

Indian Institute of Technology Madras

NPTEL

National Programme on Technology Enhanced Learning

COMBUSTION

Lecture 39

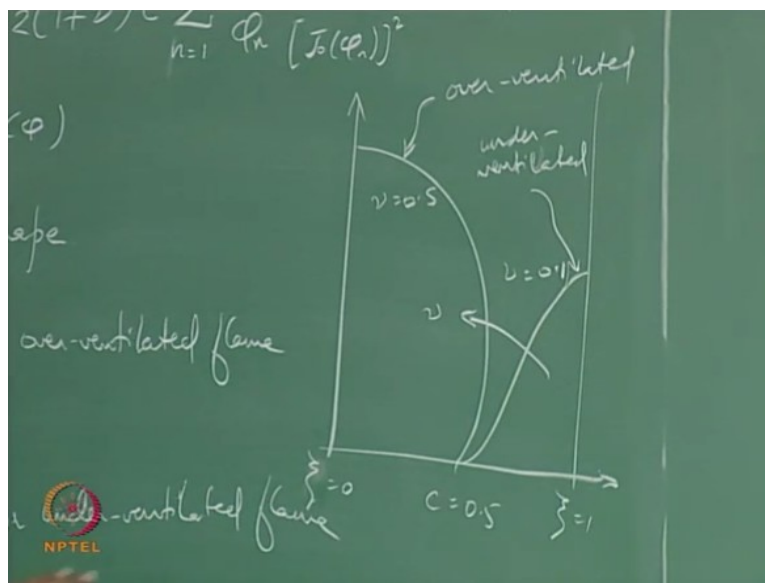
Burke-Schumann Problem

Prof. S R Chakravarthy

Dept. of Aerospace Engineering IIT Madras

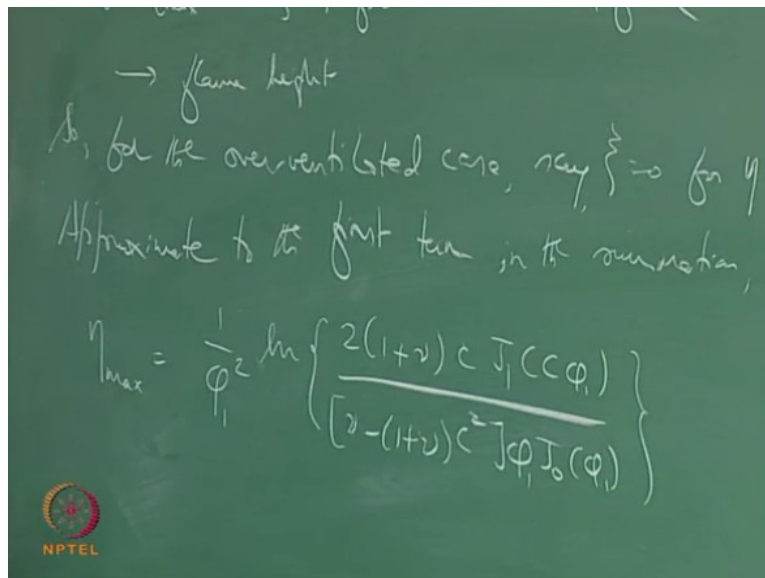
So we have been looking at the solution for the Burke-Schumann problem and what we can now figure out is we will not actually interest we are interested in the flame height surprisingly for tall laminar flames that means we still have the situation of pickling number being quite large but still not into the turbulent regime the Burke-Schumann solution gives a very good prediction of the flame height and this was significant back in 1928, if you think about it.

(Refer Slide Time: 01:15)



So the way you actually cannot find out the flame heights as I said the  $\gamma=0$  is going to give you the flame shape and of course it is going to change from an under ventilated flame on the one side to an over ventilated flame on the other side so depending upon whether it is under ventilated related you need to evaluate this expression setting  $\gamma=0$  for the  $\xi$  all right now plugging in so  $i=0$  or  $1$  right.

(Refer Slide Time: 01:59)



So with this we can get the flame shape so let us do this for at least one of those so forth so for the over liquid case say  $\xi=0$   $\xi$  that is a height of the flame so you can now plug this in here of course what happens is you know have a  $\gamma=0$  and then you have this expression which is the series  $\sum$  and we are trying to find out  $\xi$  on top of this exponent sitting there with a negative sign in each and every term right.

That is quite difficult to do so one thing you can do is to recognize how the series is going to behave and then say as a first approximation let us not worry about the series let us consider only the first term in the series right as a leading term and then the remaining, remaining terms or correction through it therefore approximate to the first term in the  $\sum$  with  $y_1$  equals 3.83 this is something that you can find out from the Bessel function tables.

So there is like a table of special functions which will give you values of these zeros so we're looking at the Bessel function of first kind the first 0 of that I will find that in the table  $\zeta_1 = 3.83$  keep this in mind and then we get  $\eta_{\max}$  all you have to do is get rid of the  $\sum$  wherever you have  $n$  you just put equal to 1.

