Indian Institute of Technology Madras

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National Programme on Technology Enhanced Learning

COMBUSTION

Lecture 44
Droplet Burning

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So this is where we are the droplet burning problem where we got the solution for β which is the Schvab's Zeldovih coupling function in terms of αT and αo the thermal the enthalpy and this is the oxidizer mass fraction in the ambience in terms of ξ works ξ is actually a non-dimensional radius which is actually the inverse of radius like as radius increases ξ going to decrease okay.

And then we are looking for boundary conditions and also an interface condition because ξ involves m. which is unknown and that is what we are trying to find out so the first thing that we want to do is to actually look at the interface flux boundary condition at the droplet surface and we are trying to actually form β which means we have to look at what happens to αT and α0 what we are doing here is to actually look for what happens to α0.

So if you now look at how what happens to Yo / vo will give you what happens for α0 and then we need to look at what happens to αT which means we have to look at the energy flux balance at the droplet surface interface right so as far as that is concerned if you now look at this basically a mass balance at the droplet surface based on convective flux at a diffusive flux.

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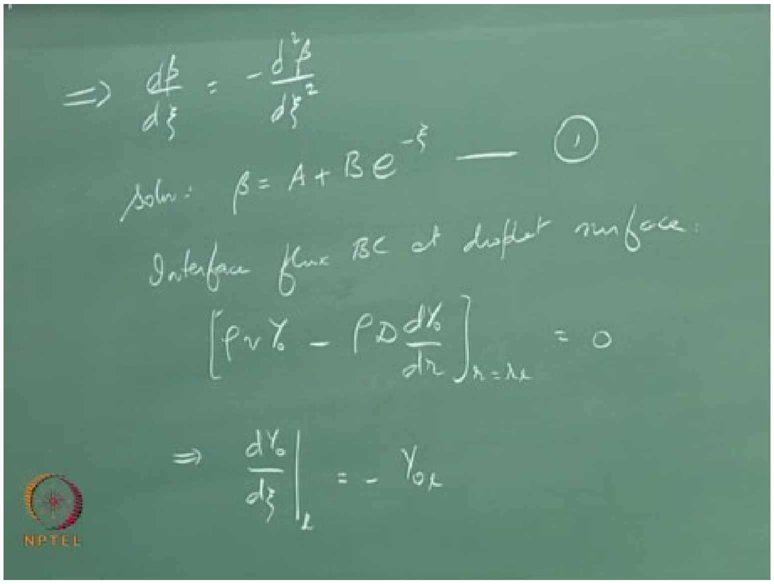

$$\Rightarrow \frac{d\beta}{d\xi} = -\frac{d^2\beta}{d\xi^2}$$

$$\text{Soln:}\quad \beta = A + Be^{-\xi} \quad \text{—} \quad ①$$

Interface flux BC at droplet surface:

$$\left[\rho v Y_o - \rho D \frac{dY_o}{dr}\right]_{r=r_\ell} = 0$$

$$\Rightarrow \left.\frac{dY_o}{d\xi}\right|_\ell = -Y_{o\ell}$$

As far as see the energy balance is concerned.

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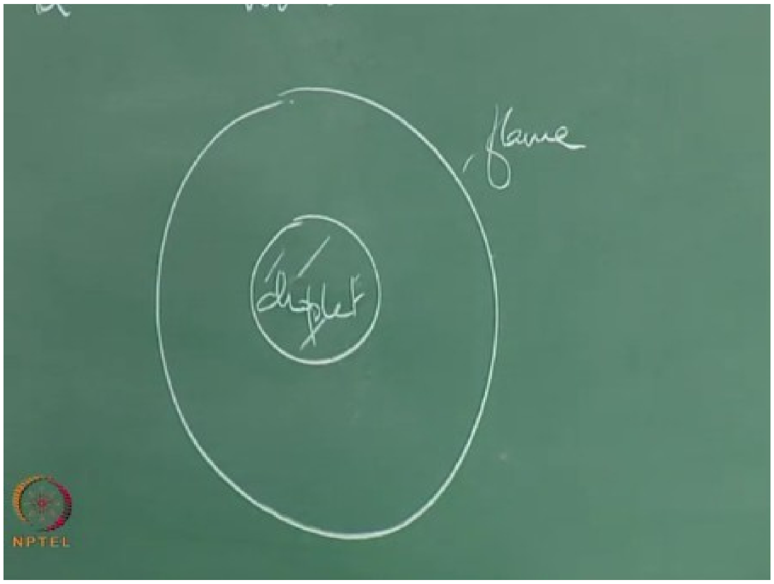

What we can understand is now if heat conducted heat conducted into the droplet must be just sufficient and I will explain what is meant by just sufficient pretty soon but sufficient is what we should start with sufficient to vaporize the fuel leaving the droplet that is to say if you now have a flame with a flame temperature and then the heat gets conducted in words radially inwards the heat conduction flux at the surface should be equal at least to the rate at which the heat is required for it to vaporize right.

And that is what is meant by saying just sufficient or at least sufficient that is to say if you if you have excess heat that is being conducted part of it in fact bulk of it is going to be taken by the latent heat of vaporization to vaporize the droplet into vapor and then you will have a further temperature gradient for the temperature to vary within the droplet okay now if you think about very small droplets what happens is you have a very quick thermal equilibrium that happens within the droplet.

So the droplet is nearly it pretty much the same temperature everywhere within with hardly any heat flux that is going on because with if you do not have any temperature gradients you do not have any heat fluxes so essentially the droplet is looking forward to only that much heat that is by conduction that is required for it to vaporize okay and the data clip that is a kind of equilibrium that is attained and this is reasonable.

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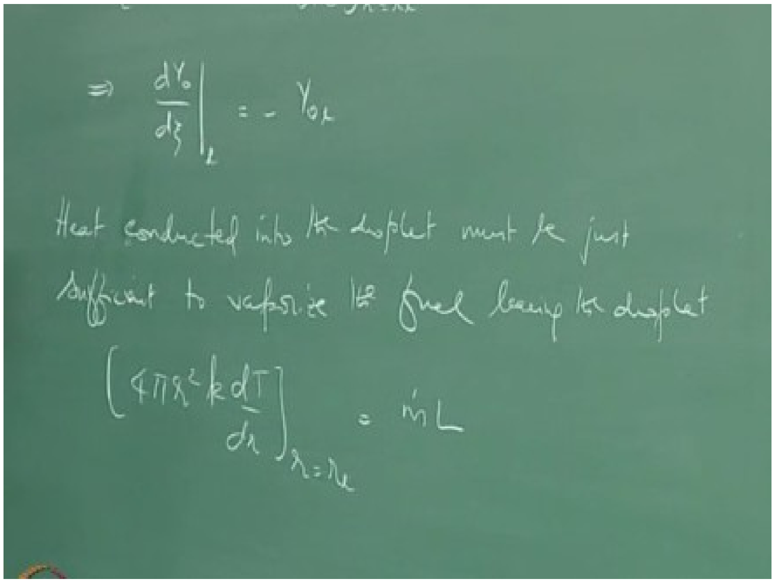

$$\Rightarrow \left.\frac{dY_O}{d\xi}\right|_l = -Y_{O,l}$$

Heat conducted into the droplet must be just sufficient to vaporize the fuel leaving the droplet

$$\left[4\pi r^2 k \frac{dT}{dr}\right]_{r=r_l} = \dot{m} L$$

So what this means is 4Πr ^{2} k dT/ dr at r = rl = m . L.

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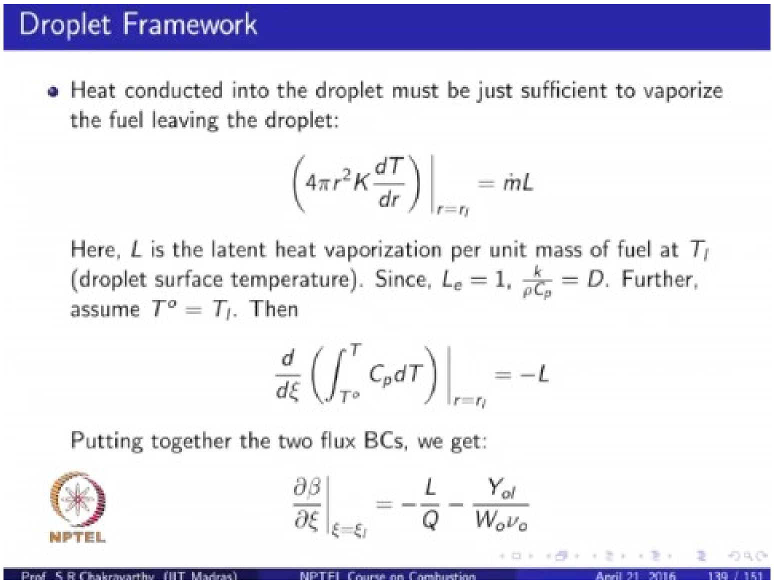

Droplet Framework

- Heat conducted into the droplet must be just sufficient to vaporize the fuel leaving the droplet:

$$\left(4\pi r^2 K \frac{dT}{dr}\right)\Big|_{r=r_l} = \dot{m}L$$

Here, L is the latent heat vaporization per unit mass of fuel at T_l (droplet surface temperature). Since, $L_e = 1$, $\frac{k}{\rho C_p} = D$. Further, assume $T^o = T_l$. Then

$$\frac{d}{d\xi}\left(\int_{T^o}^{T} C_p dT\right)\Big|_{r=r_l} = -L$$

Putting together the two flux BCs, we get:

$$\frac{\partial\beta}{\partial\xi}\Big|_{\xi=\xi_l} = -\frac{L}{Q} - \frac{Y_{ol}}{W_o\nu_o}$$

Now this is in some sense very similar to the conductive diffusive balance that we have gone through this is the diffusive part this is the convective part except what is important here for you is you have this is simply a m. times Yo okay that is like the mass flux of the oxidizer that is conducting inward in the space that is vacated by the droplet regression okay that is what that is what this dictates but here this convection is actually based on the latent heat of vaporization.

