Flight Dynamics – II

(Stability)

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Module No. # 08

Equations of Rigid Aircraft

Six - Degree – of – Freedom Motion

Lecture No. # 25

Description of Various Forces and Moments

So, we have written down the equations of motion for aircraft dynamics, and here we assume that the mass of the aircraft is, or aircraft is a rigid body, not really mass is a constant; mass will not be constant. Mass is going to change, because you have lot of fuel on the aircraft which will slowly be consumed. And there are sometimes, aircraft where you also drop things. For example, bombers will drop bombs on tank, and sometimes refueling. We also have aircraft where you refuel the aircraft, is not it? Did we take into account all those things? Not really right?

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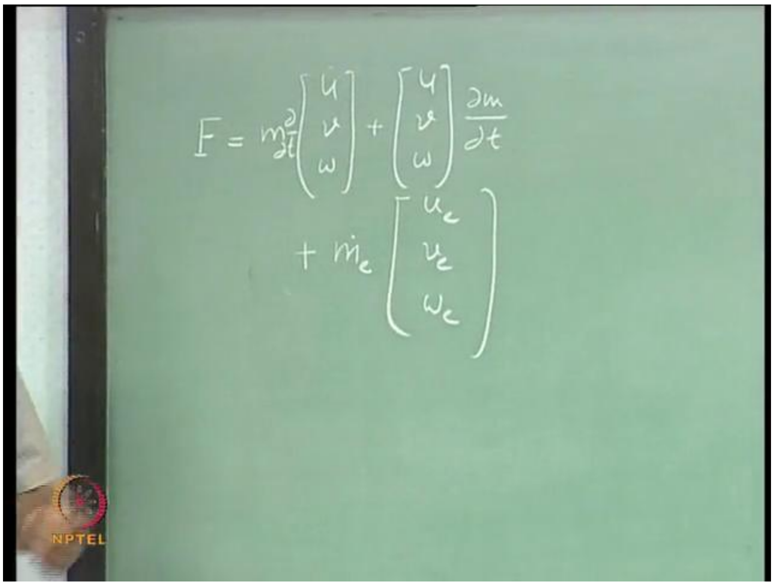

So, if you were to be including, taking all those things into account, then what you would have .....(Refer Slide above) this (Refer Slide Time: 01:32) plus mass of the aircraft changing in time. This is the, this is the mass which is internally being consumed and there will be a mass which will be exiting the aircraft, is not it? So, you also have another component which is m e dot, the exit mass rate into… (Refer Slide Time: 02:30)

Remember, u v w are velocity of aircraft along the body fixed axes. So, these exit velocities are also along the body fixed axis and m e dot is the rate of change of mass leaving the aircraft, this is alright? So, in general, this also has to be added to the Newton's law, it is going to change the momentum of the aircraft, is it not? But we have not modeled this part in our equations, only for the reason that, all of these are actually incorporated in the thrust.

So, because we already take care of the pseudo forces which are being generated when the mass is existing the aircraft through the thrust or propulsive forces, that is why we have not modeled this in our Newton's law of equations. So, our aircraft equations are you know perfect for, accurate for rigid body aircraft. This is only to; this is one question which might just come to your minds. So, that is why I was trying to answer this. So, we wrote the equations in the compact form.

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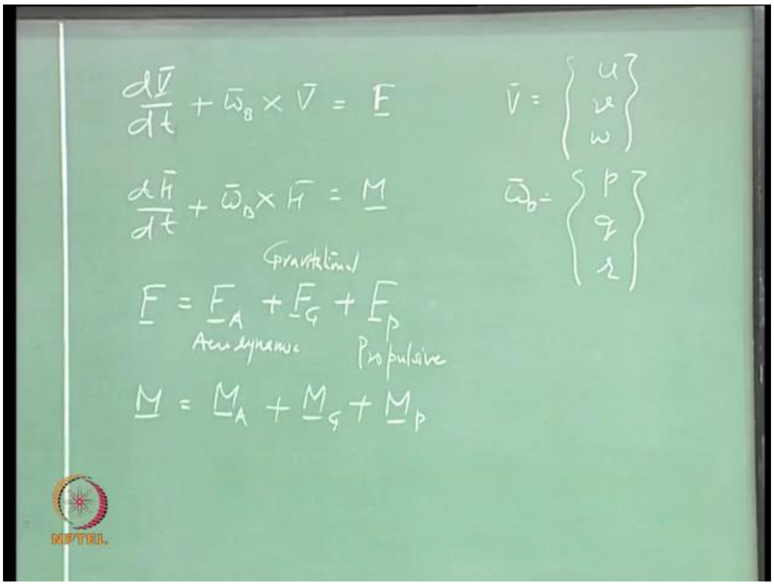

