Indian Institute of Technology Madras

NPTEL
National Programme on Technology Enhanced Learning

COMBUSTION

Lecture 50
Detonations

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So what we the last topic that we are going to take up in this course is detonations we will spend some time talking about detonation waves the structure of how we have proceeded is as follows we went through the rankine Hugoniot at some stage where we pointed out that the wrong kind Hugoniot they did the you Hugoniot curve breaks up into the upper branch in the lower branch and the lower branch corresponds to the deflagration of waves and the upper branch correspond to the detonation branch.

And would they be then we then said we will now focus on deflagration right now the major problem that we had with the rankine Hugoniot was that we were assuming that there is a inherent flame speed associated with it with which we could construct the railway line because the Rayleigh line the slope of the railway line is equal to _M 0 squared where M0 is the mass flux of the flow that is passing through the wave and that relates.

To the flame speed basically therefore by constructing a railway line we presume that we know what is it is slope and that that means we know what is the wave speed but in reality that is that turned out to be the haven value in the case of deliberations and we had to get into the structure of the deflagration wave in order to try to find out the flame speed there and so a similar situation remains for detonations as well so we will have to think about that at the moment so but basically I want to do the bottom line.

I mean odd or the or the commonality or the underlying thing about these things is we are primarily looking at premix reactants right so that was the framework in which we did this so from doing laminar deflagration which is essentially plain premix flames we then moved on to diffusion flames and so on not strictly in the framework of the rankine Hugoniot relations but then me now going through all those things.

We now come back to the detonation part of it and then we ask the first question what is the detonation wave speed right in doing this we recognize that we noticed at the time that in general even if you had what is called as strong detonations they approach the sharp and detonation unless the system is over driven and weak detonations seldom occurred because it required highly reactive mixtures for the reactions.

To occur in such a high rates that you have the lot of products that the reactions that are following the wave should happen at such high speed that when the wave propagates at supersonic speeds you now have the products that are following at supersonic speeds as well although not as fast as the waves themselves this is the wave itself so we were not necessarily concerned with weak detonations and we notice that stronger donations tend to become sharp and yoga detonations.

Therefore it makes sense for us to confine our attention to the wave speed of the Chapman joogay detonation wave.

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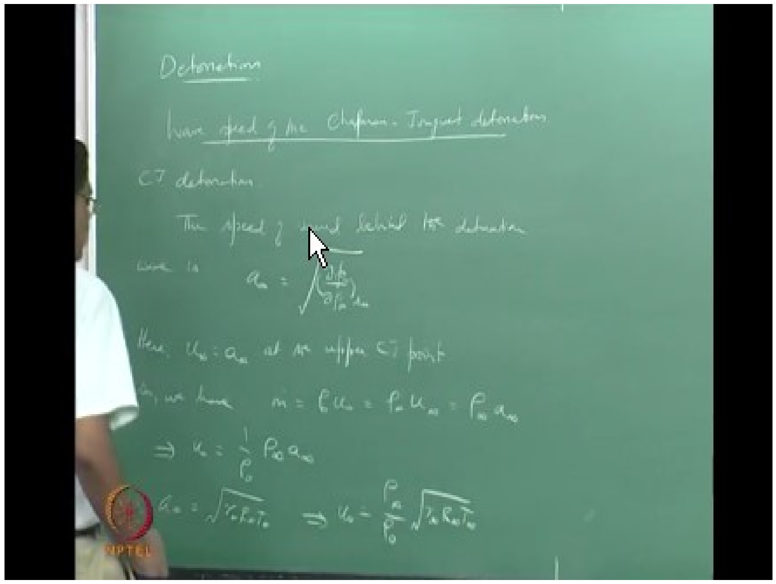

So that is what we will do now in doing this unlike in the case of deflagration were we notice that the wave speed is actually an Eigen value of a I glen value problem looking at the structure of the wave where we have to do the mass and energy balances across the or through the wave rather here we notice that the deflagration wave sorry.

They did the detonation wave is traveling at supersonic speeds so it quite does not really know what's ahead of it the information does not propagate upstream therefore in a it should be possible for us to actually try to get the wave speed without having to get into the structure so in the case of delegations we had this heat conduction that was happening upstream from the reaction zone that was heating up the reactants whereas in the case of detonation wave.

The detonation wave keeps going and it does not really know what's ahead therefore we cannot really hope to actually look at the structure in order to resolve the wave speed so there must be a trick it is involved somewhere here and I will highlight this trick as we go along but we will pretend that for the moment that we do not have to necessarily go through the structure we will just keep going looking at the kind of Franken him analysis similar.

To rankine Hugoniot alibis and then see where we can try to exploit the nature of detonation to look for the wave speed so for the Chapman youguet a or particularly sharp in Chapman youguet detonation or in this context the product speed is basically the speed of sound like for example in a in a normal shock where the downstream velocity is always subsonic when the upstream velocity is supersonic.

