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COMBUSTION

Lecture 38

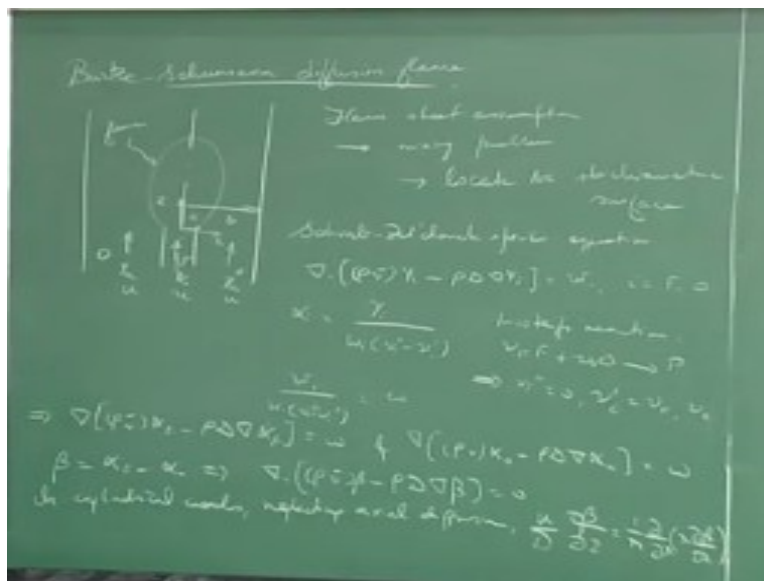
Burke- Schumann Problem

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So these stopped here, let me just go over just of what we have done so far essentially we have make it making a flame sheet assumption in this problem of co flowing fuel and oxidizer at the same velocity and so the flame sheet assumption basically is a infinite trait chemistry assumption or infinite kinetics and that basically boils down to the mixing problem.

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And we adopt this notion called the mixed is burnt approach as far as this solution is concerned, so if you now solve only the mixing problem all we need to do is to locate the stoichiometric surface in order to get the flame shape, so we just take the fuel and oxidizer species

conservation equations in the Schvab Zelovich formulation and form the α corresponding to them for a one-step chemistry of $\eta F + \eta O$ gives p and then of course you also manipulate the right-hand side to look the same in these two equations and for α and then you now have the Schvab Zelovich coupling function β that is formed as $\alpha F - \alpha O$ and then you get a homogeneous linear equation for beta and if you now unwrap this vector equation sorry vector calculus equation in terms of vector in this equation in terms are called vector identities in terms of cylindrical polar coordinates and then we also have the fourth assumption that we listed which is to neglect axial diffusion in preference to radial diffusion.

Then we are left with these two terms so these two terms basically signify the balance between axial convection and radial diffusion, so essentially the governing equation boils down to this particular balance that we have been discussing that dictates the flame shape we will talk about pretty soon we will talk about what is the consequence of retaining axial diffusion as well but just proceed this equation now is first-order in z rather than second order and that means it requires one boundary condition in Z and two boundary conditions in r and more moreover sciences the boundary conditions that they can supply or at least two at most two one order less than the leading order.

So the reading order here is second order or that means we can supply boundary conditions in the value of β or its derivative its first derivative and the first derivative in β signifies diffusion mass flux because if you now go back and see β is $\alpha F - \alpha O$ α are basically why we use with the normalization therefore a derivative in the first derivative and β will essentially boil down to indicating a first derivative in Y which essentially means diffusion mass flux in terms of the Fick's law to which we have reduced to which formulation and since the duct boundary condition is only to first order we can supply boundary conditions only in the value we can supply boundary conditions in the derivative this is going to be pretty important for us to think about when we start looking at retaining axial diffusion sometime later.

But at this stage we are recognizing that we need to have two boundary conditions either in the value or derivative for beta in our a tool to boundaries of r which are or equal to 0 and $r = B$ or $r = 0$ is the center line $r = B$ is the outer duct radius and in both the cases it turns out of course this

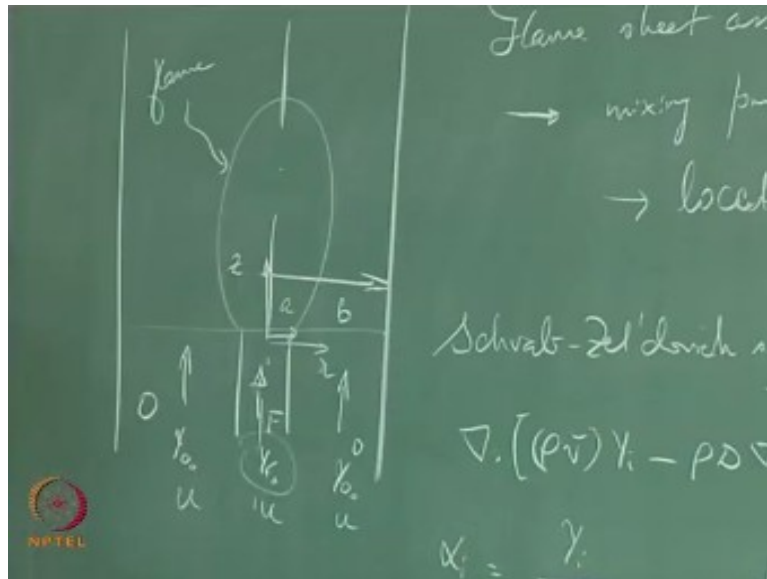
is a Norman boundary condition required for symmetry and this is a no mass flux penetrating the wall because the wall is say rigid wall its owner it is a non-porous wall and you're not looking at any penetration of mass by diffusion there therefore these are the boundary conditions and we can look at boundary conditions the first derivative.

