

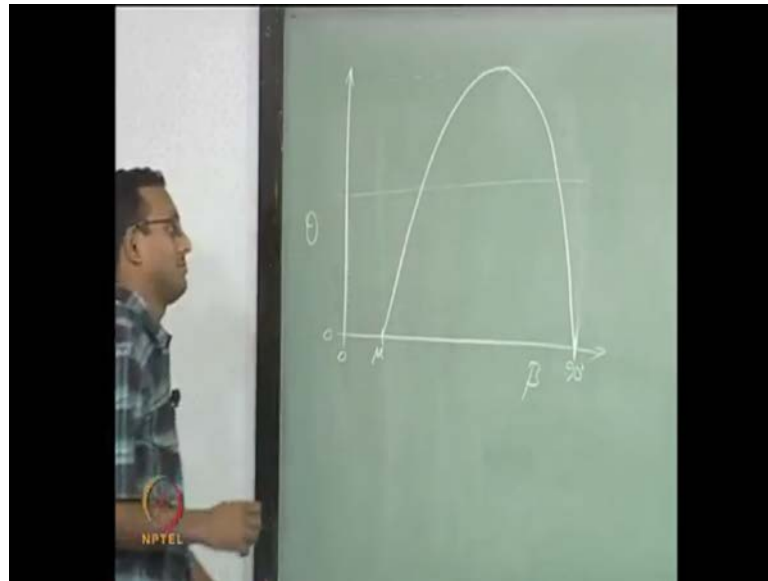
**Gas Dynamics**  
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**Module - 9**  
**Lecture - 21**  
**Oblique Shocks, Bow Shocks**

Hello everyone, welcome back, last class we were dealing with the heart of the oblique shocks. Primarily how to look at oblique shocks and what is the connection between the deflection angle and the shock angle for a given mach number etcetera. Today we will go into one aspect which we did not finish, but before that I just want to recap a little bit. We drew these pictures for any given beta and we found that, we can make same theta happen for different beta values the way we drew it if I continue the same procedure I can draw for ten more beta values but, only two of those actually, there is only specific two of those will match with the  $u_2$  normal divided by  $u_1$  normal from normal shock tables. So, all the other things we can follow the procedure and get more of them but, only two of them are exact correct solutions of which, the one with higher beta strong solution, the one with lower beta is the weak solution and we looked at in a weak solution.

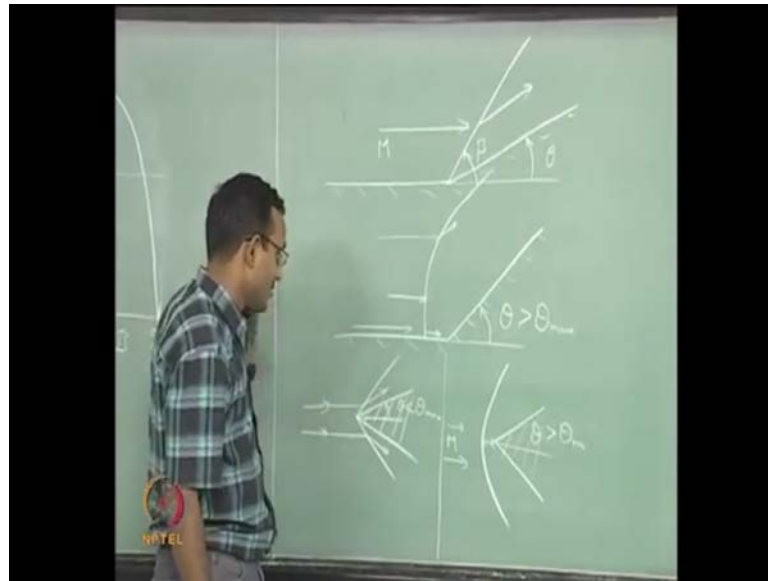
It is changing kinetic energy, very little in a strong solution. It is changing most of the kinetic energy into internal energy those are the details you saw finally, we said that we can use  $p_2$  by  $p_1$  from normal shock tables as such to get  $p_2$  by  $p_1$  for oblique shocks but, I would advise you not to use that because, it will cause a lot of confusion, that is all. It is safer to go by just finding mach number and  $p_2$  and from there  $m_2$  and  $p_2$  you can find  $p_0_2$  from isentropic tables, that is safer way to find  $p_0_2$  otherwise if you happen to have this other column, which I have in my gas tables  $p_0_2$  by  $p_1$  that will not give you the correct value. So, just to avoid confusion will just follow a single procedure which will always be safe which is we will not use  $p_0_2$  by  $p_0_1$  from normal shock tables to obtain  $p_0_2$  by  $p_0_1$  for oblique shocks can be used for normal shocks. Now, we will continue along the lines of yesterday and will go back to the original picture.

