Gas Dynamics
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Module - 4
Lecture - 12
Normal Shock Relations

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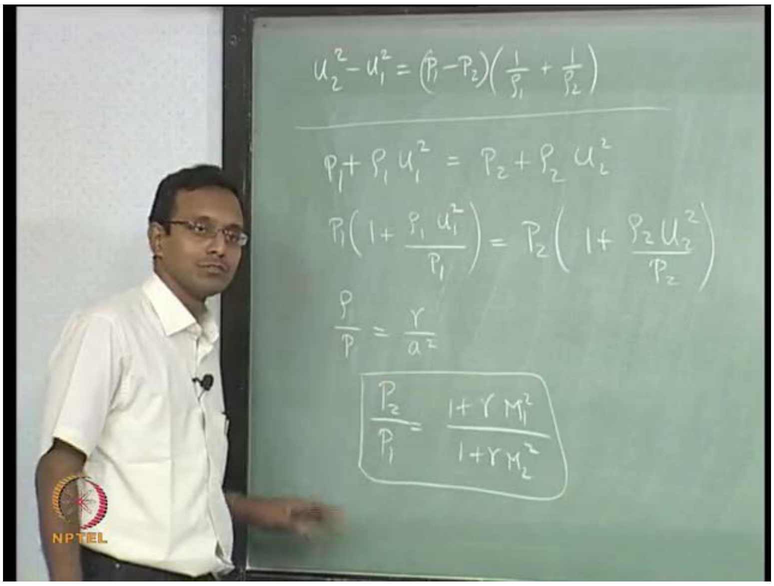

Hello everyone and welcome back. We are still in the process of deriving Normal Shock Relations. We stopped at some point there I will take one step back and then continue from there, I just want to put one more point on the board which I pointed out sometime earlier last class, which was that we had this relation and I said as a note, that if I put P 1 equal to P 2 and u 1 equal to u 2. This equation is satisfied, but if I do that my P 2 by P 1 will become 1, which will make my rho 2 by rho 1 1, which will make my u 2 by u 1 1, which will make my T 2 by T 1 1. Everything will become 1, which means no change in flow properties that is a very special case that is also a solution to this equation.

We will remember that we will go back and look at that towards the end of today's class, we will come back to you there. Now, we want to eliminate all variables pressure, temperature, density, velocity, everything in terms of Mach number, that is our ultimate goal. Of which, we started here, we had this relation constant area momentum equation a special case but it is valid for us because we are thinking across a shock, which will be a very thin layer anyway.

So, I can put a very thin control volume around it and I can say it is constant volume also. Now, if I rewrite this, this is still one equation momentum equation, I will use this sorry, it is the opposite comma b by rho is a square. It is what? I am going to use and similarly for 2. So, I will just remove this subscript, I can put 1 or 2 as subscript depending on whatever is needed, I am going to use the definition of a square and along with this the state equation both have been used here, inside here itself, if I use that this rho by P will become gamma by a square in this case, it is gamma u 1 square by a 1 square, which will become gamma m 1 square.

So, I will get to a relation, which looks like this P 2 by P 1, it will become 1 plus gamma M 1 square by 1 plus gamma 1 square by 1 plus gamma M 2 square. This is not new, you have seen this even in the last class till here, everything is last class also, we did this. This is 1 relation and this is the other piece, we have this 1, where we said there is a trivial solution sitting inside, where there is no change in the flow, here I am getting P 2 by P 1 in terms of M 1 and M 2. Now what, we want to do is, I have actually used momentum equation and state equation and a square definition here, to get to this point; now, we still have mass equation and energy equation left.

Now, I will go and look at deriving P 2 by P 1 in some other form using the remaining equations than equating that to this, that way I will eliminate P 2 by P 1 also. So, I will just have an expression for M 2 by M 1 and M 2 in terms of mass and energy equation on this side and the other side; I already have M 1 and M 2, I will equate those 2 that is a goal.

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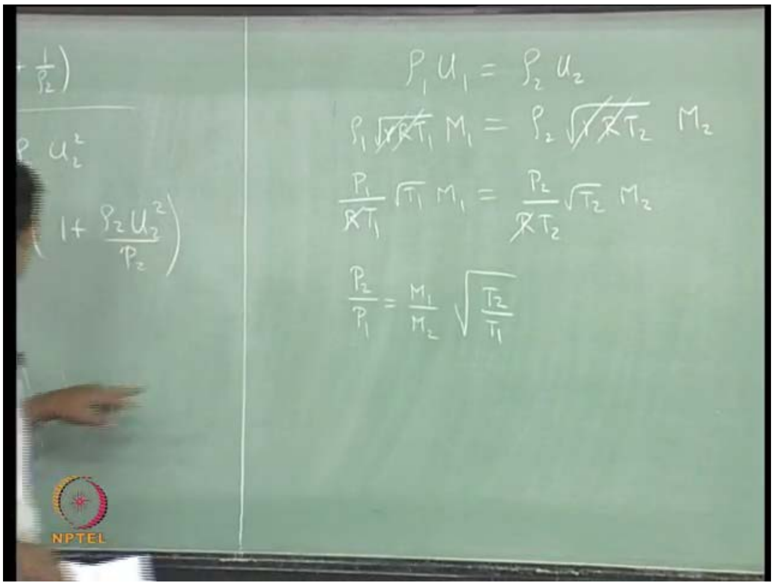

So, I am starting with mass equations, now I will write this velocity in terms of mach number definition, it should be M 1, I have this relation. Now I still have to express rho T in terms of mach numbers that is the overall goal, now I want to write this in terms of P 2 by P 1, because I have a simple expression for P 2 by P 1 in the previous board right. So, I will write rho as P by R T.

So, I will come out to P 1 by R T 1 and I will just remove this square root of gamma R from both sides, they are equal anyway the square root of T 1 and square root of T 2 still remains, where this R can also be cancelled. Now I am having a simpler expression, I will write this as P 2 by P 1 will be equal to M 1 by M 2 times square root of T 1 and the denominator finally here. And this will come out to be square root of vocalized T 2 in the numerator T 2 by T 1, I will get to this form; now I already know something about this.