Now if you now have these two boundaries subjected to Norman boundary conditions we have to make sure that the other boundary that we are looking at which is $z = \text{zero}$ for the z boundary is provided with the district condition and that is what is actually required you cannot provide a Norman boundary condition but when you now think about a axial diffusion taking an account you will also have a second-order insert which will admit two boundary conditions up to first order in beta but at that stage we cannot give normal boundary conditions everywhere, then you will have a non unique solution therefore we will think about the kind of boundary conditions admitted in that case next.

But at this stage we can only provide duration a boundary conditions for this and in β so need $1bc$ in z at $z = 0$, so at $z = 0$ which is say directly data which should be in the form of digital data, so $z = 0$ we have to now say what should be the value of beta right, so we now have to go back and look at how to form our β is $\alpha F - \alpha o$ right, and what we find is if you are now looking at $z = 0$ that is at the that the same plane as the lip of the inner duct where the fuel is coming in and beginning to mix with the oxidizer from the outer duct.

So and we have to now look at how this the β should span from 0 to b across $R = a$, so what we find is between $0 R = 0$ and $r = a$ you have only fuel at a mass fraction of YF_0 not and you do not have any oxidizer therefore your y_0 is 0 and you are therefore α_0 is 0.

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So if $\alpha = 0$ you only have αF on the left with αF , so β would be $Y F / w_f \eta F$ right, with a negative sign because you have a η in the numerator and η in the denominator. $\eta' - \eta'$ is 0 so you have a negative sign over here therefore we are going to get a negative sign there this is for $0 < r < a$.

Now if you now go back to what happens between $r = a$ to $R = B$ you have all oxidizer that means you have a Y_{O_2} over here but you have $Y F$ is 0 right, so if you now go back and see your $\alpha = 0$ with the $\alpha = 0$ and your αF would be 0 all right so if you now plug $\alpha F = 0$ and $\alpha = \alpha_0$ and α_0 is nothing but Y_{O_2} / w_{O_2} / negative sign and then you have a negative sign here as well right, so therefore you will get a $+ Y_{O_2} / w_{O_2}$ this is for $a < r < B$.

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Mathematical formulation of BS Problem

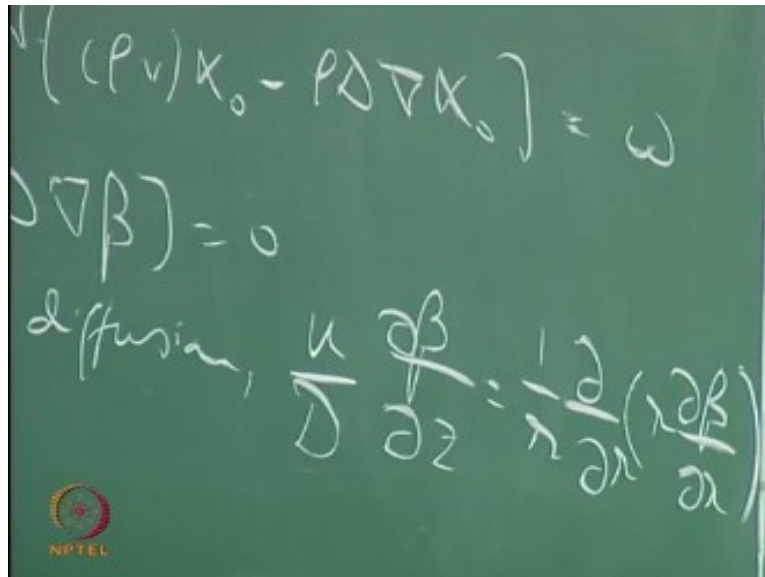
- Considering the cylindrical geometry and the assumptions, the above equation becomes:
$$\frac{u}{D} \frac{\partial \beta}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \beta}{\partial r} \right)$$
- This is subjected to the BCs:
$$\forall z > 0 \text{ at } r = 0 : \frac{\partial \beta}{\partial r} = 0 \text{ (symmetry)}$$
$$\forall z > 0 \text{ at } r = b : \frac{\partial \beta}{\partial r} = 0 \text{ (no mass flux across wall)}$$
$$\text{at } z = 0 : \beta = \begin{cases} -\frac{Y_F}{W_{F\nu_F}}, & 0 < r < a \\ \frac{Y_o}{W_{o\nu_o}}, & a < r < b \end{cases}$$



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So what this really means is you have now a jump in the value of β at or equal to a all right it is going to be continuous but that is exactly what the mixing problem is all about you have all fuels on one side you have all oxidizer on the other side and they are now going to make intermix right so and the mixing is essentially by diffusion and diffusion is essentially a transport process and like any other transport phenomena the job of diffusion is to mix things and even out discontinuities right. So it is okay to admit discontinuities at the boundary.

