

**Indian Institute of Technology Madras**

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**National Programme on Technology Enhanced Learning**

**COMBUSTION**

**Lecture 27**

**Laminar Premixed Flames**

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We have seen so far is the Rankin hugoniot relations where we are assuming that a pre-mixed set of reactants will approach a flame zone or a or a reaction sheet at a certain velocity and get converted to products as it leaves the reaction zone or the flame zone either to follow the reaction zone itself as it propagates into still reactants in a in Alaric's coordinate system if it is a detonation wave or move away from the reaction zone if it's a deflagration wave.

Now one of the major problems with that is we know really knew what is the speed at which the reactants would approach, the flame fixed or approach to flame in a flame fixed coordinate system right it is like if you knew the velocity you could you could stabilize the flame in a flame fixed coordinate system and this is what will happen, but what is the velocity? It is only if I knew the velocity I would know my  $m \dot{m}$  and only if I knew the  $m \dot{m}$  I could construct the Rayleigh line and only if I constructed the Rayleigh line can I look at its intersection at the hillgonio curve and then look for the solution but what gives me the velocity? How do I know what is the velocity with which it is supposed to travel?

All I know from this is it can travel over a range of very low velocities or a range of very high velocities right and these two ranges do not intersect at all there is like about two or three orders of magnitude difference in these in these velocities, but what is it? We never knew we just write a we just put an arrow and then say  $U_0$  and then question is what is it how do you how do you find out? Well it turns out that you cannot you cannot stop with treating the reaction zone like a black box the way we have done right.

There is obviously more to the story than what happens outside of these is the reaction zone it is not like you have some reactant status that come in into your black box and magically turn into products and we can say well let me just do the mass and momentum and energy Conservation's across this and try to understand you could understand to the extent that we have done but not anymore right, if you want to actually find out how the flame propagates or how fast the flame propagates we have to get into the flame we have to get into the open we are open up the black box and see what is happening inside.

Because that is what is actually causing the flame to propagate whatever is happening inside in side this region with all the temperature and concentration gradients and therefore heat transfer and mass transfer and so on is the one that is actually causing the propagation to happen. And that is what is dictating what the unit should be in a thin thick coordinate system and correspondingly the  $m \dot{}$  and then the Rayleigh line and then the whether it is a tangent relay or not and all those things whether DNA get a week did deflagration and so on.

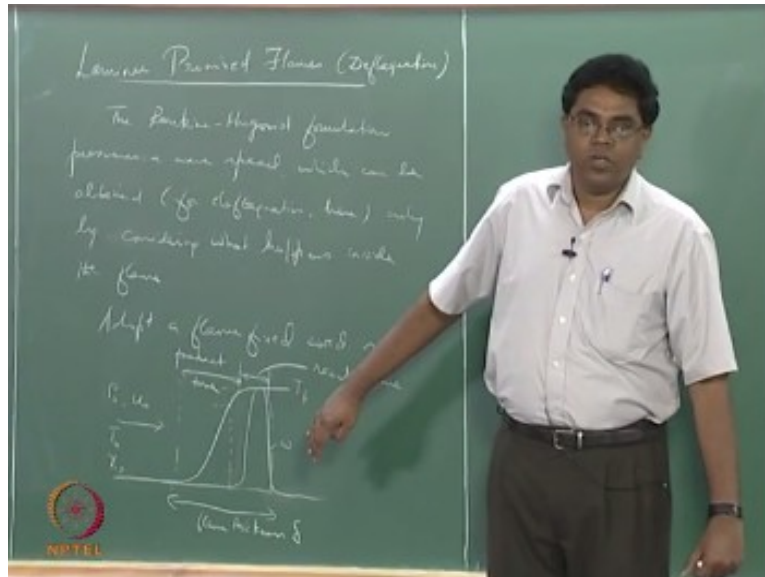
And of course what happens inside the black box that dictates the propagation velocity depends on whether it is a deflection that is along the lower branch or in the upper branch because in the lower branch we now have the deflagration that propagates in a subsonic speed that means it is actually seeing what is happening upstream there is a stream influence whereas in the case of detonation wave it is propagating at supersonic speeds so there is no influence on the upstream right because information can propagate only in one direction.

Therefore we have to treat these two cases separately when you now get into the black box and what we want to do is to start with deflagration sure enough we want to be we want to note that the Rankin hugoniot relationship we have studied so far corresponds to premix flames okay. So we are now talking about a reaction zone that is happening for reactants that are approaching the reaction zone together with each there that is reactive does not fuel and oxidizer together right.