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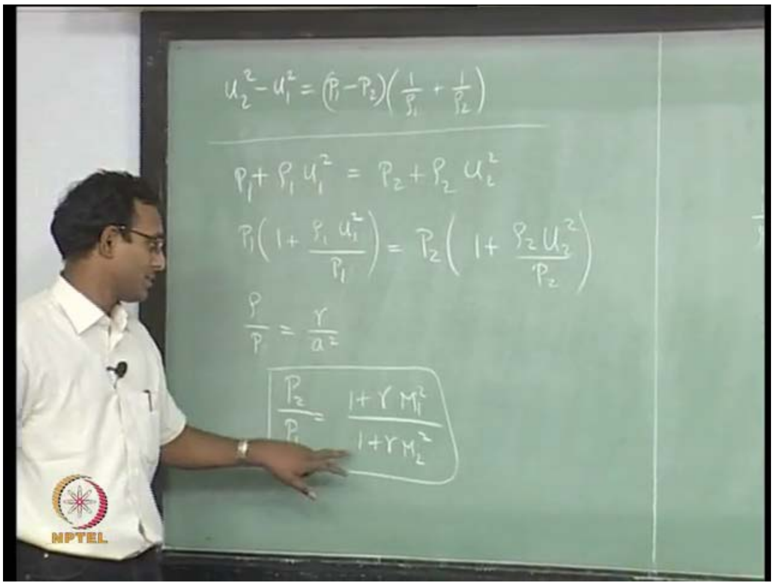

I know that this is equal to from here 1 plus gamma M 1 square divided by 1 plus gamma M 2 square, I will use this in terms of in place of P 2 by P 1. So as of now I have a relation here, if just look at terms.

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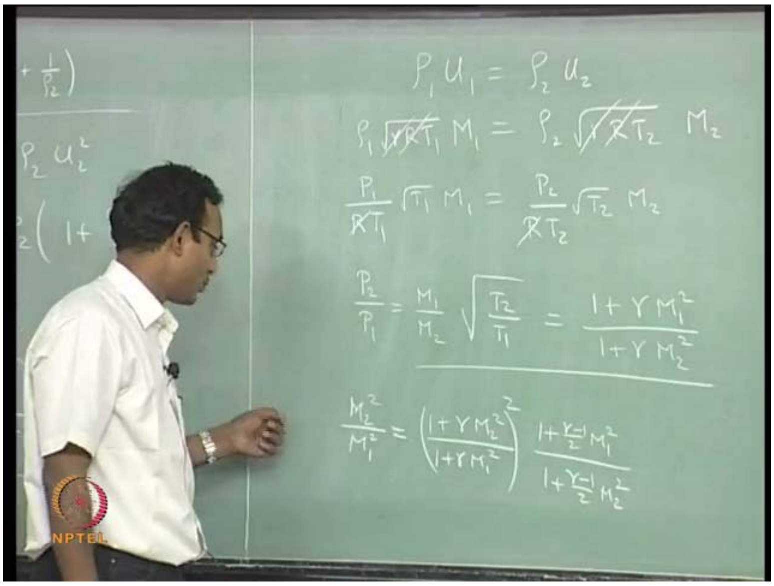

I have a relation here in terms of Mach number alone, except for T 2 by T 1. Now this T2 by T 1 by 1, I know that T not is a constant for the whole flow it. So, I can write this in terms of one plus gamma minus 1 by 2 M square. So, that is what I will do here. So, it will simplify to take a square of this, whole thing, because square root I do not like ever.

So, I will write it as M 2 square by M 1 square taking this ratio that side and everything else this side.

So, I will have 1 plus gamma M 2 square 1 plus gamma M 1 square, this whole square because I have squared the previous equation times I will have T 2 by T 1, I have squared it. So, T 2 numerator will be equivalent to T 2 by T naught divided by T 1 by T naught. So, it will be equivalent to T naught T 2 in the denominator T naught by T 1 in the numerator. So, I will write T naught by T 1 M 1 square divided by 1 plus gamma minus 1 by 2 mm 2 square.

So, I am finally, having a relation just relating M 1 and M 2, which is what we wanted, but it is still not very simple, we have to simplify this such that I can write it as M 2 equal to the function of M 1 alone, that is the overall job and that is what, we are going to do today. So, I have to solve for M 2 in terms of M 1 from here, what do, I have to do it looks like it is very easy to do. If you look at it, it is a plus b square multiplied by some b plus b or something like that, that is all, we are having here simplified form. So, I will just multiply out all the numerator multiply out all the denominators and then group all the terms together that whole thing, we are going to go through.

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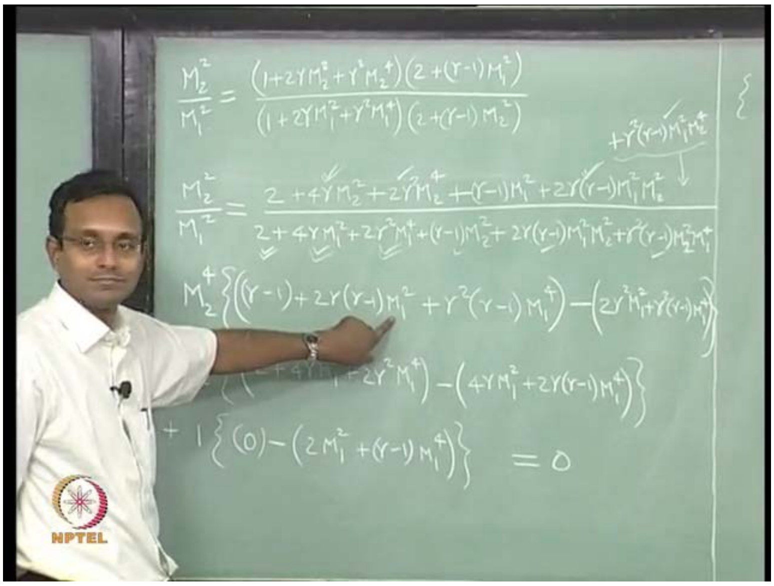

So, I will start writing, now I am going to be using more space on the board I will be more careful about the length, this term is just 1 plus gamma M 2 square, whole square that is the only thing I have written here as of now. And of course, I had the same thing with M 1 in the denominator. So, I will just write that immediately multiplied by the remaining term, I will just rearrange that as slightly multiply and divide by 2 and I will get to this form.

The good thing about this equation as of now is numerator and denominator look exactly the same expect for M 1 replaced by M 2 and M 1 M 2 replaced by M 1 we, have always been having this kind of symmetry we will just keep maintaining, it which will help us in finding any errors, if you make any, now I will have to multiply this with this a plus b plus c multiplied by d plus e, that is going to have 6 terms total. So, I will write all the terms out, I will give more space. These 3 terms are coming from 2 multiplying these 3 no I will write the terms for this M 1 square times gamma minus 1 multiplying this whole 3 terms, we will keep gamma minus 1, In bracket for now.