Because you are looking at only the first term and this thing goes to the left hand side this thing comes down and then you have this then you find it this is actually  $e$  to the negative right so if you now want to flip everything and take a natural logarithm then you will get a  $\zeta$  and  $\zeta^2$  squared  $\xi \times$  equals  $e$  to  $\max$  and therefore if you now get the fire and squared also to the denominator on the other side you will get it as  $1$  over  $\zeta^2$  natural logarithm  $2$  plus so etc twice of  $1$  plus  $\mu$  times  $CJ$   $1$  of  $C$   $\zeta^2$   $1/\mu - 1 + \mu C^2$  when this goes to the left hand side you get a negative sign.

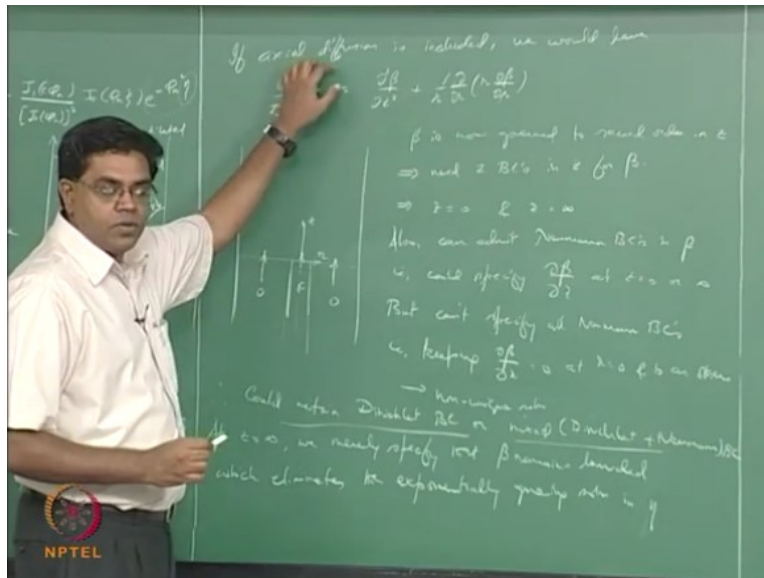
So this flips times  $\zeta_1$   $G_0$  of  $\zeta_1$  right so this just gives you a good idea of what the flame length should be but what do we learn from this okay then we know how the flame is going to how the flame, flame length is going as a matter of fact it is difficult to find out how the flame shape is going to be with this expression and as I said this is how the flame looks like but if you now look at this expression for the flame length you cannot see the dependencies in the problem right.

So how do you setup this problem you now have these coaxial pipes and then you want to send fuel and oxidizer together at particular should not say take it that I mean in each of these pipes at the same velocity that is what I meant by together right so the same velocity and the  $C$  is your variable as I said and your  $Y_{F_0}$  and  $Y$  would not order or your control variables these are the ones that you are trying to control depending upon  $Y_0$  and why we are not the  $\mu$  is going to get fixed right so you again you can do all that but how, how does it vary or for that matter is the flow velocity showing up here no because if low did this parameter did this, this equation was primarily the solution was primarily obtained for neglecting the axial diffusion all right.

So we will do a couple of things now first let us think about what happens when you now try to keep axial diffusion all right and I am not going to solve the problem I let it let you let you figure

that but I will tell you what are this what are the possible steps that can get in there and the second thing is and then what are the consequences and the second thing is let us look at the dependencies okay.

(Refer Slide Time: 06:59)



So the first thing is if you consider axial diffusion, diffusion is included rather than neglected right we would have we would have  $u$  over  $d$   $\frac{\partial v}{\partial z} = \partial^2$  equal by  $\partial z^2 \beta^2 + 1$  overall  $\partial/\partial R$  or do  $\beta$  do our previously we did not have a disturbed.

(Refer Slide Time: 07:45)

## BS Problem with Axial Diffusion

Considering axial diffusion:

- The governing equation would be:

$$\frac{u}{D} \frac{\partial \beta}{\partial z} = \frac{\partial^2 \beta}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \beta}{\partial r} \right)$$

- We need an extra BC to solve this. Consider flux conservation in z direction:

$$\left[ \rho u Y_F - \rho D \frac{\partial Y_F}{\partial z} \right] \Big|_{z=0} = \left[ \rho u Y_F - \rho D \frac{\partial Y_F}{\partial z} \right] \Big|_{z=-\infty}$$

On simplification, we get a mixed BC:

$$\left[ Y_F - \frac{D}{u} \frac{\partial Y_F}{\partial z} \right] \Big|_{z=0} = Y_{F_0}$$



We are now trying to keep this right so what is a consequence of this first of all you could say that R is still governed the second order and therefore it requires two boundary conditions the two bound reason r or r equals 0 and r equals B and I would like to supply boundary conditions there and I expect to have symmetry boundary condition at or equal to 0 so no  $\beta$  by  $\partial r$  is equal to 0 that is an condition.