So in other words you need to have latent heat of vaporization times the rate at which the droplet is regressing of all which is the same as rated which the gases are issuing out of the droplet surface.

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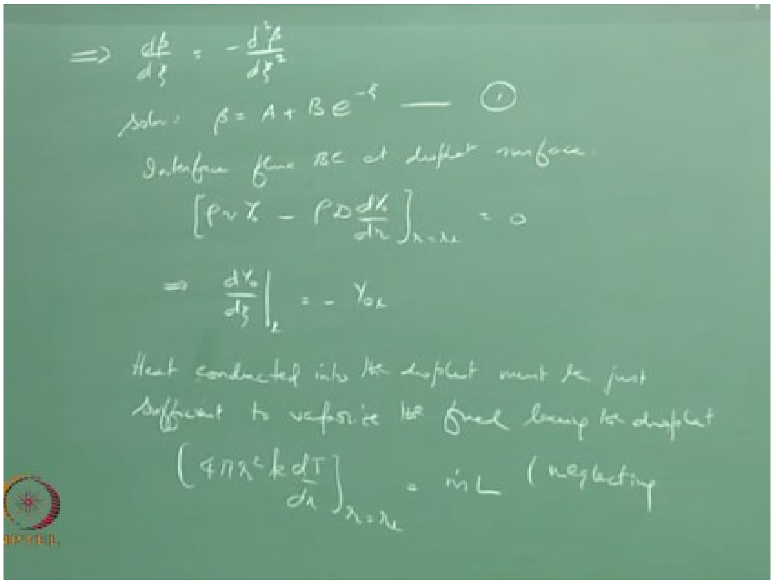

$$\Rightarrow \frac{d\beta}{d\xi} = -\frac{d^2\beta}{d\xi^2}$$

Soln: $\beta = A + B e^{-\xi}$ — ①

Interface flux BC at droplet surface:

$$\left[\rho v Y_O - \rho D \frac{dY_O}{dr}\right]_{r=r_l} = 0$$

$$\Rightarrow \left.\frac{dY_O}{d\xi}\right|_l = -Y_{OL}$$

Heat conducted into the droplet must be just sufficient to vaporize the fuel leaving the droplet

$$\left(4\pi r^2 k \frac{dT}{dr}\right]_{r=r_l} = \dot{m} L$$ (neglecting

So this is this is pretty much the same of course we are we are neglecting a few things here neglecting radiation let us say significant effect wherever you have phase interfaces some we be the notice this when we were adopting the Schvab's Zeldovih formulation with the my with the eleven assumptions and we ended neglect the here heat flux due to radiation and then at that point we noticed or we pointed out that radiation can be neglected inhomogeneous system but not in a heterogeneous system.

Heterogeneous system meaning when you have multiple phases okay physical phases so here we have a phase interface between liquid and solid gas where a radiation could be pretty important but of course we neglected for simplicity.

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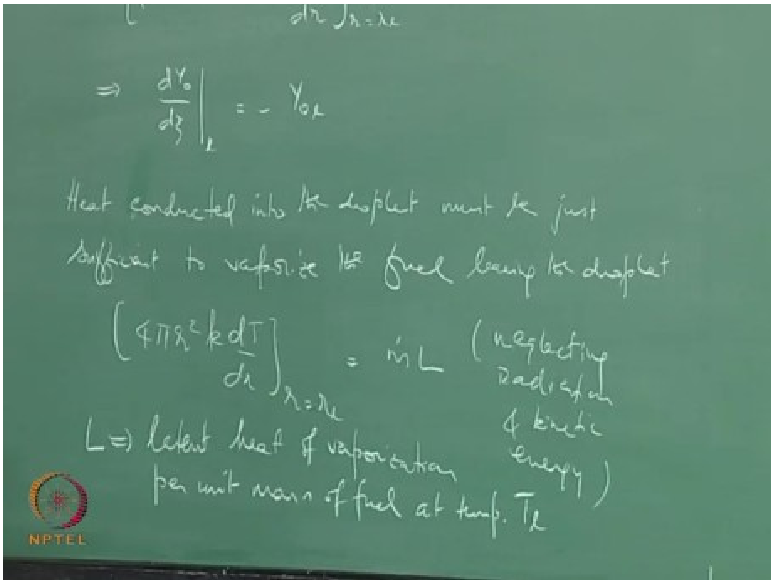

$$\Rightarrow \left.\frac{dY_o}{d\xi}\right|_l = -Y_{ol}$$

Heat conducted into the droplet must be just sufficient to vaporize the fuel leaving the droplet

$$\left[4\pi r^2 k \frac{dT}{dr}\right]_{r=r_l} = \dot{m}L$$ (neglecting Radiation & kinetic energy)

$L \Rightarrow$ latent heat of vaporization per unit mass of fuel at temp. T_l

So neglecting radiation and kinetic energy we are not worrying about this little the kinetic energy that we are going to have for the vapors that are coming out at a certain velocity because of the droplet regression that is significantly small, so this is reasonably okay to neglect it now L of course is the latent heat of vaporization they did not heat of vaporization per unit mass is the fuel at the temperature at T_L.

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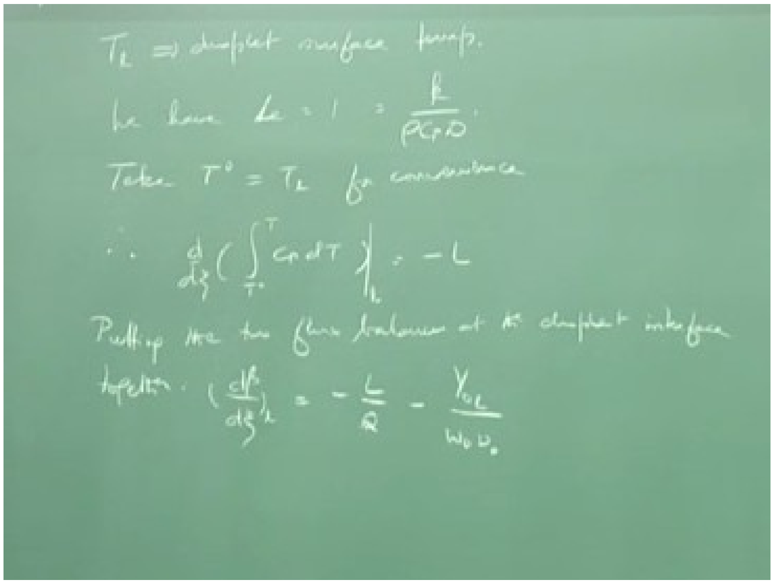

$T_L \Rightarrow$ droplet surface temp.

We have Le = 1 = $\frac{k}{\rho C_p D}$.

Take $T^0 = T_L$ for convenience

$$\therefore \frac{d}{d\xi}\left(\int_{T^0}^{T} C_p\, dT \right)\Big|_L = -L$$

Putting the two flux balances at the droplet interface together.

$$\left(\frac{d\beta}{d\xi}\right)_L = -\frac{L}{Q} - \frac{Y_{oL}}{W_o \nu_o}$$

Let us say that is adopted surface temperature T_L as I said if the dope that is reasonably small and you have a thermal equilibrium within the droplet the temperature is going to be pretty much the same at T_L whatever is the surface temperature is going to be the same uniformly throughout within it at the moment we are not really we interested in what is happening with them they drop that and that there is a reason why we are assuming this just sufficient that means we are not going to have any further heat conduction after the latent heat has been removed okay.

So now let us of course we have those number le = 1 which is k /ρ C_PD and of course with the other thing that we also do is take T_0 = T_L for convenience and this is not very difficult to swat on swallow because we have seen this before if you now take a different value and then you still insist on T_0 you can have two ∫ and we find that these things are actually showing up wherever you have αT or β showing up it is all in derivatives.