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$$\rho[(P_v)K_0 - \rho D \nabla^2 K_0] = \omega$$
$$\rho D \nabla^2 \beta = 0$$

diffusion, $\frac{K}{D} \frac{\partial \beta}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial \beta}{\partial r})$

So what is going to happen is you have all fuel all oxidizers, so the β is discontinuous at the boundary but as you now get into the domain the job of this governing equation particularly this term is to even out or smooth them out this discontinuity all right. So that is all right, so considering that we have these boundary conditions the next step that we want to do is to non-dimensionalize, non-dimensionalize equation.

Now many times as a student you feel that this is this kind of a burden like why would I want to do that can I can I just go ahead and solve this right away the answer is sometimes non dimensional no dimensionalization is done for elegance, so your governing equation will look very nice as if it has been to a hairdresser the other times most important technical reason from an engineering point of view is you can now actually come up with non dimensional parameters and the non-dimensional parameters are essentially a group of quantities, so which are now grouped together in such a way that it is okay to vary one of them and not necessarily the other or you can vary them in combinations so that the non dimensional number is remaining constant and so on.

So you can begin to see how the different parameters that are in the actual problem can group together in influencing the solution and the solution that you obtain could be just obtained for one value of the parameter as a whole non-dimensional parameters or you do not have to re do this for many different parameters which are non dimensional which are not non-dimensionalized. So it is pretty good hereto basically say that let us now say ϕ / B , now because you have a characteristic length scale in the or direction you can non-dimensionalize by that fortunately in this problem since you have neglected axial diffusion right a neglected axial diffusion it is a infinitely long duct.

So pretty much do not have a good length scale corresponding to the radial the axial direction it is almost infinite right, so what you are basically looking for is what is the diffusion length scale because that is a process that we are basically looking at in fact what we should be looking at is what is a competition between diffusion and convection or how fast are we able to diffuse as we are converting okay, it is kind of like a race between the two the question is how much could have diffused out how this way while I am converting at a particular rate that is now going to give me a certain length scale associated with this right.

So that is going to be made by saying if I now have my $\epsilon = Z$ let us say $Z / u / B^2 / D$ right, so z / u is essentially a convective length scale and b^2 / D keep in mind d has units of like meter squared per second right, so B^2 as meters meter squared therefore this has a units of time alright and so z/u also has units of time and therefore ϵ becomes non dimensional but in this the only variable is zone okay.

So the farther you go and as you convert you know how you are the more time you are taking but in this time you are also defusing this way that is because b is the length scale corresponding to diffusion because the diffusion is predominantly radial right so if you want to put these two together then this is a the $z / u b^2$ as a matter of fact you could begin to think about this s if I were to say I do not know any of this let us say I know I am not interested in thinking about the actual problem the professor asked me to non-dimensionalize ICB as my parameter ready the length scale here and that fitted and fairly well with a I canter you do we will do the same thing with ϵ I

would like to think of this is just z/d then you have all the stuff sticking out which is essentially d/ub .

Now UB/D essentially is the counterpart of your Reynolds number that you would use in momentum mixing but here this is species mixing so you would actually have to use the Péclet mass transfer number which is the corresponding part of this so this is $1/Pe$ where $Pe = UB/D$ right. So from here we can begin to think if Péclet is kind of small all right, then ϵ becomes proportionately larger okay, so Péclet is small is basically meaning that you have less convection relative to diffusion so you would expect that you have more diffusion happening this way but less convection happening that way so you should actually be able to get your flame to be shorter so that is what you should expect for smaller Péclet number.

Because you are not going too far along the convective path rather you are diffusing, so your flame should all be confined to closer to the burner because you have diffused a lot therefore a smaller Péclet number will correspond to a shorter flame effectively and we are trying to cover the shorter flame by artificially blowing up the ϵ having $1/Pe$ there is actually. So effectively this takes into account that it effect. But your governing equations do not take into account axial diffusion, so that it is something that you have to keep in mind that is what should really make the flame shorter strictly speaking how the governing equation turns out to be.

Further you can form some more non-dimensional parameters let $C_b = a/B$ and I will make a big deal about this and the next one which is $\eta = \alpha_0 / \alpha_{f0}$ okay α_0 / α_{f0} if you want to now go back and see what those are this is nothing but α_{f0} would be $Y_{F0} / W_I \eta$ with the negative sign α_{o0} will be $Y_{F0} Y_{o0} / W_o$ with the negative sign so the negative sign cancel each other all you are going be dealing with this $Y_{o0} / w_{\eta o} / y_{f0} / w_{\eta f}$ alright.

