Gas Dynamics
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Module - 4
Lecture - 13
Normal Shock Relations

Hello everyone, welcome back we were in the process of deriving relationships between variables across the normal shock, for varying Mach numbers for incoming flow that is what we were doing. In that we had a point where we had to decide which shock will actually happen either expansion shock or compression shock. And we went through entropy arguments and we showed that I will go and draw that plot here again.

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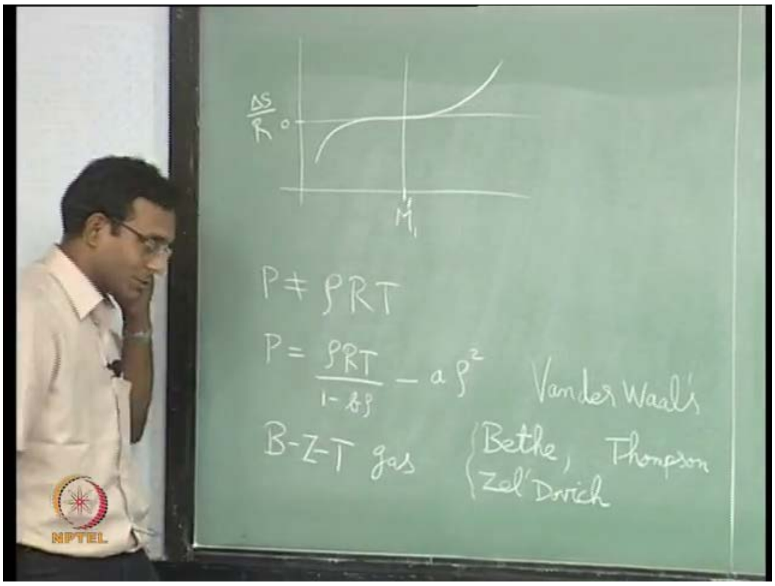

We had a plot that was looking like this, that is for mark number less than 1, we were having delta S by R that was slightly negative or more negative depending on very low mark number. For very high mark numbers more than one, we going to have delta S positive and we said that because of this reason, we are going to say only M 1 greater than 1 going to M 2 less than 1. That is the only solution that is possible, which corresponds to my gas gets compressed across the shock.

So, we are talking it is only compression shock is possible, that is what we said. And I also was very careful and telling you the statement that was when we say only compression shock is possible, I have also included this thing saying it is for our air. If I want to think about I am currently going beyond the course curriculum and just telling you if you are interested, you can go read up more. If we are in a gas which does not obey P equal to rho R T, this is what we have been using all this time we said P equal to rho R T for us our gas.

And so we got this behavior, if there is a gas where this is not true say, if I take the Van der Waal's equation which is also known to people already. So, I am using that an equation of this form where a and b are two constants for that particular gas, for that range of pressures and temperatures which we are interested in. This is going to be the equation for such gases where if I say a and b are 0, it becomes P equal to rho R T.

If there is a gas that is obeying this, then we can call it either the Vander Waal's gas or they gave a better name for it as in people who studied this based on them they get a name it is called the B Z T gas, B Z T flows, what they call for flows with this kind of gas. It is for three different people the names are Bethe, Zel'Dovich there is an apostrophe somewhere and the third guy is Thompson. I will write Thompson here, these people worked in the 1940s and he worked in 1970s, 60s and 70s and they studied flows where expansion shocks are probably possible.

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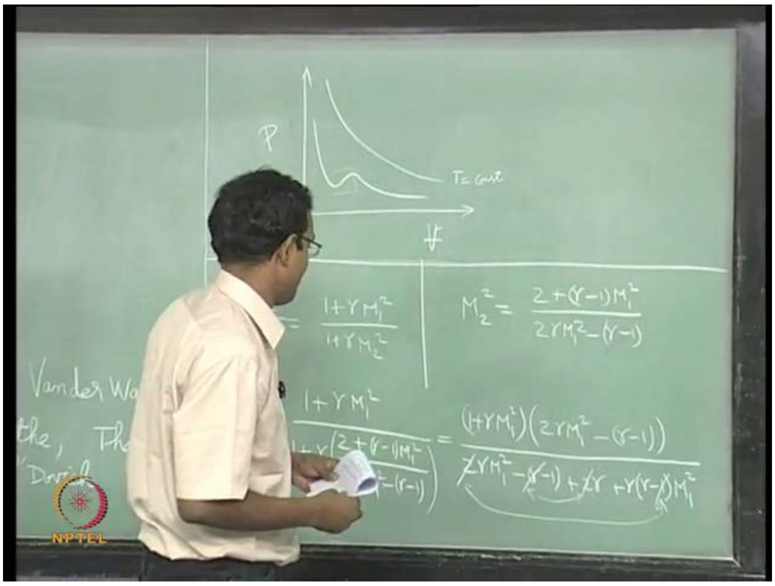

The way it works it is like this, I will go to a plot here of our state diagram I am going to use P and V, and for our common gas it is going to look like this for T equal to constant line. But, if my pressures and temperatures are very low where Vander Waal's forces are important or my gas is such that it is important, the curve may look like this. If we are very close to say condensation of water vapor, kind of ranges things may change and do crazy things like this.