Therefore a constant addition is not a problem so they can we can do this so therefore this condition then is now going to translate to because we can now take for example course take the m. down here and then k can be replaced by ρ C_PD right so you replace k/ ρ C_PD and of course Cp can be stuck with the temperature and then you have $4\Pi r^2$ / m. coming from the other side and then we look at how ξ is defined and keep noting that $4\Pi r^2$ roadies in the denominator and the way ξ goes is inverse of ρ.

So we should for example they got a negative sign here right and similarly when you now try to write this in terms of the non-dimensional independent variable oxide you should you should have a d / d ξ ∫ T_0 to T Cp dT = - L that is a reason why you got the negative sign so if you now try to say I will divide this by Q and this by Wo νo which means I go back and divide this by Wo νo and this by Q I will have wherever I have a Yo I will have a αo and wherever I have ∫ T_0 to TCp dT / Q.

I will have a αT then subtract one from the other to get αT – αo which means we get the β right so putting the two flux balances at the droplet interface together we get dβ / d ξ of course I should go back and write this at L so dβ d ξ at L = -L / Q – Yo L/ Wo ν o.

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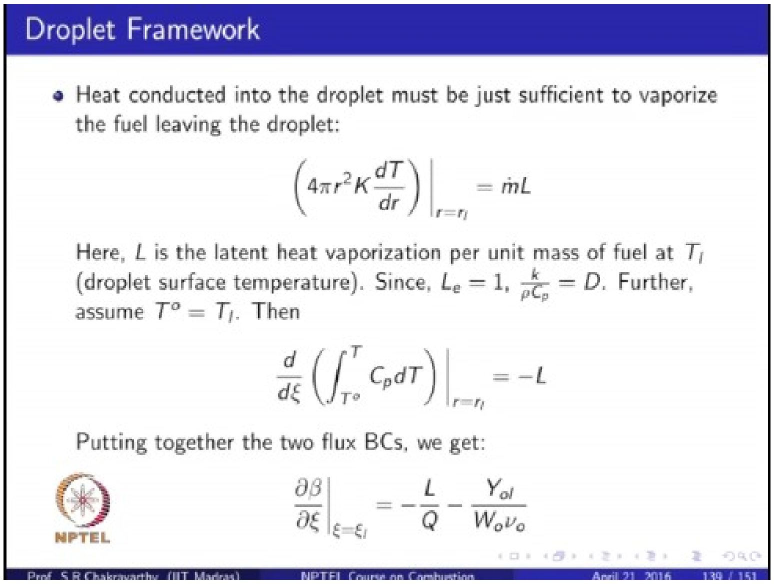

Droplet Framework

- Heat conducted into the droplet must be just sufficient to vaporize the fuel leaving the droplet:

$$\left(4\pi r^2 K \frac{dT}{dr}\right)\bigg|_{r=r_l} = \dot{m}L$$

Here, L is the latent heat vaporization per unit mass of fuel at T_l (droplet surface temperature). Since, $L_e = 1$, $\frac{k}{\rho C_p} = D$. Further, assume $T^o = T_l$. Then

$$\frac{d}{d\xi}\left(\int_{T^o}^{T} C_p dT\right)\bigg|_{r=r_l} = -L$$

Putting together the two flux BCs, we get:

$$\frac{\partial\beta}{\partial\xi}\bigg|_{\xi=\xi_l} = -\frac{L}{Q} - \frac{Y_{ol}}{W_o\nu_o}$$

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Alright now this is the interface condition in addition to this we need to have the regular delay boundary conditions right so the deliciously boundary conditions at the at the boundaries to the domain or the.

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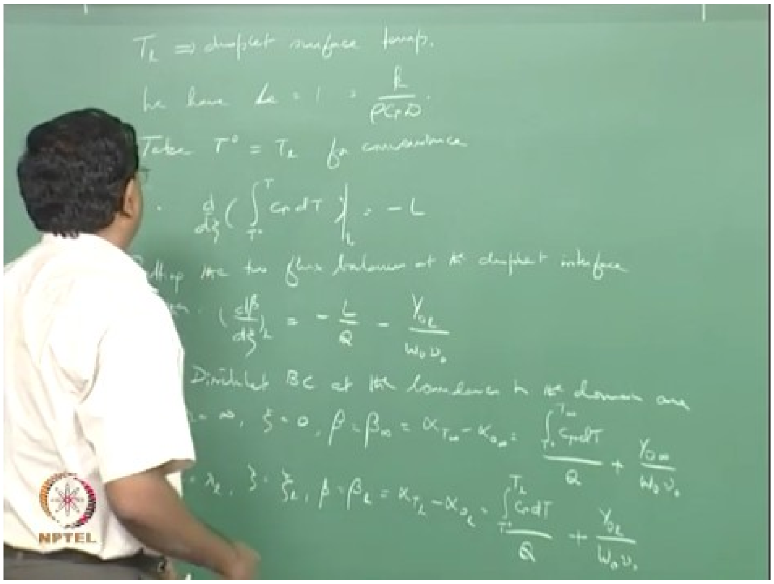

$T_L \Rightarrow$ droplet surface temp.

We have Le = 1 = $\frac{k}{\rho C_p D}$.

Take $T^\circ = T_L$ for convenience

$\therefore \frac{d}{d\xi}\left(\int_{T^\circ}^{T} C_p\,dT\right)\Big|_L = -L$

up the two flux balances at the droplet interface

$\left(\frac{d\beta}{d\xi}\right)_L = -\frac{L}{Q} - \frac{Y_{O_L}}{W_O \nu_O}$

Dirichlet BC at the boundaries to the domain are

$= \infty,\ \xi = 0,\ \beta = \beta_\infty = \alpha_{T_\infty} - \alpha_{O_\infty} = \frac{\int_{T_0}^{T_\infty} C_p\,dT}{Q} + \frac{Y_{O_\infty}}{W_O \nu_O}$

$= r_L,\ \xi = \xi_L,\ \beta = \beta_L = \alpha_{T_L} - \alpha_{O_L} = \frac{\int_{T^\circ}^{T_L} C_p\,dT}{Q} + \frac{Y_{O_L}}{W_O \nu_O}$

Little a boundary conditions domain or we should say at r = ∞ ξ = 0 okay at r = ∞ ξ = 0 right and what would that mean we would say what would be a β your β would be whatever is α TNR for at r= ∞ which means whatever is the temperature T ∞ right at at far away and whatever is Yo ∞ so for example if you have air as your ambience your y 0 ∞ will be like 0.23 because only about 23% by mass is going to be oxygen the other one the rest is going to be nitrogen.

So you have a diluents present in your Amiens in any oxidizing ambient which can be taken into account you can also take into account the ambient temperature and keep in mind if we do not have to worry about a combustion heat release then we could still solve the evaporation problem if your T ∞ is higher than T_L all right and that is going to basically mean that you have heat conduction simply from the ambience not necessarily from the flame.

So the T∞ is going to play a role and as a matter of fact you know this is actually the sensible enthalpy term and this is the heat release term so this is already showing up as a competition between the two okay so we will see distinct lot more clearly as we go along but essentially what it means is β can be = β∞ which is α T∞ - αo ∞which is nothing but T_0 to T∞ CP dT /Q – or should I say + Yo ∞ / Wovo that is the one boundary condition at r =rl we are kind of coming in words because that is how ξ increases from 0.

So ξ is now going to be = some ξL which is saying ξL = rL ∞ rl to ∞ right and so ξL and β will have some value βL which is nothing but α TL - αoL which is ∞T_0 to T_L CP dT / Q + Yo.