And we will come back to this and this quite soon we have non dimensionalized or we have non-dimensionalize Z and then we have a couple of now new non-dimensional parameters in the problem but we are not non dimensionalize β okay, so let $\gamma_b = \beta / \alpha_{f0}$ that is to say if you know where to basically go back and say as we just have to pick one of those two right we just pick α_{f0} , so β / α_{f0} would be $\alpha_F / \alpha_{F0} - \alpha_o / \alpha_{o0}$ is what we are going to so α_{f0} okay.

So you just have a αf_0 in the denominator just to go with this you can also see that this goes with this here we have divided αf_0 in the denominator therefore we are using this, so that the way the new is defined is how that way gamma is also defined okay. Now the interesting thing if you now do all these things the governing equation then becomes then becomes $\partial \gamma / \partial \epsilon = 1 / \phi$
 $\partial / \partial \phi$ of ϕ of $\partial \gamma / \partial \phi$.

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The governing eqn. then becomes

$$\frac{\partial \gamma}{\partial \eta} = \frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial \gamma}{\partial \xi} \right)$$

The non-dim. BC's are

At $\xi = 0$, $\frac{\partial \gamma}{\partial \xi} = 0$ & at $\xi = 1$, we have $\frac{\partial \gamma}{\partial \xi} = 0, \eta > 0$

Ans. $\gamma = \begin{cases} 1, & 0 < \xi < c \\ -\psi, & c < \xi < 1 \end{cases}$

So essentially we got a μ / d completely alright, so that is because that information is actually buried in eater we have scaled ϵ corresponding to the competition between axial convection and radial diffusion that is what this signifies right, so we now have a very nice-looking equation without any parameters. So this solution is now going to give you, so this equation is now going to give you a solution without any parameters except the boundary conditions are now going to carry some parameters. So what are they?

The boundary conditions are the non dimensional boundary conditions are so now we have to write in terms of γ so $\gamma =$ well let us first write the Nyman besiege for ϕ that is very easy at $\phi = 0$ we have $\partial \gamma / \partial \phi = 0$ and at $\phi = 1$ we have $\partial \gamma / \partial \phi = 0$ for all $\epsilon > 0$ right in fact we should go

back and said write here for all $z > 0$ that is true for the entire domain for the drought then, so this is the boundary condition for ϕ in so for γ in ϕ but what happens to γ in ϵ right, so $n \gamma$ we now have to translate this boundary condition in non-dimensional form that simply turns out to be 1 recall we can't this is basically this is basically αF_0 okay and then we just form a $\gamma / \alpha f_0$, so we need to get a or do we do we have a negative sign with the negative sign is $\alpha f_0 t$ okay.

So with the negative sign is αf_0 and we just divided by αf_0 , therefore you simply get a 1 and similarly if you now go back and plug in here this is actually $-\alpha_0$ okay with me the positive sign this is $-\alpha_0$ and then for trying to find the $\gamma u / \alpha F_0$ so $-\alpha_0 / \alpha f_0$ is nothing but $-\eta$ alpha F naught is nothing but minus η okay, but where are they we can now write this is actually a $0 < \phi < c$, $c < \phi < 1$ so even though your governing equation did not have any parameters there are a couple of parameters that have crept into the problem in terms of new and see through the boundary conditions all right and this is going to be very, very important in fact this is actually only one boundary that is set at, at η takes as 0 right.

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Non-Dimensionalisation

Using geometry and flow properties, we will non-dimensionalize the β equation:

- Let $\xi = \frac{r}{b}$, $\eta = \frac{z/u}{b^2/D} = \frac{z}{b} \frac{D}{ub} = \frac{z}{b} \frac{1}{Pe}$. Further, $c = \frac{a}{b}$,
 $\nu = \frac{\alpha_{F_0}}{\alpha_{O_0}} = \frac{Y_{O_0}/W_{O_0}\nu_0}{Y_{F_0}/W_{F_0}\nu_F}$ and $\gamma = \frac{\beta}{\alpha_{F_0}}$
- With the above expressions, the β equation becomes:


$$\frac{\partial \gamma}{\partial \eta} = \frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial \gamma}{\partial \xi} \right)$$

Also, the BCs become:

$$\forall \eta > 0 \text{ at } \xi = 0 : \frac{\partial \gamma}{\partial \xi} = 0$$

$$\forall \eta > 0 \text{ at } \xi = 1 : \frac{\partial \gamma}{\partial \xi} = 0$$

$$\text{at } \eta = 0 : \gamma = \begin{cases} 1, & 0 < \xi < c \\ -\nu, & c < \xi < 1 \end{cases}$$



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Basically that is the boundary that is way that is describing the problem for you if you did not have fuel and oxidizer initially unmixed entering the domain at η equal to zero you do not have a

problem okay so this essentially tells you that the fuel and oxidizer are in as they enter the domain or unmixed and that is what the problem is so all the parameters and the problem are now reduced to just these two okay.