$$\frac{d\overline{V}}{dt} + \overline{\omega}_B \times \overline{V} = \overline{F}; \overline{V} = [u\ v\ w]'; \overline{\omega}_B = [p\ q\ r]'$$
$$\frac{d\overline{H}}{dt} + \overline{\omega}_B \times \overline{H} = \overline{M} \quad (1)$$
$$\overline{F} = \overline{F}_A + \overline{F}_G + \overline{F}_P; \overline{M} = \overline{M}_A + \overline{M}_G + \overline{M}_P;$$
$$G: gravitational, A: Aerodynamic, P: \text{Propulsive}$$

Here, V is the velocity of the aircraft along the body fixed axes. V is .....(Refer Eq(1)) So, I am writing all the equations along the body fixed axes; force and the moment equations. What about this force? What is this force? This force is sum of all external forces and I have to take the component of this force when I want to write, you know, separate out the equations along each axis; then I have to take the component of these, this force, along those axes, is not it? So, let us try to write what this force is.

So, how many forces you know are going to act on the aircraft. Forces acting on the aircraft, external forces. Aerodynamic, gravitational forces, and propulsive forces. (()) It will not be external, but it will depend upon the variables of the aircraft. And this (aerodynamic) force will actually include also the forces that will come by applying the control surfaces. When you deflect the control surfaces, that force is actually changing the aerodynamic force. So, this is aerodynamic component, gravitational component and propulsive forces.

So, similarly, we will also have moments which will consist of these three different components. Now, you know that each one of these forces are convenient to write in different axis system. For example, if I look at this gravitational force, I know that, that force is acting along the Z_E axis of the earth, is not it?

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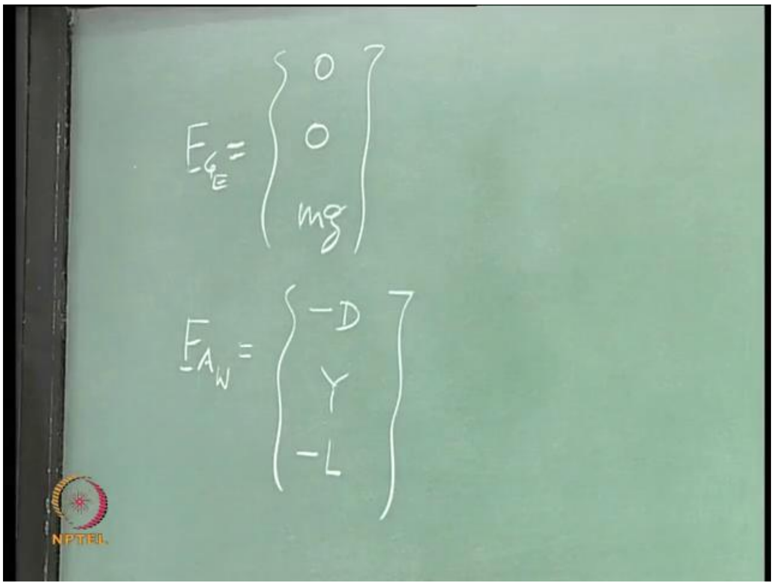

So, this F_G is most conveniently written in the Earth fixed coordinate system and, this (Refer slide above) vector is this. F_A is more convenient to write in the wind fixed axis system. You remember, we take the component of, the drag is along the velocity, or opposite to the velocity, and lift is perpendicular to that. So, this is the, this force is more convenient to write in the wind fixed axis system. We will come to these axis systems soon. Drag, the side force, and the lift. Our final aim is to write components of all these forces along the body fixed axes, when I am trying to balance the two sides, is it not? It depends, which axis system you want to write your equations into, you have to accordingly use the transformation matrices to write these forces and moments along that particular axis system.