And at the wall I have a rigid non-porous wall so I cannot have diffusion through that wall so I still have no diffusion mass flux which amounts to do  $\beta / \partial R$  is equal to 0 at r equals B at the outer wall and so I get diamond boundary condition there as far as Caesar boundaries are concerned this is now suddenly beginning to be governed by second order previously it was governed only to first order now you have actually a second order term that is governing this right.

So whatever we neglected was somewhat pretty important I mean it actually reduces the order of the equation by one which means it does not permit an additional boundary condition whereas this one demands an additional boundary condition that means you have to give two boundary

conditions in  $Z$  okay the domain for  $Z$  is not equal to  $0$  which is something that we considered earlier and  $Z$  equals  $\infty$ .

So you now can actually go all the way up to infinity here and that is where you have to supply the boundary condition what do I know about  $\beta$  are equal  $\infty$  or its derivative and now you know you have a very important situation you see because you have this government a second order not only demands to boundary conditions but it can also permit a boundary condition and the derivative okay.

So we already had two boundary conditions and derivatives here and if you now allow if this admits boundary conditions in its in derivative that means you can you can specify do  $\beta$  by doors are at or equal to are equal to  $0$  and or do  $\beta$  by doors are at  $Z$  equals infinity then you do not have a unique solution because you are you are supplying Schumann boundary conditions everywhere.

Therefore you need to supply additional a boundary condition somewhere okay now previously we could supply one lead addition a boundary condition because this is governed to first order and you could give only directly data now it is supply it is going to second order that means you can have a not only delay but also a mixture of delay annoy in what is that how could I get that so now you have a choice of different boundary conditions that you can give okay and they should mean something physically or in other words we should now interpret that physically okay.

So if you go back to your original problem right so  $\beta$  is now govern the second order and so need to be seized insert for  $\beta$  which implies  $R$  equal to  $0$  and  $z$  equal  $\infty$  okay also can admit Norman by  $\beta$  that is could specify do beta by those are at  $z$  equal to zero or infinity but cannot specify all know I mean VCS that is a keeping the  $\beta$  by  $\partial$   $R$  equal to  $0$  at  $r$  equal to  $0$  and  $b$  as others okay.

Because this would lead to a non unique solution therefore could retain the wrist leg bc or mixed just deliciously plus normal bc all right all we can say in fact at  $z$  equals infinity you don't have too much of a room to play with on what, what should be the boundary condition all we can say

and hope for ads are equal infinity is that  $\beta$  should be bounded we cannot specify values we cannot specify derivatives simply.

Because you are expecting to get a exponential solution and since you have a second derivative now you cannot readmit the plus some constant times let us say  $\emptyset N^2 e$  to the plus  $\emptyset N^2 \xi$  plus some constant times  $e - \emptyset N^2$  data and if you now admit the coefficient to  $e + \emptyset N^2$  data as it tends to infinity that solution is going to blow up so by specifying that  $Z$  equals infinity  $\beta$  should be bounded we can get rid of the exponentially growing part of the solution and retain  $1e$  the exponentially decaying part of the solution as before all right.

So at  $Z$  equals infinity we merely specify that  $\beta$  remains bounded which eliminates the exponentially growing solution in  $\beta$  in each am sorry right and retains only the exponentially decaying solution as before a matter of fact lots of boundary conditions are supposed to do boundary conditions are supposed to evaluate constants of integration that are appearing as coefficients to solutions.

And by just merely saying that  $\beta$  should remain bounded as a boundary condition at  $z$  equal to 0 we evaluate the coefficient of the exponentially growing solution and  $\xi$  as 0 so we have done the job as far as that the particular boundary condition is concerned then comes this right here is where it is important for us to decide whether we want to have add additional boundary condition or a mixed boundary condition so this question of whether we want to have a diversity boundary condition or mixed boundary condition arises primarily at  $z$  equal to 0 at the of the burner right.

That means it is now possible when you when you admit or include axial diffusion that you have a choice of boundary conditions either it could be the deletion a boundary condition that we used before which we did not have a choice about earlier or we can use the mixed boundary condition now the question is what is the mixed boundary condition really mean right so let us now think about the flow that is coming through one of these force.

And now we are admitting axial diffusion right so we now go back and look at the solution so or if you now try to map the solution and say you have a flame that is supposed to be here right

what does that mean this really means that you, you have a fuel rich region over here this is the stoichiometric surface there is a fuel pure lean region around okay and there is a progressive change in the mix diffraction from a pure fuel or more fuel in the middle progressively the stoichiometric and then filled in okay.

It is a varying region over there what that means is if you now look at it along the axial direction or the stream wise direction you have more fuel here than here right so it is sort of like as the the fuel is coming out and flows up into the end of the domain and you do not have the, the, the tube anymore it is now got into the domain it looks around and then sees wait a minute I am not there so let me go there right and then once it goes there it says then I am not there let me go there.