In the case of a Chapman you gave wave the downstream velocity is always sonic locally at the speed of sound so the speed of sound behind the detonation wave this we use the same notation as what we had before a ∞ equals square root partial derivative of pressure of the downstream pressure with respect to the downstream density at constant entropy constant downstream entropy now here U ∞ equals A∞ at the upper CJ point in the urbane curve.

So we have we m0 equals ρ 0 u equals β ∞ u∞ which is now = ρ ∞ a∞ so from u0 = 1 over ρ 0 ∞ ∞ is a equals square root of γ ∞,R ∞,t ∞ so you not equal to ρ ∞ over ρ 0 square root of γ A ∞ or T∞ right so let us call this relationship number one so here what the problem is you not a essentially what you are looking for this is very similar to what we did for the deflagration okay that is related to ρ0 also very similar to deliberations because.

They are also we were actually trying to look for are relationship for M0 versus the rest and M0 P u0 but here what we what we now find see this is the problem what we find is the wave speed which is essentially in a flame fixed coordinate system or a way fixed coordinate system it is essentially U0 depends on the downstream quantities write ρ ∞ ∞ R∞ and T ∞ and that is a problem so we now have to try to find out if we can evaluate those quantities right.

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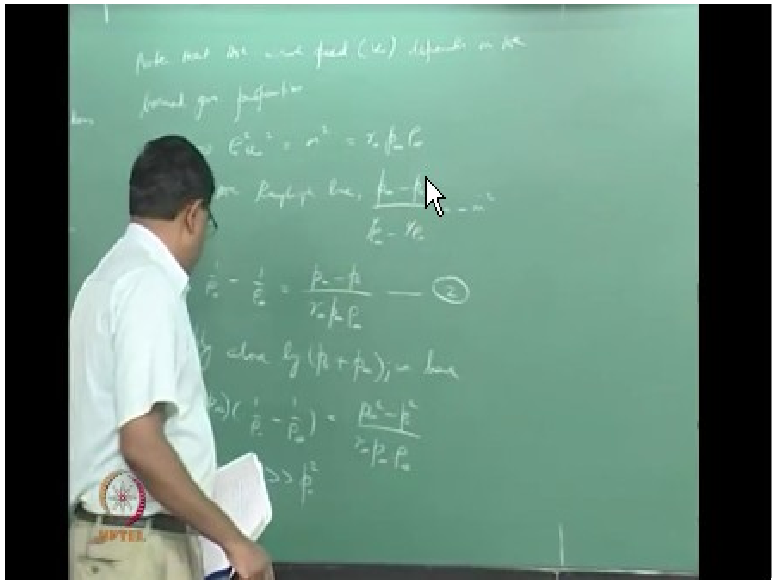

So note that the wave speed which is essentially U0 depends on depends on the burn gas properties.

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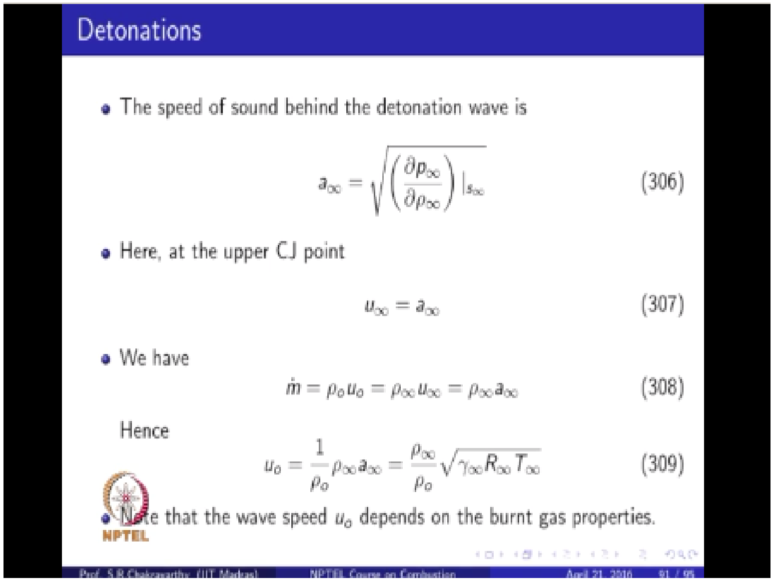

As a matter of fact if we also know that for example in the lower CJ point as well U∞ =A∞ right and M0 being =0 U0 =ρ ∞ U∞ valid regardless of where we are on the urbane curve right so if you now plug in U ∞ =a∞ that is valid for both upper CJ and lower CJ points so strictly speaking this could be a CJ detonation wave or CJ deflagration wave at this moment right so we have not quite strictly speaking exploited the fact that we are particularly interested in a detonation wave but this at this stage.

So keep that in mind and look for where we are doing this so what we want to now try to do is to explore or chase the evaluations need evaluation of the burn gas properties so let us try to do that one itself implies that ρ0 square do not squared =M0 square =∞ p ∞you can actually get this quite easily now in the road rail a line we have P ∞_ P naught divided by 1over ρ _ 1 over ρ 0 =_ M0 squared and then we plug this in here right so from this we can get 1 over ρ 0t_1 over ρ ∞ we swap this so that we can get rid.