So with this I am not going to derive the solution for you and I am going to assume that you know how to how to derive this okay and then the way to do this is you seek a product solution that means you now say γ is a function of γ is now a product of two functions one function which is a function only of η try another function that is a function only of η so you now have let us say seek, seek a product solution of the form.

Let us suppose γ equal to some sum function χ that is only a function of ξ and another function let us say I do not know what is a good symbol well could you just go back with Anglo-Saxon variable so it can simply say capital X ξ and capital Y of η alright so if you did this plug it back on back in here go through the product solution approach and substitute these boundary conditions noting that you are expecting to have a, a periodic solution in sight because you have boundary conditions there.

This is an inhomogeneous boundary condition here so you are not expecting periodic solution in η and so on you do all those things and you can now find the solution to be the solution is γ equal to $1 + \mu C^2 - \mu$ + twice of $1 + \mu$ times $C \sum_{n=1}^{\infty}$ that means it takes integer values 1 over $\emptyset n j1$ of $C n / G_0$ of $\emptyset n^2$ times $J_0 \pi n \xi e^{-\pi n^2 \xi}$.

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Non-Dimensionalisation

- Then the final solution using separation of variables is:

$$\gamma = (1+\nu)c^2 - \nu + 2(1+\nu)c \sum_{n=1}^{\infty} \frac{1}{\phi_n J_0(\phi_n)^2} J_1(c\phi_n) J_0(\phi_n \xi) \times \exp(-\phi_n^2 \eta)$$

Here, ϕ_n is the n^{th} zero of $J_1(\phi)$

- The flame surface is represented by $\gamma = 0$, because it gives $\frac{Y_f}{W_F \nu_F} = \frac{Y_O}{W_O \nu_O}$ which is the stoichiometric surface.



I do not even understand this right well let us, let us just go step by step I mean of course we are not as intelligent as we would like us ourselves to be so all these things are something that we have seen okay these are part of the parameters of the problem the first time we encounter a problem now is what is ϕ_n so ϕ_n is what is called as the n to 0 of J_1 of π where J_1 is, is a Bessel function right.

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The boundary BCs are

At $z=0$, $\frac{\partial r}{\partial z} = 0$ & at $z=1$, we have $\frac{\partial r}{\partial z} = 0, \eta > 0$

And, $r = \begin{cases} 1, & 0 < z < c \\ -v, & c < z < 1 \end{cases}$ at $\eta = 0$

Seek a product soln of the form $r = X(z)Y(\eta)$

The soln is,

$$r = (1+v)c^2 - v + z(1+v)c \sum_{n=1}^{\infty} \frac{J_1(c\phi_n)}{\phi_n [J_0(\phi_n)]^2} J_0(\phi_n \frac{z}{c}) e^{-\phi_n^2 \eta}$$

$\phi_n \Rightarrow n^{th}$ zero of $J_1(\phi)$ (Remains for 1st order)

So that is a boundary function of first order so obviously then J_0 is the Bessel's function of 0th order right now what is basis functions Bessel functions are those that look like sines and cosines in fact J_0 and J_1 would look like sines and cosines except that they do not have a constant amplitude so you will now find that they kind of keep decaying but keep alternating about the horizontal axis right so that is the kind of solution.

That you are going to get and you now have a \sum over a series of those so ϕ_n , sorry, ϕ_n being the n to 0, ϕ_n essentially means that this function keeps on alternating up and down about the horizontal axis at specific values of ϕ the argument right so when you say zeros those are the values so where are where all do you have the J is going to 0 is essentially where you get these values of ϕ okay.

So it is kind of like in sin and cos sign so if you take a sine wave you know that the sin wave passes 0 at 0 / 0 $\phi_2, \phi_3 \phi_4, \phi$ and so on so, those are the zeros of this design function so similarly we will have these what will happen is you will find these are not actually equally spaced they have a certain pattern all right so that is how the Bessel function behaves and

typically we get into a business function mainly because you are having this kind of a derivative for cylindrical polar coordinates.

And that is mainly because we using axis symmetric pipes if instead if you now had like vertical plates like channels right you will see you will not have a complicated looking derivative for the Laplacian you will simply have a partial square β divided by partial square γ divided by partial square \emptyset partial try squared and you will get sines and cosines and typically for these kinds of boundary conditions.

You should get cosine a cosine function here and that would actually imply that we are fitting strictly speaking what it really means is we are fitting this particular discontinuity by a Fourier expansion and then letting the Fourier expansion DK okay so you essentially you now have a step that is represented by a Fourier series and then it's subjected to boundary conditions of Norman BCS on either side.