Even then I do not have space let us see, if I can just squeeze this 1 term here, this whole term is sitting here, that will be the last term gamma square M 2 power 4 multiplied by gamma minus 1 M square is 1 2 3 4 5 6 terms. There the same set of terms will be in the denominator just replacing M 2 by M 1 and M 1 by M 2. So, I will try and conserve space here, only the last term will look slightly different it is M 2 square here.

It is M 1 square M 2 square M 1 power 4 here, it is M 1 square M 2 power 4, again 1 2 3 4 5 6 terms this is not enough, we have to bring it to a form even more simpler. So, what I have to do, now is take the denominator multiply, it with M 2 square take this M 1 square multiply it out with this numerator. And then take all the terms to 1 side other side should be 0, that is overall thing, we have to do lets try and do this with grouping the terms already.

So, I am going to look for all the terms that have M 2 power 4 in it every term I will have I will consider it. So, M 2 square is here any term that has M 2 square here, if I take it to that side we will have M 2 power 4. So, I am going to look for this term that is a gamma minus 1. So, I will keep that and then this whole term multiplied by M 2 square is there that M 2 square with M 2 square will give, you M 2 power 4. So, I am going to have another M 2 square is sitting here, gamma square gamma minus 1 M 1 power 4 multiplied by M 2 power 4. So, that will be this is all just coming from denominator multiplying M 2 square.

Now, I may have other terms coming M 2 power 4 from here that is this term and the last term 2 terms have M 2 power 4 and they will come to this side with a minus sign. So, I will keep another bracket here, this is all coming from numerator now for denominator I have 2 terms. So, I will put a minus and then look at what those terms are M 1 square multiplied by 2 gamma square.

So, it will be 2 gamma square M 1 square plus 1 more term is there, that is M 2 power 4 in this corner, that will be multiplied by an M 1 square plus 1 more term is there, that is M 2 power 4 in this corner that will be multiplied by an M 1 square from this side coming there, it will become M 1 power 4 gamma square times gamma minus 1. This whole set of terms are all curve versions of M 2 power 4 only, now do we have M 2 power 3 anywhere we do not have M 2 power 3 terms because every M has a square or power 4 on it.

So, is not odd power every M 2 is having square or power 4. So, there will never be a power 3 on this corner. So, there is only M 2 power 2 now. So, of course, this whole bracket plus I am writing big equation folded into 4 pieces, these are all M 2 square terms collected. Now Of course, 2 times M 2 square is directly 1 term there, 4 gamma M 1 square multiplied by M 2 square that is another term there, this is again multiplied with this another term there, this will not occur this we already used 2 terms, we already used this term also.

So, I will actually put a tick for all the terms I used already, these 3 terms are used that is 1 2 3 terms here and then on the numerator I have used this term and I used this term only the remaining terms, I need to be using, these 3 terms are all going to be multiplying this M 2 square. So, they can directly come here, I will again put another bracket for whatever came from denominator minus whatever is coming from the numerator. I am again looking for M 2 square terms, this is M 1 square going and multiplying.

So, whichever terms here has M 2 square will be used by the way, now I have used all the terms in the denominator. I will put double tick, because it is on the second iteration, second set of terms, second flower bracket, if you want to think about it. Now M 2 square will be this term and this term. So, it will be 4 gamma M 1 square and M 1 square multiplied by this, which is 2 gamma into gamma minus 1 M 1 power 4 and there is no other term left, these are all I will have, this is the next term.

Now, there is no more terms left in the denominator only thing left will be all the other terms, which are not dependent on M 2, we will put all of them together, I will just put 1 just. So, you know M 1 M 2 power 0, if you want, there is nothing coming form the denominator just to show that I will put a 0 same bracket. The first bracket is from denominator nothing is coming form the first bracket, every term is used up only set of terms left are this 2 gamma minus 1 M 1 square only 2 terms are left, that is multiplied by M 1 square and taken to the left side. So, there is a minus sign.

So, it will be 2 M 1 square plus gamma minus 1 M 1 power 4, this is the whole set of terms we have, I do not want to put a triple tick here ones that are left are the ones, we are going through and all of the terms a rewritten in such a way that everything is here then this is equal to 0. This term plus this term plus is equal to 0. So, it is an equation folded into 3 lines that is what I have right now. You want to solve this. So, we have to simplify there terms, lets start looking at the first flower bracket, I will write that part alone inside.

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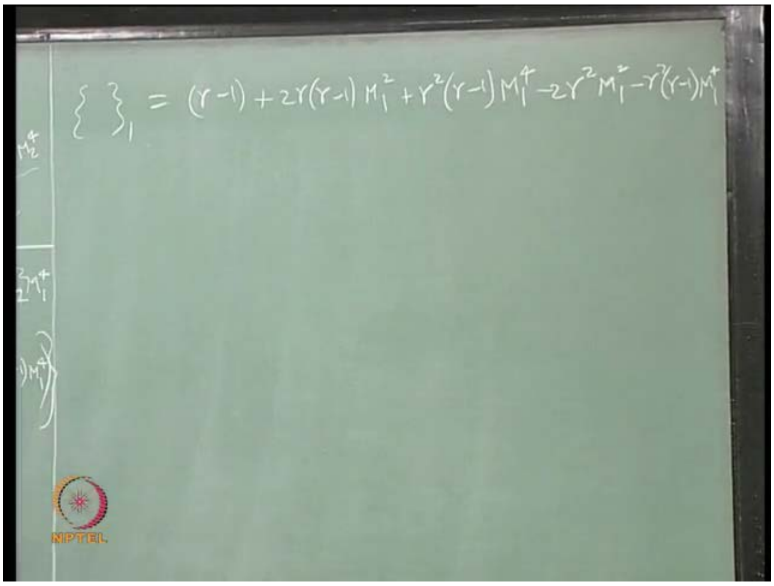

Just I will just calla at this, my first flower bracket, I want to simplify this term anything directly that can be simplified 2 gamma square M 1 square is there, it is also in this place. They are going to get cancelled, which 1 only this gamma into 2 gamma a that term will get cancelled with this term, maybe I will just directly start canceling, it will just be confusing know I will just go outside and do it again. It is not very difficult to them. This is what is in the first flower bracket.