So, now there are different ways of looking at it. One thing is, you are collecting some data in the wind tunnel; you are trying to find out the aerodynamic forces and moments and that you are doing in a particular axis system. So, if I want to keep that data as my reference axis system, then I have to rewrite these equations along that axis system, is it not? That is one reason. Second reason is, there are some axis system in which if you write the equations computation will become simpler or analysis will become simpler. So, you can do all that conveniently if you have a good knowledge of transformation matrices. I have been talking about transformation matrices in the last couple of classes. Let us try to look at how to resolve this vector along body fixed axes.

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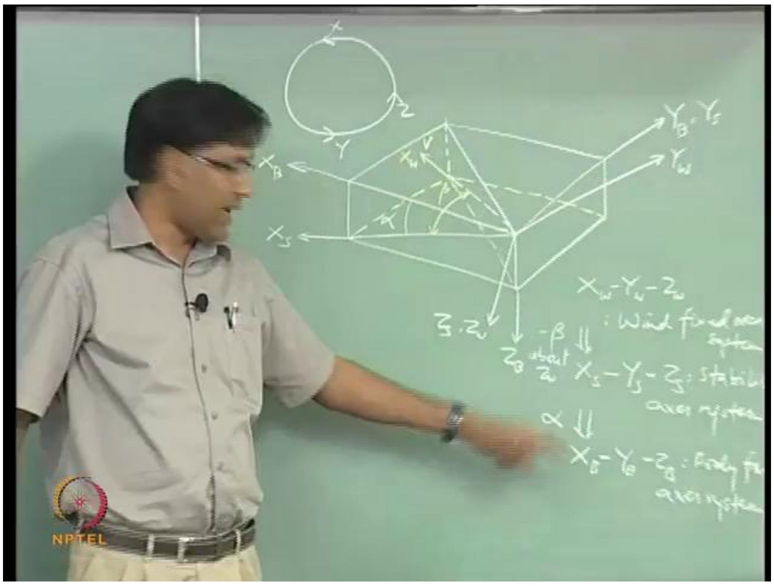

((no audio available refer time: 12:08 to 13:28)) So, if this is my body fixed X axis, this is body fixed Z axis, and body fixed Y axis. The velocity vector is along this line and that is pointing towards the X axis of the wind fixed axis system. So, the resultant velocity vector is pointing towards the X axis of the wind fixed axis system. So, if you want to know where are the other two axes of this wind fixed axis system, you can. So, let us, ... they are not going to be lying on these axes. So, we will take some right handed coordinate system. So, let us join this line and also and these two points, then you know that the, this vector, you know, the angle that it is making with this face is your sideslip angle. Everybody agrees with this? This is how we define the sideslip angle to start with. So, I should have actually drawn, joined these two points also, so that .....(Refer the slide above) you get this. So, beta is this angle - the angle that this velocity vector is making with this plane is the beta angle, sideslip angle.

Now, if I give a rotation, you know, of the wind fixed axis system about Z_W by beta, then I can come to this (stability X_S-Y_S-Z_S) axis system; I can get this X_W aligned with this line (X_S) if I rotate that wind fixed axis system about the Z_W axis, and what you get then is X_S. So, this axis system is called the stability axis system. So, let us write down this clearly, ... and when you do this, so, you have rotated this vector, you know, about the Z_W axis, XY in the X_W-Y_W plane, you are giving rotation by sideslip angle beta about Z_W axis and then you reach this .....

So, this involves rotation beta about Z_W, beta or minus beta actually, because my Y_W will be rotating towards X_W, and then, we call that rotation as a negative rotation. So, actually it is a rotation by minus beta. So, we have reached a new axis system which is … (Refer Slide Time: 19:58) So, I am going, I have, I have given rotation about Z_W axis. So, Z axis is same of the stability axis system and the wind fixed axis system, and this Y axis of wind fixed axis system is going to rotate with X.

So, finally, after this first rotation by minus beta, it is going to coincide with the body fixed Y axis. What is this angle, what is this angle? So, I have taken the component of velocity, projection of the velocity on this plane. So, now, the velocity is in this direction. What is the angle between this (X_S) axis and this (X_B) axis? (()) We are talking about the, the, wind, the velocity vector. So, that angle will be, what? The angle of attack! There are only two angles involved here - the sideslip angle and the angle of attack. So, this angle is the angle of attack.

