Indian Institute of Technology Madras

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Lecture 37
Burke-Schumann problem

Prof. S R Chakravarthy
Dept. of Aerospace Engineering IIT Madras

So in diffusion flames we could actually look at two situations that are slightly different from each other.

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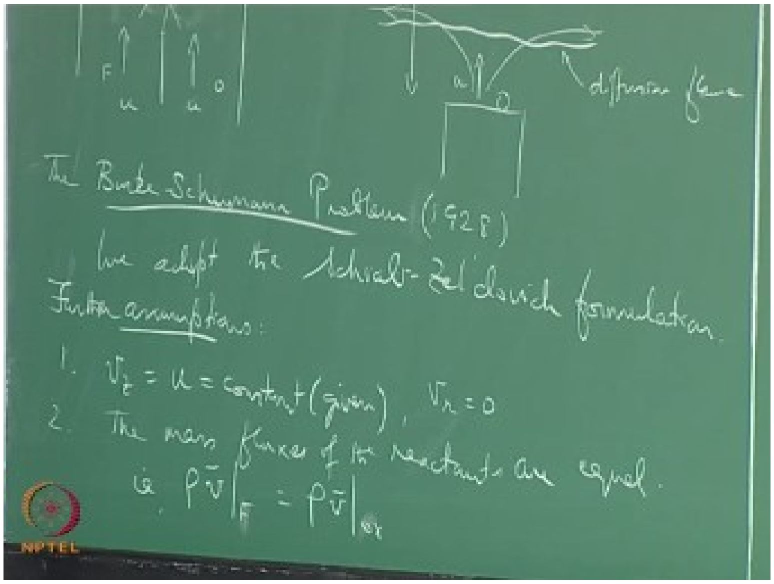

The Burke-Schumann Problem (1928)

We adopt the Schvab-Zel'dovich formulation.

Further assumptions:

1. $v_z = u =$ constant (given), $v_r = 0$
2. The mass fluxes of the reactants are equal.
i.e. $\rho \bar{v}|_F = \rho \bar{v}|_{ox}$

One is a co flow diffusion flame or the other one is a counter flood counter flow diffusion flame talk a little bit about each of these three why we need to talk about a co flow diffusion flame is because we essentially we are now talking about two different reactants that are approaching each other the question is anything when you now say flow you know talking about how do they conduct okay previously earlier we were talking about how they mix with each other mixing is eventually depending upon primarily the temp the concentration gradient that is prevailing for each species to mix into each other.

But the question the question about the configuration of the flame is how do the reactants flow to each other in the case of premix flames when we are saying pre-mixed that means the reactants were mixed with each there, so that means they flow together there is no way you are going to have the reactants flow from different sides or something of that sort right, but not the moment you are now talking about diffusion flames where the reactants or reactants are coming from coming differently and then they have to mix at the flame then you could now begin to talk about these things right.

So a co flow diffusion flame would typically configuration would be where you have for example in a duct you have a splitter plate and you have a fuel on one side and oxidizer on the other side and the splitter plate terminates at the lip of it slip and then you now have a mixing region that means as the flow is going up like this the field mix is like this and oxidizer mixes like that right and somewhere in there you are expecting a flame to happen because that is the frame is where they mix with each there right.

So this is a this is what is called as a co flow configuration there are several situations which actually mimic this for example if you now think about like a Bunsen burner right, so you have a field that is coming in the oxidizer is actually ambient it is not exactly flowing but effectively you have a flow that is being set up by the fuel and of course the products are also actually flowing outward because you are hot and in a buoyancy driven flow they basically go outward upward and therefore you have an entrainment of the air.

So the entrainment of the air is also causing a local eco flow situation as far as the flame is concerned and you will also have like a burner with an adduct where you have a fill in the middle and oxidize it in the outer any less and soon, so all these things are basically like a co flow situation a counter flow situation is where you are essentially trying to have a field come in this way and oxidizer coming this way and you now see if you did not want to have any shear layer effects in the co flow case you would now say equal velocities forward if you fuel and oxidizer that means this is coming into velocity you this is coming into velocity u same velocity.

So you do not you do not necessarily have like a shear layer between the two and you are focusing mainly on the species mixing rather than any momentum mixing and all that stuff. Similarly you can also think about in the counter flow situation a counter flow at the same velocity for example, so if you not think about this in typical examples would be like what is called as an a post z so if you now have like a z here and the jet here this is like the nozzle for the z or the z mouth if you will and then essentially they are they are now apart by about a distance H let us say right and then they are coming in with a certain velocity.

So what this is going to do is you are going to have a flow that is going to go like that right and as the fuel is flowing like this and oxidizer is flowing like this they are mixing with each other and when they mix with each there somewhere and they are not necessarily in the middle of the flow but somewhere in there you are going to have a flame that is that is a diffusion right, and this is a flame where you can actually hope to establish something that is very flat the flame would be quite flat because the mixing is going to happen equally on both sides except now the problem is you do not have a uniform velocity field for the flame in fact you are not going to have what is called a high strain rate here and then the frame is actually being stretched because of the flow wanting to actually go this way on neither side.

As a matter of fact you could also have a premix frame configuration here in the opposed z a burner how would you do that you could now have a fear and oxidizer coming from the same nozzle let us say from the bottom and then you now send a z of inert gas let us say nitrogen right and what would this do it will now create like a stagnation point flow so this is essentially what is called a stagnation point flow because by symmetry this would this would be as if like you had one z that is impinging on a wall.

So if you now have a z that is impinging on a wall it just spreads out and you have a stagnation point right at the middle right, so this creates like a stagnation point flow in the middle essentially the both the flows together now or like mirror images of each other if the velocities are the same and so you have a stagnation point flow and you could have a pre-mixed flame which is flat which is subjected to a strain right that is an experiment to actually investigate the effect of the strain rate on the premix flame.

But the case of diffusion flame you can actually hope to get a flat diffusion flame as opposed to in this case what is going to happen as the frame could actually be curved depending upon whether you have awe will talk about this in greater detail pretty soon depending upon whether living over the stoichiometry of the mixture right, so you could actually have a dilute mixture for the fuel or dilute mixture for the oxidizer for example if you use air then it is already diluted with lot of nitrogen for the oxygen and then it depends on the stoichiometric coefficients in the reaction like the stoichiometric coefficients means if you know we need no oxidizer for a given amount of fuel for stoichiometric reaction and so on so these sinks will actually dictate what the shape of the flame is but you cannot really expect to have a have a flat flame okay.