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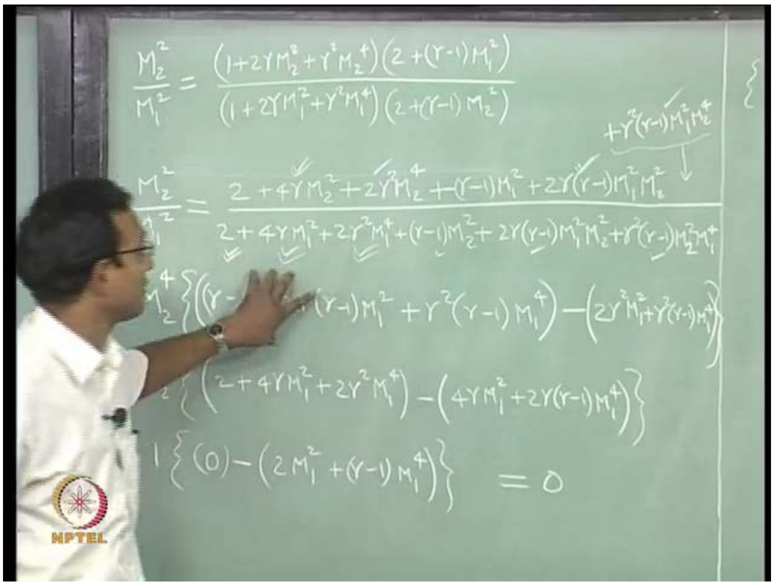

And I am just writing whatever is here there, now will just simplify the portion.

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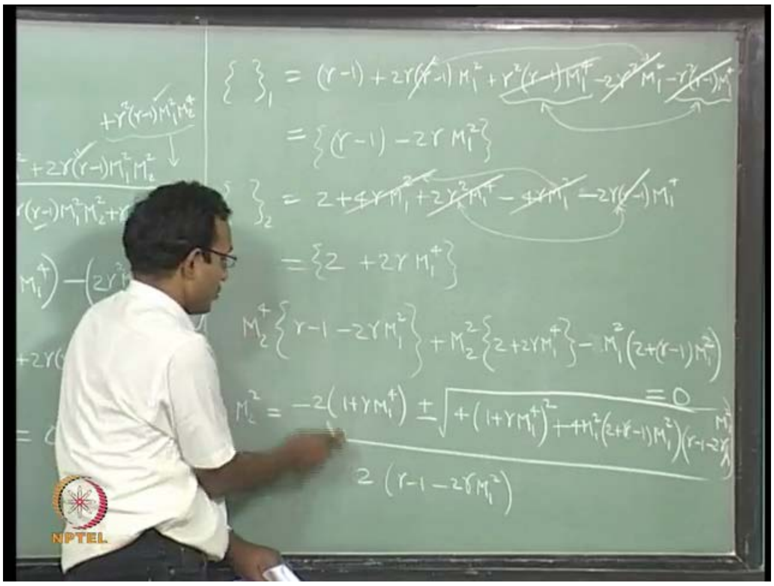

Of course, now I can cancel this and this and just, so canceling things correctly I will connect these 2 lines. And this is 1 cancellation am doing. Now, I am supposed to be looking for this term M 1 power 4 term and that is here, it looks like they will exactly cancel each other. So, I will just remove these 2 terms together cancelled the full term.

So, what is left is much simpler. So, I am left with gamma minus 1 and there is this minus 1 times 2 gamma M 1 square. So, what I am left with is gamma minus 1 minus 2 gamma M 1 square, this is my first flower bracket in this big expression, now I will go and pickup the next 1. Of course, we have to write the full expression with M 2 power 4 later, we will go pickup the next 1. So, the next flower bracket is going to be this is the full expression and we have to simplify this, if I look at this 1, I can directly cancel 4 gamma M 1 square with 4 gamma M 1 square, I will again keep that connection line.

So, that we know that, we are canceling correct terms and then now I am seeing 2 gamma square M 1 power 4, 2 gamma square M 1 power 4 of course, the minus 1 times 2 gamma is still remaining. So, I will just cancel this with this, I connect 2 together. So, the remaining terms, they are just going to remain and that is going to look like 2 and then minus 1 times minus 2 gamma. So, it will become plus 2 gamma M 1 power 4, this is your second flower bracket, this is what I am having here, third flower bracket is nothing great no need to simplify this.

So, much we just keep it like this nothing is going to get cancelled, we just keep it like this. Now I will just write the full expression for M 2 power 4 equation M 2 power 4 times first flower bracket, which is gamma minus 1 minus 2 gamma M 1 square plus second flower bracket times M 2 power 4 M 2 square sorry. And that is 2 plus 2 gamma M 1 square M 1 power 4 sorry and plus the last term. But, it has a minus sign in front of it I will put this minus 1 outside and also I will pull out a minus M 1 square.

So, just so it looks at least it looks 2 plus gamma minus M 1 square, we will keep it that way. So, it will become 2 M 1 square minus 2 M 1 square times, no there is no 2 sorry, if I pull out a 2, then it must be gamma minus 1 by 2. We will keep it like this minus M 1 square is what I am pulling out, we will keep 2 plus gamma minus 1 M 1 to the power 4, this whole thing equal to 0, whatever I wrote in three lines has now become almost 1 line equation. That is what I have brought it down to now if I look at this expression, it is actually M to power for M 2 square no function of M 2 equal to 0, M 1 square thank you is actually M 1 square, I have pulled out M 1 square there, should be M 1 square, I have to write here, I copied it wrong.

So, now, we have an expression M to power 4 multiplied by something constant as far as M 2 is concerned, it is a constant, M 2 square and then just a constant equal to 0, this is a quadratic in M 2 square. So, we have to write expression for M 2 square from what, we know from quadratic equation, we are going to solve quadratic equation. It will be minus B plus minus square root of B square minus 4 A C by 2 A is the solution for, which equation A x square plus B x is C equal to 0. So, we are going to use that whole thing here. So, you have to try and group things, but we will group that after I write it once here M 2 square, that will be my variable here right for quadratic equation M 2 square is my variable, M 2 square is what is here, M 2 square is given by minus of the second coefficient 2 into 1 plus gamma M 1 power 4 plus minus square root of this square B square right.

So, it will be I will take out the 2, because it is going to get cancelled soon nice, I have squared the 2 and put a 4 here, remaining things are sitting here minus 4 times first coefficient into last coefficient, that is what it is supposed to be. So, if I look at 4 times this into this with a minus sign, there is a minus sign here, they will go away and it will become plus put plus 4 M 1 square into 2 plus gamma minus 1 M 1 square. This thing multiplied by the first coefficient gamma minus 1 minus 2 gamma M 1 square, I will just write this M 1 square above and then close bracket around this.