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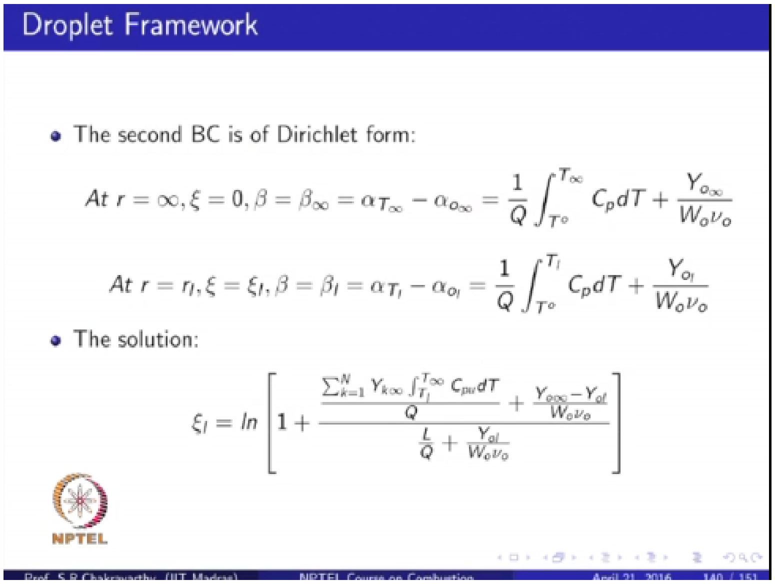

Droplet Framework

- The second BC is of Dirichlet form:

$$At\ r = \infty, \xi = 0, \beta = \beta_\infty = \alpha_{T_\infty} - \alpha_{o_\infty} = \frac{1}{Q}\int_{T^o}^{T_\infty} C_p dT + \frac{Y_{o_\infty}}{W_o \nu_o}$$

$$At\ r = r_l, \xi = \xi_l, \beta = \beta_l = \alpha_{T_l} - \alpha_{o_l} = \frac{1}{Q}\int_{T^o}^{T_l} C_p dT + \frac{Y_{o_l}}{W_o \nu_o}$$

- The solution:

$$\xi_l = ln\left[1 + \frac{\frac{\sum_{k=1}^{N} Y_{k\infty}\int_{T_l}^{T_\infty} C_{pu} dT}{Q} + \frac{Y_{o\infty} - Y_{ol}}{W_o \nu_o}}{\frac{L}{Q} + \frac{Y_{ol}}{W_o \nu_o}}\right]$$

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Wo No we have already seen that if you now take T_0 =T_L for convenience this would not have any contribution but that is alright let us just at the moment were worried only about βl that is not worried about how it is exactly defined we will ultimately see that what matters is not exactly each of these β∞ - βL in which case we can now put these things together and these ∫s will now go together and then you will get an ∫ goes from T_L to T∞.

Okay so the T the T superscript not gets out of the way anyway so good that means we are now ready to attack the solution better will be armed with these so let us do one by one so let us say how do I number it is yeah I think I should I should mention what is ξl here for the record where ξ l is m . ∫ r_L the ∞4 Π r^2 ρd to the negative 1 dr okay and I think we should probably call this as to right.

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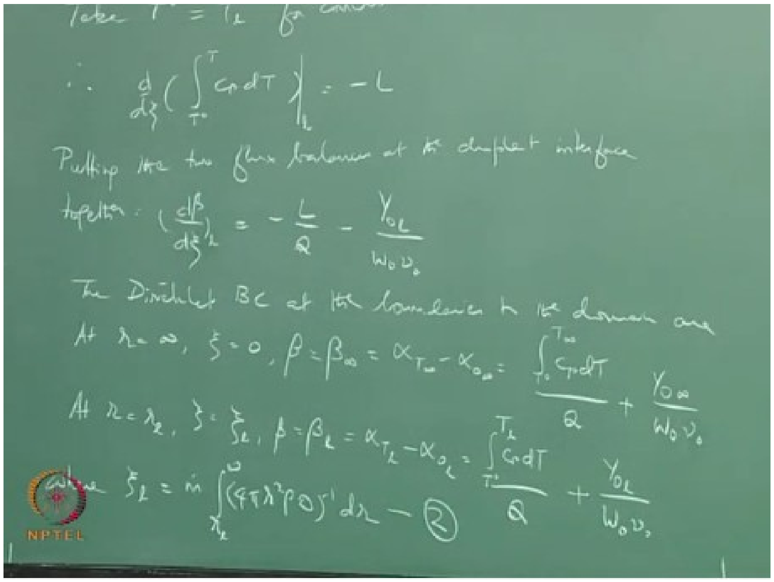

Take $T^{\circ} = T_l$ for

$$\therefore \frac{d}{d\xi}\left(\int_{T^{\circ}}^{T} c_p\,dT\right)\Bigg|_l = -L$$

Putting the two flux balances at the droplet interface together: $\left(\frac{d\beta}{d\xi}\right)_l = -\frac{L}{Q} - \frac{Y_{O_l}}{W_O \nu_O}$

The Dirichlet BC at the boundaries to the domain are

At $r = \infty$, $\xi = 0$, $\beta = \beta_\infty = \alpha_{T_\infty} - \alpha_{O_\infty} = \frac{\int_{T^{\circ}}^{T_\infty} c_p\,dT}{Q} + \frac{Y_{O\infty}}{W_O \nu_O}$

At $r = r_l$, $\xi = \xi_l$, $\beta = \beta_l = \alpha_{T_l} - \alpha_{O_l} = \frac{\int_{T^{\circ}}^{T_l} c_p\,dT}{Q} + \frac{Y_{O_l}}{W_O \nu_O}$

where $\xi_l = \dot{m}\int_{r_l}^{\infty} (4\pi r^2 \rho D)^{-1}\,dr$ — ②

So we call this is one and okay now.

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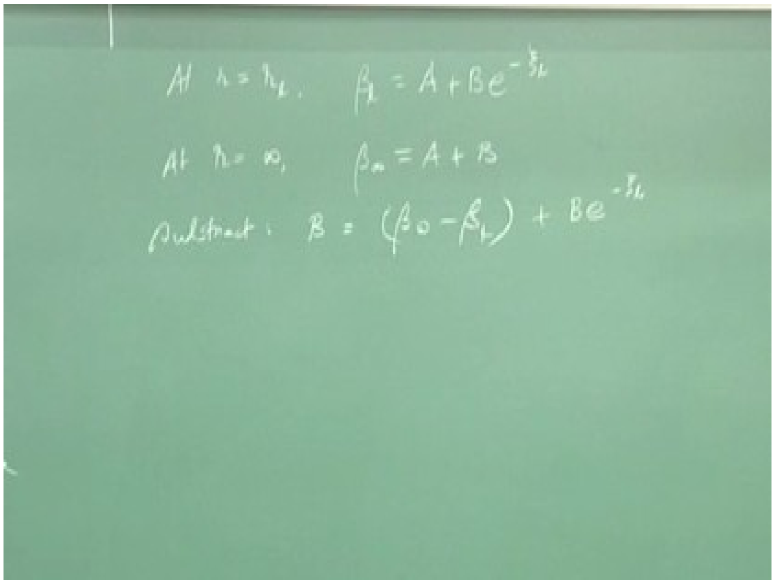

At $r = r_L$, $\beta_L = A + Be^{-\xi_L}$

At $r = \infty$, $\beta_\infty = A + B$

Subtract: $B = (\beta_\infty - \beta_L) + Be^{-\xi_L}$

So the first thing we can do is at the r = r_L β_L = a + b e $^{-\xi L}$ r= ∞ β∞ is simply = A + B because ξ = 0 so e to the 0 is going to be 1 and subtract we get rid of A and actually you know by now you should get the idea of what the drift is we are not really interested in solving for α mean bit as it off okay that is done that is not really our problem a problem is to find ξL so that from ξL we can get an idea of m .

And m. is what we are basically looking for we are not looking for the actual profiles so the real problem is what is the rate at which the droplet is shrinking okay everything else is a detail and so we will solve the we will keep our focus on the problem all these details like how exactly β is varying that means how is the temperature going to vary how is oxidized a concentration going to vary and all these things are all exam questions or PhD qualifying exam questions and so on okay.

So that is for you to figure out but we will we will focus on our problem that means at the moment we can just subtract these two get rid of the A we are not going to bother about evaluating it for ourselves and we are simply going to get B =β ∞ - βL + Be $^{-\xi L}$ right that is true so B= β∞ - βL + B.

Right then how question is what is Bβ ∞ - βL but β∞ - βL is nothing but you just have to this is this is β∞ this is βL so this is β∞ this is βL subtract one from the other so you get Σ K=1 to n.

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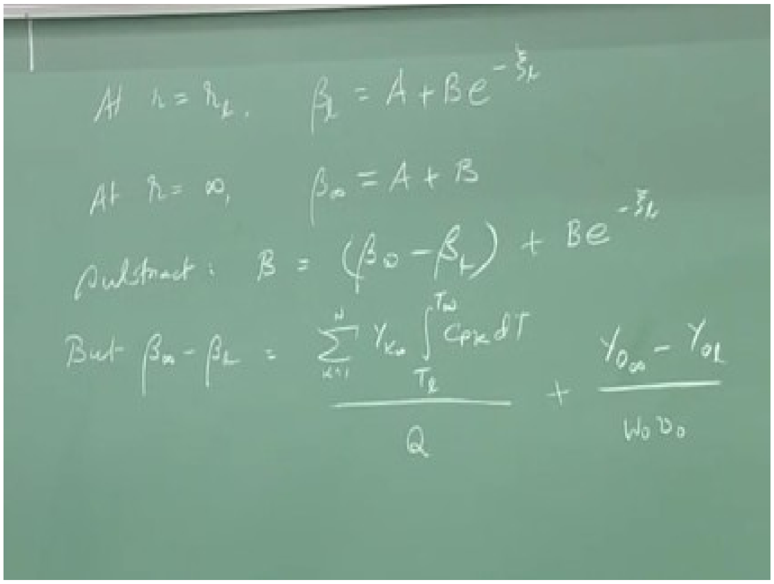

At $r = r_L$, $\beta_L = A + Be^{-\xi_L}$

At $r = \infty$, $\beta_\infty = A + B$

Subtract: $B = (\beta_\infty - \beta_L) + Be^{-\xi_L}$

But $\beta_\infty - \beta_L = \dfrac{\sum_{k=1}^{N} Y_{k\infty} \int_{T_L}^{T_\infty} C_{pk}\, dT}{Q} + \dfrac{Y_{O\infty} - Y_{OL}}{W_O \nu_O}$

Yk ∞ T_L to p ∞ Cpk dT that say that is a fancy way of writing CP simply you could have taken the Yk inside and but you want to show the ∞ very clearly /Q and the reason why we are not really worrying about YkL is because as I said here if $T_L = T_0$ this is not going to contribute right and + Yk sorry Y from Yo∞ - YoL / Wo vo right so that is what we get then we have to use the interface condition.