I am not sure whether air with this little percentage of water vapor will it affect, I do not think it affects, but I may get such a curve for special gases. Typically, the refrigerants which people use will have such characteristics, anyways in this range the gas may behave differently because this is the opposite slope of d P by d rho. I will get here compared to all the other places this behavior is similar to this behavior, this behavior is similar to this behavior. But, in this small zone alone if my temperature and pressure and volume or pressure and temperature are such that my gas is operating in this range.

Then I may have expansion shocks and only compression waves are spreading away from each other, that is also possible. This is of course, is not in the scope of this particular course, I am just telling you there are things that are out there which we do not deal with in our course. That is the only thing I will say about this then we would not talk about B Z T flows any more. It is there somewhere, we are there are some gases where we can have expansion shocks and compression fans.

Of course, you do not know what they are right now, so I should not be talking about it more. Now, we will go back and derive every ratio variable which you have been deriving all this time in terms of M 1 and M 2, in just terms of M 1 alone just one variable. So, we had this variable P 2 by P 1 which is 1 plus gamma M 1 square by 1 plus gamma M 2 square, I had this.

Now, I will substitute the M 2 square which we got last class, I wanted to write it separately, so I will put a line here and write it here. M 2 square we got it to be 2 plus gamma minus 1 M 1 square divided by 2 gamma M 1 square minus gamma minus 1. I had this expression now I want to substitute this M 2 square inside here. So, P 2 by P 1 will now become 1 plus gamma M 1 square divided by 1 plus gamma times.

That expression in here 2 plus gamma minus 1 M 1 square divided by 2 gamma M 1 square minus gamma minus 1. Now, I want to simplify this it will come to very simplified expression. So, I know that already, so I am going to say I want to simplify this, so what I will do is I will take this denominator multiply it with this 1 and take that whole term to the numerator up there it is all I am going to do then it will become 1 plus gamma M 1 square times 2 gamma M 1 square minus gamma minus 1 divided by this multiplied with the 1.

So, it is going to be 2 gamma M 1 square minus gamma minus 1 plus 2 gamma plus gamma times gamma minus 1 M 1 square, this is what I am getting here. Now, we want to simplify this expression further, I had this particular form, if I look at terms in here, if I group it in a particular form. It will come out to be nice, but let us say I will go one step slower, I will just try and simplify it by the way it is already. What can I simplify here, minus gamma plus 2 gamma I can simplify that I am going to link these two.

Alright that can cancel each other of course, there is a plus 1 here nothing can be done with it currently. There is a gamma nothing can be done, 2 gamma M 1 square is there, here there is minus gamma M 1 square, so that can be simplified a little bit. I will remove that 2 with this minus 1, I will again link these 2 this is how I cancelled it. So, the remaining terms just stay, there is a gamma M 1 square there is a gamma square M 1 square and then there is a gamma and there is a plus 1. Those are the 4 terms that I have in the denominator.

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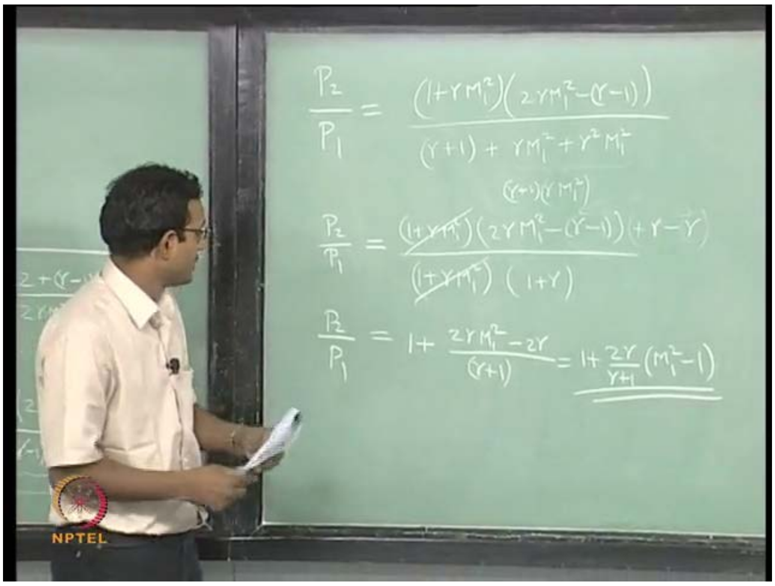

So, I will write this continue it here, I will keep the numerator as is that is the same numerator. Now, in the denominator I will write it in a different form, so it will become a gamma plus 1 plus gamma M 1 square plus gamma square M 1 square. This is what I have, I wrote it in a different order that is all I have done. So, this now if I look at these two terms this will just be gamma plus 1, because there is a gamma here there is no gamma here.

That is the only way I look at it gamma by M 1 square can be common, this is what it comes out to be. So, now, I can write this whole expression as P 2 by P 1 equals I get to this particular form, so basically have just pulled out 1 plus gamma common here, remaining will be 1 plus gamma M 1 square. Now, I am just directly going to cancel these two, whatever I have left is my final answer.

I can leave it like this or I can simplify it one more step which is what I will do. So, I am going to look at this term, this is 1 minus gamma plus 2 gamma M 1 square. I want to make it 1 plus gamma on the top, that is what I want to do. So, I will add and subtract gamma here minus gamma will go with this already existing gamma here, so that will become minus 2 gamma.

The remaining I will write it one step here plus gamma minus gamma I have added these two terms here now, and I am going to group this and this together. That will give me a minus 2 gamma, and the remaining term will gamma plus 1. So, I will write it as 1 plus 2 gamma M 1 square minus 2 gamma divided by gamma plus 1, this is what I have which can further be simplified as is 1 plus 2 gamma by gamma plus 1 times M 1 square minus 1.