What is that? That is axial diffusion okay. Now of course you might think wait a minute do not we have all the flow kind of coming up yeah that is, that is what we had in premix flames also right so but at that time what were what happened maybe now decided that you are going to have a flame in the property and if the reactants are coming in and then they all getting converted to products and when the products are formed they suddenly get formed and then look around.

And then fine it is all products over here no products over here can I diffuse backwards right and it tries to diffuse against the current, current meaning the convection right and it succeeds to some extent just as well as the heat gets conducted even as the convection is actually carrying the enthalpy this way if the heat could penetrate why cannot mass right it is after all both of them or transport processes similarly here you could have a current that is, that is setting up set up, set up upwards.

But you could have a, a reverse diffusion that is for the fuel but think about the oxidizer the oxidizer does not even have to fight the current right the oxidizer is here and it finds wait a minute there is more fuel over there right can I now go, go up right so along with actually convecting it begins to diffuse up right so these axial diffusion processes the question is how good are they is a question of how well are they competing with convection.



So when you do not have a large convective effect that means you are fairly low velocities right in simple English huh then you can you can now expect the axial convection axial diffusion to be more predominant all right so this is a problem where the convection could predominate axial diffusion but balances radial diffusion alright so the balance between radial diffusion and convection is, is the centerpiece of this but alongside at low convective effects.

You could have a significant contribution from axial diffusion as well if you now think about that so you now say fuel is coming out like this refusing like this and then going backwards oxidizer comes like this diffusers like this and or other even carry and carry and diffuse like this and then goes backwards if it did then the question is how good is additionally data correct we were imposing saying at the at the tip of the burner.

You have one leaf you will here and one Lee oxidizer here that is what we did right but that need not be the case you could have the incoming fuel get contaminated by oxidizer that is diffusing from the other side right or products as a matter of fact and similarly oxidizer so what can I expect to be sure that I do not have any contamination far upstream because you have a convection.


There is a certain length scale associated with the diffusion in competition with the convection right and beyond that length scale you can hope to have pure fuel and pure oxidizer but how do I know what that is unless I solve and for me to solve I need to know the boundary conditions so why do I go in search of the boundary do I want to now take like a minus infinity to plus infinity insert for the domain when you in fact I am interested in what is happening here.

(Refer Slide Time: 22:12)

Flux boundary condition

$$\left[ \rho u Y_F - \rho D \frac{\partial Y_F}{\partial z} \right]_{z=0} = \left[ \rho u Y_F - \rho D \frac{\partial Y_F}{\partial z} \right]_{z=-\infty}$$

$$\rho u Y_F \Big|_{z=0} - \rho D \frac{\partial Y_F}{\partial z} \Big|_{z=0} = \rho u Y_{F_0}$$

$$\Rightarrow Y_F \Big|_{z=0} - \frac{D}{u} \frac{\partial Y_F}{\partial z} \Big|_{z=0} = Y_{F_0}$$


That is where what is called as a flux BC comes in the picture right so we could now think about a flux boundary condition which is essentially a species balance that is integrated over a control volume for each of the species from minus infinity for dessert to over here knowing that this is going to be all fuel alright so if you now take the governing equations and integrate within this control volume what you can now expect is we now know that you can say  $\rho u Y_F - \rho D \partial Y$  right at  $ZR$  equal to zero should be  $\rho u Y_F - \rho \partial Y F$  by those are at that equals minus infinity there is hardly anything that is going on along the walls you neither have convection or diffusion along these walls right so the, the control volume here that we are looking at is having exchange only at the surface.

And the surface at infinity far upstream far upstream we know that you do not have you help your fuel and therefore you do not have any fuel concentration gradients so you do not have any diffusion to talk about right so this goes away and you can directly now say this is equal to so you can say  $\rho u Y_F$  at  $z=0 - \rho D \partial y_f / \partial Z$  at not equal to 0 equals  $\rho u Y_{F_0}$  not right and then I can say  $Y_F$  at  $z=0 - \frac{D}{u} \partial f / \partial z, z=0$  equals  $Y_{F_0}$ .

(Refer Slide Time: 24:37)

## BS Problem with Axial Diffusion

Considering axial diffusion:

- The governing equation would be:

$$\frac{u}{D} \frac{\partial \beta}{\partial z} = \frac{\partial^2 \beta}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \beta}{\partial r} \right)$$

- We need an extra BC to solve this. Consider flux conservation in z direction:

$$\left[ \rho u Y_F - \rho D \frac{\partial Y_F}{\partial z} \right] \Big|_{z=0} = \left[ \rho u Y_F - \rho D \frac{\partial Y_F}{\partial z} \right] \Big|_{z=-\infty}$$

On simplification, we get a mixed BC:

$$\left[ Y_F - \frac{D}{u} \frac{\partial Y_F}{\partial z} \right] \Big|_{z=0} = Y_{F_0}$$