So what we have now is we have B = this + Be to the – ξL with which you can evaluate B all right but is we are not really bothered about as I said evaluating be we are we are not interested in evaluating a as well you are not interested in finding β as what we are really interested in is to apply the interface boundary condition, so that we can even eliminate B okay so fortunately the interface boundary can endure interface flux condition put together in terms of β has it in terms of derivative.

So if you have it in terms of derivative A automatically drops out you try to take a derivative here A does not even show up B shows up okay and between that and this we will eliminate B as well okay that is it what we are going to do so using the interface flux balance right we get $Be^{-\xi L}$ = L / Q dβ ξ is going to have a -Be to the -e ξ right and evaluated at L right so we should have a negative sign.

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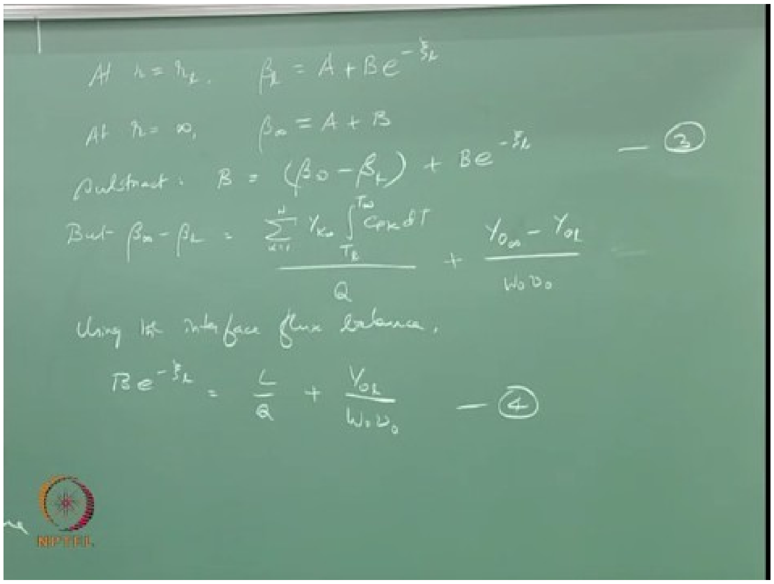

At $r = r_L$, $\beta_L = A + Be^{-\xi_L}$

At $r = \infty$, $\beta_\infty = A + B$

Subtract: $B = (\beta_0 - \beta_L) + Be^{-\xi_L}$ — ③

But $\beta_\infty - \beta_L = \dfrac{\sum_{k=1}^{N} Y_{k\infty} \int_{T_L}^{T_\infty} c_{pk}\, dT}{Q} + \dfrac{Y_{O\infty} - Y_{OL}}{W_O D_O}$

Using the interface flux balance,

$Be^{-\xi_L} = \dfrac{L}{Q} + \dfrac{Y_{OL}}{W_O D_O}$ — ④

But it also had a negative sign here which I am going to turn it as positive right, so we now have this as L / Q / + YoL / Wo vo if you call this as your 3 okay I guess this is your 3 and this is your 4 then what you do is we now use a 4 and we now use 4 and 3 together in order to get e to the -X ξ and use what is the definition of ξL that is what we do.

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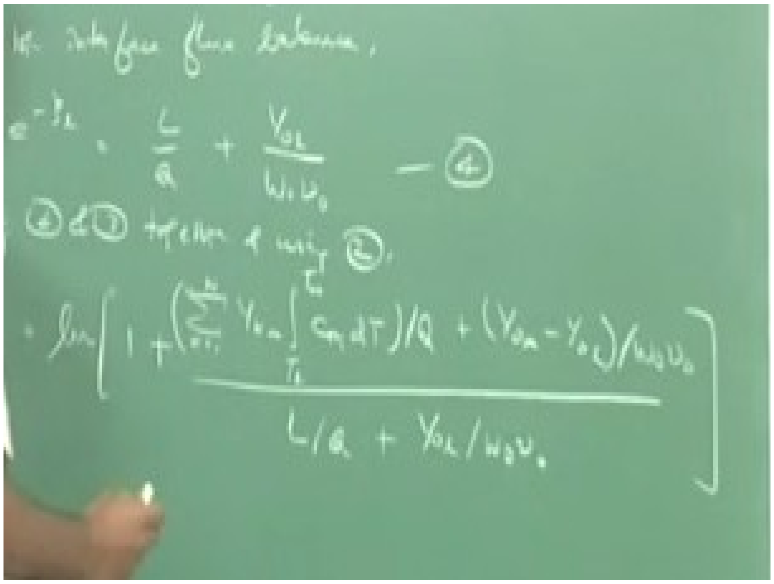

So putting 4 and 3 together and using 2 right we get ξ L = natural logarithm you got the natural logarithm because between these 2 if you now try to get rid of B you will be working with e $^{-\xi L}$ okay and if you now try it extracts I will out of it you will get a natural logarithm okay so natural logarithm of 1 + Σ K=1 to n Yk ∫ ∞ TL to T∞ CPi Dt/ Q + Yo∞ - YoL / Wo νo so that is coming from here / L/ Q + Yo L/ W No that is coming from here okay and this 1 + was coming because you had a B and a B there.

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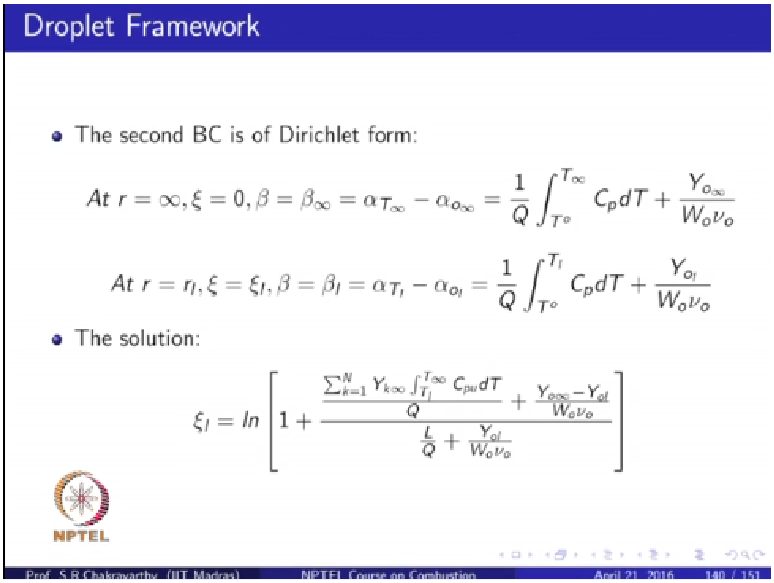

Droplet Framework

- The second BC is of Dirichlet form:

$$At\ r = \infty, \xi = 0, \beta = \beta_\infty = \alpha_{T_\infty} - \alpha_{o_\infty} = \frac{1}{Q}\int_{T^o}^{T_\infty} C_p dT + \frac{Y_{o_\infty}}{W_o \nu_o}$$

$$At\ r = r_l, \xi = \xi_l, \beta = \beta_l = \alpha_{T_l} - \alpha_{o_l} = \frac{1}{Q}\int_{T^o}^{T_l} C_p dT + \frac{Y_{o_l}}{W_o \nu_o}$$

- The solution:

$$\xi_l = ln\left[1 + \frac{\frac{\sum_{k=1}^{N} Y_{k\infty}\int_{T_l}^{T_\infty} C_{pu} dT}{Q} + \frac{Y_{o\infty} - Y_{ol}}{W_o \nu_o}}{\frac{L}{Q} + \frac{Y_{ol}}{W_o \nu_o}}\right]$$

So you could have a 1 + so that is that is the mathematics and then in this you now views 2 because ξL is this definition so you now get a in dimensional form.