That is your final result, it is extremely simple this is equal to P 2 by P 1, I will write it here again very simple form easy to remember. Now, will do the same thing for rho 2 by rho 1, because I know this is going to give me a simple enough expression. We are going to start with what we have derived already.

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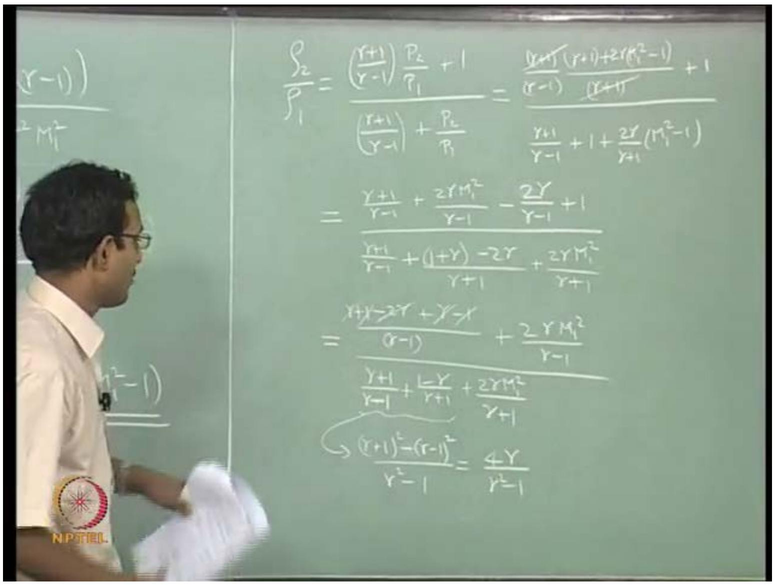

We have derived this long back, at least three classes before I believe it, it will just come out to be this form. Hopefully, you remember that we derived this long back it, when we started with normal shock we derived this the very first class of normal shocks. Now, I want to substitute that P 2 by P 1 which we just derived in here, and it will become P 2 by P 1. I do not want to write it as 1 plus 2 gamma M 1 square, so I will write it as gamma plus 1 divided by gamma plus 1.

This way I can cancel the gamma plus 1, that is why I wrote this way. And the other one will be gamma plus 1 by gamma minus 1 plus 1 plus 2 gamma by gamma plus 1 M 1 square minus 1. Here, I am retaining the original form which we had P 2 by P 1 which we derived just now, the numerator I have multiplied this gamma plus 1 in there and put common denominator gamma plus 1. That is what I have I can cancel this gamma plus 1 directly there, so I am going to have a slightly different expression which I have to rearrange eventually.

So, I will start pulling out gamma plus 1 by gamma minus 1, that is 1 term plus 2 gamma M 1 square by gamma minus 1 minus 2 gamma by gamma minus 1. And then plus 1 is there I will just keep it like that currently divided by now I have two rearrange this whole group of terms. So, I will write it as currently gamma plus 1 by gamma minus 1 remains the same plus, I will pull out this minus 1 with 2 gamma by gamma plus 1 along with this 1.

So, it will become 1 plus gamma minus 2 gamma divided by gamma plus 1 plus 2 gamma M 1 square by gamma plus 1. It is all just algebra I am just rewriting things in one way or the other, now I have to simplify this I will look at the numerator terms first I will keep the M 1 square terms separately. I will link all the other terms together, so that is going to have a common denominator gamma minus 1, numerator is gamma plus 1 for the first set. And then I will take this minus 2 gamma and then this 1 can be written as gamma minus 1 by gamma minus 1, so it will be gamma minus 1.

This is all I am having now if I look at it I will write the remaining one term also, then I will write I will start cancelling things 2 gamma M 1 square by gamma minus 1. Denominator will come and write later, but as of now numerator everything goes away, this whole term goes away only this term remains that is the nice thing about it. Now, we will go to the denominator, I will write it one more step like this gamma plus 1 by gamma minus 1 plus I will simplify this.

This will become 1 minus gamma by gamma plus 1 plus 2 gamma M 1 square by gamma plus 1, this is what I have now. Now, I have to simplify this set of terms, if I make this multiply these two together it is going to have gamma square minus 1 as denominator. And numerator will be gamma plus 1 square plus alright I am having 1 minus gamma and gamma minus 1. So, I am going to have a minus sign of minus 1 square, this is what I will get I can write this as minus of gamma minus 1. And then gamma minus 1 multiplying gamma minus 1, I am getting gamma minus 1 square, I am just looking at this 1 term right now in the denominator.

I will write the whole thing later as of now I will just look at this term, that is coming out to be this. Just this 1 term is this which, now if you look at it, it is going to be a square plus b square minus 2 a b. And you will find that a square and b square will get cancelled with this a square and b square only thing remaining will be this will have a plus 2 gamma minus minus 2 gamma. That will give you four gamma total for gamma by gamma square minus 1, this is what I will have for just this set of terms remaining term exists still I will write this whole thing in the next corner again.