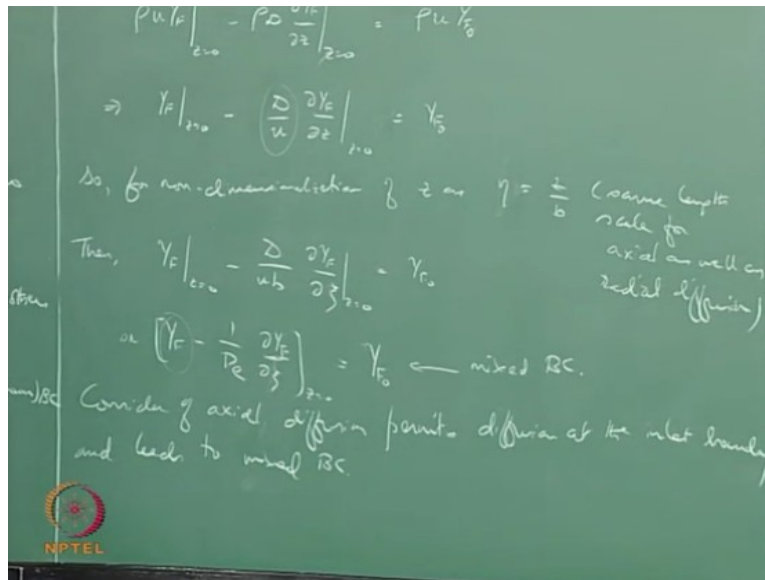
So what has happened now previously we ignored this term we did not consider any diffusion that was happening axially across the inlet to your domain we simply said  $Y_F$  at are equal to zero is why if not but now you have to subtract that amount that is actually if you diffused you see and what is that going to be like in fact if you think about axial diffusion including axial diffusion then the problem becomes somewhat symmetric in both Z and the R you do not have to worry about Z an R.

That means if you now thought that the length scale of your or dimension was be the burner width what you are essentially saying is it is diffusing along the radial direction just as well as it is diffusing along the axial direction at least you begin with the way that is a consideration we will re-evaluate how much this is versus that okay and that will be done by the pecllet number so if you were to now say that z is also going to be of the order of B and then say so for non dimensionalization non dimensionalize of Z as  $\xi$  equals Z by be okay same length scale for axial as well.

As diffusion this was not the case before we did not have an axial diffusion length scale unless we actually started looking at what is the flow versus the diffusion length scale so we have to do something like  $Z/u/ b^2/d$  and then we came up with a new non dimensional number and sorry a coordinate  $\xi$  last time but if you do this then  $Y_F$  ads are equal to 0 minus you now get  $Ab$  by out

here right so we can have  $Y F_0$  or  $Y F - 1 / do$  why I am sorry do I YF x duct I at z equal to  $0$  equals  $Y F_0$  right now this is what is called as a flux boundary condition.

(Refer Slide Time: 27:25)



And it is a mixed boundary condition it now has a linear combination of value and derivative together right this is sort of like why  $ayf$  plus  $d$   $dy$  of  $x$  dokes I where  $a$  is equal to  $1$   $B$  is equal to minus  $1$  over  $peg$  leg its linear combination that is what is a mixed boundary condition so the summary of what we are talking about is that an  $Z_i$  consideration of axial diffusion permits, permits diffusion at the inlet boundary right and leads to flux sorry a mixed boundary condition.

Now mixed boundary condition is all right because it still involves some later right yes I am sorry that keep making good mistake sorry I need to grief sorry right as a matter of fact  $\xi$ ,  $\xi$  is equivalent of  $X$  and  $\xi$  is actually equivalent of what it is, it is  $Z$  it is actually  $H$  so that is just by the Confucian we should have been using  $\beta$  all right so all right so now question we can ask two questions one all right I have axial diffusion okay but let me insist on having only the delay boundary condition as before that is possible mathematically that is correct okay.

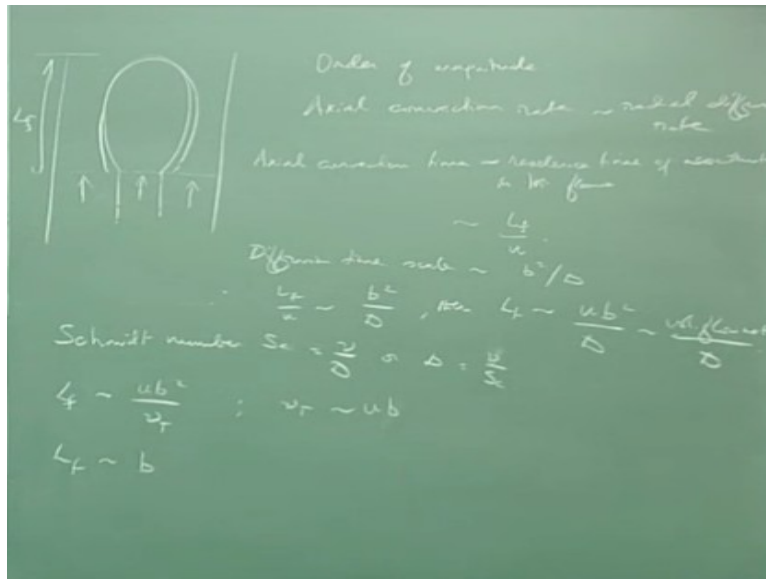
It may not be physically correct in some situation but mathematically that is okay, you can do that what would be the effect of considering axial diffusion ok but just add additional a boundary condition as before the answer is if you that now begins to depend on a new parameter which is called peclet number before we did not have pick a number we had a non dimensional governing equation without any parameters.