So prim exclaims all right but we are actually looking at deflagration. So let us let us confine ourselves to deflagration now certainly premix 2 deflagrations and of course we want to also talk

about laminar in the sense we don't want to get into turbulent effects at this stage right. So this is what we want to do.

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So they so we want to actually see what happen inside the flame what we mean by inside the flame whatever we have neglected so far what did we neglect so far? We neglected anything that had any gradients right not only not only did we neglect chemical reactions but we also neglected regions that were corresponding to having a temperature gradient or a concentration gradient or velocity gradient and so on all greens were all now shout into does this region right.

So question is if you are now thinking about a deflagration how does it really propagate, anybody, can we can we think about this? So let us do a thought experiment that is kind of destructive, so let us suppose that this room is filled with methane as well as air of course right however you are sitting here right and then I go and ignite one corner and what is going to happen now? The flame now begins to propagate to the rest of the room yeah of course with windows open we cannot expect pressure rise to happen but if it is not if the windows are not open you could have a pressure rise and you could have a detonation transition and so on.

So let us not get into that yet right, so let us suppose that we have the windows open and the pressure remains constant more or less or there's like a mild pressure rise across the flame or a pressure decrease re across the flame to market a deflect market as a deflagration then now we have a deflagration wave that is propagating towards you huh how does that look like? Would you be sitting there wondering why the deformation propagating in the first place or you is want to get up and run okay, unfortunately this is just a thought experiment.

So can you think about why we are actually having a flame propagate why it propagates, so you have let us say still reactants out there and then the flame is approaching it right and I have ignited it somewhere so let us suppose that that has happened quite aback it is not having any bearing on the flame propagating at the moment here you know have a flame it is not going to go from here to there why, you think about this? Well as the flame comes near you what do you feel because of the heat right.

So as the flame gets near you feel the heat right so what happens so what does it mean to say some someplace upstream of the flame is getting heated up what happens downstream of the flame it is already hot right, so we now got the products right and the products are at ideally speaking the adiabatic flame temperature and recall how he got the adiabatic flame temperature we got adiabatic flame temperature therefore the product because the products had fairly low negative or actually fairly high negative heats of formation or low heats of formation that is why that is why there are products right.

And they had exchanged their heats of formation for the sensible enthalpy and therefore their sensible enthalpy has become high because their heats of formation or algebraically low right and then they became hot so you know how a hot products over there hardly any temperature gradient there that means you do not have too much heat that is getting conducted there, so if you now have a wall that is kind of suddenly getting heated up and the temperature is high what do you have for upstream you know have a fairly large temperature gradient that is actually pointing the in the upstream direction that means all the heat really wants to get conducted upstream of the flame and heat up the reactants and what do the reactants do, when you when they get heated up?

They start reacting like crazy why like crazy because strictly speaking if you now follow the Arrhenius law you have the cold boundary difficulty that they should be reacting even you are sitting there in the cold reactants okay at any temperature strictly speaking but the reaction rate is so low that the heat that is released is going to be at such a low rate that it just going to get completely dissipated away most of the time where you are until the flame hits you right. But as the flame is approaching and it is conducting heat upstream right and it is heating up the reactants the reaction rates are beginning to look appreciable particularly when your temperature reaches beyond a certain point dependent upon the activation energy.

So you know that the Arrhenius law is like earthy has goes as earthy exponential of minus  $e$  to the  $e / RT$  right where  $E$  capital  $E$  is the activation energy right. So if you now look at that particular expression what happens is larger the  $e$  greater is the temperature to which you want to take your reactants but you will also have the sudden rise in reaction rates much more steeply but with any further increase in temperature when compared to if you are  $e$  were capital  $E$  were low right.

So it with capital  $e$  being low you do not have to increase the temperature to a whole lot for your reaction rates to become appreciable but it is going to grow a little bit more gradually as you further increase the temperature right. So that is the highly nonlinear sensitivity of the reaction rate to temperature because of a high  $E$  that's how it happens and therefore you now have the chemical reaction rates beginning to become appreciable therefore the heat release is not going to is also now going to become appreciable if the chemical reactions are going to consume the reactants and how did you decide where the flame was wherever the reactions are happening see if the reactants had got consumed the reactions they now have moved forward and started reacting started consuming fresh reactants at a higher temperature.

And then they release heat on conduct so essentially the way it is going on is you go through a high temperature patch that you started out with when you ignite it let us say it is like a matchstick I am creating a high temperature patch that locally increases the temperature, so that the reaction rates become appreciable and once the reaction rates become appreciable the heat release rates become appreciable and this actually conducts heat upstream and heats up fresh

reactants consumes the current reactants therefore the reaction zone advances into the region with which it has heated up upstream in the fresh reactant mixture and therefore it propagates.