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This is our shock angle, there is our 90 and 0 here, this is your deflection angle theta and there is no limit for this. I will just draw a particular case i drew slightly over beyond this something like this and we said that for any given theta there will be two solutions the weak solution and the strong solution and we found that this was mu for that particular mach number for which you have drawn the line and this is 90 degree, which corresponds to normal shock solutions. Now, we have to look at there is a particular theta above which there is no solution for oblique shocks, there is no solution for a beta value above this, that means there is something special happening above this point what will happen. So, we will go draw that picture of that oblique shock happening.

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If I consider this is my theta typically, we will have a shock like this with an angle beta and flow vector will come like this will turn and go parallel to this wall. I am saying it is a wall but, it need not be wall it could be a stream tube, it could be anything just all I care about is the streamline turning like this, and the wall is very easy to work with. So, we will just remember it as a wall it is easier to tell.

Now, I am deflecting and it is a sudden corner, people call it as sudden change in angle. It is a slope is discontinuous there and this is what I will have and I have drawn in such that, this sector is very long and this sector shot, it is consistent with theory, this is for some particular  $m$ . Now, if I for this particular  $m$  curve, if I go look at that theta, beta for a particular mach number I will find that, it has a theta critical theta maximum, what will happen if I will keep on increasing this theta. I will draw one more let us say that, the deflection angle for the wall is theta greater than theta max. If such a thing happens then the shock will not be along this line from this corner.

But, it will be detached from this corner and it will sit like this. It will be close to a normal shock at the bottom and as it goes out, if it will turn out like a bow shock, we saw an example, of a bow shock in one of the videos some classes back. So, it will look something this. Now, the flow here is going to come straight. It is going to see, that it is almost a normal shock actually very close to the wall. It is a normal shock, the flow will just go straight and the flow here will be subsonic as we know from normal shock, flow

downstream is subsonic and somewhere here if I pick this point, it can oblique shock locally with that particular beta value and here that will be a deflection like this, when I go far away. If I go up to very long distances out I may go to a point where this angle is very close to mach angle for that particular mach number that is, it is very weak wave finally, starts as a very strong wave for the given mach number.

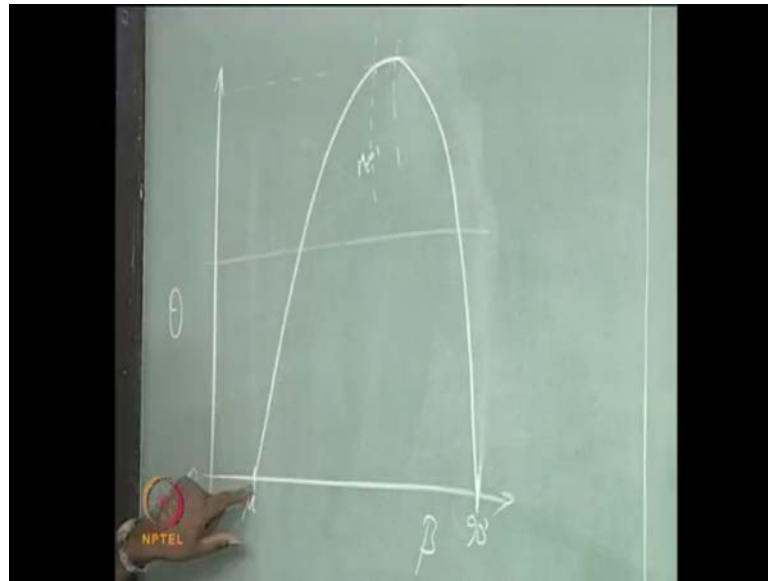
This is the strongest possible compression wave that is your normal shock as it goes out. It is getting weaker and weaker it goes to a point where  $p_2/p_1$  equal to 1 at very far distances from this change. I am telling you again from this change only everything matters, very close to this change there is a normal shock far away from the changes effect is lesser  $p_2/p_1$  effect will be lesser. It is a simple way of looking at it, if I look at it from a point of view of there is a wedge shaped body in flow considering 2 d flow, if I have some particular semi-angle less than  $\theta_{max}$ .