But now what will happen is you will have a  $1$  over pick square quiet showing up and clearly what that means is as peclet number is large the diffusion effect relative to axial convection is going to be small okay that means axial diffusion is important mainly for small peclet numbers right so small pick numbers so peclet number here is obviously  $u_b / d$  this is the mass diffusion counterpart of Reynolds number that I pointed out yesterday.

And what that means is it small pick lay numbers your convection is not as convection is not predominating over diffusion okay diffusion is quite important and therefore you will now end up with aflame that is a bit fatter and shorter right so this is for so this is the way you are going to get things to go as peclet number increases and the Burke-Schumann solution that we saw so far neglecting axial diffusion is in the limit of infinite peclet number alright.

So the job of axial diffusion is to make the flame shorter because you have more mixing happening as well as racially and that therefore you get a shorter and a bit fatter flame correspondingly then the question the next question that we have to ask is well fine now I have axial diffusion and then I look at the problem and then decide that I will not have a mixed boundary condition which means I want to have a flux boundary condition like this.

(Refer Slide Time: 32:48)



What do I do how does the flame look like right the answer is if you now had a, a Burke-Schumann solution with axial diffusion and there is a boundary conditions if you had a flame that looked like that your flux BCS are going to actually make sure that you your force your domain starts here right your flame starts some of that that means the flame is no longer going to be attached to the rim of the burner right.

Why was it attached to the remove the burn up before because we did not permit axial diffusion across the interface sorry across the across the inlet and therefore when the fuel and oxidizer met the first opportunity at which they could meet was right at the burner lip and they meted stoichiometric, stoichiometric proportions there and then on in a certain curved manner and therefore the stoichiometric surface starts from the lip of the burner like what we have shown here.

But when you have axial diffusion taken into account you can now permit mixing to happen this way across upstream of the inlet to the domain and therefore you could find the fuel and oxidizer being in Turkey metric proportions away from the lip of the burner and that is possible and we are not strictly speaking drawing what is happening inside because you are not solving for it our boundary condition is still applied 1ly here and our solution starts only into this domain right.

So we do not know, we do not know exactly what is happening but this is permitted okay so this is the consequence of having axial sorry flux boundary conditions along with axial diffusion taken into account in the case of the Burke-Schumann in general or if you look at the literature the, the, the term Burke-Schumann stands for infinite chemistry right that means we have not really bothered about solving anything to do with finite rate chemical kinetics.

So you can have for example Burke-Schumann flame like look like you have a spray of droplets and the droplets are burning and what it means to basically that as the droplets evaporate and, and the vapor from the droplet is mixing into the oxidizer let us say you are looking at fuel, fuel droplets fuel spray right wherever you find fuel vapor in stoichiometric proportion with the oxidizer ambient oxidizer mixing into this uniform a flame.

That is a Burke-Schumann okay and that is what you would call as a Burke-Schumann flame right you can also have something called a Burke-Schumann diffusion flame or a Burke-Schumann flame so what that would mean is I could I do not have to have walls working Truman were very clever you see they, they made these walls so that they do not worry about entrainment from the surroundings right this is a purely species mixing and convection problem they isolated the most important aspects of this very, very well okay so you could now think about a jet flame in which you have only the fuel coming out you have oxidizer everywhere right.

And it now gets entrained as well as diffuse and in this flow field you could now think about wherever the field and oxidizer are present in stoichiometric proportions and that would be your Burke-Schumann flame so on the one hand the Burke-Schumann problem would actually mean this problem a Burke-Schumann flame in the literature now comes to mean any stoichiometric surface that is now coincident with a Burke-Schumann flame.

That is essentially what it means that means we are adopting the infinite rate chemistry assumption or the flame sheet assumption or the mix burnt approach or whatever it is you can you can call it in any other way anyway this is what we are talking about the second thing that we decided to talk about when we looked at this expression was what about the dependence on

the flame height right do we get these get these dependencies we do not get to see that right there okay.

And then part of the reason was tell number did not show up because we had we had supposed infinite pecllet number there this kind of a formulation you could hope to actually begin to see pecllet number show up in, in places and that would actually now denote the flow velocity relative to diffusion right and then you can begin to see what is the effect of flow velocity but still the expressions are going to be so complicated you do not get the physical feel right.