I do not have space for the final thing is 2 gamma M 1 square there, that is basically this term there at the end square root extends all the way up to there this is just the numerator. I have to put divided by 2 a, 2 into just the first coefficient gamma minus 1 minus 2 gamma M 1 square, I said that 2 is going to get cancelled here. So, it is a nice point to cancel it, I will just take this to divide the numerator with it, this 4 will come out of square root and become 2.

So, I am just canceling this 4, this 4, this 2 with this 2 all that is cancelled, now the remaining thing is my solution, it is not easy to solve. But, that is my solution, I have M 2 square in terms of M 1 square or M 1 power 4, that is what I have in a way this is the solution, we will try and simplify this a little more, we will just go and look what is inside this square bracket inside this square root and see what happens inside here, I will go to the other corner.

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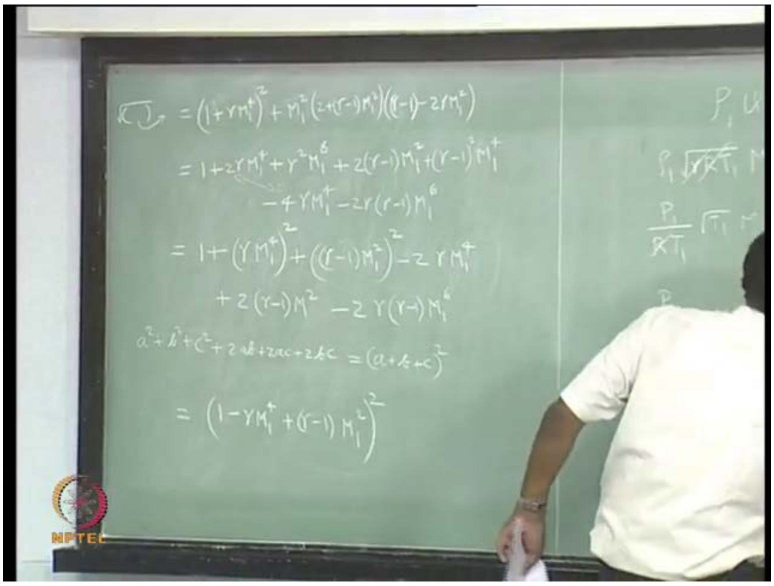

So, the term inside the square root term, whatever is inside only, I am writing whatever is inside the square root, it is going to look basically I will write the first full term inside square plus M 1 square times 2 plus gamma minus 1 M 1 square multiplied by gamma minus 1 minus 2 gamma M 1 square. This is the term that it is present, I just want to multiply it all out and write out the full expression and then we will start grouping them again as of now it is a plus b whole square.

So, it is going to be 1 plus 2 into this 2 gamma M 1 power 4 plus gamma square M 1 power 8, you have gone all the way up to 8 now plus I had to multiply things, I will keep this gamma minus 1 as a group, because it is easy to work with gamma minus 1. We will keep it like that for now, this gamma minus 1 is kept as a group it could be just gamma multiplied separately 1 multiplied separately, we will keep it as a group it is simpler to work with.

So, I am taking this and this multiplied with M 1 square that is 1 term, next term will be gamma minus 1 multiplied with the second 1 multiplied with M 1 square, which will be your M 1 power 4 into gamma minus 1 square. I do not need a bracket here, it is just M 1 power 4, that is just gamma minus 1 multiplied with these 2, now I have the remaining 2 terms, I will write that minus 2 gamma M 1 square multiplied with 2, it will become minus 4 gamma M 1 square into M 1 square M 1 power 4.

I am starting to write in the second line, it is still just one line equation multiplying this with this will give me minus 2 gamma into gamma minus 1 into M 1 square M 1 square M 1 square M 1 power 6. I have M 1 power for M 1 power 8, M 1 power 2, M 1 power 6, these are the whole set of equations you have, now look at these 2. So, these 2 can be replaced with 1 term, which is minus 2 gamma M 1 square M 1 power 4 sorry. It will just come out to be that now, we will rewrite that, I will group my terms in a nice way.

So, that I can simplify this further, I am going to write it as gamma M 1 power 4 whole square, which is what it was originally, this term I did not write this minus 2 gamma 1 power 4 yet. I will write it next, but before that I will write this gamma minus 1, M 1 power 4 terms. I will write that as gamma minus 1, M 1 square whole square of course, one can be written as 1 square. I do not want to write it that way, we will just keep it like this.

Now I will write this minus 2 gamma M 1 power 4 that is 1 term, now the remaining 2 terms are this 1 and this 1, M 1 square term and M 1 power 6 term. So, I will put that plus 2 gamma minus 1 M 1 square minus 2 gamma minus 1 M 1 power 6, these are the terms I have written it in a nice way, because I want to simplify. So, when I write it like this, if I look at this term this can be obtained by multiplying this with this without the square, it is gamma M 1 power 4 multiplied by 1 multiplied by 2 will give you this term.

Of course, there is the minus sign remember the minus sign also then I look at this 1, this is gamma minus 1 M 1 square only inside the square that term, multiplied by this 1 square, where 1 is there multiplied by 2. This is the plus sign that is here and then I have this term, which is gamma M 1 power 4 multiplied by gamma minus 1 M 1 square that is giving you M 1 power 6 multiplied by 2 and there is a minus sign.

So, it is of the form a square plus b squared plus c square plus 2 a b plus 2 a c plus 2 b c alright. This is of this form, this is equal to a plus b plus c, whole square that is what it will come down to of course, you can multiplied this.

And find out that is correct it is correct, you will get to this particular form. So, now, my a happens to be 1 and then of course, these 3 terms are a square b square c square terms now 1 of the terms is having minus sign, which one will it be, it will be this wherever there is gamma M 1 power 4, there is a minus sign, wherever there is gamma minus 1 m m square there is no minus sign.

So, I am going to say the minus is for these, that will be minus for gamma M 1 power 4 term. So, that b square the b is actually minus gamma M 1 power 4, b square happens to be this correctly nothing wrong with it. So, I will write this as 1 minus gamma M 1 power 4 plus this term, gamma minus 1 M 1 square, this whole square this is what is inside my square bracket, inside my square root. So, my whole square root expression will just be without the square, it just simplifies to that, now we will go and write M 2 square in terms of M 1 square again.