So we are now beginning to look at some two or three things one heat released by chemical reactions to heat conduction upstream three propagation right what a dollar Swiss thus these three because these are the three elements that are again coming back again and again in combustion you see the heat release due to chemical reactions is because of chemical reactions. So that is the term in an energy balance that is going to come up because of chemical reactions the heat conduction upstream is essentially the diffusive component right that is a transport phenomena that is going there that is that is essentially corresponding to diffusion of heat right.

And then finally you know how the propagation you can think of that as a convection in a flame fixed coordinate system as if you were to write over the flame you will see that the reactants are approaching the flame at the flame velocity that is Carissa that is basically meaning that the reactants are having a convection of their in a sensible enthalpy a lot of course sensible enthalpy along with the heat standard heat of formation right.

So if you now look at this is like a energy balance if we have to think in our minds there we are where the reactants are actually converting into the flame at their enthalpy getting the heat conducted into them from the flame because of chemical reactions, so we now have a energy balance that we need to think about containing three aspects convection diffusion reaction right. So pretty much anything that you can think about in combustion we will have to deal with these three balances in these three right and that is exactly what we figure out just by thinking about what happens right.

So let us kick any can we now draw this picture so let us suppose that I still adopt a plane fixed coordinate system right, so it is not as dramatic as like a flame propagating towards you or something because we were wrestling the propagation holding it there and what we are saying is if I now have a cold reactants that are coming in at  $p_0$   $u_0$   $T_0$  pressure is not a major consideration here the pressure is hardly going to change just mildly decrease in a deflagration so let us not worry about it you notice what is something that we are to find out okay.

So although I write this  $U_0$  there is something that we need to find out okay but still it is coming with a certain unit whatever it is right, and so if I were to plot my  $T_0$  I saw a temperature  $T$  it starts from  $T_0$  for the reactants and of course you can say  $Y_0$  for the reactants it starts for this and then gradually increases until it reaches the  $T_F$  the flame temperature. So let us say that that is like  $T_\infty$  is essentially  $T_F$  here for standing for the flame temperature or the adiabatic flame temperature specifically for ideal conditions without any heat loss to the surroundings strictly speaking you know you do not really have much heat loss to the surroundings all right.

Because the products as I said were fairly had a high temperature and relatively uniform temperature therefore there is hardly any conduction going on beyond that point okay, most of the conduction is going towards the reactants in fact this is the reason why it is adiabatic so long as you have heat conduction going only to the reactants okay. So the reactants take that heat and convert their standard heat of formation into a higher sensible enthalpy of the products okay, that is an adiabatic process you do not have to worry about the fact that you had a heat conduction it is not really a heat loss it is after all going to the reactants it is kind of like you a mass a lot of property and give it to your kid okay that is not philanthropy right.

So okay so you know you know had this  $T_F$  right and then the temperature increases right, so what we said was the reaction rate has an Arrhenius dependence on temperature and it becomes fairly sensitive to temperature only when the temperature becomes above a certain point okay. So if you were to plot the reaction rate it is not going to really rise a lot until you reached a fairly high point in temperature let us suppose let me sum over here okay, so this temperature is only badly lower or maybe I am exaggerating maybe that is suppose that I want to mark this point you see this temperature is not very low when compared to that with the final temperature and that is when the reaction rates the beginning to increase and how do they increase them you just go up right.

So this is the crazy reactions if you are talking about right, and then why does it fall we ran out of fuel okay, so we just completely consumed the fuel so that there is nothing to react right or other of the reactants any of the reactants you could have one of those deficient reactants typically we are thinking about like a fuel lean situation there where you have plenty of oxidizer

but that is not necessarily true all the time but we essentially looking at the deficient reaction running out and then that dot abruptly stops the reaction zone okay.

So this is  $\omega W$  whichever way you want to call it based on mass balance or on a mass basis or a molar basis okay question, what is the same here? You would like to think the same is essentially a reaction zone and therefore the reactions are primarily happening here because that is why the reaction rate so significant course made the  $\omega$  could be plotted in any scale it does not necessarily have to go over here but there is sort of like normalizing all of them from something zero to one or something like that okay so everything goes from bottom to top that is it.