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$$v) c \sum_{n=1}^{\infty} \frac{1}{\phi_n} \frac{J_1(c\phi_n)}{[J_0(\phi_n)]^2} J_0(\phi_n \xi) e^{-\phi_n^2 \eta}$$

(Remains for 1st order)

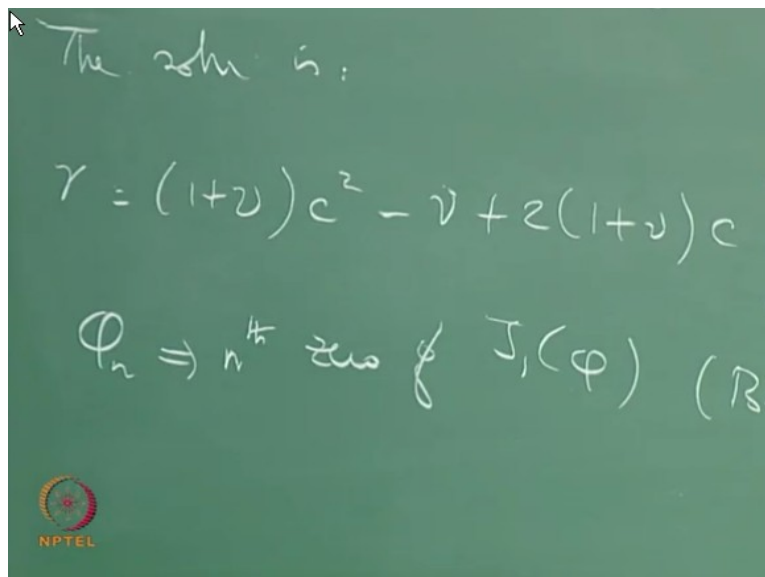
And then as you keep on mixing or nor this, this particular function now changes and essentially the, the Fourier representation keeps changing as you as you go along until you get a more and

more uniform variation in the mixed refraction in the key instead of a Fourier series you now have a series that is based on Bessel functions and that is possible anytime you have you are looking at functions that are orthogonal to each other like in terms of sine's and cosines you will have orthonality investors function also you have orthonality so based on these you can actually form a series that look like Fourier series.

But not necessarily in sines and cosines but some other orthogonal basis functions that is what we have done here okay or that is how it comes out to be you need when you now pursue the product solution how do you know it is a product solution because you now see the side dependence actually here and the ξ dependence here it is a product okay.

So you know you do not have sigh and a term mixing up with each other there they are separate but they are multiplying the functions containing them are multiplied with each other and we also see that you $e^{-\xi^2}$, $e^{-\xi}$ and square data that is exponentially decaying term ξ so what that basically tells us is as you now go to e to ∞ further and further from the burner this entire term is going to go away.


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The solution is:

$$\gamma = (1+\nu)c^2 - \nu + 2(1+\nu)c$$

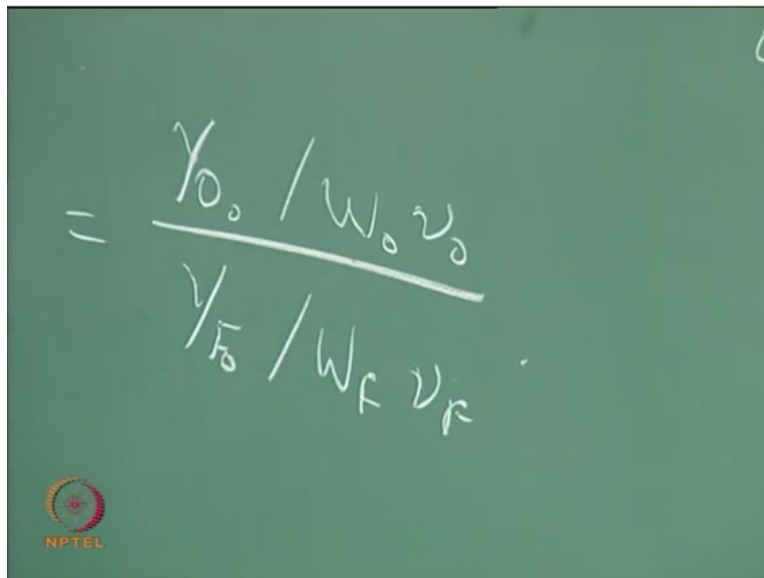
$\phi_n \Rightarrow n^{\text{th}}$ zero of $J_1(\phi)$ (Bessel function)



And you are now going to get a, a γ that is only this, this term all right and γ is nothing but β normalized and β is nothing but a difference in normalized mass fractions and normalized mass fractions are this so essentially it tells you that as you go further and further out the mass fractions are going to differ by a constant all right and the constant depends on two things μ and C so now we have to think about what does what does μ and C .

Basically mean they mean something very interesting μ is telling us $Y_{O_0} / W_{O_0} v_0$ or divided by $Y_{F_0} / W_{F_0} v_F$ this has got nothing to do with the geometry of the problem it is got to do with our Inlet mass fractions of oxidizer and fuel and their respective molecular weights and their stoichiometric coefficients in the reaction in the single step stoichiometric reaction ok this is a stoichiometric reaction that means you do not have either fuel or oxidizer left over as part of the products.

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$$\mu = \frac{\gamma_{O_2} / W_{O_2} v_{O_2}}{\gamma_F / W_F v_F}$$

So what are the stoichiometric coefficients required for them to react with each other completely right that is what is contained in this so this is a mixture quantity and it is a Inlet quantity because it has Y_{F_0} and Y_{O_0} see on the other hand is purely a geometric parameter it tells you how small or large is the inner duct relative to the auto duct okay so now you care to think about many

different combinations is it possible for me to have a very small inner duct relative to a very large outer duct but send a high concentration of fuel inside when compared to a very diluted concentration of oxidizer outside that is suppose that that is one possibility.