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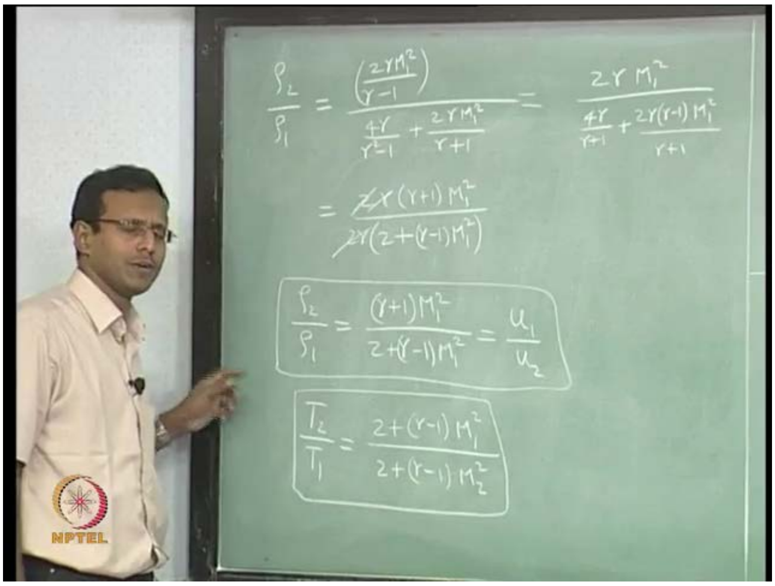

We already found that the numerator is very simple 2 gamma M 1 square by gamma minus 1. Now, we have simplified the denominator and it looks like 4 gamma by gamma square minus 1 plus 2 gamma M 1 square by gamma plus 1. This is what I have, now I will take this gamma minus 1 to the denominator and multiply it throughout, that is a next step. If I do that when I multiply that gamma minus 1 with this term gamma minus 1 into gamma plus 1 is what gave this.

So, it will become gamma plus 1 in the denominator here we will write it one by one. So, it is going to be 2 gamma M 1 square divided by 4 gamma by gamma plus 1 plus 2 gamma into gamma minus 1 M 1 square by gamma plus 1. This is what I have, now I can take this common denominator gamma plus 1 and put it up there. So, I will rewrite it as 2 gamma into gamma plus 1 M 1 square divided by I am having 4 gamma plus, 4 gamma actually I will just take out the 2 gamma first.

So, that I can cancel it with the numerator, 2 gamma into 2 plus gamma minus 1 M 1 square we will write it like this. This is what I have, if I write it up to this point now I can cancel this 2 gamma with this 2 gamma and I do not think I can simplify this any more. So, that is my final expression I will write rho 2 by rho 1 is equal to gamma plus 1 M 1 square divided by 2 plus gamma minus 1 M 1 square.

It is so happens that this is also equal to u 1 by u 2, why from mass equation alright. So, this is another set of relations we have given the M 1 I can find the rho 2 by rho 1. I can find u 1 by u 2 I already wrote an expression sometime back for the third word P 2 by P 1 in terms of M 1 square, that is also given. So, the next thing left is T 2 by T 1, I tried deriving T 2 by T 1 in any simple form, but it is very, very complex.

So, we will just keep the simplest form M 1 square on the numerator, and this is the only form I will keep. Where, is this coming from I am going to say T naught 2 equal to T naught 1 and I am using energy equation, basically T naught 2 by T naught T 2 T naught 2 by T 2 is 1 plus gamma minus 1 by 2 M 2 square. Similarly, T naught 1 by T 1 will be 1 plus gamma minus 1 by 2 M 1 square and I have just rearranged that two such that it looks like this, this is another expression. So, now after this whole set of expressions I just want to have in your notebook one-page which will have all the formulae together for normal shocks. Like, we did for isentropic relations we want to have one-page just all the possible relations for normal shock.

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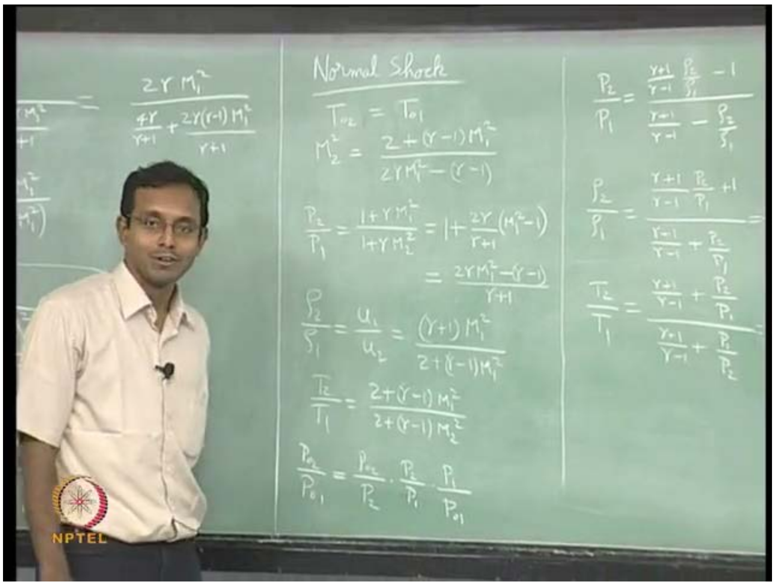

So, first thing I can directly say T naught 2 equal to T naught 1, we proved this long back we will just keep it that way. Next main thing I will divide the board somewhere here because I want to write some parallel set of relations there. M 2 square we derived this towards the end of last class and we used to this extensively today, 2 gamma M 1 square minus gamma minus 1 that is the next relation. And then we just derived a whole set of relations.