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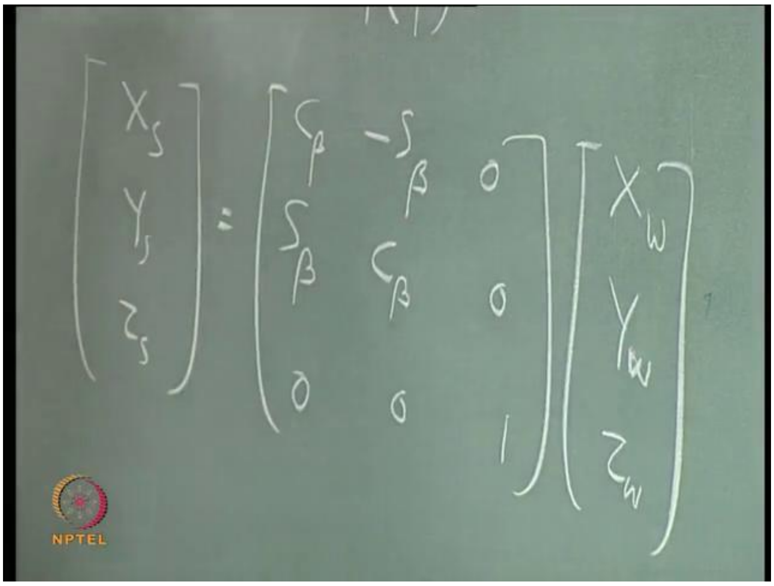

Now, in the second rotation, you have to rotate this X_S-Y_S-Z axis system about Y_S so that now you are coinciding with the body fixed axis system. So, second rotation is alpha, and this alpha is going to be positive because Z_S is going towards X_S. We are going to follow this, keep this in mind all the time. If this X axis is moving towards Y, then it is a positive rotation about Z. So, in this case, now we are rotating about this new Y_S, or new Y axis, by an angle alpha. So, in this case, Z is moving towards X when I am trying to rotate this up. And when Z moves towards X, then you have a positive rotation. Now, we want to find out the transformation matrices and those are going to include alpha and beta. So, first rotation is this minus beta, second rotation is alpha.

So, let us write down the transformation matrix for the first rotation ((audio not available Refer Time: 23:47 to 25:00)) . ..... So, this was the first rotation, which is actually minus beta. So, I am rotating Y towards X, (Refer Slide Time: 25:40) Is this clear?

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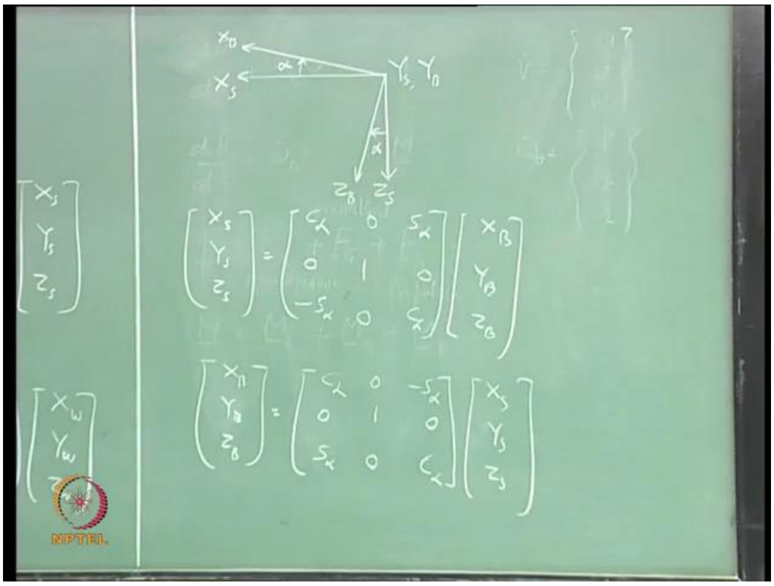

$$\begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_W \\ Y_W \\ Z_W \end{bmatrix}; \begin{bmatrix} X_W \\ Y_W \\ Z_W \end{bmatrix} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix}$$

(2)

$$\begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix}; \begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix}$$