And so let us suppose we draw this picture and we take we tend to think that this is where the flame is supposed to be but that is not what we decided when we did the Rankin hugoniot the Rankin our black box was to consider every gradient that means the flame strictly speaking extends all the way up to here from here right. So that is really your flame right and this is this refers to the flame thickness  $\delta$ , so what is happening here upstream of where the reactions are happening essentially you have a preheating right so the reactants is simply getting heated up ahead of when they react yeah they are just cooking just getting ready to react.

So we can now differentiate these as the preheat zone and the reaction zone right higher the  $e$  the activation energy smaller and smaller is the reaction zone as a fraction of the flame thickness right, so rudimentarily many textbooks would simply show like a half the flame thickness is like a preheat zone and half the flame take flame thicknesses as a reaction zone but that is not necessarily true for high  $E$  okay. So many hydrocarbons have activation energy that is significantly large it takes on the cannot say it is infinite but when it is large a therefore we get into these thin flames in fact what happens is when you now have large  $e$  the flame thickness itself decreases because larger the  $e$  smaller this region smaller this region higher is the  $W$  right.

Because all the reactions have to actually happen within a very thin zone maybe the reaction rates became significant all of a sudden and then there is like a huge amount of reactions is going on per unit time right. So when you now have a large heat release that means a large temperature gradient has to be set up to dump all that heat upstream, so this is something that we have to be



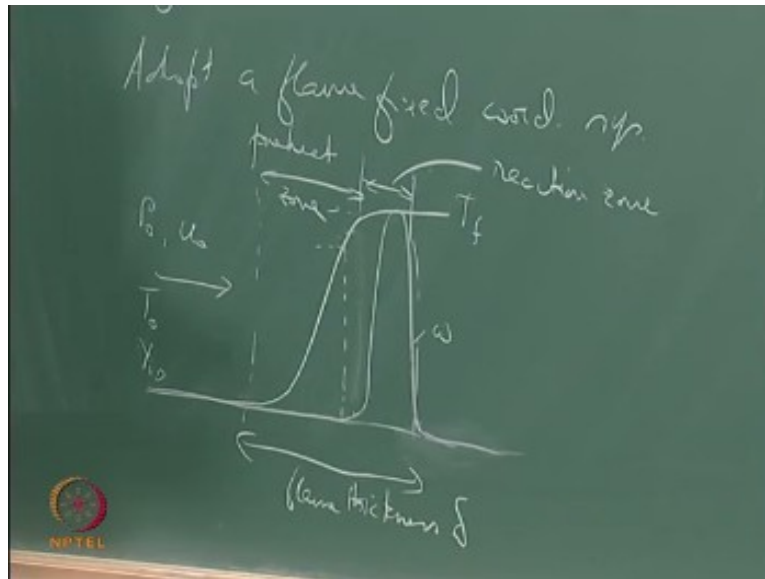
very careful in thinking in an adiabatic flame if you have a very large chemical heat release rate in the reaction zone it should correspond to a large temperature gradient.

So that the heat conduction is happening at exactly that high rate okay and think about the gradient now the gradient is caused by a temperature difference happening over a distance that is what a gradient means right, the temperature difference is not too much different if you have a that is high okay, you control your  $T_0$  and that is like let us say 300 K 500 K something like that and  $T_F$  is thermodynamically determine okay that has got nothing to do with the kinetics so  $T_F$  is like typically 2400 K or thereabout depending upon the reaction that you are talking about right.

Therefore that difference is not going to change a whole lot at all, so what can now change to cause a steeper temperature gradient the only possibility is if this preheat zone thickness came down you see, so because you had a high  $e$  and therefore got a fairly low thickness for the reaction zone correspondingly the preheat zone will become thinner all right. But as the  $e$  increases the fraction of the flame thickness that the reaction zone occupies become smaller that is what I was they need to say okay.

So as  $e$  increases the reaction zone becomes smaller the preheat zone becomes smaller okay the flame becomes smaller on the whole but the fraction of the flame that the reaction zone occupies becomes smaller alright, so this is what is going on as you now think about how the reaction rate increases with temperature through the activation energy okay fine.