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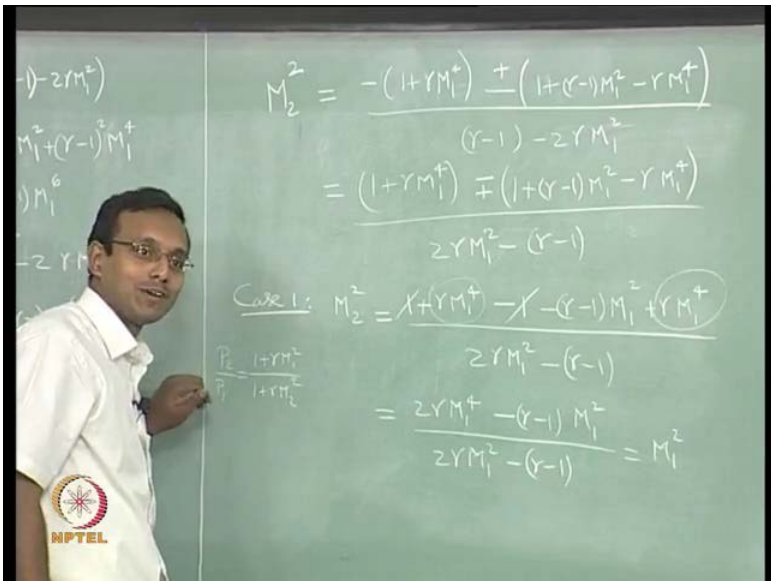

So, my original solution to my quadratic that expression M 2 square equal to it will become plus minus that bracket, which we just found out without the square term, because square root removes it minus gamma M 1 power 4 gamma minus 1 M 2, M 1 square, this divided by 2 time's, 2 is already cut off right. So, what will have will be gamma minus 1 minus 2 gamma M 1 square, this is what I should get from the quadratic thing. There is this coming from that corner of the board, this equation we just did what happens to the square root and we gave one bracket to the square term fully remaining terms, I am just writing as is in there.

So, I am coming to this point, we just have to simplify this for case 1 and case 2 plus and minus those are the 2 things left of course, if I want, I can rewrite this a little bit more multiplying numerator and inaugurate with minus 1. So, it looks little easier to work with. So, I will do that ideally, I should write it as minus plus, if we want to worry about, which solution is first and stuff.

We would not worry about it really since, I multiplied the denominator with minus 1, it will become 2 gamma M 1 square minus gamma minus 1. Now we have 2 cases, we have to solve for let us pick case 1, I will pick the first sign lets say I will pick the minus sign M 2 square is going to be equal to 1 plus gamma M 1 power 4 minus, I will rewrite the whole thing. So, I will just multiply with minus 1 each of the terms. This is what I have, can I simplify anything, there is nothing to simplify except this 1 minus 1 just that can be removed nothing else can be removed. But, if I look at this expression, I can simplify it a little bit of course, I can add these to together this term and this term are exactly the same.

So, I will write it as 2 gamma M 1 power 4 minus gamma minus 1 M 1 square divided by 2 gamma M 1 square minus gamma minus 1 look at the numerator and denominator the answer is supposed to be M 1 square that is all I will have. So, I am getting M 2 square into M 1 square, what are the possible solutions, M 2 equal to M 1 or M2 equal to minus M 1 minus M 1 does not make any meaning for us, except for I can say that the flow has reversed, which cannot happen for us.

Because, mass from this side going into the shock, mass from that side coming into the shock, what happens to all the mass is not conserved. So, that is not possible. So, the only solution will be M 2 equal to M 1 from here, but it comes out to be a simple solution, if M 2 equal to M 1, I already wrote this initial thing noises P 2 by P 1 equal to 1 plus gamma M 1 square by 1 plus gamma M 2 square. Now, if M 1 equal to M 2 or M 1 square equal to M 2 square, this whole thing will become equal to 1, P 2 by P 1 is 1, we already saw that, if P 2 by P 1 is 1, rho 2 by rho 1 is 1, t 2 by t 1 is 1, u 2 by u 1 is 1, that is there is no flow change at all across my wave comes down to that.

So, basically this is the case, which I pointed out at beginning of today's class where, we said if I set P 2 equal to P 1 and u 2 equal to u 1. The solution is satisfied, the equation is satisfied, there is the trivial solution hidden inside your equation and this comes out to be that trivial solution. That is one solution where, my flow equation are satisfied with no change in the flow, which is not what we are looking for, we are looking for a sudden change in the flow, we are looking for it. So, we are picking the other solution, we will not consider case 1, they will go to case 2 and consider only case 2.

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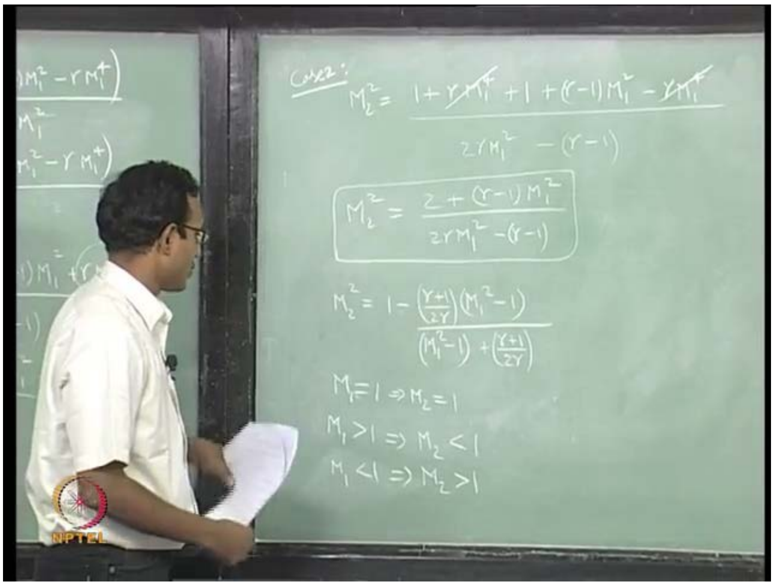

So, my case 2 is the same thing with a plus sign. So, I will write that gamma M 1 power 4 plus 1 plus gamma minus 1 M 1 square minus gamma M 1 to power 4, this is my case 2, here gamma M 1 power 4 will get cancelled. Now, the remaining terms remain it will become 2 plus gamma minus 1 M 1 square. So, M 2 square equal to 2 plus gamma minus 1 M 1 square divided by 2 gamma M 1 square minus gamma minus 1, cannot be simplified any more, this is just your relation between M 1 and M 2. Now I can rearrange this, I would not go through the algebra here it is not important, I can rearrange this and go to a point where, it looks like, I can get it to this form do not worry about how I got to this just rearranging this.

