speed this is typically what you are expecting when you are now looking for a detonation wave that is not a CJ detonation so if you now have a any detonation right.

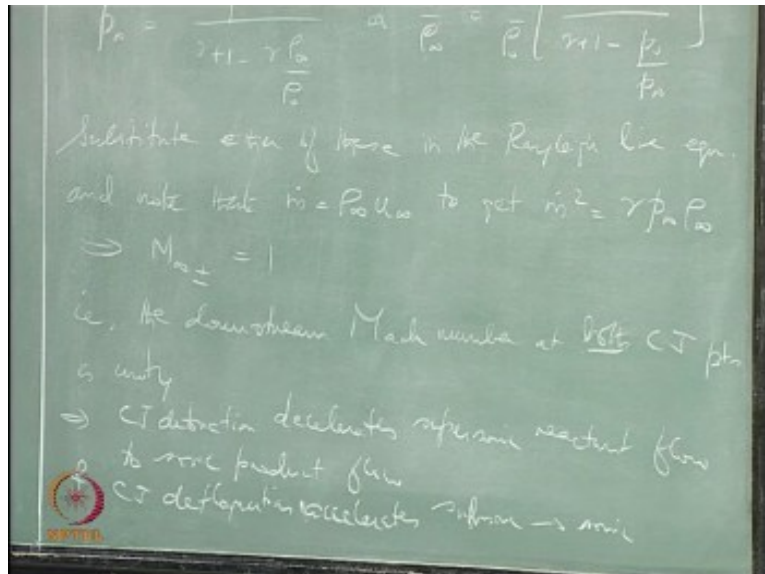
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You can now have solution here and here and this is something that is going to correspond to a very high pressure and very low density sorry, very high density as well with very low one over density therefore we are expecting it to get a lot compressed and therefore lot decelerated so it can get down to subsonic speeds over here and on this side it is not going to be that much compressed therefore it is not that much decelerated so you could expect that this is not decelerated down to subsonic speeds but it is supersonic yet less than slower than this the incoming supersonic speed.

And therefore we need to anticipate that at the CJ point the outgoing wave is actually going at sonic conditions right, so this is what we need to anticipate and let us see if we can if that is going to work out.

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So downstream Mach numbers at the CJ points right, so what we then get is we had this expression that we have this expression by matching the slopes of the Rayleigh and the Hugoniot curves right, for either the P_{∞} or the $1/\rho_{\infty}$ which we substituted earlier in the Hugoniot curve to get the coordinates of the CJ points all right and from there we now equated plug that value in the valley to get the \dot{M} and from there we now went to the upstream Mach number but we can do something different here.

We have at the CJ points $P_{\infty} = P_0 / \gamma + 1 - \gamma \rho_{\infty} / \rho_0$ or $1/\rho_{\infty} = 1/\rho_0 [\gamma / \gamma + 1 - P_0 / P_{\infty}]$ I think this is slightly different from what I just said a minute ago but it is possible for you to derive this by dividing the numerator and the denominator appropriately by I will say in P_{∞} and $1/\rho_{\infty}$ and so on so it is supposed to have these, what you did was you actually plug this into the Hugoniot curve expression and got the quadratics and solve them to get the coordinates for the LCJ and UCJ.

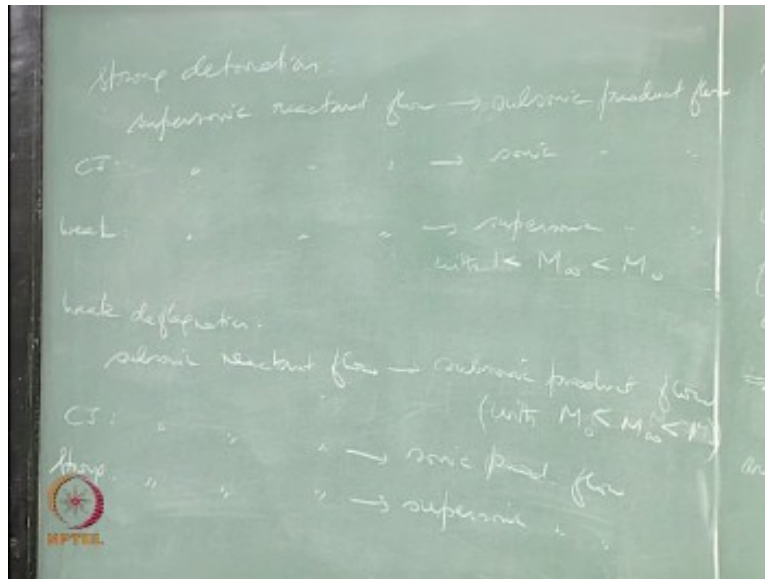
But if you can now go back and actually plug these in the Rayleigh line itself right, so substitute either of these in the Rayleigh line equation and note that this time we are interested in noting that \dot{M} is not $\rho_0 U_0$ because we are interested in the downstream conditions so \dot{M} is actually $\rho_\infty u_\infty$ so note that $\dot{M} = \rho_\infty u_\infty$ right, you should get right, so to get simply get $\dot{M}^2 = \gamma P_\infty \rho_\infty$. Now this is not very different from what we had over here you see you got as I told you we could have written this $M_0 M_\infty = u_\infty / a_\infty$ equals $u_\infty / \sqrt{\gamma P_\infty \rho_\infty}$ and then notice that plug this back in this here and then you can get $\dot{M}^2 / \gamma P_\infty \rho_\infty$;

If you got that and then by substituting this expression you also got this that simply means that $M_\infty = 1$ right, so this is simply going to mean that $M_{\infty \pm} = 1$. Now in a mathematical sense it is not surprising because we plugged it we plug this expression in a Rayleigh line which is essentially linear right that is just a straight line therefore we are not expecting any quadratics right, so if you do not have a quadratic then we cannot get two solutions we should get only one solution.

And that is only one solution okay, we strictly speaking should not even write a_\pm we are not even at the stage of distinguishing between \pm both of them are the same, so that means we are basically looking for the same answer for the downstream Mach number and either of these points right, that is to say the downstream Mach number at both CJ points right, at both CJ points is unity this implies the CJ detonation decelerates supersonic reactant flow to sonic product flow.

And CJ deflagration accelerates subsonic reactant flow to sonic product flow I am just going to abbreviate there because it is pretty obvious now, if that is going to be the case so the next step that we do is to then say that if you are a strong detonation.

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Then strong detonation means supersonic reactant flow becomes subsonic product flow right, CJ detonation is what we just saw supersonic reactant flow becomes sonic product flow and weak detonation becomes supersonic react and flow all right, but still supersonic product flow with $M_\infty < M_0$ you still have a deceleration, weak deflagration means subsonic reactant flow becomes subsonic reaction flow still becomes subsonic product flow with $M_\infty > M_0$ still less than 1 right, so we strictly speaking say with $1 < M_\infty < M_0$ the previous case here we should say with $M_0 < M_\infty > 1$ right.

What we should say CJ deflagration we soft subsonic reactant flow become sonic reactant flow sorry product flow and strong deflagration subsonic product reactant flow becomes supersonic product flow which is impossible because it violates second law which we have not explicitly considered mathematically and we will not do that just accept the state that this is not going to be possible physically. We will stop here for the day and we will try to wrap up this one on Monday.

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