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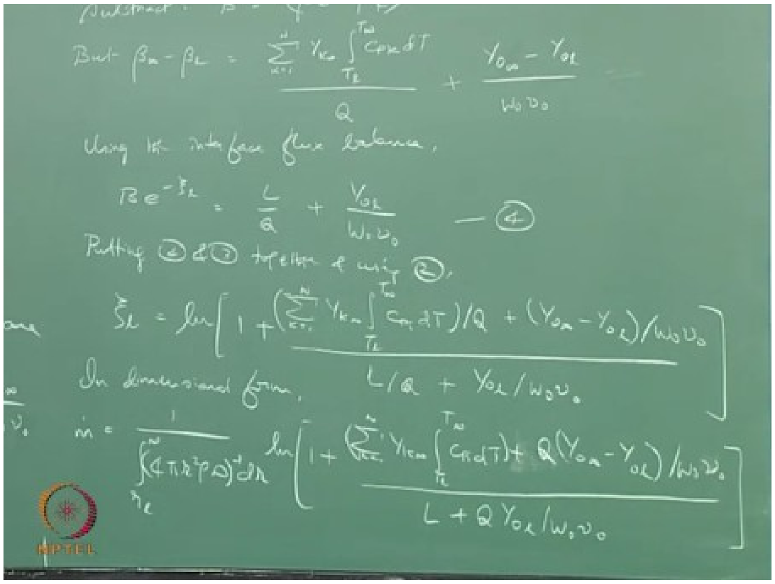

But $\beta_\infty - \beta_\ell = \dfrac{\sum_{k=1}^{N} Y_{k\infty}\int_{T_\ell}^{T_\infty} C_{pk}\,dT}{Q} + \dfrac{Y_{O\infty}-Y_{O\ell}}{W_O\nu_O}$

Using the interface flux balance,

$B e^{-\xi_\ell} = \dfrac{L}{Q} + \dfrac{Y_{O\ell}}{W_O\nu_O}$ — ④

Putting ④ & ③ together & using ②,

$$\xi_\ell = \ln\left[1 + \frac{\left(\sum_{k=1}^{N} Y_{k\infty}\int_{T_\ell}^{T_\infty} C_{pk}\,dT\right)/Q + (Y_{O\infty}-Y_{O\ell})/W_O\nu_O}{L/Q + Y_{O\ell}/W_O\nu_O}\right]$$

In dimensional form,

$$\dot{m} = \frac{1}{\int_{r_\ell}^{\infty}(4\pi r^2\rho D)^{-1}dr}\ln\left[1 + \frac{\left(\sum_{k=1}^{N} Y_{k\infty}\int_{T_\ell}^{T_\infty} C_{pk}\,dT\right) + Q(Y_{O\infty}-Y_{O\ell})/W_O\nu_O}{L + Q\,Y_{O\ell}/W_O\nu_O}\right]$$

In dimensional form m. is going to be1 / ∫ rL to ∞ 4 Πr^2 ρd to the negative 1 dr natural logarithm just for the record got to write the whole thing again 1+ Σ k =1 to n Yk ∞ PL / T∞ CPi dT/ Q+ Yo∞ - Yo L / wo v0 /L stop we could do a little bit of manipulation if you now try to multiply Q throughout both numerator and denominator you could write this as + Q and this would be + QYo L / Wo v0 of course in the final videos we can edit out this time it took for it to write A and then we can just show the final expression written again.

And here we will basically point out that we are multiplied the numerator and denominator throughout by Q so that the Q shows up here and here instead of at the bottom of this in this so that is a final expression for m. and as usual we do not understand this okay, so we get some complicated looking expression which just does not make sense we started out thinking we wanted to get the d^2 law okay the d^2 law is essentially saying that if you know or looking at the rate at which the evaporation happens the time of evaporation is proportional to square of the diameter.

And we notice that the square of the diameter is going to be sort of like the surface area and we also then notice that if you now look at the m. the mass flux the square of the radius shows up which means the square of the diameter shows up and this in fact is the surface area 4 Πr^2 or it is quite a surface area of the droplet so all that seems okay and now we got n m. how in the world are we supposed to get the d^2 law from this okay.

So or how are we supposed to now find out how the d^2 varies with time for the droplet regression okay we have adopted a quasi-steady assumption by which in the instantaneous snapshot at a particular time the m. is like this without any time dependence any further okay the time dependence if you have gotten rid off is the smalltime scale that that it takes for the gases to just diffuse in convent at a particular snapshot freeze-frame time.

And from this time for you to get to the next time when the droplet regresses significantly takes a lot of time and then will now look at the bitch now and then do this balance and then figure out what the current m . as and so on that is what we are trying to do so if you want to try to understand this further it is it will be helpful if we do some more simplifications okay so let us try to do that.

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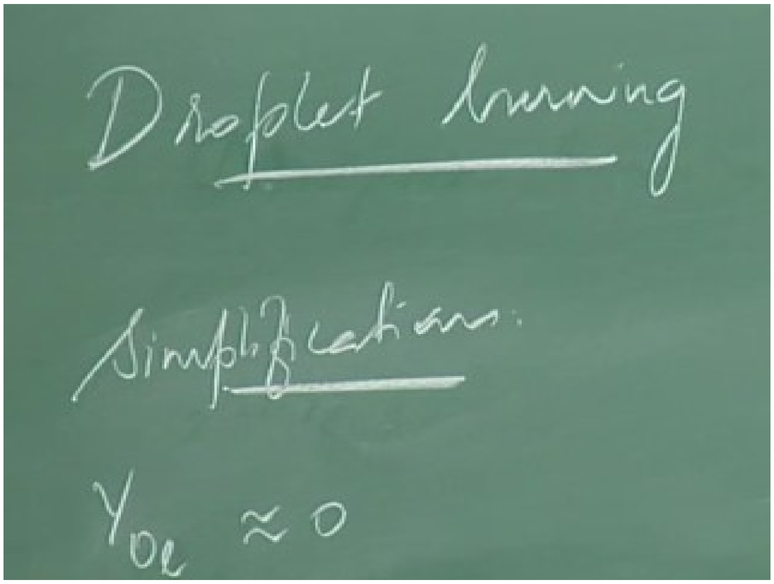

Droplet burning

Simplifications:

$Y_{Ol} \approx 0$

So let us try to do some simplifications here the first thing that we will say is Yol is approximately = 0 that means we are saying that the droplet the deoxidizer as it diffuses all the way to the droplet surface is pretty much of 0 concentration or put it another way all the species that you have at the droplet surface is going to be mainly fuel vapor so if all of it is going to be completely displaced all oxidizer.

So you do not really have any oxidizing species this is one way of looking at it in a in a mostly evaporation situation in a combustion situation it is not more convenient because we expect that the fuel vapor and the oxidizer vapor are going to diffuse against each other radially and meet at stoichiometric proportions where the flame is and then get consumed there right so if you if you know took a magnifying glass and looked at this particular sheet we should now be able to find out how these things gradually decline the oxidizer constantly.

The fuel concentration gradually declines up to this flame and then rapidly declines because of consumption and the oxidizer concentration gradually declines as you go radially inward because of diffusion and then rapidly decline across the flame right.

And therefore at the droplet surface you are not going to get any oxidizer at all so Y0L can be assumed to be more or less 0 without much of a problem the exception see when it anytime you make a assumption which is simplifying you have to look at how good it is or when would it not be held and the answer is this is not really quite true near extinction conditions that is when the droplet is going to actually extinguish right you have a rather weak flame and you have ample opportunity for the oxidizer to diffuse past the flame in spite of going through reactions and consuming getting consumed quite a bit because it is not getting completely consumed right.

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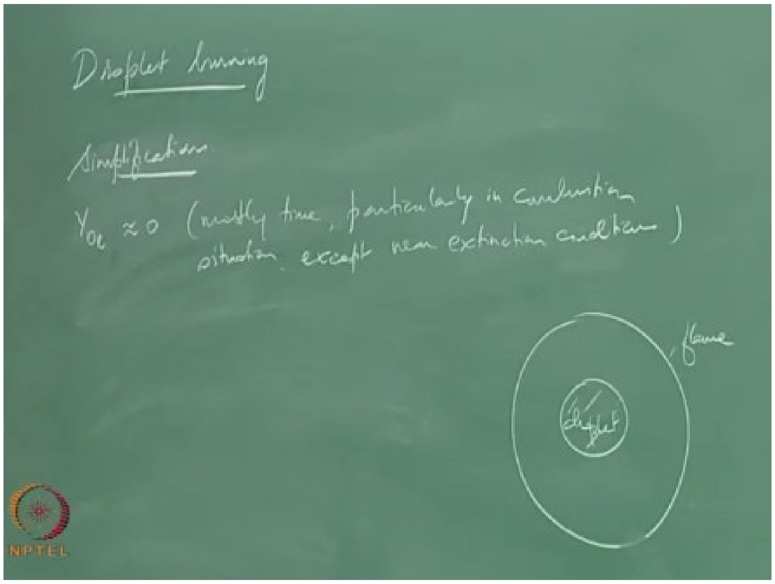

Droplet burning

Simplifications

$Y_{OL} \approx 0$ (mostly true, particularly in combustion situation except near extinction conditions)

So yeah this is mostly true a particularly in combustion situation except near extinction conditions so that is one simplification what would that give me that will get rid of the YoL here right that we get rid of the YoL here so it is kind of like we could send this to a hairdresser and get it to be nicely dressed up and look like a very simple expression right so that we are trying to get rid of some of these terms that way.