Of the negative sign in the M0 ∞ that sets coming from here right now let us call this 2 and this will be pretty interesting and important at some stage in the future from now on what we are going to do something else that that could be a little bit boring all right but we will come back to this and say well we can forget about everything that I said which is kind of complicated and boring but we can just go back and look at this and then see.

If we can exploit something here so little let me just go through whatever needs to be gone through and then we will come back to this so here what we want to do is multiply about by p 0 + p ∞ so we have a p0 + P∞ times 1 over ρ 0_ 1 over ρ ∞ equals we now get p ∞ squared _ p 0 square divided by α ∞ T ∞ so why did we do this is where we are now going to make the point that we are looking at a detonation wave we already made the point.

That we are looking at a Chapman and you give wave right and we pretend that we do doing detonation but not quite yet until now right so what we are what we want to do is to notice that across the detonation wave the pressure increases manifold right so many times and therefore P ∞ is larger than P 0 quite larger than peanut actually right now what we do not want to do at this stage at this step is to say that p ∞ is much larger than p 0 so I use their the words carefully there is aid quite larger than not really much larger than.

So if you now say quite larger than right then we can we can then easily say that p ∞squared is much larger than peanut okay p0 square so what we are going to right now do is to say suppose the ∞ squared is much larger than p0 square this is much less of an approximation when compared to directly supposing that p∞ itself is much larger than p 0 we will find is this is not going to really help us directly it is going to lead us to go into some circles and then we will come up with a iterative scheme of solving.

This right so that is the boring part that I was just talking about and then we go through all that and then I am going to come back and say wait for an engineering purpose can I relax and say I do not want to actually x p 0+ p∞ here and then get this peanut squared to be considered to be much larger than p0 squared directly into can I say if p 0t its p∞ itself it is much greater than V 0 again instead of looking at the squares I will do that quite some time later okay but this is the point where we are beginning to talk about detention.

So it is essentially the idea and this allows us to get around the structure of the wave okay by doing by recognizing this we do not have too bothered about getting the structure tubes unlike we did in the definition but they are the physics demonic it okay the upstream conduction demands that we consider the structure of the wave here we did not have to do that instead we actually short-circuited by going through this approximation so now let us suppose.

We will now continue to assume that p∞ squared is much greater than p0 squared and that is going to imply.

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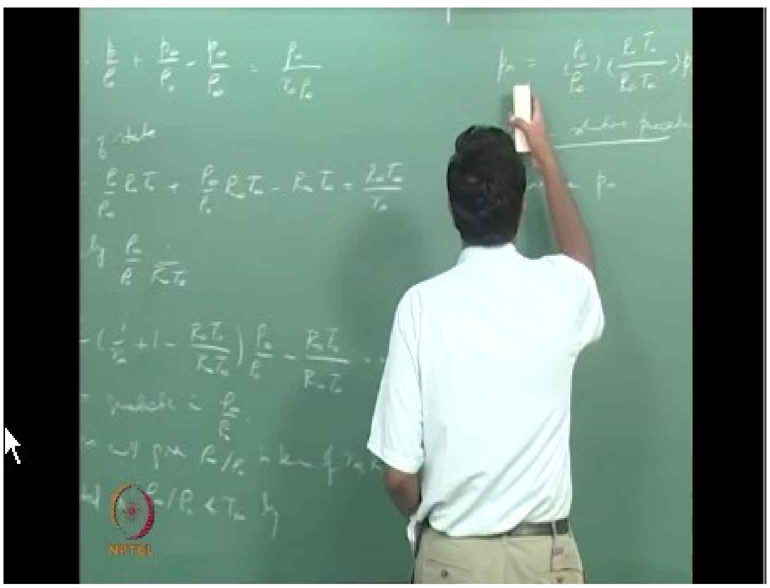

That the p naught divided by ρ 0 _V 0 divided by ρ∞ +∞ divided by ρ0 _P ∞ divided by ρ∞ that is opening up the speared theses here that is = P ∞ divided by α ∞ ρ ∞.

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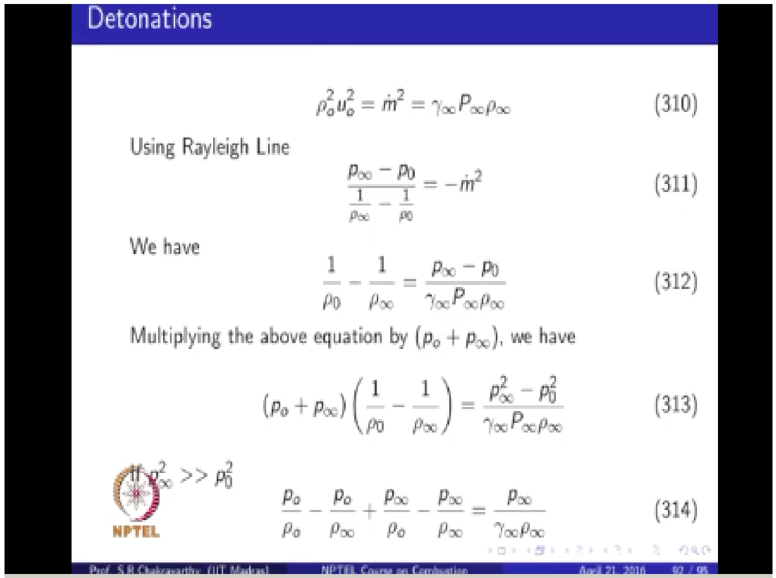