I will write the whole set of them here P 2 by P 1 one nice form for P 2 by P 1 is 1 plus gamma M 1 square by 1 plus gamma M 2 square. This is a nice form, so I always want to keep track of this another form which we derived today is this. Now, this can also be rearranged in one form, which we actually did today during one of the derivations it came out to be this. I am writing this also because this numerator is shared by the this thing as a denominator, that is the only reason why I will keep this. This denominator is same as this numerator that is the only reason I like this form also.

In case sometimes I have M 2 square multiplying P 2 by P 1 in any relation, I may need to use this kind of form. That is what ideally you can use any form it will automatically rearrange itself that is the beauty of math. Anyways, now the next thing is rho 2 by rho 1 we know that it is equal to u 1 by u 2, which we just now derived, but anyway I will write the expression here for completeness sake gamma plus 1 M 1 square divided by 2 plus gamma minus 1 M 1 square. And we said that there is no special relation for T 2 by T 1 I will write that expression also here.

These are the set of relations we have one more I have not written here, I will come back to it, but before that I will also write the other set of parallel relations. One ratio in terms of other, supposed to be rho 2 by rho 1 that is 1 relation rho 2 by rho 1 in terms of P 2 by P 1. Of course, it can be obtained by rearranging the top 1, of course, this is equal to u 1 by u 2, T 2 by T 1 in terms of P 2 by P 1 can be written in a different form.

I do not think I derived this particular relation, but it is not very difficult to get if I take this rho 2 by rho 1 and then I take this P 2 by P 1 I am going to say T 2 is P 2 by rho 2, T 1 is P 1 by rho 1. So, if I use this divided by this actually I do not need to do that I want everything in terms of P 2 by P 1. So, I am going to take this multiplied this with P 1 by P 2, that will give me I want the reciprocal of this multiplied by P 2 by P 1 that will give you this relation.

You can go write it, it is not very difficult I will just write that expression here, P 2 by P 1 times rho 1 by rho 2 I am using this rho 1 by rho 2 from here reciprocal of this multiplied by P 2 by P 1. Then I will get to this form, this is another form which you can get. Now, other than this I want to write one more expression here, we have not yet derived it, but for completeness sake I will put it here and then I will derive it now.

One simple thing is P naught 2 by P naught 1, this is not very difficult to derive I will just write it here we use particular technique which is very common in gas dynamics. We have expanded this particular ratio into multiply a product of 3 ratios this is a cycling product of ratios that is what we have done. We look at it, I have expression for this in terms of 1 plus gamma minus 1 by 2 M square to the power gamma by gamma minus 1. Similar, expression for this just the reciprocal of that and then P 2 by P 1 we have a relation here up already. So, I can get this P naught 2 by P naught 1 already. Of course, I can write that expression, but you know that expression, I do not need to write it separately.

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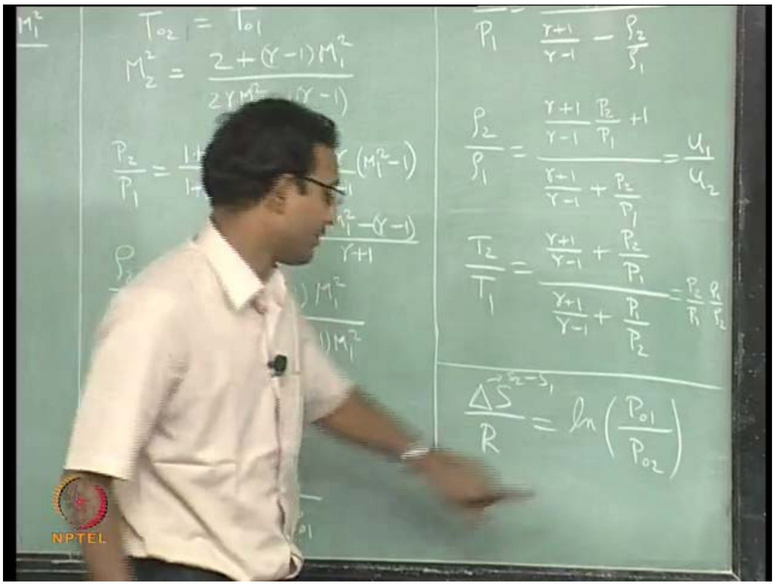

One more remaining expression which belongs in this column I will write here is delta S by R, the entropy change is equal to log of P naught 1 by P naught 2. Where, this delta S is actually S 2 minus S 1 is coming out to be log of P naught 1 by P naught 2. Now, this is the only thing which I have not derived we will derive it now not very difficult to derive it is easy enough.

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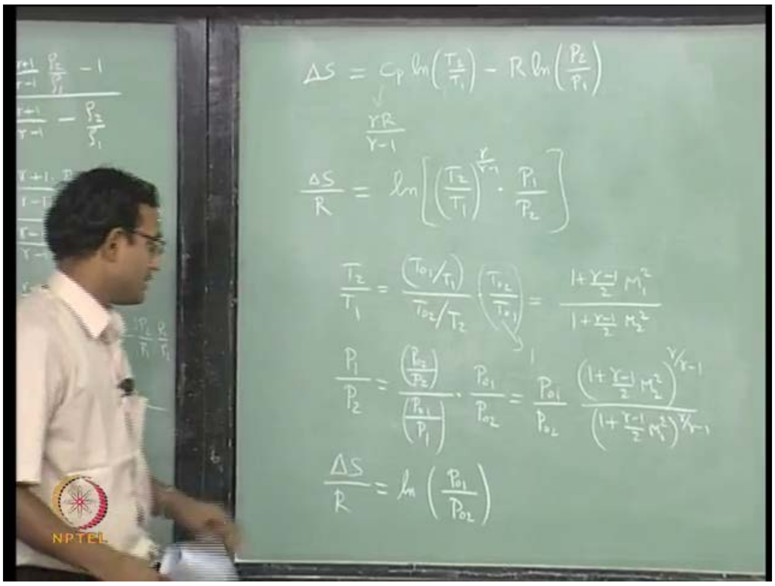