I can get to this I have to multiply and divided by appropriate terms and can subtract appropriate terms to get to this form, it is possible to get here, not much of use just for one simple reason, I am going to keep it. You can figure out how to get to this it is not very difficult to get. Now if I set my M 1 equal to 1 here, what will happen, this becomes 0 that does not change anything, I am going to get overall M 2 equal to 1, M 2 equal to 1, if M 1 equal to 1 M 2 equal to 1.

Now, I will look at the next case, I will write down all the three cases, M 1 equal to 1 implies M 2 equal to 1, if I pick M 1 greater than 1, if M 1 is greater than 1, then this quantity is positive, this is positive. So, I am going to get some quantity, which is positive subtracting 1, subtracting from 1. So, this will be less than 1, M 2 will be less than 1 square root of that will also be less than 1.

So, I am writing it like this, but there is one more possibility M 2, if this is greater than 1 sorry, I have to write it as other way, I will write it as M 1 less than 1, M 1 less than 1. You can show that, if this is less than 1, this whole term will become negative and multiplied by minus 1 will become positive. So, 1 plus some quantity square root of that that will be more than 1, I will get to this form, this is also possible. So, now, I have. So, many solutions M 1 equal to 1 M 2 equal to 1 is not in any great solution, it is just your mach wave, why do I say that it is a mach wave that is a wave. That is going through and it is going at speed of sound 1 more thing I have to look at I already remove it is here.

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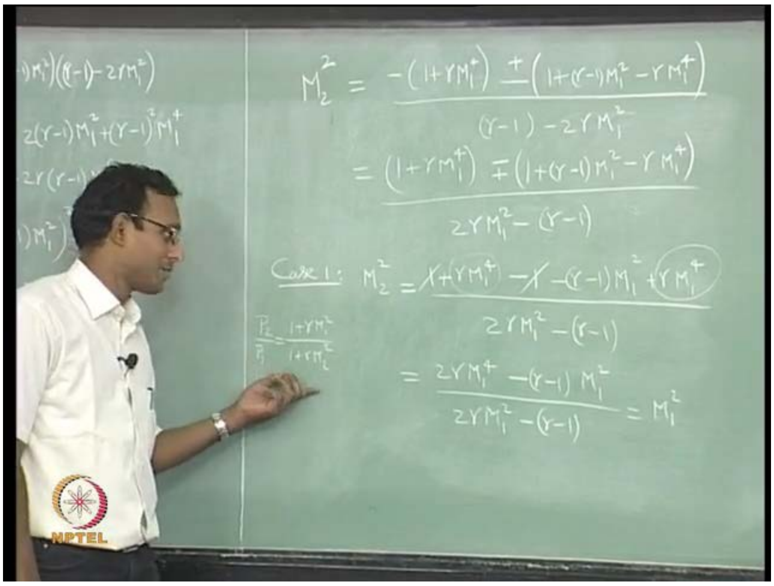

We will go back here, if I look at that M 1 equal to 1 and M 2 equal to 1, I am going to get P 2 by P 1 equal to 1, which means there is no change across that particular wave that is your simple acoustic wave called your mach wave. That is also a possible solution, but it is naturally coming out of expression, I do not need to worry about it anything special, that is just your mach wave nothing great, that we have to understand that is also a possible solution.

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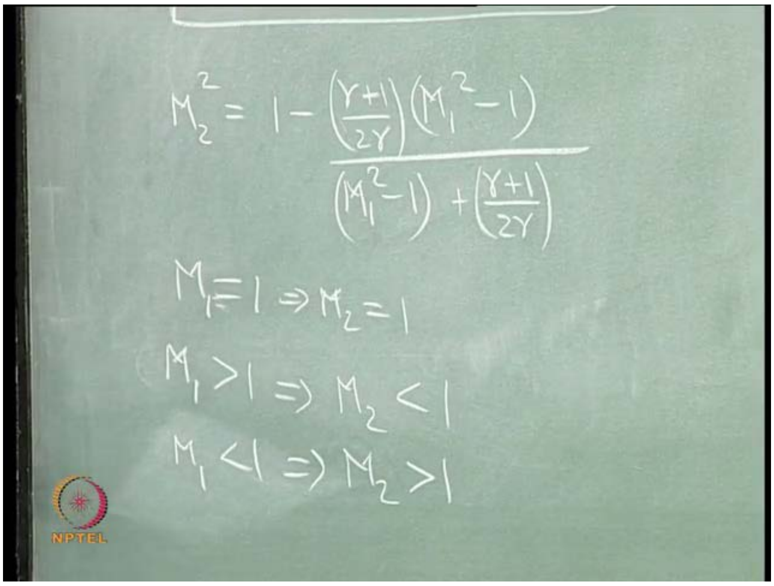

If I think about that particular solution, it is not doing anything to the flow, it is just acoustic wave, moving at speed of sound at every point, that is all this is. But, this 1 and this 1 will be very different speech of the waves, you are going to look for this. So, now, only one of them will be possible in real life it cannot be that supersonic flow goes to subsonic and subsonic flow goes to supersonic. If I look at my pressure, I will write that same expression here.

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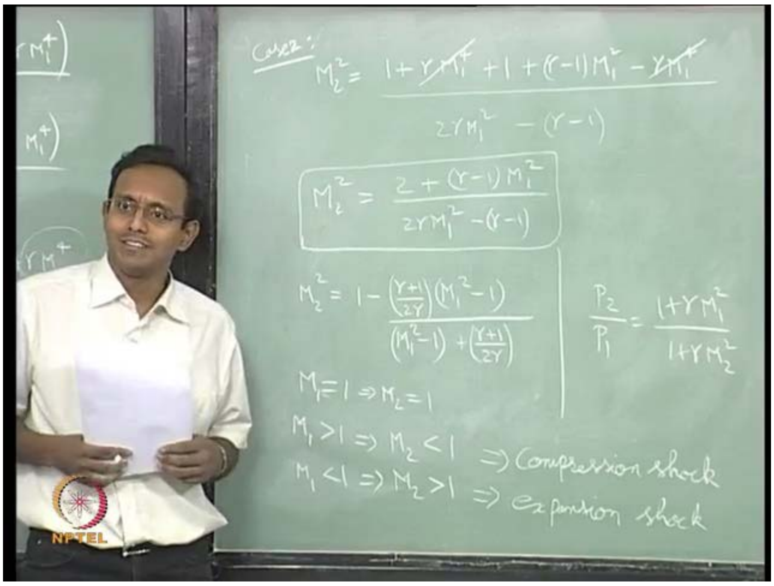

If I have this expression and I say supersonic to subsonic, this is more than 1, this is less than 1, I am going to get this P 2 by P 1 more than 1. Because, this is a big number this is a small number, I am going to get this whole thing greater than 1, if I pick this case what does that mean, I am going through a compression. I will call this the compression shock, my wave I already called it as a shock.