That means we have got rid of the P 0t squared so we remain with p∞ squared one of the painfully is there gets cancelled with the one at the bottom. So you get money this or the right hand side and so it use the equation of state or not T0 _ρ0 divided by ρ∞ or not T 0ρ ∞ by ρ 0R ∞ T ∞_ R ∞teen it means wherever we have a p ∞ by ρ ∞ we use in our ∞ we not run out p 0 over ρ 0we use or not he not but when you have this next kind of quotient.

We now take the ratios of densities and then use the equation of state wherever applicable and that's going to mean that on the right hand side you also write R∞ to ∞ divided by α ∞ so multiply this is the kind of boring part I was talking about so it may basically some algebra that we go through do not R∞ T ∞and that is going to get you a quadratic you might recall doing something like this with the urbane curve and they rankine are they on the Rayleigh line put together earlier.

On to get the upstream Mach numbers but here our goal is slightly different so we get ρ 0 ∞bro not the whole square _ 1 over α ∞+1 _R 0 T0 divided by R∞T ∞ times ρ ∞over ρ 0 _R 0T 0 divided by R∞ T∞ equal to 0 let us call this 3 now this is a quadratic in ρ ∞ over ρ not right now but you see what about are the coefficients the coefficients involves α R ∞ or not T 0R∞T ∞

right so those are the coefficients sir so this solving this will give ρ ∞ you overrule not in terms of α R ∞ or not t 0 R∞ right.

So course we are still chasing that means we wanted to go back here and then say can I get ρ ∞ well you can but it's going to be in terms α ∞ R∞ T∞ we need to know those things as well right so that is okay so we are not introducing anything new we are trying to count only the old things so that is all right so we can easily now so also use a p∞ p ∞ is related to a ρ ∞ a Byron R and T ∞ by essentially the ratio of equations of equations of state at the product and reactant conditions

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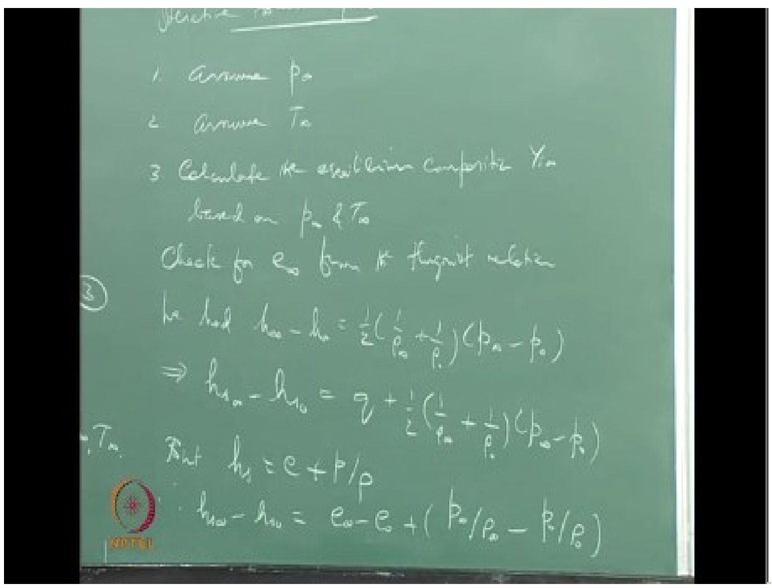

So that is p ∞ =0 divided by $\rho\infty$ not t 0 divided by R∞ T∞ times P. So let us call this for the reason.

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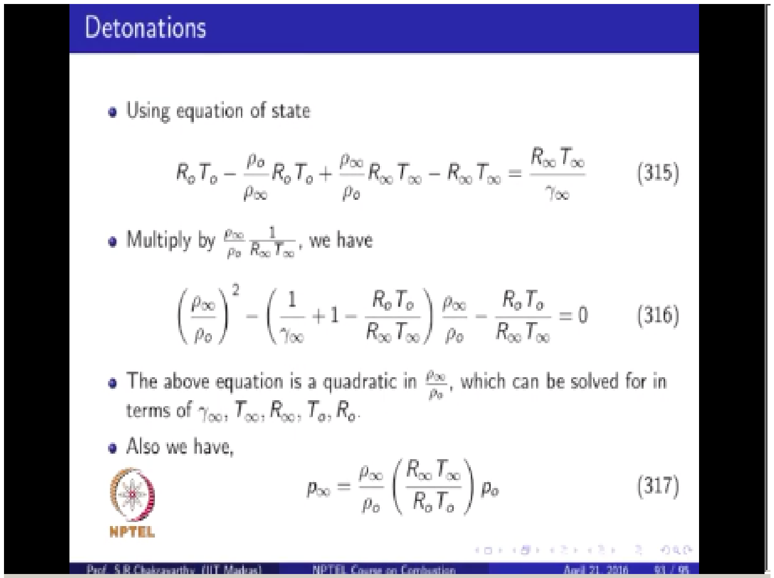