Then, I will have an attached shock like this is my body and there will be attached shock like this, the streamline there is coming here will turn around and go like this, here the streamline will come here and go like this is just mirror image problem, I will not worry too much about this turn you can convert it to be minus theta a minus beta it will work fine. So, that is this case, if I have theta greater than theta max here, and I have a problem like this, flow incoming is having a mach number and it is uniform and what we will see will be a bow shock. Similar to what we saw over in front of a sphere in the video two weeks back two classes back it will look something like this and there is a particular detachment distance, this detachment distance depends on the gamma value depends on how long is this straight-line is it going forever, if this is going forever.

It will be a particular value, if it is stopping somewhere after some distance like in a missile or something, if it turns becomes straight after some time than this will be a little closer and it is related to where the shoulder is shoulder of this wedge. We would not worry too much about it is not related to our problem. Now, I want to look at let us say this particular case I will pick, what is happening in here? when I go to some case here, last class we said that strong shocks do not have occur in steady problems, that is only for attached shocks. Now, I will qualify that statement again now, last class, we said we have strong shock solution and weak shock solution. Strong shock solution is proven to be unstable and it will not occur in a steady problem, that assumed that I am going to have a deflection like this, where it is an attached oblique shock if it is a bow shock it.

So, happens that, I am having a normal shock here and as I go up it is going to go to from ninety degree beta angle to something less than ninety slightly less than ninety. So, it is going to go from ninety to eighty five, seventy, seventy five, sixty whatever all the numbers will be there and far away if I extend this, very far away, somewhere far out it will become mach angle for this particular mach number.

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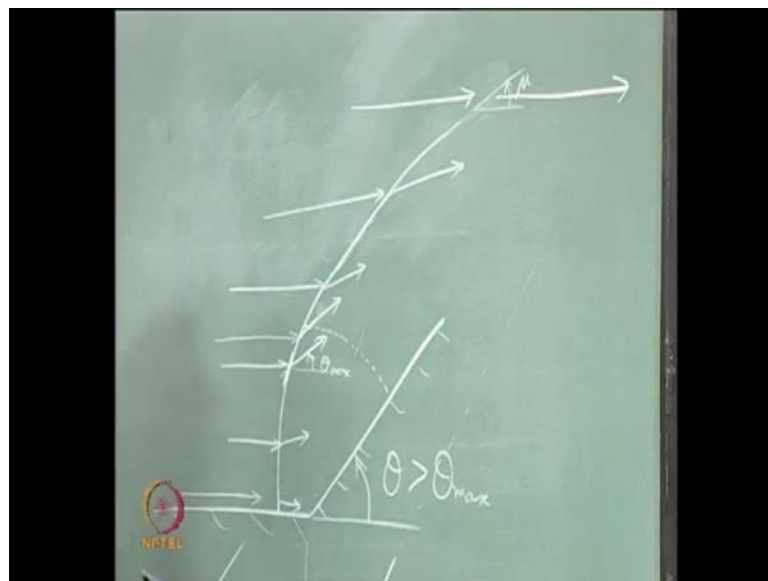
Which means, if I go back to that curve again for my bow shock at the centre line it is here and it is sweeping through the whole curve for that particular mach number, it is having all the compression waves possible in this curve, it is going to have all solutions in this. That is one observation you want to make one more thing, I will tell from here is if I look at it starts with  $M_1$  normal very high and it goes all the way to  $M_1$  normal will become 1.

That is, it is not going to have any  $p_2$  by  $p_1$ , and it is going to go through the whole sequence  $M_1$  normal very high to  $M_1$  normal nothing  $M_1$  normal equal to  $M$ . It will start with, that is 90 degrees right  $\sin \beta$  will become one and I am going to end with  $\sin \beta$  equal to 1. So, I will end up with  $\sin \beta$  will become  $M$  by  $M_1$ . So, it will become  $M_1$  normal will become one, we will end with that. if I look at  $M_2$ , the mach number behind the shock, it starts with subsonic, it is going to go we already had this dotted line we drew last time it was going like this, we said this is  $M_2$  equal to one line

and we said that this is theta max line, we had two curves going across different mach number lengths. So, if I think about it, it starts with very low subsonic.

This is the strongest shock, the very low subsonic it is going slightly higher and higher mach number and up to here. It is becoming  $m$  equal to 1 at this point that particular beta value, it will be  $m$  equal to 1. After that it is decreasing, it is further increasing to supersonic values up to a point where it becomes finally,  $m_2$  equal to  $m_1$  goes all the way to there that is mach wave with there is no change. So,  $m_2$  equal to  $m_1$  becomes that goes from the whole spectrum. The next thing I want to look at, theta value this is more difficult. Theta value is 0 initially when near the centre, as it goes up it is increasing goes up to the maximum value possible for that mark number theta max and then it drops again and goes to 0 again. Now, I want to draw this picture actually, I will have the picture, I will erase the top picture I do not want to draw this so much.