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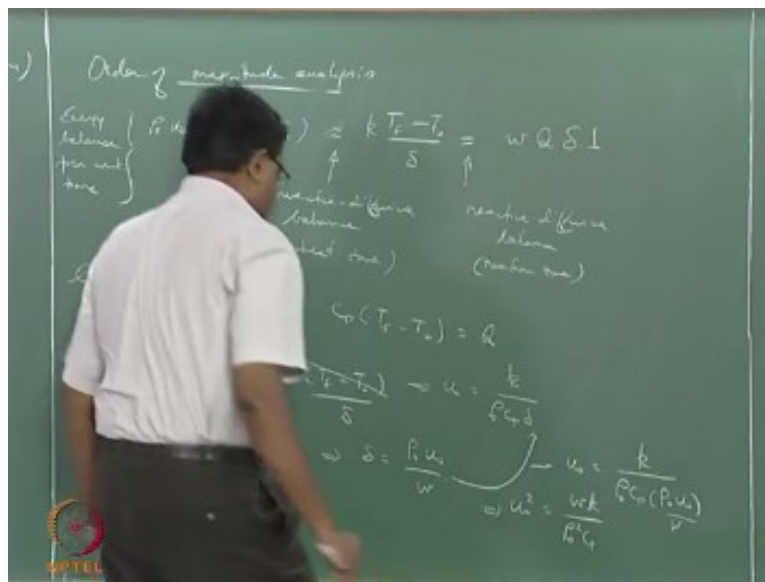


So can we do the energy balance that we were talking about akin can we now start doing like a that is the posters okay or order of magnitude order of magnitude analysis or whatever we just know danced about or for this flame propagation in the room and thought about how this exactly propagates can we now put it in some sort of mathematics right, so can we can we write like a energy balance, so we talked about an energy balance of convection diffusion and chemical reaction okay.

So the way we want to actually do this is one of the major problems as I said with combustion that is we have to actually take in account these three things convection diffusion and reaction and typically the reactions are the ones that are that post's the major hurdle in dealing with this. So you will find this time and time again that we try to avoid dealing with all three of them at the same time that's what is called as a mixed problem okay because you are now mixing up all the other three issues of convection diffusion and reaction is it possible for me to isolate only two of those at a time the peens can I deal with the convection and diffusion alone without reactions in a region where no reactions are hardly happening right, is it possible for me to actually deal with only diffusion and reaction where you do not have to worry about convection it is just a passing gases they do not really contribute to convection of enthalpy all right.

So that is what I am going to actually do now think about think about so when I now look at this region I find that the reactions are happening only here out here there is some energy balance going on you have species that are coming in with a certain enthalpy and they are all actually feeding into this feeding into this right but the rate at which this is coming it should actually balance the rate at which the heat is being supplied, so that the temperature can rise from  $T_0$  to  $T_F$  right.

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So there are two or three different balances that we can write for example we are not when we now say, if you now look at the  $\rho u_0$  that is your  $m \dot{C}_p (T_F - T_0)$  that is the rate of rate of change of enthalpy right that came primarily from the heat that was given in the chemical reactions or let me take a step back that is primarily coming from the heat conduction that is happening from here to there right, the first place so as the reactants are approaching the flame the first thing that they notice is the heat okay.

So and in that heat is still basically getting conducted, so if I were to write this as  $K$  approximately all these things are proximate we will do only like an order of magnitude idea so the temperature gradient here can now be written as  $T_F - T_0 / \delta$  right, and this heat conduction

has to match the rate of heat conduction has to match the rate at which the heat is released right, so that is equal to I could write a wiggly equal to but I have already done that I mean this is this is the Wiggly equal to is to basically mean we were we were supposed to write  $K \Delta T / \Delta X$  but  $\Delta T / \Delta X$  is approximately approximated by a temperature difference divided by the distance as if you had like a straight line all the way okay we ignored this little gap here where this curving is happening and we supposed to this is about the same as this we said that this is about the same as that we did we did a lot of approximation say okay.

And so how does he how does how does this heat get it conducted because you have a heat that is being released you do chemical reactions and that is happening when you now have so much mass of reaction reactants that are actually getting consumed per unit volume per unit time per unit time alright this is per unit time okay this is per unit time because this is actually  $\dot{M}$  all right, so  $\rho_0 Q_0$  is essentially corresponding to  $\dot{m}$  so this is per unit time this is per unit time also so  $W$  corresponding to a volumetric mass change mass consumption per unit per unit time okay we have to multiply this by the heating value of the reaction so this is like joules per kg this is per kg per meter per second per meter cube okay so this becomes like  $\text{kg}^2/\text{s}^2$  per meter cube we now have to multiply by a volume right.

So the flame thickness is  $\delta$  over which the volume over which let us suppose a first approximation the reactions are happening I know I am not even going to distinguish between the chemical so the reaction zone and the preheat zone as a first step but we know better we know that this is where the reactions are happening I still do not know exactly how the portion does this thing okay, I said that this is going to be thinner if you had a high  $e$  so obviously it depends on  $e$  in some sense okay I do not know how okay let me not worry about it at the moment.