We number these equations is we will now go through an iterative procedure iterative solution procedure the first thing we do is assume a p ∞ we do not know what p ∞is but let us assume it and yellow make a mistake right p =ρ RT so we need to have p ∞ equals the right I want to make sure I do not make that mistake but I did so zoom to ∞three now next thing we do is calculate the equilibrium composition why I ∞ based on p ∞ and t ∞ let us assume that is yeah once you get to step 3 you have a pain Florien T ∞ to work with.

Which you now use the aquarium composition there and then so once you do this you know use the energy balance and to check whether this is going to resolve things so check for infinitely from the he go near relation in fact we have the mass balance here and the railway line is a combination of statements of mass conservation and momentum conservation that is what we have been working with we have no used energy conservation yet so we will try to now try to use that as a check right.

So we will we will try to use the he go near relation we had h ∞_ H = one-half 1 over ρ∞ + 1 over ρ∞ times p ∞ _ p 0 and which means the sensible and therapy part of it HS ∞_h is not so what we do is we now split this in a sensible enthalpy and heat of formation so the difference is in the heat of formation of the products and reactants altogether is the heat release right so that is Q so that would be Q +one half 1 over ρ ∞ + 1 over ρ 0 times P ∞ _ P 0here then.

We want to use the internal energy that is about HS a sensible enthalpy is equal to internal energy + P over ρ there for HS ∞ _H =e∞ -+p∞ over ρ∞ _p a not over ρ0 okay let us now substitute this of the substitute this.

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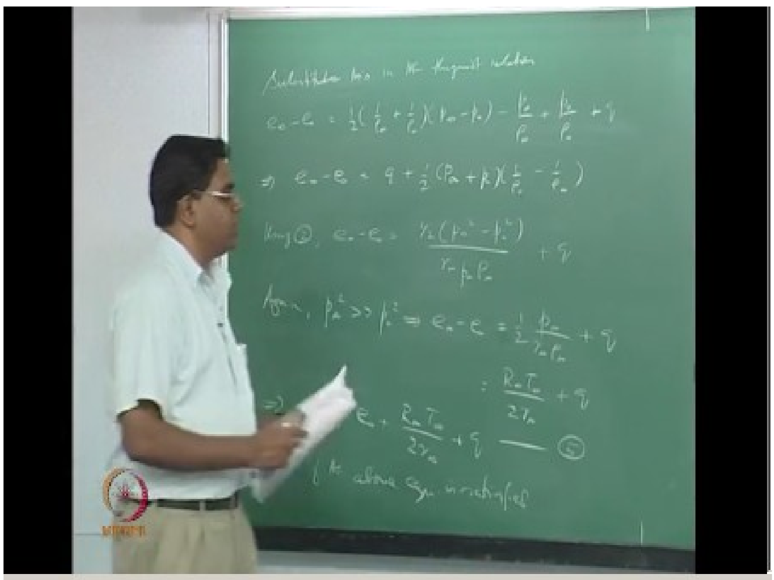

In the he go no then we get ∞_Z 0 = one-half 1 over ρ ∞+ 1 over ρ 0times P ∞_ P 0_ P ready divided by ρ ∞+ P 0 divided by ρ 0 + Q so simplify this you have some opportunity to cancel things therefore we get E∞_ E0+ Q + one-half p ∞ + P 0 times 1 over ρ 0 _ 1 over ρ∞ so here we can now go back and use this relationship number two and what you get you get a p ∞+ P 0over here that is what we try to multiply but then if you now use 1 over ρ∞ know 0 - 1 over ρ ∞ as this you naturally get a product of P ∞+V 0 times P ∞_ P 0 which is they need to p∞ squared _ P 0 square right sousing to ∞_E 0=one-half p ∞ squared _ P 0 squared divided by α∞P.

And fill the ρ∞+ Q here again you now try to approximate saying p ∞ squared it is much greater than P 0 square and therefore that is going to need us to e ∞_e 0that is equal to one-half p ∞

divided by α ∞ρ ∞+ Q or +Q and P ∞ over ρ ∞ is 0 ∞T ∞therefore this is R∞ T∞ divided by 2 α ∞+Q which is a V ∞ =E0 + R ∞ T ∞ divided by α X will be + Q so if you call this five then we have assumed a T ∞ so the equilibrium composition is also not going to give you a A α∞ and R∞ right.