So if you want to have and the reason why we would like to have a flat film for example is if you want to treat it in a one-dimensional manner right, but there are ways of on-dimensionalizing these flames and so on we will discuss those things in greater detail but effectively you could think about a two configurations like a co flow on account of low geometry which makes more sense in the context of diffusion flames.

Now what we will do now is actually pick the co flow geometry and then look at what is called as the Burke Schumann problem, now the book human problem is a is a very fundamental problem in fact it was it was presented in the first symposium on combustion in 1928 by Burke Schumann and was subsequently published in the Journal of industrial chemistry, so it is a very celebrated piece of work and it is had numerous citation sense subsequently and lots of different extensions to this problem this problem and the name the face Burke Schumann flame connotes a certain awesome set of assumptions that in that they are a ten bodies and it refers to certain limits of like very high activation energy infinite rate chemistry and so on. So the Burke Schumann problem is as follows I will describe the problem first.

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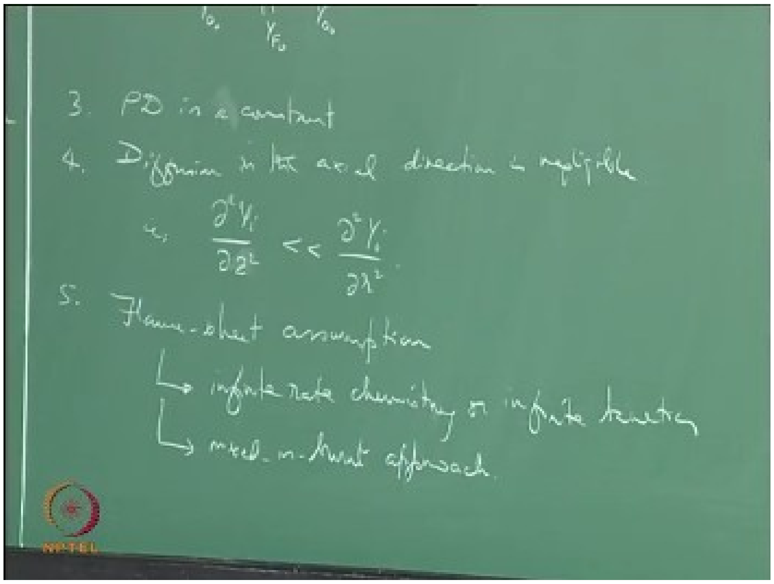

Y_{O_0} Y_{F_0} Y_{O_0}

3. ρD is a constant
4. Diffusion in the axial direction is negligible

i.e. $\frac{\partial^2 Y_i}{\partial z^2} << \frac{\partial^2 Y_i}{\partial r^2}$.

5. Flame-sheet assumption
    - ↳ infinite rate chemistry or infinite kinetics
    - ↳ mixed-is-burnt approach.

And then w start the assumptions that are involved in this the Burke Schumann problem is essentially that off two concentric ducts you could do this problem in a Cartesian coordinate system where you have like two channels right. So there is like infinite plates and then is also infinite plates, so these are essentially two quad zero channels or you can also do this in the in an axe-symmetric manner that means you are actually having two circular pipes that are concentric to each other and the original Burke Schumann problem was done as a pipe problem so it is an axe symmetric problem and the inner pipe terminates at a particular point the other pipe extends to infinity.

So the inner pipe is a semi-infinite pipe it is coming from infinity below and up to this point the outer pipe is infinite all the way right only on either side and you have a fuel coming in at a certain a certain mass fraction YF 0 inside on the only inner duct and you have oxidizer coming in the outer duct at a oxidizer mass fraction of y o 0, so these are the inlet mass fractions alright and they are also coming in at all the flow of both the flows are actually the same velocity u so it's actually uniform velocity and we are basically supposing that you have a uniform flow everywhere and at this velocity all right we are going to adopt the Schvab Zeldovivh formulation where we are going to solve one is a species equation and the energy equation perhaps we will actually find that we will also make other assumptions that requires us to solve only the species equation and not the energy equation.

And we can decouple them therefore we are not going to solve the momentum equation we are going to prescribe the velocity field and the velocity field if you will prescribe here is this uniform velocity everywhere for every any point in the flow for any mixture whether you have only reactant only product a mixture of reactant and product only product or only fuel only oxidizer among the reactants whichever way right. So everywhere you are basically going to say the same velocity exists now of course is not the this is not a good assumption okay for several reasons you could first of all think wait a minute I am looking at pipe flow should not actually have something like a parabolic velocity profile if it is laminar right and then parabolic velocity profiles for the outer flow and so on.

Then what about the nozzle boundary condition or these are these walls for this velocity profile let us not worry about all those things that is all a fluid mechanics person's problem okay, I am not going to worry about all these things I am going to worry only about the combustion problem the second thing is we are we are not going to bother about the density variation with temperature.

So the density varies with temperature then as if as there is a flame and the flow goes through the flame it expands because of thermal expansion and therefore when the density changes the velocity will change as well right and then you the velocity field will now depend on the flame but that is now like a convoluted problem I need to prescribe a flow field in order to actually work out the flame and the flame is now going to change the flow field so what do I prescribe so that is a problem that I that I will have to reckon with if I have the couple the flow and the combustion problems together right.

If I want to keep them separate I will simply have to prescribe a velocity field in spite of the flame that means I have to I should not worry about the effect of the flame on the flow field this is reasonable because up to pretty much up to the point where the flow is heating the flame it is not expanding so and it is a convective situation of all the all the flow that is going out, so all the expansion is happening downstream of the flame say it say it is an approximation all right but still it is a reasonable one okay.

So this is typically the problem the question that we are asking this problem is what is the flame shape and this is very important when compared to what is the flame structure okay in the context of premix flames I have clarified the distinction between aflame shape and a frame structure. So when we are looking at one dimensional premix flames right we were we were saying let us look at the preheat zone let us look at the reaction zone let us look at how the temperature increases and then tapers off and how does the reactant concentration falls in the preheat zone and then tapers off at a low value all these things and how does the reaction rate grow and all those things right.