And  $\delta \times \text{one}$ ,  $1$  is like for a unit cross sectional area all right perpendicular to the board if I did this what am I looking for I am looking for  $u_0$  but you now have additional things that are coming up there is like a  $\delta$  that is coming off  $T_F$  is something that I think I can find out I can give you  $T_0$  and  $C_P \rho_0$  all these things I can take not a problem but there is a delta that is

coming up fortunately we have now two equations two unknowns you see this is one equation and this is another equation alright.

So the two unknowns are you not in  $\delta$  okay and of course the nice thing is the  $\delta$  does not show up in the sorry  $u_0$  does not show up in this you can actually try to find the  $\delta$  over here and use that in this expression for finding the  $u_0$ , so that is how you're going to go before we just proceed the way that I just described I want to point out that this is what is now called the convective diffusive balance right.

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**First Approximation of Flame Speed**

- The Rankine-Hugoniot formulation presumes a wave speed which can be obtained (for deflagration) only by considering what happens inside the flame.
- Consider the flame fixed coordinate system with reactants' speed  $u_0$ . Other properties be  $\rho_0$ ,  $T_0$ , etc.
- Convective-diffusive balance in preheat zone:  
$$\rho_0 u_0 C_p (T_f - T_0) \simeq k \left( \frac{T_f - T_0}{\delta} \right)$$
- Reactive-diffusive balance in reaction zone:  $k \left( \frac{T_f - T_0}{\delta} \right) \simeq wQ\delta$ .
- Global energy balance:  $C_p (T_f - T_0) = Q$
- Hence,  $u_0 = \frac{1}{\rho_0} \sqrt{\frac{wk}{C_p}}$ . This is the first approximation of laminar flame speed.

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This is primarily happening in the preheat zone we could say that the enthalpy flux that is coming in to the preheat zone primarily goes primarily this is coming from the heat conduction in the preheat zone because there are hardly any chemical reactions happening there. So if you know think about only an energy balance it is sufficient for us to consider only the convection and the diffusion right, whereas here this is what is called as the diffusive or to have it rhyming with this we can say this is reactive diffusive zone or reactive diffuse of balance so this is the reaction zone right.

So many times the technical way of for actually talking about a preheat zone essentially is to say that this is a convective diffusive zone and this is the reactive diffusive zone why do not think how to worry about convection here? Because we know not talking about any appreciable change in temperature in this there is hardly any change in the enthalpy flux there is a so much amount of enthalpy that is coming in at this temperature it is pretty much leaving at pretty much the same enthalpy there is hardly any change that was caused in the enthalpy flux convection right.

So we did not have to worry about that all we had to worry about was as the chemical reaction heat release was happening it is getting dumped by conduction upstream alright, so that is the reason why we had we could live without a convection showing up in this balance and so effectively what it means is we are dealing with two of these three processes at a time here we are not really dealing with reactions here we are not dealing with convection all right. So that is a step that we do in dealing with chemical so we're dealing with combustion we look at the look at how this zone clarifies isolate where the reactions are happening try to look at what happens elsewhere and look at balances without the chemical reaction there and then try to match it up with the balances that are happening in the chemical reaction zone checking the chemical reactions in account.

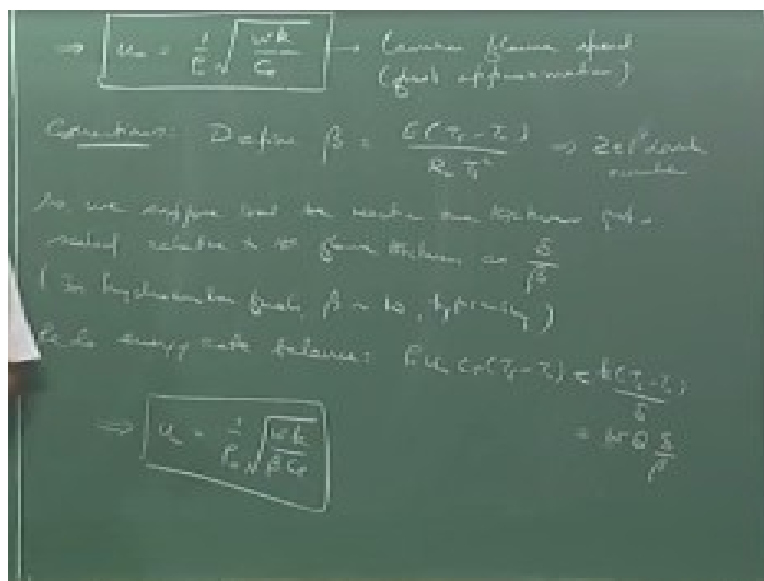
So this is how we do this so let us just try to find out how to how to go about this, so there is also a global energy balance that we can this is a per unit time okay, so this is energy balance per unit time right there is a global energy balance that we have already done to begin with global energy balance which is basically to say that the reactants had a standard heat of formation and sensible enthalpy the products have standard heat of formation and sensible enthalpy these two sums should be equal in an adiabatic situation right which means the differences in the sensible enthalpy of the reactants and the products should be equal to the negative of the differences between the form a standard heats of formation of the products in the reactants right.