So, I am calling it a compression shock, if I look at the other 1 M is low M 2 is high, this is low, this is high, I will go through an expansion process, it is going from subsonic to supersonic. And I will get this to be expansion shock only 1 of them is possible alright, although I know that have to actually know this only from physical intuition, I cannot know it just by looking at the expression as far as the expansion is concerned, it can support both, maths says everything is possible.

So, now, we have to say is this available in nature is not available in our air, I am being very specific by saying, it is not available in our air, we will go deal with that next class. So, what, we have to do is go and look for what will happen really. So, how will I figure out that this is not possible only this is possible only way to do it is in nature is to go for thermodynamics and say entropy, what is happening to entropy, that is the only way to say something will not occur in nature.

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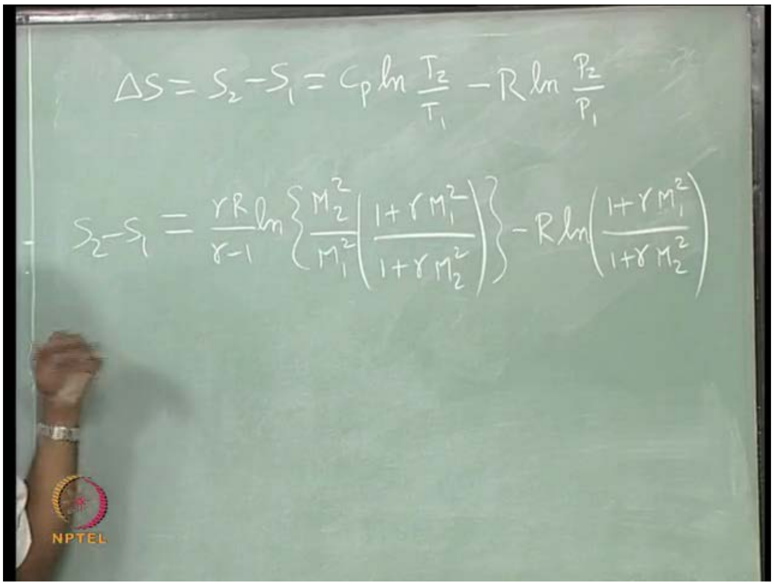

So, I am going to write delta S 2 minus S 1 is equal to we know this expressions, I am just writing it as is you know how to derive it also, we did it in the first few classes, we have this expression. Now, we have expressions for P 2 by P 1 in terms of M 2 and M 1, we have t 2 by t 1, in terms of M 2 and M 1 and C P, I can write it as gamma r by gamma minus 1, if I do all that together, I will get to form where, my S 2 minus S 1 is equal to this.

But, now if I substitute M 2 in terms of M 1 here, it will become very, very complex expression and now we have to figure out whether there is more than 1 or less than 1 for each of those individual cases, that is very difficult to do. Instead, we will go and do something different, it can be simplified, but we would not worry about that all I did was I took a graphical method, now we will go to that screen there.

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We will go to that screen, now I have plotted this expression delta r by s, which we found out, in here and as a function of Mach number. I am plotting from subsonic to supersonic M1 actually. This is just M 1, I do not know whether M can be subsonic or supersonic. So, I am plotting it for all the cases as a function of delta s by r, I am plotting as a function of M 1 for various gamma values, I do not know whether gamma is also matric. So, what I see is the blue line corresponds to gamma 1.1 mach gamma can go only from 1.667 to 1, somewhere there 1.67 is your noble gases or inert gases more atomic species. So, that is sitting something like this.

It is on the bottom end here and it is also on the bottom end here and what we are seeing predominantly is delta s by r equal to 0, when n equal to 1. Of course, we know already because when M 1 equal to 1 M 2 equal to 1, there is no change in the flow s delta is a 0, otherwise I am seeing that delta s is positive for M 1 greater than 1, delta s is negative for M 1 less than 1, that is the important thing, we are looking at here. So, I am going to say from this plot of course, I have used all the expressions math, I cannot directly argue out on experiment board. So, I used the plot.

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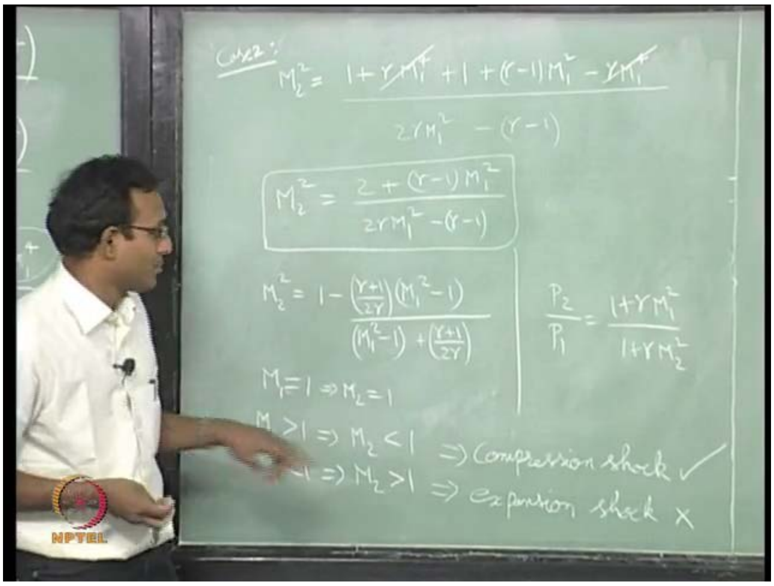

So, Now, I am going to say here, that this expansion shock is not possible only compression shock is possible. Because, in nature only delta s more than 1, more than 0 sorry, more than 0, will be the only thing that will occur in nature, it can also be equal to 0, which will be this case M 2 equal to M 1, that is a very special case. So, now, we have come to a point where, we can say that compression shock is what is possible in our air, I am again specifically choosing the words, we will look at why next class.