The other possibility I now have a fairly fat inner duct and just a little bit bigger outer duct that means most of the outer duct is contained by the inner duct and you are essentially trying to send a lot of fuel but let us suppose that the fuel is highly diluted but the oxidizer is highly concentrated what kind of flames will we get right on the one hand you are trying to send a very concentrated amount of fuel but in a very small region.

On the other hand you are sending in a lot of fuel all right but it is actually highly diluted it would you would you happen to have the same kind of flame right so that is what this, this is going to dictate okay so what are we going to look for in this what we want to see is interestingly this is now a γ that is a function of ξ and ξ that means we should be able to plot γ in this domain in were the mixing is happening and all we are looking for is look for $\gamma = 0$ right as the stoichiometric surface which is essentially the flame shape.

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$$\Rightarrow \frac{Y_F}{Y_O} \Big|_{\gamma=0} = \frac{W_F \nu_F}{W_O \nu_O}$$

The image shows a green chalkboard with a handwritten equation. On the left, there is an arrow pointing to the fraction $\frac{Y_F}{Y_O}$. A vertical line is drawn to the right of this fraction, with the label $\gamma=0$ at the bottom. To the right of the vertical line is an equals sign followed by the fraction $\frac{W_F \nu_F}{W_O \nu_O}$. In the bottom left corner of the chalkboard, there is a small circular logo with the text "NPTEL" below it.

So that means if you are able to plot this particular function in the domain and have a contour map of γ you now look for the $\gamma=0$ control the contour of $\gamma=0$ corresponding to $\gamma=0$ so what is $\gamma=0$ mean right, $\gamma=0$ means or $\beta=0$ if $\beta=0$ then $\alpha F = \alpha O$ back here right so $\gamma=0$ means $\beta=0$, $\beta=0$ means $\alpha f = \alpha O$ and αF is nothing but $-Y_F / W_F \mu F$ which is equal to $-r y_0 / W_O \mu O$ all right so what this basically means is we want our Y_F / Y_O along $\gamma=0$ to be in the ratio of $W_F / W_O \mu O$ right.

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Non-Dimensionalisation


• Then the final solution using separation of variables is:

$$\gamma = (1+\nu)c^2 - \nu + 2(1+\nu)c \sum_{n=1}^{\infty} \frac{1}{\phi_n J_0(\phi_n)^2} J_1(c\phi_n) J_0(\phi_n \xi) \times \exp(-\phi_n^2 \eta)$$

Here, ϕ_n is the n^{th} zero of $J_1(\phi)$

• The flame surface is represented by $\gamma = 0$, because it gives $\frac{Y_f}{W_F \nu F} = \frac{Y_o}{W_O \nu O}$ which is the stoichiometric surface.



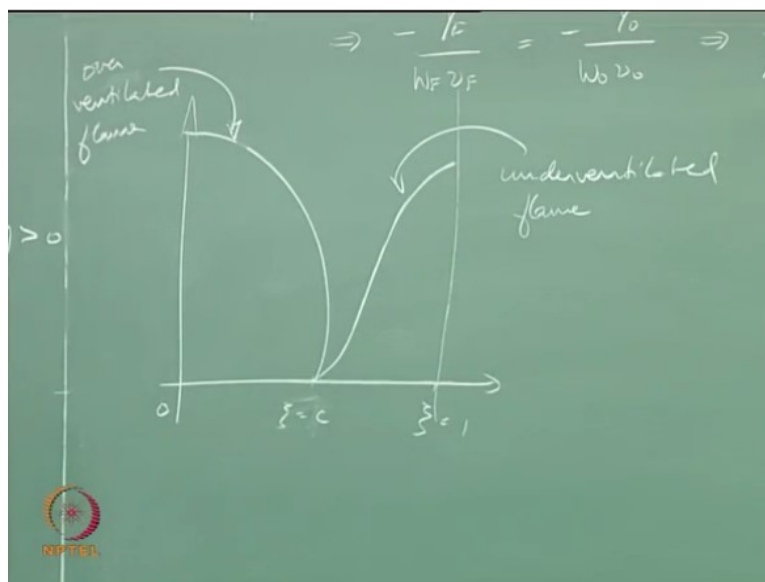


What does that mean if I were to simply say $\mu f / \mu O$ that would mean the stoichiometric ratio in terms of moles right all I have done here is to multiply $\mu f / w_f$ and multiply $\mu r / w_o$ so I am essentially looking at the mass ratio okay so this is the mass ratio of stoichiometric proportions of fuel and oxidizer in a reaction force truck for the fall for complete reaction to happen and what you are basically saying is $\mu f / \mu O$ anywhere in the domain is in this ratio you have a stoichiometric surface.