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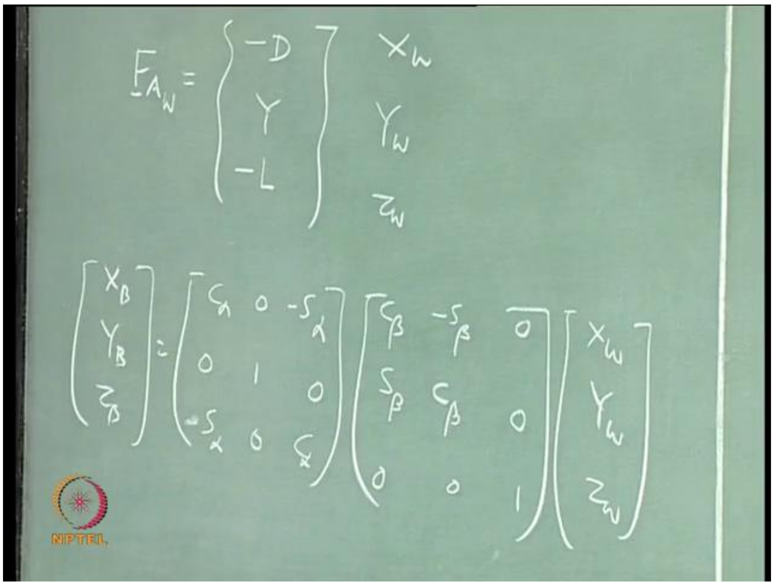

What do we want to do? We want to write this force, you know, this vector along body fixed axis. So, these are given along X_W, Y_W and Z_W and I want to write the component of this vector along body fixed axis system. The relation between this, ... (()) relation between the components of a single vector, we are talking about one single vector and writing the components of that vector in two different axis systems. So, in this case the relation between the component of a vector in the body fixed axis system and the components of the same vector in the wind fixed axis system is given by this relation.

So, this is, this is true for any vector, is not it? It is not only the X Y Z coordinate; it is true for any vector. So, now you have to, if you want to find out what is the, what are the components of this aerodynamic force which is written in the wind fixed axis system. What are the components of that force along the body fixed axis? What you need to do is, just need to substitute this with this vector.

$$\begin{bmatrix} F_{AXB} \\ F_{AYB} \\ F_{AZB} \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -D \\ Y \\ -L \end{bmatrix} \qquad (3)$$

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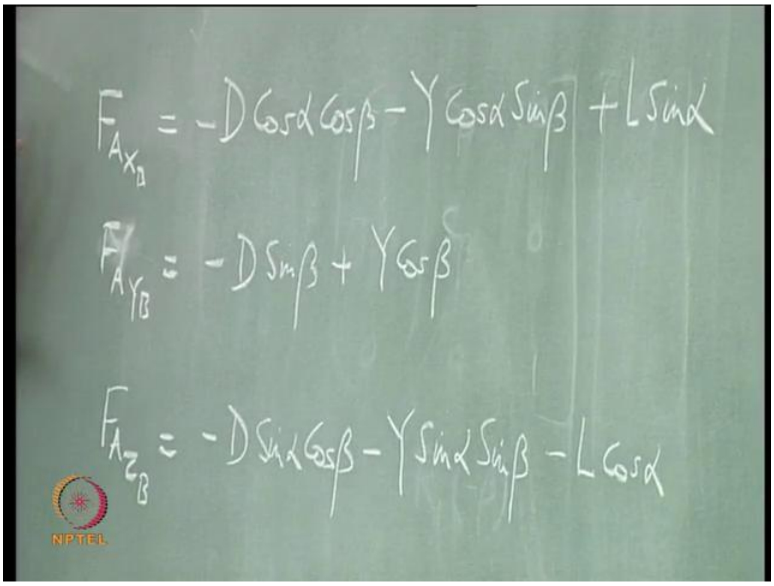

So, that you can, if I want to now write, what is F_A in the body fixed axis system ......(Refer Eq(3)) into this? This is going to be here and write this in the expanded form. I am multiplying these two matrices. So, F_A component along X_B is …(Refer slide above) (Refer Slide Time: 32:29) Is this clear? Component of aerodynamic forces along body fixed X axis, Y axis and Z axis. See you have to repeat this exercise now for finding the component of this vector along, you know, we are writing the equations along the body fixed axis.