Now that is looking at the structure when you now look at the structure of the flame that means you are looking at this patient variation in temperature profiles concentration profiles and reaction rate profiles and so on right, shape of the flame in the context of premix flames was something to do with like a conical flame about a Bunsen burner or a wedge flame in a in the case of a ramjet and so on. So that is like a gross thing and it is almost like we are looking at a sheet of aflame wherein you the structure really happens right and then the way we were actually handling it is by looking at the G equation and how the flame propagates with a certain flow speed and balance is the normal component of the flow those things that is what we were doing.

Here again we are so it is sufficient for us the wave Burke Schumann post this problem to work out the shape of the flame okay, and so what we have what we are expecting is the shape of the flame to look somewhat like this or somewhat like that right or somewhat like this I do not know that is something that I am going to work out so if you can imagine like a like a candle flame is something that is like a teardrop kind of shape right you could also have like a tint shape flame or if you had a fuel if you had a lot of fuel that is trying to actually mix with the oxidizer the frame could open up right many times when you keep in mind when you wave in your when you are looking at flames that are opening up it could be like a z flame.

So a z flame is where you could have like a very high velocity fuel and very low velocity oxidizer then you are mixing up in your mind the shear layer effects like momentum transport because of velocity gradients and you have a jet that is coming up entrainment and all those things okay it is all possible to think about all these things that we are talking about in that context also. But the fact that you are having a uniform velocity is to remove any effects of sheer layers from this and we are looking at only species mixing and no momentum if mixing effects no thermal mixing effects none of these things is brought into picture right.

So it is a very nicely post problem in the sense it clearly focuses on primarily two things one is that there are there are only two things that are happening here primarily one you have a axial convection of species and when I say species we are talking about reactants that are separate to begin with and then mixture of reactants somewhere in the middle and products also coming up as we go and then fully products are later on, so you see it is not as nicely done as a one-dimensional premix flame previously.

So we had our reactants hear you had a flame you had products there right there is more of a black and white thing again with very little fuzziness if you now go from here to there you have and then keep scanning across you will find all kinds of things that are happening up from the point where you had separate reactants and complete mixture products at the top right, so we have to now evolved how this flame shape is happening so one of the things that is happening regardless of whether it is reactants that are separate or products that are combined is axial convection of species and it is balanced by radial diffusion of species okay.

You also have some axial diffusion all right, I will explain this a lot more carefully some little later but the two primary things that are of most importance in a workshop in diffusion flame or axial convection and radial diffusion. So this is the balance that we are basically talking about which is How well do the species defuse while they are busy conducting right this is what is going to dictate how the flame shapes up right, so the flame shape is essentially dictated by the balance of axial convection and radial diffusion.

So once you get the flame shape the question of industrial interest for you is what is the flame height right, so for example if you now want to design a furnace or a burner or a combustor and so on and you want to light up a flame at the burner you want to know how long you want how the combustor walls right. So you want to be able to cover up until the tip of the flame at least, so that you can ensure complete combustion within the combustor so the type will be the height of the flame is a practical interest right.

So the way Burke Schumann actually try to get the flame height was to actually solve for the flame shape and once you get the flame shape we now find out where it actually stops the center line or at the walls or what is the vertical distance over which it stands and that is the height of the flame right so you can you can obtain this so this is basically the way the problem is posed and the solution procedure is to go through the shape of the flame. So the adoptee they adopt the Schvab's Zeldovich formulation, so right and the Schvab Zeldovich formulation recall comes with about 11assumptions right.

And we did this adoption even when we started analyzing the premix flame structure and we went through some additional assumptions maybe about two or three right, so here again we will have to go through additional assumptions some of which I have already stated so the first assumption so for the resolutions for the resumption number one if you now set up a coordinate system that is centered at the mouth of the fuel dock in the middle and then say call this R and call this Z then let us say the VZ component of velocity is U which is a constant right.

Given unlike in the case of MX flames where this is not given this is actually an Eigen value you need to find out what is the flow velocity at which you need you have a flame that is stabilized we do not have to worry about the flame stabilization problem in the case of in the case of the diffusion flame as it is posed by were consuming and that is very important for you to think about, we will have to see later on what the stabilization mechanism s and we will bring in some elements of premix that is coming up that the reason why we called it is not exactly entirely diffusion flame we would simply call it a non pre-mixed flame.

But so we will not worry about stabilization issues this is given and correspondingly we say vr = 0 that means you do not have any radial velocities strictly speaking this is not true if you were to allow for the density to vary with temperature you will now have the flow diverge similar to the way the we saw in the conical premix flame and that will acquire a radial velocity component but we are disregarding this right.

The second assumption is the mass fluxes are equal that is a ρ v = ρ v oxidizer right, now putting things together what this basically means is that if the velocities of the two fluids are the same right and this is the velocity field that is given their that and being so fuel and oxidizer all of them have both of them have the same velocity if you see if the velocities are the same and the mass fluxes are the same that imply me means that the densities are the same and densities being the same is essentially what it amounts to saying the density does not vary right.

So the density does not vary not only because of temperature but also because originally the densities were the same so you do not really have to worry about any change in density in the first place and how does how does that visit is it possible is it possible to have a fuel and oxidizer of the same density right the answer is in the laboratory it is possible to mix the fuel and oxidizer with different diluents since such a way that you achieve the same density for these diluted mixtures of fuel and oxidizer right.

So we say YF0 and YO0, so in this you have basically fuel with a diluents because 1- y YF 0 should be that element in the outer annular to abduct you are having 1 - y we are not I said no it is a diluents and such a way that the densities are the same right.