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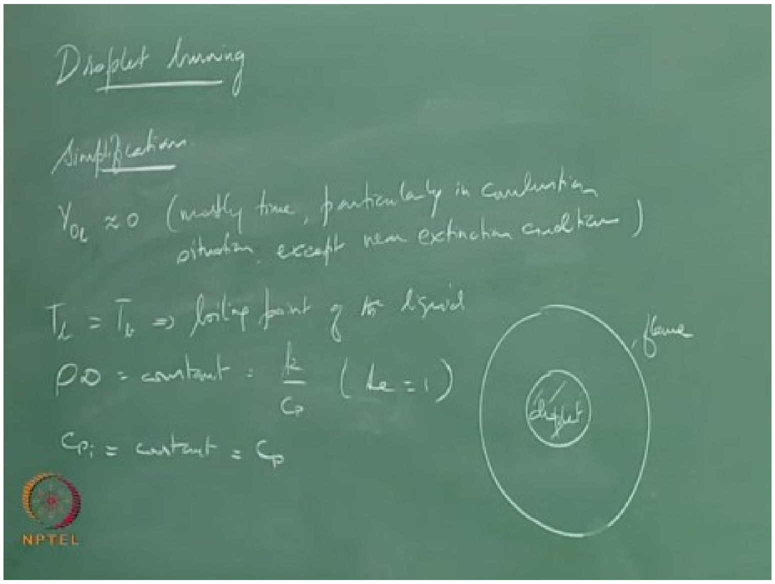

Droplet burning

Simplification

$Y_{OL} \approx 0$ (mostly true, particularly in combustion situation except near extinction condition)

$T_L = T_b \Rightarrow$ boiling point of the liquid

ρD = constant = $\frac{k}{C_p}$ ($Le = 1$)

C_{P_i} = constant = C_p

Let us also assume that Tl = TB which is the boiling point of the liquid that is a simplifying assumption in reality we have to actually look at something called a saturation temperature and so on which depends on the ambient what you call the ambient composition of its vapor and soon so you have to look at vapor pressure balancing the actual pressure and all those things now we let some very worried about all that will simply say T_L the droplet the temperature is going to be the same as the boiling point.

And let us assume that ρD is a constant which is = k / CP okay because L = 1 and suppose CPi = constant = CP.

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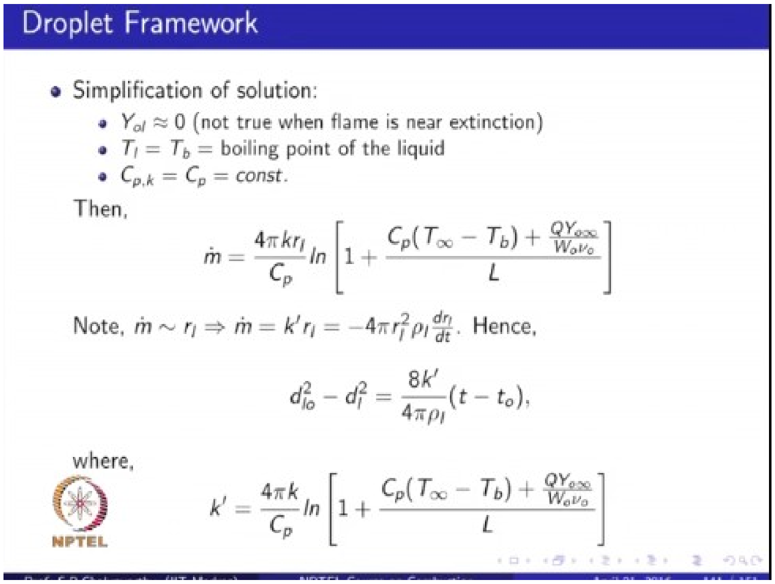

Droplet Framework

- Simplification of solution:
  - $Y_{ol} \approx 0$ (not true when flame is near extinction)
  - $T_l = T_b =$ boiling point of the liquid
  - $C_{p,k} = C_p = const.$

Then,

$$\dot{m} = \frac{4\pi k r_l}{C_p} ln \left[1 + \frac{C_p(T_\infty - T_b) + \frac{QY_{o\infty}}{W_o \nu_o}}{L} \right]$$

Note, $\dot{m} \sim r_l \Rightarrow \dot{m} = k' r_l = -4\pi r_l^2 \rho_l \frac{dr_l}{dt}$. Hence,

$$d_{lo}^2 - d_l^2 = \frac{8k'}{4\pi\rho_l}(t - t_o),$$

where,

$$k' = \frac{4\pi k}{C_p} ln \left[1 + \frac{C_p(T_\infty - T_b) + \frac{QY_{o\infty}}{W_o \nu_o}}{L} \right]$$

All CP are the same and it is constant right so make these assumptions and what do you have we get so with which we can now try to evaluate this ∫ also right so ρD gets pulled out of the ∫ and you can you can you can integrate dr / 4 Πr ^{2} right that is not a problem and then substitute the limits you get m .= 4 Π kr L /CP that is what ρD has.

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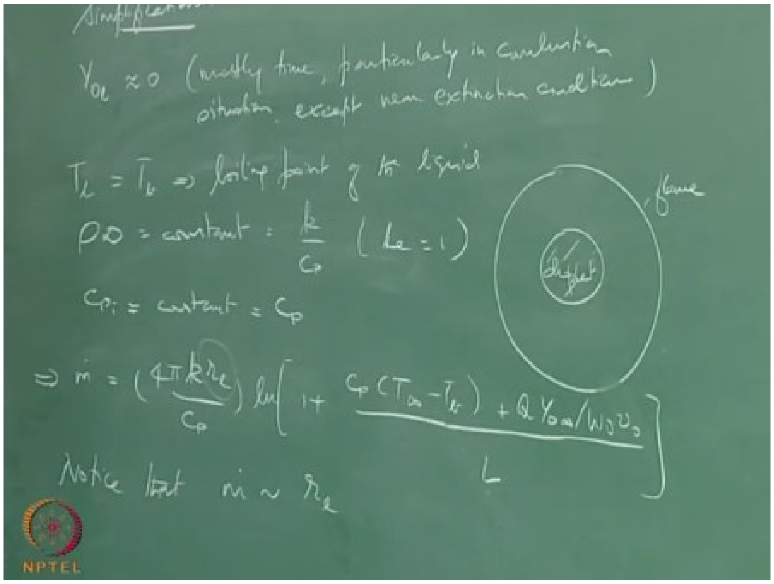

Simplifications

$Y_{O\ell} \approx 0$ (mostly true, particularly in combustion situation except near extinction condition)

$T_\ell = T_b \Rightarrow$ boiling point of the liquid

ρD = constant = $\frac{k}{c_p}$ $(Le = 1)$

c_{p_i} = constant = c_p

$$\Rightarrow \dot{m} = \left(\frac{4\pi k r_\ell}{c_p}\right) \ln\left[1 + \frac{c_p(T_\infty - T_b) + QY_{O\infty}/W_O\nu_O}{L}\right]$$

Notice that $\dot{m} \sim r_\ell$

Now become times natural logarithm1 + CP can now evaluate the ʃinside as well as simply T ∞-TB + QYo∞ / should say we have a Wo ν0 /L right okay you still have some expression but this is a little bit more manageable we can we can try to understand something more let us just think about it for a minute okay what this means is this is the sensible enthalpy okay for the gas going from the boiling point of the droplet to the ambient temperature.

So this is the sensible enthalpy rise of the gas that is coming out of the droplet should actually be corresponding to the heat of combustion so for the moment let us not worry about the heat of combustion okay let us suppose that we are thinking about evaporation that means you have an ambient hot gas that is trying to evaporate the droplet and that means it is simply giving rise to a sensible enthalpy rise by L so the L is what is actually is the latent heat that is required first of all for you to vaporize.

And then the next thing is you have to raise the temp sensible enthalpy okay so the m. is dictated by how much you are able to evaporate versus how well you are able to rise a sensible enthalpy of whatever we have evaporated if you now throw in combustion you now have this term in addition okay so the heat that is available to you goes first to evaporate the droplet and second to raise the sensible enthalpy okay so you can see how these three processes essentially heat release that is available to you during combustion can go on to raise a sensible enthalpy in competition with evaporating the droplet okay.