So if you remember I already gave the transformation matrices, know which will, which you can use to write down the components of any vector in the, in the, Earth fixed axis system know, or component of any vector. The relation between the components of any vector along Earth fixed axis system and body fixed axis system can be written using the three rotation matrices that we talked about yesterday - psi ψ, theta θ, and phi ϕ rotation matrices. We can use that and write the component of this vector now along body fixed axes.

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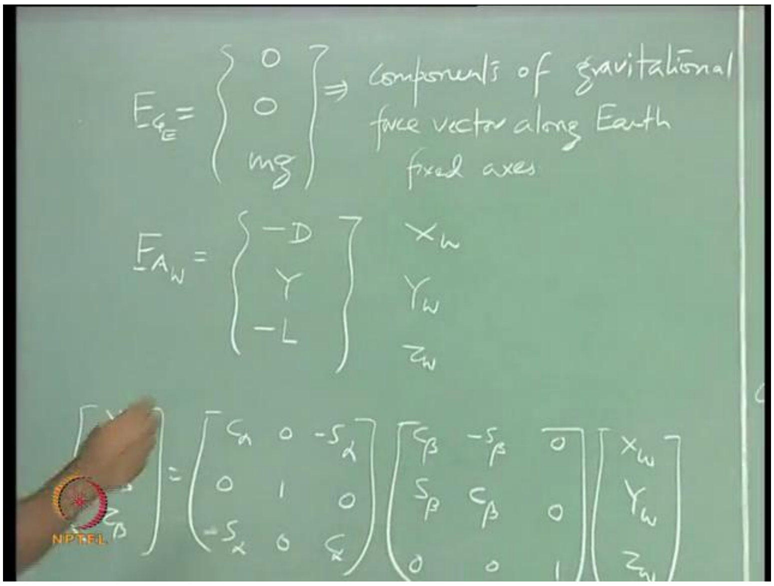

So, I have F_G written as this. Now, I want to find out what is F_G in the body fixed axis system. So, what do I need here? Actually, I do not need all the terms. I only need the terms in the last column, because these two components of this vector are 0. So, I will not write this; you know what this is. Only write this column. ......(Refer slide above) ((audio not available refer time: 36:46 to 38:44)). So, clearly here you see that if you are talking about defining your Earth fixed axis system on the surface of Earth somewhere, then the Z axis will be pointing towards the center of Earth and X axis is pointing towards the North Pole and Y has to be completing the axis system, has to be normal to X and Z.

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We know the D is drag; Y is side force; L is lift. So, you know how to resolve any vector now along different axis system.

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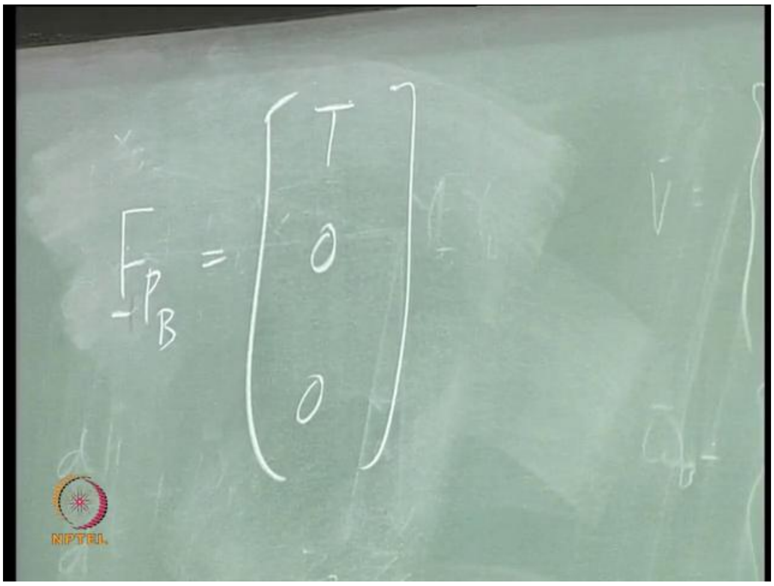

Thrust is now, we are going to assume thrust to be along body fixed X axis. So, we will take it like that and .... so I am assuming that F_P which is the propulsive force is along body fixed axis. So, this thrust is along the body fixed X axis. Other two components are 0. So, any, any, questions? We can stop for today.