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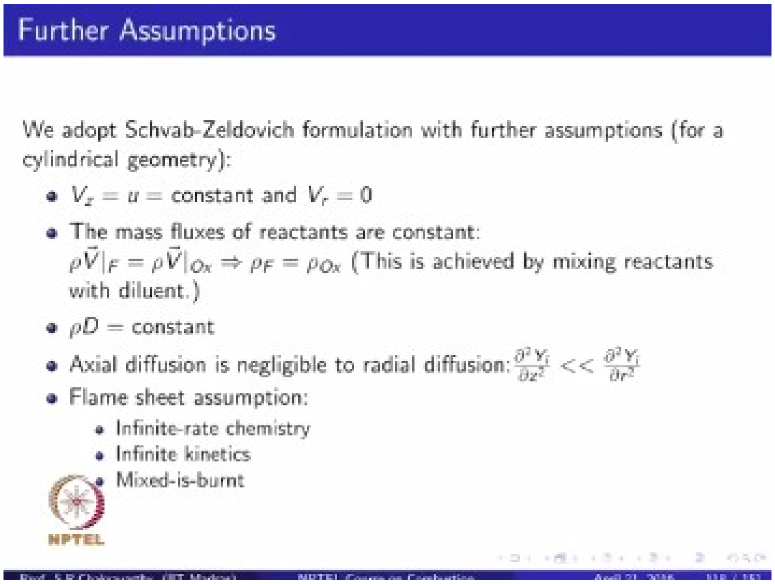

Further Assumptions

We adopt Schvab-Zeldovich formulation with further assumptions (for a cylindrical geometry):

- $V_z = u =$ constant and $V_r = 0$
- The mass fluxes of reactants are constant: $\rho\vec{V}|_F = \rho\vec{V}|_{Ox} \Rightarrow \rho_F = \rho_{Ox}$ (This is achieved by mixing reactants with diluent.)
- $\rho D =$ constant
- Axial diffusion is negligible to radial diffusion: $\frac{\partial^2 Y_i}{\partial z^2} << \frac{\partial^2 Y_i}{\partial r^2}$
- Flame sheet assumption:
  - Infinite-rate chemistry
  - Infinite kinetics
  - Mixed-is-burnt

The next assumption is certainly a simplifying assumption which is ρ D is a constant and this simplification is required for from a mathematical stand point right and of course what this means the density is constant d must be the same everywhere okay that is there is part of the subs elder which formulation anyway right. So we are not looking at variation in D across the fourth assumption is diffusion in the axial direction is negligible diffusion in the axial direction is negligible that means we are saying it is negligible relative to the radial diffusion okay.

So the pains we are saying wherever we have a ∂ Z^2 is much less when compared to partial squared Y I over partial R^2 right, now this is to basically say that we are interested only in retaining the radial diffusion the convection is at zero because we have disregarded any radial convection by this assumption right. So we are prescribing a velocity field that is only axially convective and we now permit only radial diffusion.

So that we can clearly see the balance between axial convection and radial diffusion I want to point out that this assumption is not necessary you could keep the radial diffusion and sorry we could keep the axial diffusion and show that the axial diffusion is important when you have a small peclet number or it is negligible truly when compared to the radial diffusion at four large peclet numbers and large could be for laminar flows greater than 10 is large enough to10 10 10 20 or 50 maybe is large enough right, but less than five is something that you have to keep and what I will do is I will keep talking about the consequences of retaining the axial diffusion but I would solve that problem all right and it is interestingly think in nineteen this was actually published 1928 as I said I think it is 1982 or 84 there was a combustion science and technology paper by Chunkin law that actually publishes the solution with axial diffusion retained oaky.

So the interesting thing about these things is as and when people make more and more assumptions long later there could be somebody who relaxes an assumption and then you know you now have a expanded version of this or a more general version of this solution that is possible as a matter of fact we be talking about a Burke Schumann problem erupting the Schvab Zeldovich formulation which essentially is a steady-state a problem right.

So you are looking at a steady state situation that means this flame is not going to change shape with time and very recently in about three four years ago we relax the steady state assumption and actually did an oscillatory diffusion Burke Schumann flame all right, so what I mean to say is you could relax many of these assumptions and solve the Burke Schuman problem, and these things over a period of time begin to come into your exams and homework problems and assignments and all those things right.

The fifth assumption is perhaps the most important and insightful assumption that needs to be considered what we are thinking here is we say we were the right words yes it is called the frame sheet assumption right flame sheet assumption means similar to how we adopted the G equation where we said the frame is essentially going to be a sheet it is a surface of discontinuity between reactants and products in the case of the frame explain as I said in this problem we are interested only in the shape we're not interested in the structure.

So it is sufficient for us to think about like a sheet right, therefore we want to actually make the assumption that the flame is going to be a sheet the question is what is it that we are going to do in the equations that implements this assumption right what is the consequence of this assumption in turn in terms of for analysis what does it really mean okay physically what this really means is this directly corresponds to what is called as infinite rate chemistry infinite rate chemistry or infinite kinetics infinite kinetics the other name for this is what is called as the mix dis brunch mix just burnt approach.

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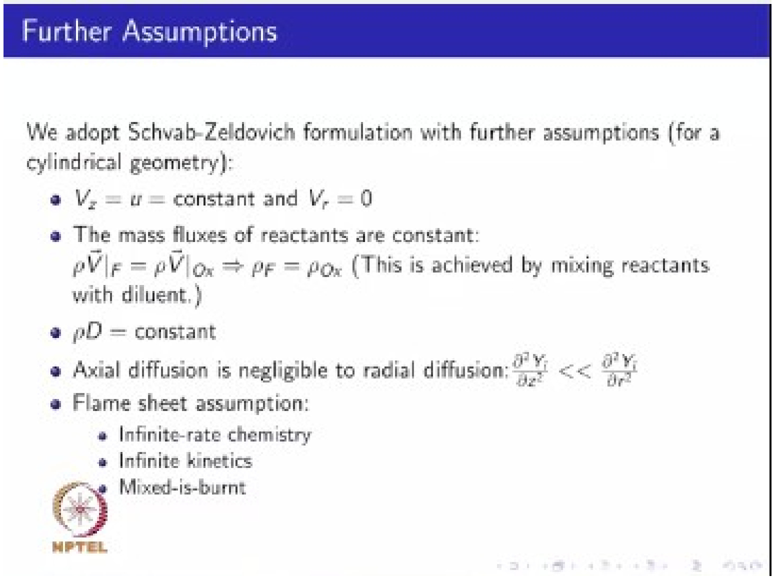

Further Assumptions

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  - Infinite-rate chemistry
  - Infinite kinetics
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So this requires some explanation what we are basically saying is and this is very important this assumption means.