And that is what we are calling as  $Q$  the last thing that I said the negative difference of the Heat standard heats of formation of the products in reactants is essentially the  $Q$  right, so we should now be able to say that  $CP(T_F - T_0) = Q$  this is on a per unit mass basis okay for strictly speaking

I should basically take a certain amount of reactants a mass of reactants and say  $m C_p \delta T = M \times Q$  because  $Q$  is essentially heat release per unit mass okay joules per kg therefore we can again I can say this per unit mass and so how do I how do I now a factor in factor these things into account.

So if I now say that  $\rho_0 C_p u_0 (T_f - T_0)$  is approximately  $K (T_f - T_0) / \delta$  the  $T_f - T_0$  gets cancelled it is as if it did not matter what the temperature difference was as far as the convective diffusive balance was concerned and what you then get is you are looking for you not sir that therefore you get  $u_0 = K / \rho_0 C_p \delta$  right, and then we say that  $\rho_0 u_0 Q = W Q \delta$  right then what do I get I can cancel the  $Q$  and I get  $\delta = \rho_0 u_0 / W$ , so plug those in here to get  $u_0 = K / \rho_0 C_p$  alright and  $\rho_0 u_0 / W$  this will be you can take the unit to the left hand side you get a  $u_0^2 = w k / \rho_0^2 C_p$  alright.

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Which implies which implies  $u_0 = 1 / \rho_0 \sqrt{W K / C_p}$  this is the first cut estimate that we have had for the flame speed, so this is essentially the laminar flame speed okay first approximation what is mainly the first approximation here is mainly the first approximation is we have assumed

that the  $W$  is happening over the entire  $\delta$  we did not know how to split the same thickness into a reaction zone and a preheat zone we just said that the reactions are happening everywhere okay.

So obviously we now get a we should actually get more like a rate prediction of what is going on interestingly this gets all the dependences right okay, so what we can understand is if the reaction rates are very high right if the reaction rates were very high then your flame speed will be high okay you can also get an idea of what the thickness looks like we get we get like a  $\delta = \rho_0 u_0 / W$  right, and then what happens is when the reaction rates are very high they same thickness decreases so this is also coming from saying that if I had a large  $e$  I would have this zone decrease and then the reaction rates that shoot up because all of them all of it is supposed to happen within a short distance therefore you have a high reaction rate.

So that means a thinner flame essentially is going to propagate faster that is what this mean this really means right, if you have a high thermal conductivity it is going to propagate faster because the upstream heat conduction can happen faster right if you have a lot of  $CP$  that is bad news for the flame propagation because as the flame is trying to conduct heat up stream into the cold reactants the reactants are simply soaking up all that heat without a rise in temperature that is what  $CP$  means okay  $CP$  basically means the amount of heat that is required for a unit rise in temperature if you have a lot of  $CP$  that means you take up a lot of heat without showing up in terms of temperature right you are a very calm person you are pushed around and you still smile okay you do not really heat up and show your anger okay that that means you are a high  $CP$  guy okay.

And then that is bad news for the chemical reactions because the temperature needs to rise for their rate to increase right therefore if you have a high  $CP$  you are going to get a fairly low you not and therefore so and then similarly for the  $\rho_0$ ,  $\rho_0$  is essentially like a mass inertia  $CP$  is like thermal inertia this is like mass energy you have a lot of mass per unit volume to heat up alright. So obviously it is going to take a while for the flame to go when you now say are you not as getting low so you get all the dependences all right.



Let us now start correcting this a little bit from what we what we can think about so Corrections I will finally lead up to a point where we can actually construct the exact expression for you not taking a lot of if a lot of Corrections and therefore additional effects into account which is only about the  $\sqrt{2}$  factor away from the analytical result okay, so and I would like to go through this because this is a lot more physically intuitive so we think about what happens and then construct these equations in some sort of a phenomenal what is called as phenomenological manner okay.