So it is easy for us to actually look at what happens in a, in a order of magnitude manner so order of magnitude all we are saying in this problem is a axial convection balances radial diffusion right so what that means is you can now look at the axial diffusion axial convection timescale an axial convection time scale is essentially let us suppose that you now have the flame length as  $L_f$  so axial convection time is essentially the residence time of reactants in the flame.

(Refer Slide Time: 39:15)

**Order of Magnitude Analysis**

Let's obtain flame length  $L_f$  from order of magnitude analysis:


- Axial convection rate  $\sim$  radial diffusion rate
- Also, axial convection time scale  $\sim$  residence time of reactants in the flow  $\sim \frac{L_f}{u}$
- Radial diffusion time scale  $\sim \frac{b^2}{D}$
- Thus,

$$\frac{L_f}{u} \sim \frac{b^2}{D} \sim \frac{d^2}{D} \Rightarrow L_f \sim \frac{ud^2}{D} \sim \frac{\text{volume flow rate}}{\text{diffusion coefficient}}$$

Here,  $d$  is fuel duct width.

- Let's introduce Schmidt number  $S_c = \frac{\nu}{D}$ .  $\nu$  is kinematic viscosity.

Thus,  $L_f \sim \frac{ud^2}{\nu} \sim \frac{ud^2}{\nu}$ . (Here we considered a turbulent flow, where  $\nu \sim ud$ ). Thus,  $L_f \sim d$ . This implies flame length is independent of flow velocity but only dependent on fuel duct width!



The residence time is actually a very, very basic and important idea that you will find happen think thought about in design of combustors and so on so whenever you want to actually design



the length of a furnace right particularly for diffusion flames previously we saw what happens when you have and soon or after burners the flame is here the pre-mixed flame is held at the flame holder and then it gets inclined of trying to burn into the reactant flow.

So that the, the normal component of the flow velocity balances the flame speed of course it could be turbulent flame speed so you get the shape of the flame and therefore you get the length of the combustor all those things right essentially the idea basically there is what is the residence time of the reactants within this region okay.

So there what happens is you are looking at a left who are you so if this is the length of the flame then as the flow goes along the along this it is going to be there for so long while it is there it is now diffusing this way right so the diffusion time scale is as we saw yesterday we have  $B^2$  over  $D$  right therefore, therefore if you now say  $LF$  over you is of the same order as  $v^2$  over  $D$  then  $LF$  which is the same as  $\xi_{max}$  here okay.

So, so long as we were doing a heavy duty mathematics we were we were using Greek like  $\xi$  either  $\xi_{max}$  and all those things but this is just order of magnitude just thinking about this so we just you started using your left right there is the same thing so  $LF = ub^2/d$  right now this is an order of magnitude idea and it works reasonably well not just for Burke-Schumann problem of the Burke -Schumann geometry where you have coaxial ducts but even for jet flames.

So if you have a jet that is coming out of a fuel jet that is coming out of a pipe of certain radius or diameter  $D$  you could still say do not worry about the fact that you have to use the outer down the outer duct diameter the order is not going to be significantly different if you use the inner duct diameter all right so in a jet diffusion flame you have only one duct you do not have the outer duct it is just in training and diffusing with atmospheric air.

Therefore you could simply say you, you small  $d$  square divided by capital  $d$  where small  $D$  is like the diameter of the jet right or the inner pipe that is fine now what that means is two things you can look at one the diameter squared is proportional to the cross-sectional area right and the cross-sectional area times velocity is essentially the volume flow rate like what you would

measure with a flow meter let's say like liters per minute or meter cube per minute or whatever it is right.

So this is basically volume flow rate divided by the diffusion coefficient so you can look at it in two ways one either a say velocity burner diameter split or volume flow rate if you look at it the first way you learn two things one the flame height is going to be proportional to velocity and proportional to square of the duct diameter all right if you look at it the second way all you get is the flame is going to be proportional to the volume flow rate alright.

So larger faster the flow that is coming taller the flame is going to be or larger the duct diameter much taller the flame is going to be or whatever is more the volume flow rate larger the flame longer the flame is going to be right that is what it means whatever we have done we have been talking we whenever we talk about flow velocity we had to bring in the pecllet number in our in our discussion and so on.

But to do that and then I also said pecllet number is kind of like Reynolds number and soon but we never talked about turbulence right here we are not really worried about that so where is the question of that that and that why are we talking about it the reason why we are talking about it is as we keep on increasing the velocity at some stage or keep on increasing the volume flow rate or your time duct diameter right at some stage.