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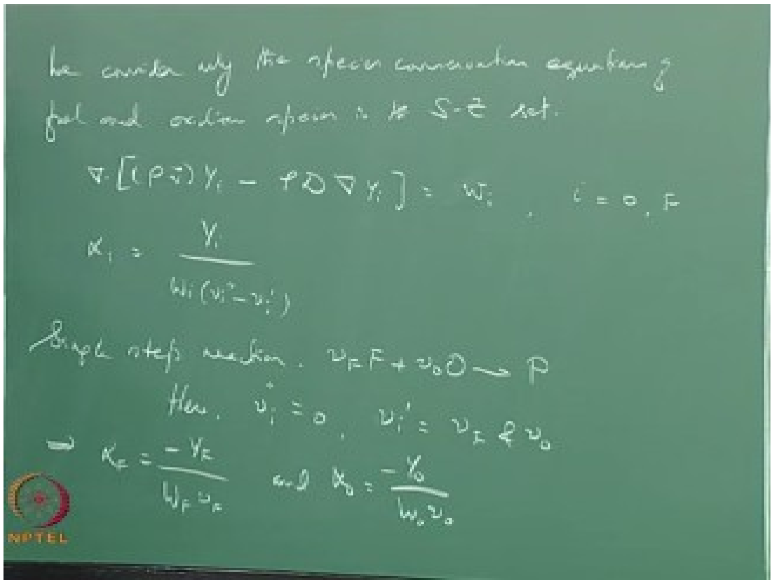

We consider only the species conservation equations of fuel and oxidizer species in the S-Z set.

$$\nabla\cdot\left[(\rho\vec{v})Y_i - \rho D\nabla Y_i\right] = w_i \quad , \quad i = O, F$$

$$\alpha_i = \frac{Y_i}{W_i(\nu_i'' - \nu_i')}$$

Single step reaction. $\nu_F F + \nu_O O \rightarrow P$

Here, $\nu_i'' = 0$, $\nu_i' = \nu_F$ & ν_O

$$\Rightarrow \alpha_F = \frac{-Y_F}{W_F\nu_F} \quad \text{and} \quad \alpha_O = \frac{-Y_O}{W_O\nu_O}$$

That the reactants react instantaneously, when the mix in stoichiometric proportions okay, and again that needs to be further explained what we mean by stoichiometric proportions is you now have the fuel and oxidizer that are mixing right. If you now think about even like a split a plate problem let us suppose you do not even have these walls right you simply had only a sped up late and this is a semi infinite domain that is a semi infinite domain and you had a plate that just stopped here and you had all fuel coming in this side all oxidizer coming in that side this is to say we are focusing on one lip here right.

Let us not worry about what is on the other side then slowly what is happening is as the fuel and oxidizer were actually flowing out they are beginning to mix right, so as they begin the mix if you are some are very far away you are not going to know that they are mixing that means it is all fuel if you are on this side very faraway you know go to know that it there they are mixing it is all oxidizer but as you keep going from here to there all oxidizer becomes all feel through a state where you have a mixture right.

That means there is going to be some particular contour where the affair then oxidized or actually finding themselves mixed in stoichiometric proportions and what we are saying in this assumption is the flame is going to be a sheet that is coincident with the stoichiometric contour in the mixing field right so let us just write that, the flame is a sheet coinciding with the structure metric contour in the mixing field we will elaborate this is a lot more after we go through the Burke Schuman solution but at the stage I think I will just give you a peek of what is going to happen let us suppose that we do not have this assumption and we have what is called as a finite feed chemistry or finite kinetics okay.

And that is what is reality mean reality if you know work out the arena is law or the law of mass action for the reaction rates you are going to get some number right then that number is going to be large alright for most combustion things but it is still funny let us say that say it is finite as the fuel and oxidizer are getting together right you are going to have no more reactions happening if you find sufficient amount of fuel and oxidizer at a place because the law of mass action requires for the reaction rates to have concentrations of both fuel and oxidizer if you went little bit further away and you found that you know you had only fuel and not much oxidizer that is mixed in you are going to have the reaction rates drop substantially.

If you go on this side and now you find that you have more of oxidizer and you didn't have hardly any fuel mixed or to all the side you are not going to have concentration of feel the reaction rates are going to drop substantially as a matter of fact if you think about the premix flame you had a certain reaction zone thickness which was restricted because you are running out of the diffusion fuel, that means the reaction constant reactant concentration comes almost too do right here the reactant concentration comes to 0 from opposite sides for the two reactants then both the reactants were actually coming from the same side of the flame here the two reactants are coming or diffusing from opposite sides of the flame.

So both reactants will have to actually go to zero concentration on both side and then that that is going to restrict the region of your reaction zone right, where is it going to be like highly reactive? So if you now think about the reaction rates they are kind of hitting 0 at these ends where you are running out of fuel on one side and running out of oxidizer on the other side and you now have a peak in the reaction rate along the stoichiometric surface right, that is where they are available in equal abundance so to speak right.

Then we now say well let us want to have a frame sheet I do not want to have thick thing right, so we want to have a frame sheet I know how to collapse this to a particular day to a sheet right why would I collapse it obviously at the stoichiometric surface. Therefore we can imagine that this is a remarkable insight that Burke Schumann had back in1928, when you know there was a too much combustion theory that was going on at the time right.

So the flame sheet obviously now can be convincingly thought of as coinciding with the stoichiometric surface in the mixing field if you now assume this we have a extremely simplified situation which is we do not have to solve the combustion problem at all it is sufficient for us to solve the mixing problem it could be as if we are doing a cold flow mixing without igniting so you just have to send the fuel in it is going to send this in the oxidizer there they mix and then they will they will mix it a stoichiometric proportion wherever they are mixing instructor metric proportion is where the flame is supposed to be right whether you have it or not okay, I just want to find that.

Therefore I do not have to actually worry about the energy equation along with the species equation in fact it is sufficient for me to actually worry only about the species equation of the fuel and oxidizer around here even have to worry about other species right, and see how these species dynamics is going to work out for fuel and oxidizer alone I do not worry about energy. So in the Schvab Zeldovich order which equation set of n + 1 equations I will just focus only on two equations which correspond to the species conservation of fuel and oxidizer alone and then work out the mixing problem right.

So this is how this is going to happen, so the Schvab Zeldovich said we just consider the species conservation equations of fuel and oxidizer species only and first of all for any species your Schvab Zeldovich formulation is divergence of ρ v y i- ρ d grad YI = ω I or I think we are we are the simple WI right and what you're going to consider here is i = 0 and f.