So means we think about the phenomena and then set up these equations as opposed to well the equations are only derived you know what the equations are you have 5 and + 6 equations to deal with okay and then we can we can appropriately simplify and pick and choose some of these equations to deal with and all those things to do the short  $\delta$  which and then have again  $n + 1$  equations to confront and all those things we know all that stuff okay.

So we will do that and we will find out that finally we have only about a  $\sqrt{2}$  difference between what we end up with all these corrections that are physically employed okay, so how do you want to do this it turns out that what we want to do is we want to make sure that the  $W$  is factored in  $1/d$  in a small region that means you cannot put a  $\delta$  over here but it has to be like a fraction of the  $\delta$  question is how do you develop that fraction we know that the fraction is going to depend on the activation energy right.

So higher the activation energy we have thinner this region right so there must be like a measure of activation energy that is factored into this right, so define  $\beta$  as  $e \times T_F - T_0 /$  or  $u T_F^2$ .

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## Second Approximation of Flame Speed

- Correction: Zeldovich number definition

$$\beta = \frac{E(T_f - T_o)}{R_u T_f^2}$$

Scale reaction zone thickness relative to flame thickness as  $\frac{\delta}{\beta}$

- Redoing energy rate balance:

$$\rho_o u_o C_p (T_f - T_o) \simeq k \left( \frac{T_f - T_o}{\delta} \right) \simeq w Q \delta. \text{ This gives } u_o = \frac{1}{\rho_o} \sqrt{\frac{w k}{\beta C_p}}.$$



The way this is defined as pretty important, we were simply said  $E_f / R_u T_f$  and that that ratio is important because it shows up that way in the Arrhenius expression right but we also want to factor in like a temperature difference right, so the temperature difference is  $T_f - T_o$  for a given  $T_f$  okay, so if you now say  $e / R_u T_f \times (T_f - T_o) / T_f$  you get this right. So this is actually factoring in what happens in the Arrhenius expression and also the initial temperature as a fraction of the final temperature.

So this is typically referred to as the Zeldovich number or many times in this particular context it is also called as Zeldovich scaling factor, because you are going to try to use this as a factor to scale our length scales okay. So when you now say you want to have the reaction zone so we suppose that the reaction zone thickness get scaled relative to flame thickness as so, the means if the if they react if the flame thickness were  $\delta$  then the reaction zone thickness is like  $\delta / \beta$ .

So typically you know for hydrocarbon fuels  $\beta$  is about 10 typically I will try to give you exact number some of the time okay on what is the  $e$  is like and so what will be the  $T_f$  like and so on so we can actually factor in and find out that betas are of the order of 10, 10 exactly means that it is about 1 order of magnitude more than one that says it when you are now doing an order of magnitude analysis we are looking at things like 1, 2, 3, 4, 5, all these things are like up the other

one okay even like 6, 7, is of the order of 1 but anytime now you exceed 10, 10 to 100 counts as like a drift magnitude of 10 so that one order of magnitude more than 1.

So that is up that is the way you want to look at it okay, so this means that we are talking about the flame thickness typically being about a more refined attitude more than the reaction zone thickness or vice versa okay so for high activation energies essentially you get your reaction zone thickness to be quite small but as a fraction of the flame thickness itself, and now let us go back and do what we wanted to do. So we now say the now this is fine okay so if you want to now do this so redo energy rate balance energy rate balance the pins  $\rho_0 u_0 C_p (T_F - T_0)$  that stays as it is and  $k (T_F - T_0) / \delta$  you might quibble here you should say wait it cannot be  $\delta$  it has been little smaller it is like do not be like  $1 - \beta$  okay and since everyone  $1 - 1/\beta$  over beta that is how it is going to be okay.

And since  $\beta$  is 10  $1/\beta$  is like 0.1 is much smaller than one on an order of magnitude basis so let us not worry about it essentially what you are saying is if this is going to be so small the temperature is risen almost close to  $T_F$  almost at the end of the flame and therefore this is like roughly how the heat conduction should still behave and that is now going to be equal to  $Q$  sorry  $W = Q \delta / \beta$  this is exactly where I want to really bother about it it's not showing up in comparison with one okay, it is showing up as it is and now I have to I have to take that there if I now do all these things so I go through the same rigmarole right and what should I what should I get I should now get my  $u_0 = 1 / \rho_0 \sqrt{w k / \beta c_p}$ .

So that means that the second estimate that we have for the flame speed is little less how much is how much is it less it is like  $\sqrt{10}$  less that is about so this was that treats predicting this about three times more still within an order of magnitude three is right order of the order one okay, so this is this is okay but if you want it further refinement here we have okay, let us go on with a further refinement tomorrow at 10 o'clock.

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