I just have to start with the second class of our thermodynamics review, delta S is c p log T 2 by T 1 minus R log P 2 by P 1. We had this form already and we are going to say calorically perfect gas kind of assumptions this c p can be written as gamma R by gamma minus 1. Once, I have this I can pull out an R and put it below delta S, so I will rewrite it as delta S by R equal to log of gamma by gamma minus 1 is in front of log I can take it inside the log by putting power of T 2 by T 1.

So, I will make it T 2 by T 1 to the power gamma by gamma minus 1 minus another log is there. So, it will become reciprocal of P 2 by P 1, so it will be P 1 by P 2, I made it into 1 log that is all I have done. Now, I have expressions T 2 by T 1 and P 2 and P 1 by P 2, we just have to put the appropriate expressions and it will be comfortable enough. And if you use a wrong expression I will get a very complicated relation, you have to use the correct one, so that it looks simple enough alright.

So, I want to make everything in terms of stagnation pressures and stagnation temperatures, because I know something about the stagnation temperatures they are equal. So, I will write this T 2 by T 1 as I will write it as T naught 1 by T 1 divided by T naught 2 by T 2 multiplied by T naught 2 by T naught 1, this is correct. Now, what I want to do is I will write an expression for this in terms of M 1 I will write a expression for this in terms of M 2.

That will come out to be and I know that this is equal to 1, T naught 2 equal to T naught 1 across a normal shock. So, I can just use this, now I will go look at the remaining terms alone that will be 1 plus gamma minus 1 by 2 M 1 square divided by 1 plus gamma minus 1 by 2 M 2 square, I have this. Now, I have to look at P 1 by P 2, this can be written in terms of stagnation pressures again, that is going to become P naught 2 by P 2 divided by P naught 1 by P 1 multiplied by P naught 1 by P naught 2.

I will write it like this, currently we will keep it like this P naught 1 by P naught 2, we do not we know that it is not equal to 1. It is something other than 1 will keep it, but the other two expressions I can write in terms of 1 plus gamma minus 1 by 2 M square, P naught 1 by P naught 2 times 1 plus gamma minus 1 by 2 M 2 square to the power gamma by gamma minus 1 divided by 1 plus gamma minus 1 by 2 M 1 square again to the power gamma by gamma minus 1.

Now, if I go back to my delta S by R expression I have a T 2 by T 1 to the power gamma by gamma minus 1, that is T 2 by T 1 to the power gamma by gamma minus 1 will be this. This to the power gamma by gamma minus 1, while this is P 1 by P 2 expression is here which is also having almost this to the power gamma by gamma minus 1. Look at carefully M 2 square is the denominator here in the numerator, when I multiply these two will get cancelled.

Similarly, M 1 square will get cancelled with this 1, so finally, I will be left with just P naught 1 by P naught 2. So, that is how I am deriving there is delta S by R will finally, just come down to log of P naught 1 by P naught 2. This is S 2 minus S 1 is P naught 1 by P naught 2 that is what you will get. And of course, you should know from here that P naught 2 is always less than P naught 1, because we want entropy to be increasing. Delta S has to be positive which means this log has to be positive which will happen only if the ratio is more than 1, so I can tell that P naught 2 is less than P naught 1 directly. So, this is the derivation for the previous page I gave you a formula sheet, now we have come to a point where we have to get more physical feel for stuff.

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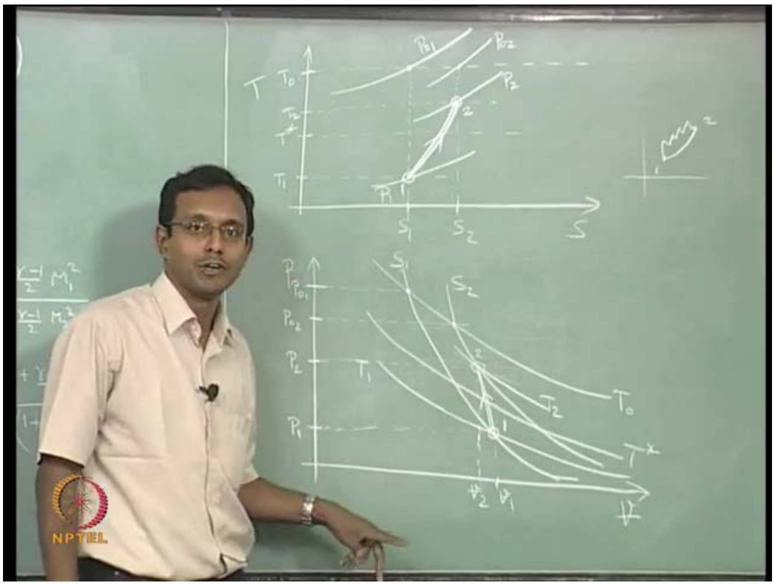

So, we will go and plot thermodynamic process, I am going to look for thermodynamic process here I am looking at T s diagram. So, first thing I will mark will be a T 0 because I know it is a constant, now I will mark a P 0 1, this is T 0 T 0 1 or T 0 2 both are equal I just put T 0 now I will mark a P 0 1. That will be some curve you know that constant pressure curves on T s diagram will be exponential curves. It is going like this, I will put it as P 0 1, that is pressure equal to P 0 1 will be this curve.