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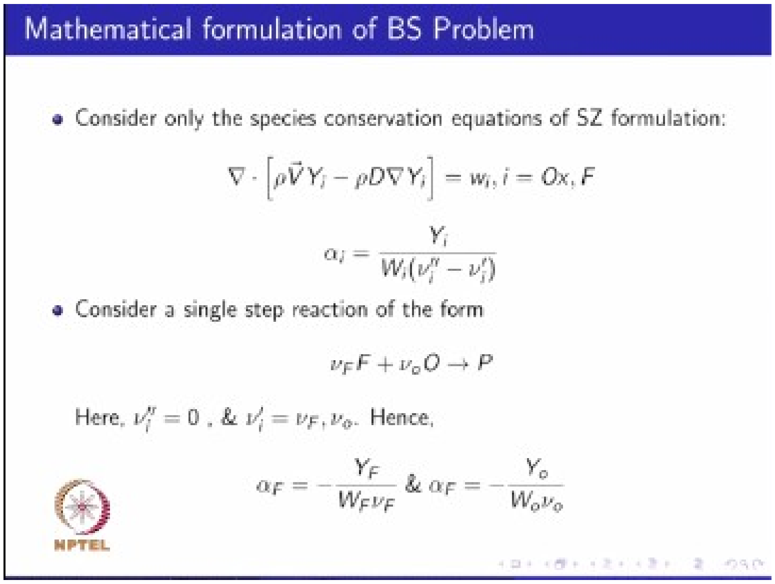

Mathematical formulation of BS Problem

- Consider only the species conservation equations of SZ formulation:

$$\nabla \cdot \left[\rho \vec{V} Y_i - \rho D \nabla Y_i\right] = w_i, i = Ox, F$$

$$\alpha_i = \frac{Y_i}{W_i(\nu_i'' - \nu_i')}$$

- Consider a single step reaction of the form

$$\nu_F F + \nu_o O \rightarrow P$$

Here, $\nu_i'' = 0$, & $\nu_i' = \nu_F, \nu_o$. Hence,

$$\alpha_F = -\frac{Y_F}{W_F \nu_F} \; \& \; \alpha_F = -\frac{Y_o}{W_o \nu_o}$$

This is a step before we form the α and the β if you go back to the Schvab Zeldovich formulation right, so for the sake of refreshing your minds this is the species equation the Schvab Zeldovich species equation okay, and then we form that α such that you will now divide these by something and get only ω here and that that is common for all the right-hand sides and we would also do a particular alpha t for the energy equation should not go to consider here, so that you will still get a ω there and then you can start subtracting one from the other two and by forming β which are like α - α one or something of that sort.

We will try to do something like that here right now but at the moment we are going to consider one day i = o and f you know we do not worry about anything else at the moment, so we know from the α so let us say we have a α I which is y I / WI you I single prime now of course the Schvab Zeldovich formulation is for a single step reaction, so single step reaction we are looking at is yff + μ oo gives products we are interested only along this only the only the stoichiometric surface and the stoichiometric surface this is the stoichiometric reaction right.

So μ F and μ o or stock a metric co efficient of F and oh okay, such that you get only products you do not get anything more right, and therefore in this case if you know compare this with the template reaction that we have which is like μi’ σ = I to N, μi’ = σ I = 1 to N, μi’’ = and that is why you will get a μi’’ – μi’ for the α definition here μi = μi’’ or = 0 and the μi’ are essentially μf and μo okay.

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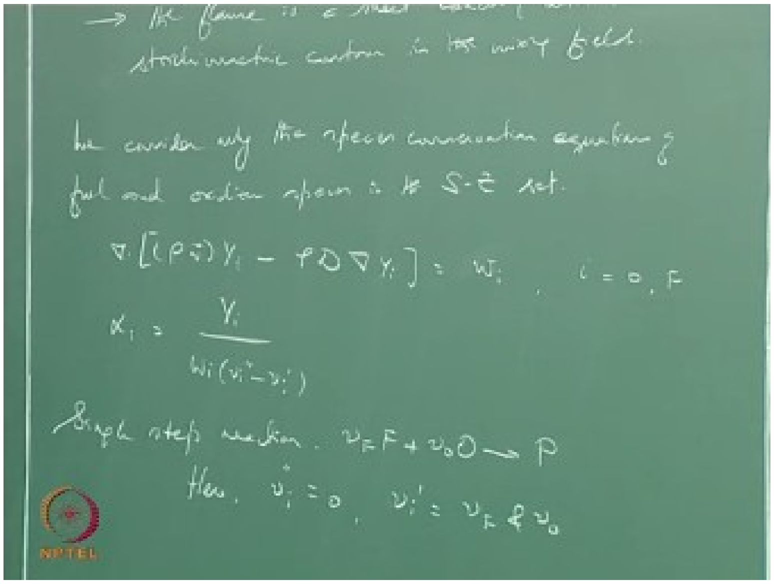

→ the flame is a sheet [illegible]
stoichiometric contour in the mixing field.

We consider only the species conservation equations of fuel and oxidizer species in the S-Z set.

$$\nabla\cdot\left[(\rho\vec{v})Y_i - \rho D\nabla Y_i\right] = w_i \quad , \quad i = O, F$$

$$\alpha_i = \frac{Y_i}{W_i(\nu_i'' - \nu_i')}$$

Single step reaction. $\nu_F F + \nu_O O \rightarrow P$

Here, $\nu_i'' = 0$, $\nu_i' = \nu_F$ & ν_O

Which means α f = - yf / wf μf and αo = -yo / wo μo all right.