So the this is what is actually dictating this m . now what we are interested is in trying to find out what is the relationship between or how is this rL going to change in time okay that is what we are really interested in so for this purpose we can we can see that.

So note notice that m . is now linearly proportional to rL right so say m. = k’ r L the reason why we are using k’ k is reserved for the evaporation constant in the d^2 law and that is what we are trying to deduce okay we are trying to reduce the d ^{2} law and therefore we want to see how what is the dependence of the evaporation constant on the system parameters namely like the ambient gas composition ambient gas temperature the boiling point of the liquid the latent heat of the the delivered and so on.

Right so we want to know what the k is and so we now keep this as the k’ so where k’ is = accept or l we have to write everything else which is 4 Πk / CP natural logarithm 1 +CP T∞ - TB + QYo∞ / Wo νo/L right so keeping there is a k’the next few minutes we are simply going to ignore all this all the stuff and go and basically say m. is = k’ rL and let us see what that means right.

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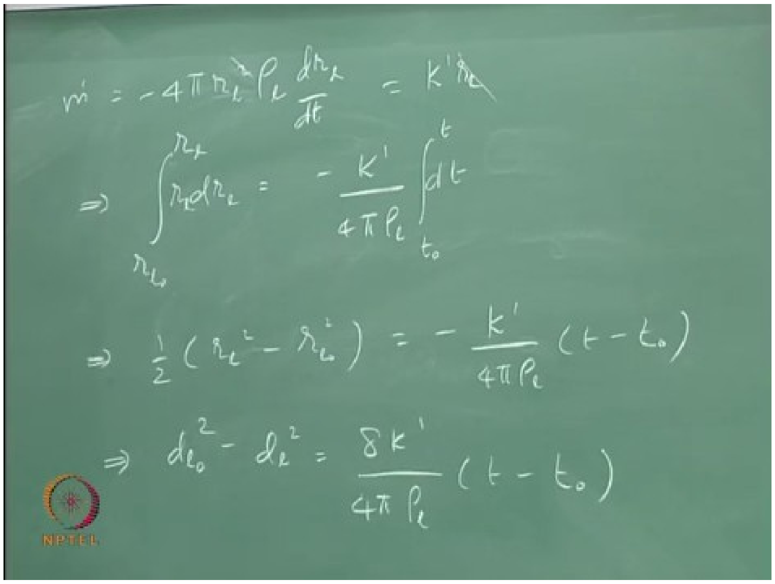

$$\dot{m} = -4\pi r_L^{\cancel{2}} \rho_L \frac{dr_L}{dt} = K'\cancel{r_L}$$

$$\Rightarrow \int_{r_{L_0}}^{r_L} r_L dr_L = -\frac{K'}{4\pi\rho_L}\int_{t_0}^{t} dt$$

$$\Rightarrow \frac{1}{2}\left(r_L^2 - r_{L_0}^2\right) = -\frac{K'}{4\pi\rho_L}(t-t_0)$$

$$\Rightarrow d_{L_0}^2 - d_L^2 = \frac{8K'}{4\pi\rho_L}(t-t_0)$$

So m . m . on the other hand is negative 4 ΠrL^2 ρL d rL /dT right this T is now at the time scale of the droplet regression it is not a shorter time scale of the gases equilibrium in diffusion and convection so this is the kind of time that you are looking for in the d^2 law so this is now equal to k' rL which means we can now cancel the rL^2 one of the rL and rL^2 with this and from this we can now write rL drl = negative k' / 4 Π ρl dt all right then just go back and write this with let us say rL0 to rL any rL starting from T0 to any T and that is now beginning to show up as the d^2 law.

So this is 1/2 rL^2 - rL_0^2 or okay let us just keep it that way and we have a negative sign to fix things so we have a 4 Π ρl t- t_0 and of course now this negative sign will allow us to flip things because rL0 is actually greater than order therefore we want to keep things positive and keep in mind rL is nothing but dl over 2 so rl^2 will be dl^2 /4 and so the 4 and 2 are going to get together to make an eight and then of course you have a 4 Π that is going to go away and k' also has a 4Π.

So do not get mean hurry to cancel that with a 8 okay so dl_0^2 - dl^2 i= 8k'/ 4 Π ρl t – t0 so now it is beginning to look like the d^2 law bye couple of minutes in about 10 minutes ago this did not look like the d^2 law at all okay but that is the d^2 law there and so we have the d^2 law.

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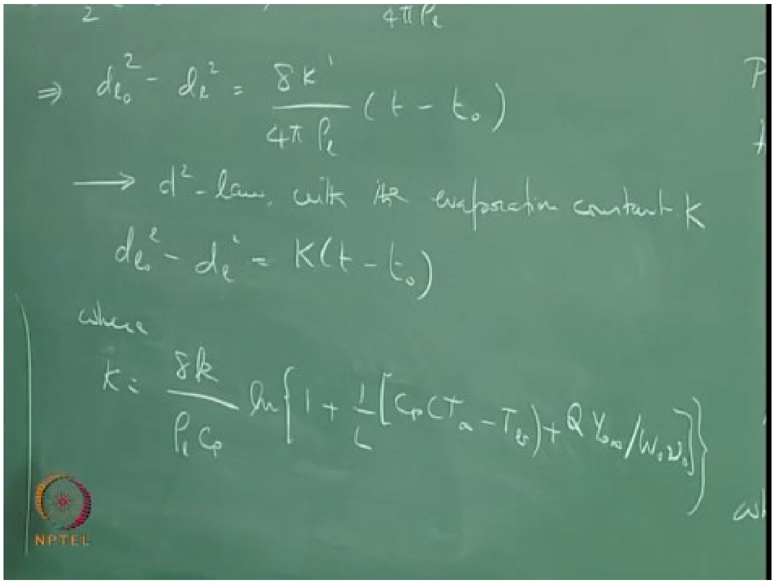

$$\Rightarrow d_{\ell_0}^2 - d_\ell^2 = \frac{8k'}{4\pi\rho_\ell}(t-t_0)$$

→ d^2-law, with the evaporation constant K

$$d_{\ell_0}^2 - d_\ell^2 = K(t-t_0)$$

where

$$K = \frac{8k}{\rho_\ell C_p}\ln\left\{1+\frac{1}{L}\left[C_p(T_\infty - T_\ell)+Q\,Y_{O\infty}/W_O\nu_O\right]\right\}$$

So with the evaporation constant k right so what is the d^2 law just for the sake of completeness d^2 dl0 ^{2} = k times t - t_0 k = say of evaporation constant k right where k = these are put things together 8 k'/ 4 Π ρL and then of course you have F so you just divide this by 4 Πρl and then pull through in an 8 and you are simply going to get a 8 k /ρl CP natural logarithm 1+ 1 /l C_PT∞ - TB+ QY0 ∞ / Wo ν 0.

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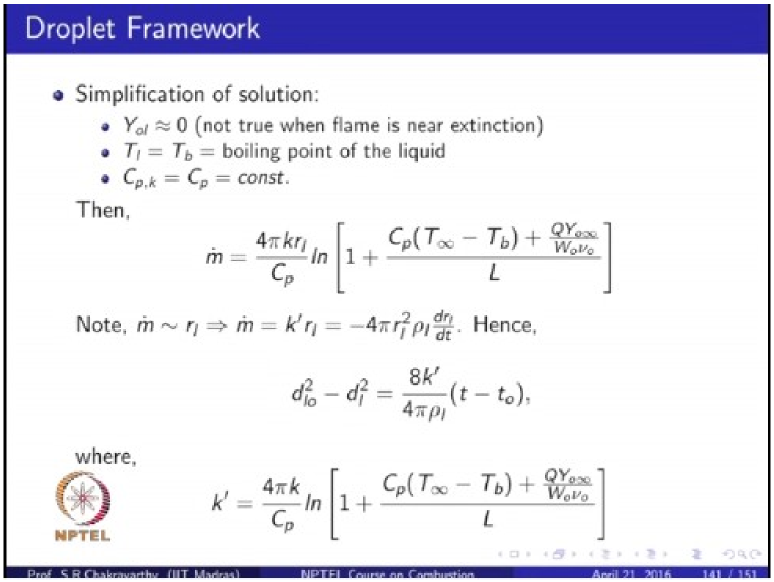

Droplet Framework

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where,

$$k' = \frac{4\pi k}{C_p} ln \left[1 + \frac{C_p(T_\infty - T_b) + \frac{QY_{o\infty}}{W_o \nu_o}}{L}\right]$$

Right and as I said all the dependences that we are looking for over in the evaporation constant you can also use this for evaporation as well as combustion all you have to do is neglect the heat of combustion for getting the evaporation if we want to keep the combustion then keep the heat of every heat of combustion and you will get an additional evaporation rate okay so that means because the combustion it burns faster it regresses faster that is what it simply means so but we should stop doing droplet burning and we will see what else we need to do next week.

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