So now you have for the assumed value of P ∞ and T ∞ you get α∞ and R∞ with which you can check right so check if the above equation is satisfied right so once you know that this is satisfied you are okay with the choice of T ∞ that you made instep two all right of course you still have the outer loop of having assumed α p ∞ so once you do this solve for so this is there is a step three that we have just finished.

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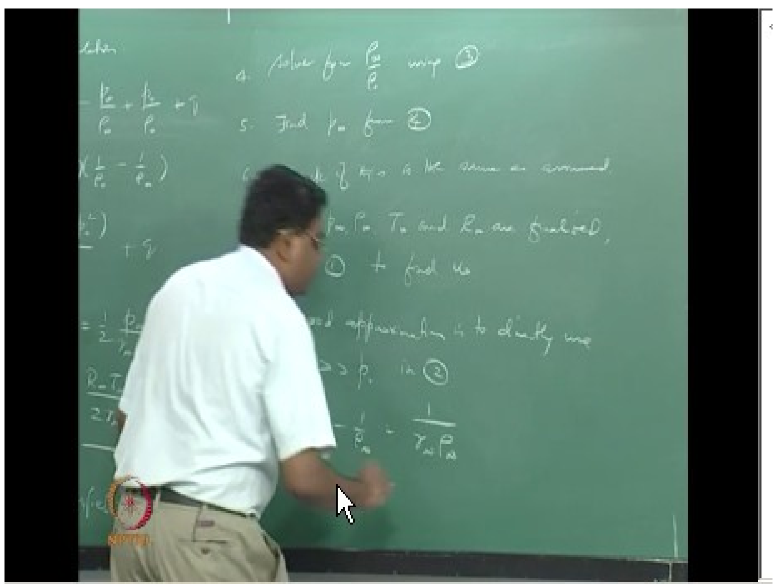

Step 4 would be solved for ρ∞ over ρ not using the quadratic equation that is number three right and step 5 would be fine p ∞ from for that is here right then6 check if this is the same as assumed right. So whenever we say check we would like to do too we would like to think about two things one it is always but within a certain tolerance okay so you're not going to exactly match maybe you can match but to the first decimal place second decimal place third decimal place or as a percentage whatever it is you want it you want to hold to a tolerance and if it is satisfied then you proceed.

If not then you have to repeat the inner loop right so in this case for example you now assume T ∞ and then you go through this check and if you are satisfied within the tolerance you proceed if you are not satisfied within the tolerance right then you assume a different T ∞ in step to go through the check for the assume P∞ and then once you have converged on a T ∞ then you proceed this step for step π and then check so in your check of course it is within a tolerance and within the tolerance.

If you have not satisfied then you have to go back and change your p ∞ go through the whole thing again right so you do you do this of course it is a bit of a tedious process so once you do this so finally step 7 would be once p ∞ ρ∞ T ∞ and R∞ or finalized use one to find you not that is almost like the post-processing right now of course we find that this is kind of like a loop within a loop and it is going to take a while for you to converge.

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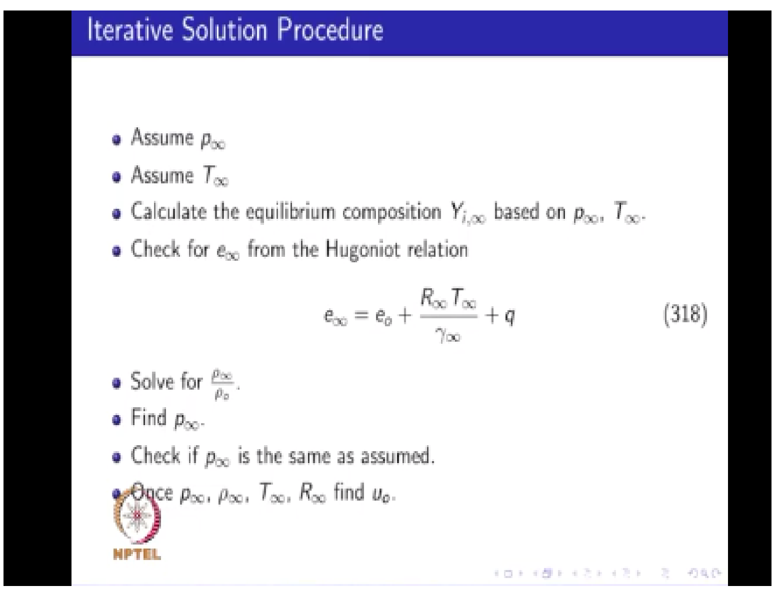

So instead of going through this the next best thing that you can do as I pointed out earlier is do not push yourself as you p ∞ squared is much greater than p 0 square directly say can I adopt p ∞ itself as much greater than p 0 right so a good approximation is it strictly use p ∞ itself as much greater than p 0 in to right that is what I that is what I said we will go back to so that implies 1 over ρ0_1 over ρ∞= the earlier had p ∞ squared _P 0 square divided by α P ∞ ρ ∞ we threw away p 0 squared in preference to p ∞ squared cancel one of those paint-filled ease at the top at one at the bottom and so on but now.