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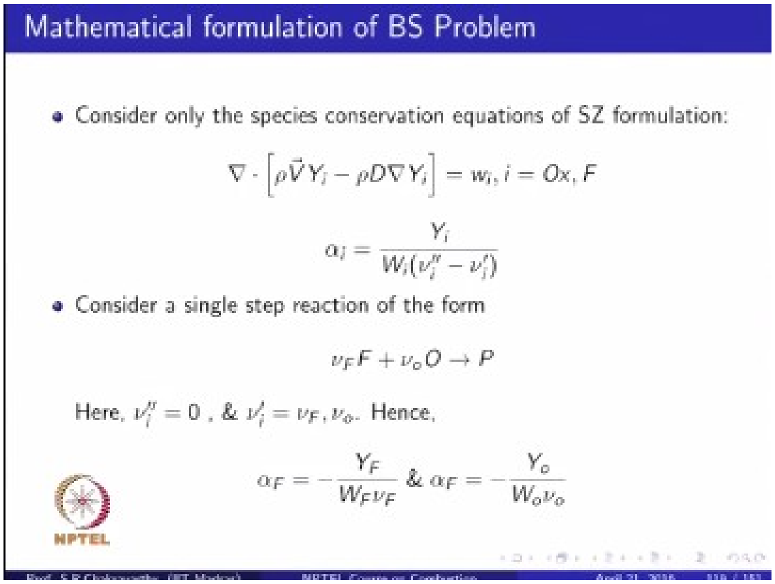

Mathematical formulation of BS Problem

- Consider only the species conservation equations of SZ formulation:

$$\nabla \cdot \left[\rho \vec{V} Y_i - \rho D \nabla Y_i\right] = w_i, i = Ox, F$$

$$\alpha_i = \frac{Y_i}{W_i(\nu_i'' - \nu_i')}$$

- Consider a single step reaction of the form

$$\nu_F F + \nu_o O \rightarrow P$$

Here, $\nu_i'' = 0$, & $\nu_i' = \nu_F, \nu_o$. Hence,

$$\alpha_F = -\frac{Y_F}{W_F \nu_F} \; \& \; \alpha_F = -\frac{Y_o}{W_o \nu_o}$$

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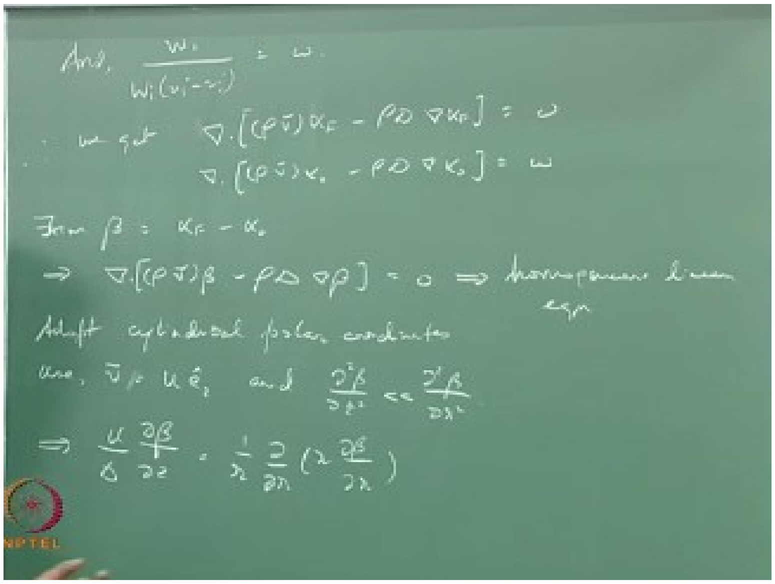

And, $\frac{w_i}{W_i(\nu_i''-\nu_i')} = \omega$

∴ we get $\nabla\cdot[(\rho\vec{v})\alpha_F - \rho D\nabla\alpha_F] = \omega$

$\nabla\cdot[(\rho\vec{v})\alpha_O - \rho D\nabla\alpha_O] = \omega$

Form $\beta = \alpha_F - \alpha_O$

$\Rightarrow \nabla\cdot[(\rho\vec{v})\beta - \rho D\nabla\beta] = 0 \Rightarrow$ homogeneous linear eqn

Adopt cylindrical polar coordinates

Here, $\vec{v} = u\hat{e}_z$ and $\frac{\partial^2\beta}{\partial z^2} \ll \frac{\partial^2\beta}{\partial r^2}$

$\Rightarrow \frac{u}{D}\frac{\partial\beta}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\beta}{\partial r}\right)$

And W I / w capital W I µi’’ - µi’ = ω right, therefore we get ∇ ρ v YF sorry you can begin we can directly write α f - ρd ∇ f = ω and ∇ ρ V vector as for ρ - ρd gradient α 0 = ω. So now form β, this is the last step in the Schvab Zeldovich formulation in this case we now say that β = α F - α 0 if you do this then we get ∇ ρ v /β – ρ d ∇ β = 0.

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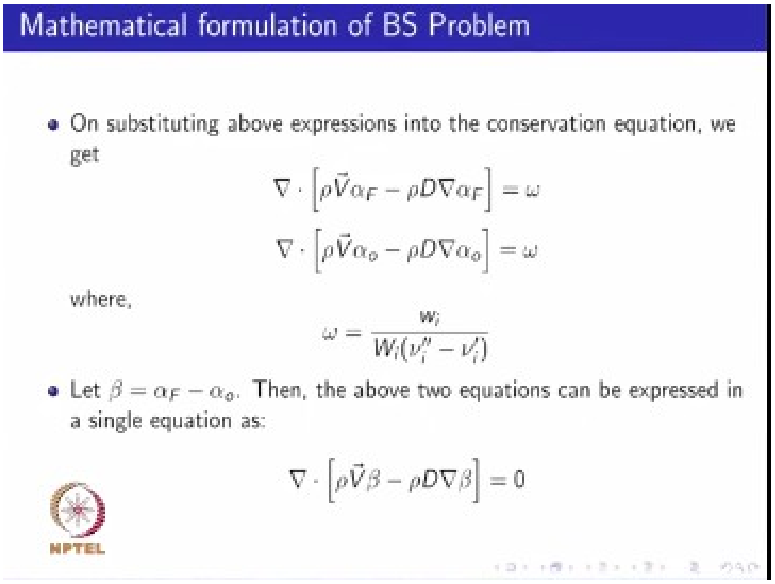

Mathematical formulation of BS Problem

- On substituting above expressions into the conservation equation, we get

$$\nabla \cdot \left[\rho \vec{V} \alpha_F - \rho D \nabla \alpha_F\right] = \omega$$

$$\nabla \cdot \left[\rho \vec{V} \alpha_o - \rho D \nabla \alpha_o\right] = \omega$$

where,

$$\omega = \frac{w_i}{W_i(\nu_i'' - \nu_i')}$$

- Let $\beta = \alpha_F - \alpha_o$. Then, the above two equations can be expressed in a single equation as:

$$\nabla \cdot \left[\rho \vec{V} \beta - \rho D \nabla \beta\right] = 0$$

This is amazing because this is a combustion problem where we do not have to solve we do not have to consider chemical reactions okay, and look at what we have been doing so far the last time we solve the Schvab Zeldovich formulation for a premix we looked at the preheat zone separately from the reaction zone and in the preheat zone we had a convective diffusive balance which means we had to consider convection and diffusion only and the reaction zone we had a reactive diffusive balance which means we had to consider only the diffusion and reaction only.