Now, I am telling that for this T 0 T star is here and I am saying my flow incoming is supersonic which means my T 1 will be somewhere lower. This is my T 1 line and I know this is a stagnation condition, which means this is the entropy value. So, my P 1 must pass through this point that will be another exponential curve going like this, this is my P 1 curve. So, my actual state on this diagram is P 1, T 1 which will be this point that is my initial point.

That is where I am from here I am going to a subsonic line that is I am crossing this line and going above. So, let us say this is my T 2 I am crossing my T star and going to some higher temperature as am doing that I am also increasing entropy, so I will put a entropy line here. So, this will be my new state and so I am going to put a P 2 line through that curve through that actually, this is my P 2 curve.

So, this is my actual state T 2 S 2, I will call this S 1, this is S 2, T 2 S 2 will define my state right, two thermodynamic variables or I could use T 2 P 2 that will define my state any ways. So, I am having these two this, this is my state 1 and this is my state 2, one more thing missing is P 0 2. It is just extending it through an imaginary isentropic process from 2 to 0 2 that will reach there.

So, I will draw another curve here that is might P 0 2, it is already seen directly that my pressure drops P 0 2 should be less than P 0 1 graphically also you can see that. Now, my actual process of shock is this line I will draw double line. So, that I will tell that this is my shock, this is the actual process for the shock. When, I say a process I do not know what is exact path it is taking all I know is it starts with 1 and ends with 2, I have drawn a curve like this maybe somebody else will draw a curve like this.

I want to disturb this picture I will just take another 1 and 2 I have drawn a curve like this, maybe somebody else will draw a curve like this. Both are valid and my opinion, because we know only starting point and ending point, I know how it goes through this currently, both are equivalent in my opinion. So, what we can say is from here the gas jumps from this state to this state within a thin layer and we do not have details of what is it doing inside the thin-layer, all we know is it is jumping from here to there.

So, ideally we should not be worrying about what is the path it takes there is no path inside the shock, it is a very thin layer, so that is the detail I have here. Now, we will go for P V diagram of course, every time I draw a T S diagram I will immediately go to P V diagram you have to get used to this. You should be able to handle the problem in both the coordinate systems, now here I have to draw first a constant temperature line my T naught line which will be a hyperbola.

So, it is going to look something like this T naught equal to constant, that is the T naught line. Now, I have to say this is my P naught 1, I will pick a P naught 1. Wherever, this line goes and meets my T naught 1 that is my stagnation 1 condition 0 1 condition will be T naught 1 and P naught 1, wherever that meets that is my stagnation condition. Now, from there I have to go to different temperatures, what did we do here we went to T star, so I will draw T star curve which will be a lesser temperature than that.

It will be another hyperbola this is my T star line. Actually, what I want is T 1 line, it should be below this that is all I want, so I will draw a T 1 line. Once, I have drawn a T 1 line I need to extend it further, I will extend this T 1 curve all the way up to their, this is my T 1 curve. I know the stagnation condition, I have to go to the static condition. How should I go, I have to go along isentropic curve.

So, I will draw isentropic curve through this point stagnation condition which will be having a more steeper curve that is why it is interesting. It will be a curve like this of course, it will never cross 0, it is going to turn and go become something like this. Now, so this is my state 1 and if I draw a line along this that is my P 1 line, and of course, if I want density 1 it will be reciprocal of this particular V 1 that is all.

Now, I have this is my state 1 and I am saying it is having same T naught that is going to have a lower P naught after it crosses. So, I will mark a P naught 2 and same T naught and P naught will mean this is the new stagnation condition P naught 2 P naught two that is this point. Now, I know my final state T 2 must be sitting somewhere along this isentropic line, this is my S 1 this is my S 2 line I have to draw S 2 line, it will be a line like this. Now, I have to tell my solution is somewhere between T naught 2 T naught and T star, so let us pick line as my T 2.

No, I have drawn it wrong, it should be in between that these 2 other two curves. So, it will be more steep something like this, that is my T 2 curve. So, I am getting a P 2 line, I am just checking whether I have drawn the curves correctly, it is close enough to say that. I have to make sure that it is compression alright, I should not pick a crazy delta P that it will look like expansion that is all. So, currently I have made sure that it is close this is my state 1 and V 2 is less than V 1, which says I am going through a compression process everything is matching.

So, now, my state process is something from 1 to 2, again I will draw double line because there are so many curves. This is my jump across the shock, it is going from this point 1 to this point 2. We do know, how it goes all we know is this is the starting point, this is the ending point intermediate path we do not know we leave it like that for now. So, at any time you should know that in a P V diagram my correctly going to go up and to the left for a shock and in a T S diagram it is going to go up and to the right.

It is easy to tell shock heats the gas and it also increases entropy, that will tell you it is going up and to the right, in a P V diagram it is going to compress the gas and increase pressure. Volume decreases, pressure increases it is going to the left, left on top easy way to think about this. Now, we want to do some analysis with expressions we got today, so maybe I will there is just enough time to show some plots, so I will show you the plots then we will do the analysis next time.