We have a p ∞_p 0 divided by α p∞ ρ∞ p 0 is thrown away in comparison with T∞ the painfully directly gets cancelled with the one at the bottom so you are left with only one over α ∞through ∞ right so this means that ρ ∞ over ρ not is simply =α ∞+ one divided by government philly HS ∞ _

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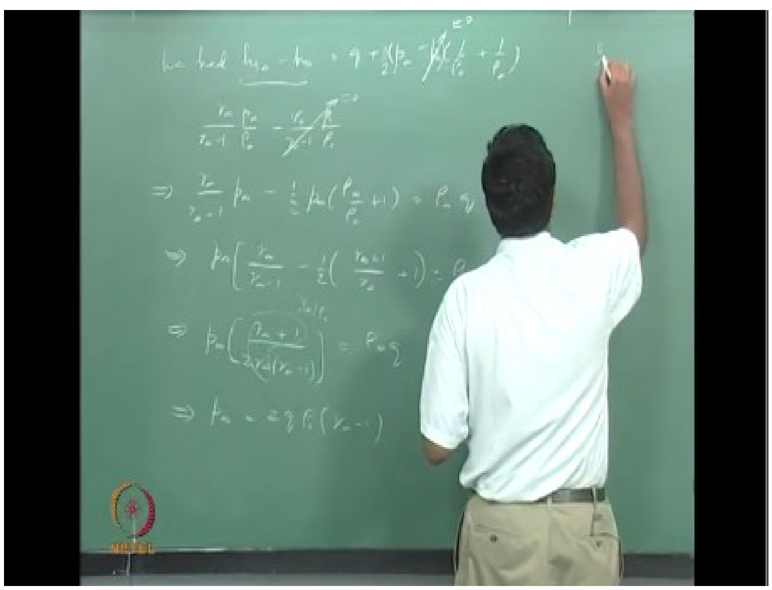

HS not it is not = Q +one-half p ∞ _ p 0 times 1over ρ0+1 over ρ∞ in fact we have we have just written it here still around so here what we want to do is to this is something that we did during the racking Hugo new days. That is we want to write this as α∞ over α∞_1 p ∞ over ρ ∞_α0 t divided by α0_1 p 0 over ρ not the way we get this is we now say hedges ∞ is CP ∞ t ∞ see p ∞

is α ∞ α ∞ or ∞ divided by α ∞ _1 and then he had a t ∞ or ∞ to ∞ is p ∞ divided by ρ∞ similarly for in not conditions so if you now do this and then the next step you do is you notice that you are not going to have a big difference in α ∞ and α not between the two okay to think about this α is basically ratio of specific heats ratio of specific heats depends on.

Whether the gases are monatomic or diatomic or polyatomic whether it is linear polyatomic or non-linear polyatomic and so on now if it is monatomic you are going to have something like so if it is diatomic you are going to have something like 1.4 if it is mono atomic it is going to be like 1.6 67 and so on and if it is polyatomic you may have something less than 1.4 right 1.3 so all these things are around of the order of 1 if you know think about orders of magnitudes all of the order of 1 and mostly for mixtures of gases.

In both cases it is going to be in general I polyatomic system and so we expect that we do not even have this variation right so we do not expect a big difference between α ∞ divided by α in _ 1 and α n0 divided by α 0_1 so for an order of magnitude point of view we notice that just like how we now threw away p0 itself as opposed to p ∞ right we now say let us get rid of this entire term all right so if you now do and similarly you can do.

It here as well in preference to p∞ so wherever we find pain-free _p0 kind of thing you know get rid of this so with this we now say α ∞divided by γ∞ _1 times p ∞ _one half p ∞ times ρ∞ divided by ρ naught + one equals P ∞ q some mores amplification well basically you can pull out p ∞ all right and then plug gear ρ ∞ you over ρ 0 as α ∞ + one divided by α ∞ here and pull this p ∞ out you had something in terms of γ ∞ here there is also going to be in terms of γ∞ so you'll now have one big function that is safe that is γ∞ that's in terms of γ∞.

So quickly going through that /come on 20_ 1 _ half ∞ + one divided by γ ∞ + one equal to P ∞ quick q and then you put things together so we get pay ∞ times γ ∞ + 1 divided by 2 γ∞ times γ∞ _ 1equals P ∞ q but then keep in mind that this itself after the simplification still has luminance of going to leave over P not right so and this P ∞ then gets cancelled with that right so then you get P∞ =to 2 q 2 q ρ not times ∞ _ 1 now if you did not make a big fuss about having to find γ_∞ of course you can assume equilibrium find the composition and soon but the idea was you do not go through all that right that means you.

Now say let us not worry about the variation of α between reactants and products you assume some α that is common and unknown then you directly get a value for P_∞ just knowing the heat release and the density of the reactants right the interesting thing here is Q is like joules per kg and P not as kilograms per meter cube so P naught Q is like joules per meter cubed so the is basically something like a volumetric heat release of the reactants right that means the denser the reactants greater the pressure all right the means denser.