The full combustion problem in general always has a minimum of convection diffusion and reaction these are the three elements that constitute combustion primarily right we escaped trying to solve all three of them together at the time by dividing it into two parts where we consider two of them to get at a time and here on the whole we are evading the consideration of reaction rates completely and we are looking at it is only a convective diffusive balance.

Except here we are going to specifically say it is a axial convection versus a radial diffusion right, so it is a two dimensional problem rather and therefore that allows us to actually make these distinctions between axial convection and radial diffusion but there is a one-dimensional problem. So we did not have that liberty okay, so this is the this is a hallmark of classical combustion solutions we try to avoid situations where we have to deal with all three of them together that is what is called as a mixed problem okay.

So we try to avoid these mixed problems you always consider two processes at a time balancing each other wherever that is possible, and you now get in this context a homogeneous linear equation right and we now adopt cylindrical polar coordinates so far we have not done that in the governing equations adopt cylindrical polar coordinates use V vector is equal to you e cap and $\partial^2 \beta / \partial z^2$ much less when compared to those squared $\beta \partial r^2$ that is going to happen here in the second term right.

So with all these assumptions you are simply going to have $u / d \partial \beta / \partial z = 1 / r \partial$ derivative with respect to r of $r \partial \beta r / \partial r$ this is the odd component of the Laplacian cylindrical polar coordinates and so it is a bit complicated there because of the cylindrical polar coordinate, now for the first time you need to worry about boundary conditions we got the simplified governing equation which is this and let us now look at the boundary conditions okay.

The boundary condition that we require is in β okay, so we need boundary condition for β β is now our single variable and it is a single equation that is governing it and we need boundary condition for β in r and z okay and since its governing govern to second order and ZR you need two boundary conditions and since it is going to first order insert you need to bond you need only one boundary condition insult right.

So if you now look at the boundary conditions that are supposed to be in our for the two boundary conditions nor we have to look at what is the domain the domain for us is going from $r = 0$ to R = let us say you want to now distinguish between the inner duct radius, so let us call this a and the outer duct radius it is V, so what domain is going from $R = 0$ to $R = B$, so we have to supply a boundary condition at $r = 0$ right and that is the symmetry boundary condition.

So b bc at for all z right at $r = 0$ $\partial \beta r / \partial r = 0$ this is the symmetry boundary condition all right, now what is partial what is the first derivative of β actually implies physically if you now go back β is nothing but $\alpha F - \alpha 0$ and what is the α? α is why I have divided by WF new F and swim similarly α o, so this ultimately goes back to taking partial derivative of R of Y F and Y of Yo with respect to r right, so partial derivatives whoa with respect to our right.

So partial derivatives of mass fractions among to diffusion fluxes right, so what we are saying by symmetry that ∂ β or ∂ R should be equal to zero means that you do not have diffusion across the center line because of symmetry right. So whatever is happening here should actually have be the same thing that happens there and if you now have a diffusion over here that means that also means that you allow for diffusion from that side to this side that indicates in accumulation which is not possible right.

So you need to have that is what symmetry boundary condition is all about it is physical okay and that R = B you have a wall and when you have a wall you cannot have species diffuse species is not going to convert to the across because you have only u that is like kind of like the no-slip book I should say no slip this is like a in viscid flow that is slipping past the wall but it does not penetrate the wall by convection but it cannot defuse this is a diffusion problem therefore we still have to see that ∂ β or ∂ r = 0 this is wall basic okay no diffusion across the wall.

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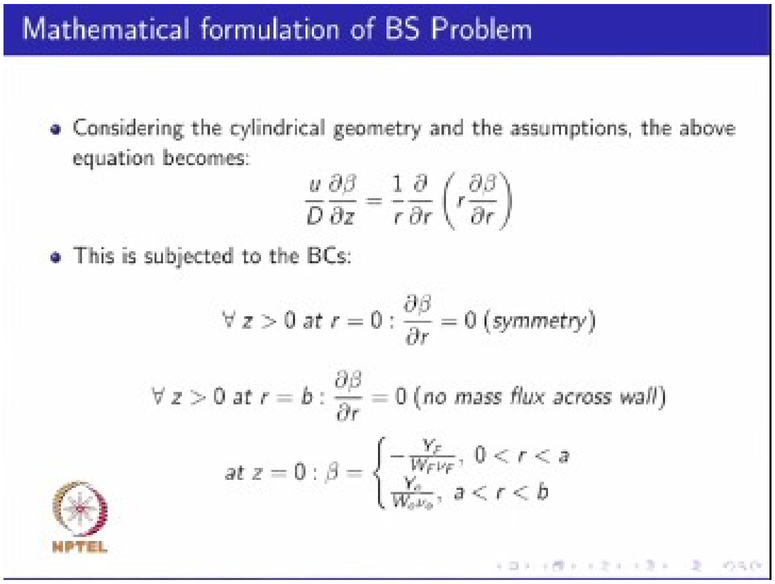

Mathematical formulation of BS Problem

- Considering the cylindrical geometry and the assumptions, the above equation becomes:

$$\frac{u}{D}\frac{\partial \beta}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \beta}{\partial r}\right)$$

- This is subjected to the BCs:

$$\forall\, z > 0 \text{ at } r = 0 : \frac{\partial \beta}{\partial r} = 0\,(symmetry)$$

$$\forall\, z > 0 \text{ at } r = b : \frac{\partial \beta}{\partial r} = 0\,(no\ mass\ flux\ across\ wall)$$

$$\text{at } z = 0 : \beta = \begin{cases} -\frac{Y_F}{W_F \nu_F}, & 0 < r < a \\ \frac{Y_o}{W_o \nu_o}, & a < r < b \end{cases}$$

Now what we have seen here is we have two Neumann boundary conditions right, we to count on the third boundary where we need to have a boundary condition which is z = 0 to provide something directly, so that we will be able to pick the value of the problem if you are not when we see here as well then you your solution will not be unique right now to get a unique solution we should look for some boundary condition on at the burner look like your H z = 0 which we will consider tomorrow.

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