You are now getting to turbulent flows and then what happens well if you now have turbulent flows the diffusion that is going to happen is going to be a turbulent effusion right and there is not going to be molecular diffusion anymore so we have enough factor in the turbulent diffusion so the way you can do this is you can now look at the Schmidt number  $SC$  is essentially equal to  $\mu$  over  $D$  or  $D$  goes as a  $\mu$  over  $SC$  right.

o now for a constant victim but let us not worry about how the Schmidt number goes typically we do not have to worry about that and therefore  $LF$  goes as let us say you  $B$  squared divided by  $\mu$  right so if new changes  $d$  changes through Schmidt number therefore if you want to have

talked about  $d$  as a turbulent mass diffusivity you can think about it in terms of turbulent kinematic viscosity and a turbulent kinematic viscosity is a flow dependent parameter right.

And the way it goes is this goes is  $ub/d$  right the greater the velocity more the turbulent viscosity greater the diameters is because it depends on Reynolds number right so if you now think about this then  $LF$  simply becomes proportional only to  $b$  that means in fact I think, I think it feels lot more comfortable you can say also  $LF$  then is  $u d^2$  divided by  $D$  where  $D$  is fuel duct diameter I think it makes a lot of sense to talk about our in terms of in the context of jet flames and keep it as  $d$  rather than  $b$  because I am not thinking more about the outer duct diameter anymore.

Mainly talking about the fuel duct diameter so what does tells us is the flame, the flame length is no longer going to be dependent on the flow velocity it simply is going to scale only with the duct diameter right so if you look at the data what you should find is if you know plot your  $LF$  versus  $U$  what you will find here is  $LF$  is proportional to  $U$ ,  $uv^2/D$  or  $ud^2/d$  so this is linearly increasing the  $D$  with you right so for small you, you get a linear increase alright but then you get into your transition region where you transition the turbulent flows and you know have a transition.

That now makes it insensitive ultimately to why is that because you have more and more mixing that's happening and as more and more mixing is happening you are having the fuel and oxidizer burn in stoichiometric proportions much closer to the burner and that does not make that, that becomes insensitive to reflow because more and more flow more and more burning can happen and all these things are happening within a very short within a pretty much constant distance right.

(Refer Slide Time: 48:15)

## Order of Magnitude Analysis

Let's obtain flame length  $L_f$  from order of magnitude analysis:

- Axial convection rate  $\sim$  radial diffusion rate
- Also, axial convection time scale  $\sim$  residence time of reactants in the flow  $\sim \frac{L_f}{u}$
- Radial diffusion time scale  $\sim \frac{b^2}{D}$
- Thus,

$$\frac{L_f}{u} \sim \frac{b^2}{D} \sim \frac{d^2}{D} \Rightarrow L_f \sim \frac{ud^2}{D} \sim \frac{\text{volume flow rate}}{\text{diffusion coefficient}}$$

Here,  $d$  is fuel duct width.

- Let's introduce Schmidt number  $S_c = \frac{\nu}{D}$ .  $\nu$  is kinematic viscosity. Thus,  $L_f \sim \frac{ud^2}{\nu} \sim \frac{ud^2}{\nu_t}$ . (Here we considered a turbulent flow, where  $\nu \sim \nu_t$ ). Thus,  $L_f \sim d$ . This implies flame length is independent of flow velocity but only dependent on fuel duct width!



So that is essentially what is going on as far as turbulent mixing and burning is concerned and therefore you, you, you, you, you have a pretty insensitive flame linked to the flow velocity so the, the interesting picture I have in my mind about these things is, is about our Cotton's that we watch when your kids like you, you have this dragon that that spews fire right like the spit fire dragon there is a full moon then you now get as far out of it they can clearly see from there that the greater the velocity greater the flame length right.

And of course there and then the dragon really wants to hurt you and then keeps increasing the velocity and if the flame length does not, does not increase anymore because it is become turbulent and it starts blinking what the hell am I not able to be make any impact so it starts opening up with more and more in the flame length increases and it let it attacks the enemy right so we can learn quite a bit of diffusion watching cartoons thank you very much.

## Production and Post Production

M V Ramchandran

G Ramesh

K R Mahendra Babu

Soju Francis

S Subash

R Selvam

S Pradeepa

Ram Kumar

Ram Ganesh

Udaya Sankar

Robert Joseph

Karthi

Studio Assistants

Krishnakumar

Linuselvan

Saranraj

NPTEL Web & Faculty

Assistance Team

Allen Jacob Dinesh

P Banu

K M Dinesh Babu

G Manikandansivam

G Prasanna Kumar

G Pradeep Valan

C Rekha

J Salomi

P Santosh Kumar Singh

Sridharan

P Saravana Kumar

S Shobana

R Soundhar Raja Pandian

K R Vijaya

Administrative Assistant

K S Janakrishman

Principal Project Officer

Usha Nagarajan

Video Producers

K R Ravindranath

Kannan Krishnamurthy

**IIT MADRAS PRODUCTION**

Funded by

Department of higher Education

Ministry of Human Resources Development

Government of India

[www.nptel.iitm.ac.in](http://www.nptel.iitm.ac.in)

Copyrights Reserved