The reactants it compacts more heat within it k per unit volume and that means greater the downstream pressure infect we have been exploiting the notion that the pressure itself is much greater than the initial pressure and that is really the hallmark of detonation waves so what's actually great about detonation waves is not as much about the km/s kind of velocities but what is what is utilized in detonation wave in terms of applications is the pressure.

So you have this huge pressure buildup behind the wave and as the wave propagates it now has rarefaction that follows it which sucks everything in and then destroys the materials that it passes through or the or the matter that it passes through so here what we are basically seeing is if you were are actually talking about gases all over but if you now think about reactants that were actually in solid form and correspondingly.

You gave rise to some heat okay because of the gas evaporating and then reacting sorry the solid evaporating and giving rise to the interpretation wave in the gas phase alright then the volumetric heat capacity heat of the of the reactants is a lot higher and therefore solid detonation reactants give rise to much higher increase in pressures right so you get you get this physically meaningful relationship from making this approximation here and when proceeding instead of getting stuck in this iterative loop so finally.

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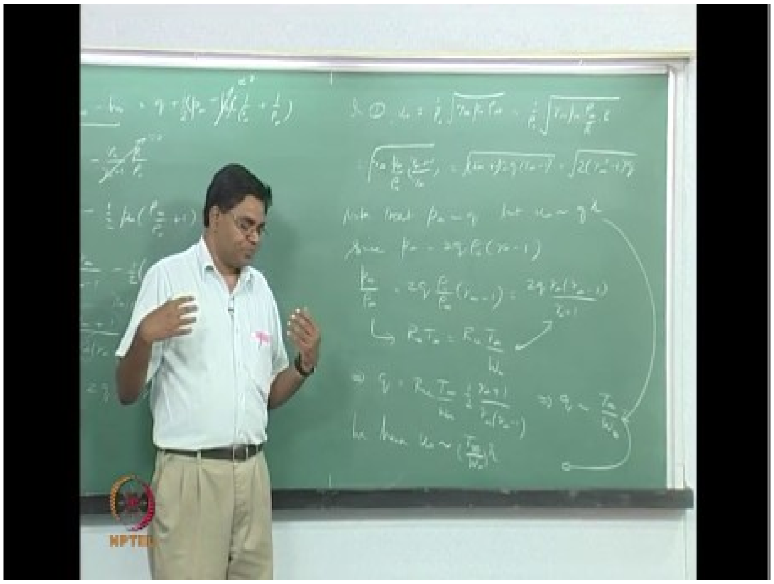

So now you want to go back to one and say U0=1over P 0 square root of γ ∞ p ∞ road finery that can be written as 1over P 0 square root of α∞ p ∞ P ∞ over P 0times P0 and therefore we can write this s still we got their own on there and so we can write this as square root of α∞ p ∞ divided by P 0timesγ ∞ + one divided by γ ∞ cancel the γ ∞ equal to γ ∞ + one and so the γ∞ gets cancelled and p∞ divided by ρ0 is to q times γ_ one that is equal to square root of two go ∞ squared _ one times Q so what we can see is now that p ∞ goes ask you but you knock goes is square root of Q.

So you do not quite get you know the same effect out of the heat release and let us just push this a little bit more and we are pretty much there to see the result here so since P ∞ equals 2 q PP 0 times α ∞ _ 1p ∞ divided by ρ ∞ is equal to 2 q mu 0 divided by ρ∞ times α ∞ _ 1then whenever you see a row in fillory over ρ not plug in a row ∞ α ∞ + one divided by α∞ so you say to q times α∞ α energy_ 1 / comment will be + 1 and that gives you q equal to Sophie∞ over P ∞ to start with is what is this face this is nothing but R ∞ T ∞ which is are you T∞ divided by W ∞ right.

So from here we can get q trying to put these two together right q is are you $T\infty$ divided by $W\infty$ times one-half $\gamma\infty$ + one divided by $\alpha\infty$ times $\gamma\infty$ minus one so from here what we can see is Q goes as the ∞ divided by $W\infty$ and then putting these two together right we have you not essentially goes as Tina sorry $T\infty$ divided by $W\infty$ De Hoff right now for those of us who are familiar with rocket propulsion we get a very similar result for the exit velocity of the rocket from a guest dynamic nozzle undergoing supplied by combustion gases.

At a temperature $T\infty$ a molecular weight of gases being $w\infty$ right and this is what translates to something called a specific impulse now if you detonation engine pulse detonation engine the detonation wave keeps on propagating out and the this at the speed and what we find is that the speed of the wave is also directly proportional to $t\infty$ divided by $W\infty$ loop of the same dependence so effectively a pulse detonation rocket engine is not going to have a specific impulse that is very two different from a ordinary chemical rocket so there is no way of beating around nature in this case right so you are pretty much going to get the same result as we can see from this you.

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