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So, I will go to this plots in here what I am showing you here is a P 2 by P 1 and T 2 by T 1 on the same axis. While, the solid lines are given by P 2 by P 1 this three lines here the yellow is not very clear this line, that is your T 2 by T 1 for one of the gamma values I believe. This is having highest compression, I am not able to see the color I think this is yellow is that right. So, that is 1.67 gamma equal to 1.67 T 2 by T 1 curve looks like this, I have drawn this for three different gamma values.

Gamma equal to 1.3 1.4 and 1.67 that is why it is coming onto be, and similarly here 1.3 1.4 1.67 that is what I am having here. What we are seeing first observation should be that as my mark number increases, I have drawn from 1 to 10. As, my mark number increases both my ratios increase, that is if my shock strength is higher, I am going to have higher and higher pressure ratio or temperature ratio. Next detail I can say that pressure ratio is much higher than temperature ratio, that is a next observation.

If I have drawn a density ratio on the same plot, it will go in between these two sets of curves, but I am drawn it. It will look more confusing, so I did not draw it there. We look at rho 2 by rho 1, but we will look at it later. So, if I have drawn it that will go somewhere in between these two, actually you know that rho 2 by rho 1 times T 2 by T 1 is your P 2 by P 1 right. So, when I multiply this curve and some other curve here, it will give me this curve.

That why it should be I have not drawn the rho 2 by rho 1 curve in the middle here, that is all I have done. If I look at the effect of gamma value, in my gamma value is low for the same R and T my speed of sound will be lesser. What does that mean if my speed of sound is lesser, my gas is very compressible. And if not my gas is less compressible, what we are finding is for a given mark number if my gas is less compressible I am having better compression, it is the pressure increases more.

If not pressure increases less, that is what we are seeing actually, this is for this particular case temperature also does exactly the same thing. As, I become more and more compressible, the gas that gets compressed less. I can give you reason for this, but I do not think it is useful in this particular course, it is something to do with the internal energy. Gas will rearrange that compression, that is the pressure energy in other internal energy modes that is what is happening inside. When, gamma decreases your C V and C P values will increase and that is what is causing, this no more details than this will go to the next plot.

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I will go to the some other plot and come back to this, now I am looking at M 2 versus M 1 I have also given u 2 by u 1 ratio in this dotted line. We will first look at M 2 versus M 1, what we are seeing is as I increase my mark number my M 2 drops. It drops steeply up to around mark three after that when I go to very high mark numbers say around mark 6 and above does not look like M 2 changes much, it is almost a constant.

In fact, we can show asymptotic analysis will show you that it will go to some particular number, we will look at numbers, but I will do that next class. We would not do it now, but I will look at u 2 by u 1 here. We are again seeing that u 2 by u 1 is dropping and then becoming constant for hypersonic velocities, hypersonic mark numbers. When we go a very high mark numbers see above 7 or, so there almost the constant u 2 by u 1 is a constant, M 2 is a constant for each of the gamma values any gamma values for that instance.

Now, we will look at details of what happens when gamma changes, if my M 2 if my gamma is 1.67 my M 2 is the highest. When my gamma decreases, that is my gas is more and more compressible my mark number drops. That is what we are seeing, if my mark number drops my velocity decreases my temperature increases. That is what decreases my mark number or a can think about it as R changes, but we would not worry about R change currently will have simple enough case. So, if my gas is more and more compressible I can go to law mark numbers, that is what we are seeing. And here you will say that if my gas is more and more compressible I will get even more velocities compared to the previous velocity, that is what I will see here.

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Now, we will go look at the next this curve I have already seen this one P naught 2 by P naught 1, the stagnation pressure drop. What I am seeing here is P naught 2 by P naught 1 is dropping very steeply for low mark numbers, that is 1 to 4 or so it is dropping very steeply. After that, the ratio is going to somewhat constant it is almost 0, but I will tell you if I plot it in log scale it will keep on dropping, it will never go to a steady-state it will never go to asymptote. It will keep on dropping that will be the case here, and if I look at again gamma high-value, it is having less drop gamma low value it has more drop. This is again related to internal energy adjustment inside the gas and because of that your entropy increases which is equivalent to say that your P naught drops.

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Only one more plot that is your delta S by R which we just derived an expression for, we look here if my gas is more compressible, then I am having a condition. Where, if my gas is more compressible, that is gamma is lesser I am finding that delta S is higher. There is more internal energy rearrangements, because of which you are getting more entropy, I cannot prove this in this course that is beyond this course. If you know molecular gas dynamics we can talk about it currently just assume that I am right that is all. When the gas is more compressible I will get higher entropy for the same mark number, we looked at effects only based on mark number as of now.

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Now, the only thing left to do in this set of things is gone look at this M 2 what is this asymptote value for a given gamma value. For a given gamma it is 1 particular asymptotic value, you can look at that and we can show that P 2 by P 1 or T 2 by T 1 will not go to any asymptote.

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Those are the things we can show, and what happens to this particular condition when M equal to 1, we will look at that also. What should be the value for P 2 by P 1 when M equal to 1, it just becomes 1. We should be able to get from our relations, we will look at all that next class at the beginning we will do that. We will hopefully remember all these plots and we will just start with equation next time see you people next class.