And that is what the flame is that is what the frame sheet assumption is that is what infinite chemistry means that is what makes this burnt approaches all about right so effectively it all translates you are saying just look at the $\gamma=0$ control alright further let us look at go back and see what μ is in the in the right of what we have done right.

So this implies we can write or you can simply write this equal to μ_0/μ_{fo} right / $w_o \mu / w_f \mu_f$ so what new signifies is the incoming oxidizer fuel ratio by mass to the stoichiometric oxidizer fuel ratio by mass right so effectively new tells us something about what is coming into the domain relative to the stoichiometric mixture ratio and see tells us how much of what you are getting alright.

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So what you are then thing is we can we can now begin to do a few things here one how does the height of the flame get determine right so how does the frame look like what, what will happen if you now try to plot this contour for $\gamma=0$ right so depending upon the values of your μ and C right so if you now say this is your $\xi = 1$, $\xi = 0$ and you now fix your equal to C , $\xi = C$ that is where the inner tube ends and then the outer tube is, is there you could now get a flame that either looks like this or in fact we have a wall there you could have a flame that looks like that as you now change your μ .

So for different values so if you now fix your C at a particular value and change your new you can get the flame to close over the fuel or you can get the flame to open up and close over close

over the oxidizer open up from the middle and close over the oxidizer all right so this is now called the over ventilated flame and this is what is called the under ventilator right what does that mean you are essentially having a concept of ventilation.

As in opening up more air or so when you now say over ventilated that means you have lot of air when compared to fuel so the, the flame is actually closing over the fuel and consuming all the fuel and we now say under ventilated that means you do not have enough air so the fuel is spreading out and then mixing with whatever little added can and then having and meeting it at stoichiometric proportions along this line that closes over the oxidizer and opens up over the fuel.

So by this what it means is if you now have a flame that looks like this you now have a fuel rich region over here or pretty much whole field in the, in the actual case and outside is going to be oxidizer because the flame is separating the fuel and oxidizer and getting them to meet at stoichiometric proportions in between all right so if you now have a under ventilated flame you are simply going to have a lot of fuel that is going to go out unreacted over here.

So when would you expect a fuel which sorry when we expect a under ventilated flame or a over ventilated flame so now think about those two limits that we were talking about if you now have a very short duct for the inner duct we are very, very small duct for the inner duct then compacted out a duct and you are having infinite domain pretty much no matter how concentrated the fuel is relative to how dilute the oxidizer rich you are very likely to have a if you a flame that is closing over the field.

That means it is going to be an over ventilated case if you now have a very, very broad inner duct when compared to the outer duct being just a little bit bigger you are very likely to have a opened up flame that is under ventilated right no matter how much you are diluting things but there is an effect of that and that effect is going to come up when you have your a fifty-fifty kind of situation that is your neither too small or too large.

The message here is the, the slot width is going to dictate whether the flame is good going to be over ventilated relevant and related more than new in other words the far-field behavior of the flame is more sensitive to see rather than new so what is it above new in this then you have to ask yourselves look at this picture this flame if you are no similar here you are just beginning to look at how this mixing is happening and then the flame is shaping up notice them a big in eclipse pretty much the same.

They are going like that over the oxidizer starting from the field right but then if you keep going that went that way this one this way and only now call those were not related that fundamentally did but why did they start going like that in the first place right what cannot you have a flame that what that was like this that could have been a possible solution as well so what is the difference between this and this, this flame started going over the fuel and closed over the field where as this flames started going over the oxidizer and then closed over the fuel.

This flame goes over the fuels and closes over the fuel so you now have multiple possibilities you could have a flame that starts going over one of the duct and then closes over the other or it goes over the same that encloses over the same duct right so what governs this is new when they are just beginning to mix and react they are just beginning to consume the reactants and it is a final large-scale availability of reactants.

That is going to shape up how the flame is going to end up and as we go along and the last scale availability is dictated by the see how much fuel you have on the whole versus how much oxidizer you have but locally as they just begin to mix and react they are not worried about that they are not worried about how they are going to end up and all the less deficient species is consumed reactant is consumed.

They are going to first be dictated by what is the new show, how concentrated the oxidizer is relative to the fuel as it comes in relative to what their stoichiometric ratio demands so if you had a fairly concentrated fuel but coming in a very, very small region it is going to actually go out in search of the oxidizer because oxidizer is dilute all right and ultimately you are as you go along

you are having a short fuel duct is a small steel duct so you are running out of fuel and then it closes over.

So again you can see that the new dictates the near-field behavior of the flame and the sea dictates none both of them out together it says like it, it is difficult to separate them where you look at the sensitivity right you look at the sensitivity of this the, the far-field behavior of the flame is more sensitive to see the near-field behavior of the flame is more sensitive to new all right and then from this you can also find out what should be the shape of the flame.

So what should be the combination of new and see together that will give rise to a optimally ventilated flame right a flame that is kind of like that but I just went too fast I did not tell you whether it is going like that or like this right that depends on the new what I am talking about is for a given new what should be the C such that the flame goes vertical and as it good ∞ we will talk about it later.

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