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So, I have this picture, where I am having theta greater than theta max. So, I am having a detached shock sitting here and let us say it is going up all the way to oblique whatever mach wave somewhere here, let us say this is my mach wave length, if this is my  $\mu$  than the streamline that is coming here, does not change it just goes straight, that is far away from the change and very close to the change. We said it is 90 degrees beta. So, it is going to be just going straight normal shock somewhere here, it has small theta.

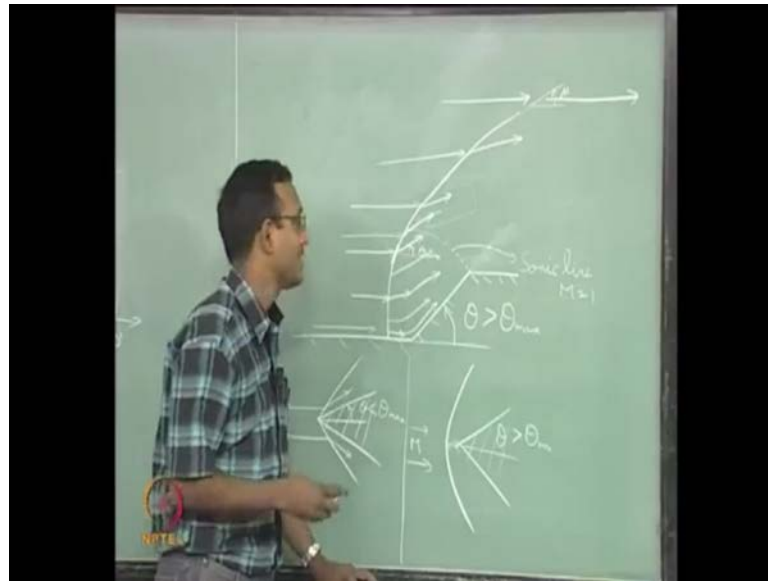
So, it will turn a little bit let us say this is my beta corresponding to theta max. This particular thing let us call it. I will just put this arrow and call it theta max, the deflection that particular line I do not know where it is. It is somewhere, that particular line will be theta max after that. Now, I am going to have again decreasing streamline angles that are fine, I will leave that we will draw the next one with a lesser angle something like this. Ideally I am supposed to draw these vector lengths also correct we know that this will be the lowest and as it goes up the velocity increases. So, vector lengths should be increasing all the way up to their where it will become same lengths all the way together.

Now, if I look at this area, this one line has the highest theta on both sides. It has lowered theta, this one will want to go and crash into the streamline. There is an extra compression happening after the shock, there is something more happening after the shock, here such things are also possible in most cases somewhere up to here will be subsonic flow inside this region. Why am I saying the theta max will be in the subsonic range? We will go back to the curve.

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Theta max is sitting in a point where  $M_2$  is subsonic, that is all I am saying and slightly below theta max. On the beta lower side is where you will have  $M_2$  equal to 1 happening. The only point where  $M_2$  equal to 1 touches the theta max is at  $M$  equal to infinity. We would not go there and of course,  $M$  equal to 0 also. Then there is no flow, no shock we would not go there.

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So, I am having this whole region inside here, inside this curve all the way inside this whole range is all subsonic flow, all this time when I had this nice attached shock condition. The flow could adjust to this change. That is changing the streamline direction by just having one attached to shock. that was in off just change in angle of the flow direction and that was enough just having one oblique shock, if I had the change too strong then it has to change very fast which is not possible for a fluid. So, what it does is it makes the flow go to subsonic has a velocity is very slow and temperature is very high. So, speed of sound is very high velocity is very low. So, it will have a very nice feedback in here to adjust to the situation. What will happen after this? The flow has become subsonic. Now, this subsonic flow knows where the fluid is, if I think about the steady-state where the wall is sorry, where the wall is, if I am thinking about steady-state problems, we are considering only steady-state problems.

At final stage, we will deal with shock forming in front of oblique shock, in front of wedge towards the end of this course. Currently no and what will happen is this, if I just look at this region of flow after the shock, it looks as if there's subsonic flow over a wedge, what should we do? It starts turning like this and go very slowly. It will start turning, it is subsonic flow anyway it can find its own way to adjust. It will do something like this and adjust, it will have region of contracting stream tubes what does that mean subsonic and stream tubes contracting area, decreasing subsonic mach number will increase, that is also happening in here.

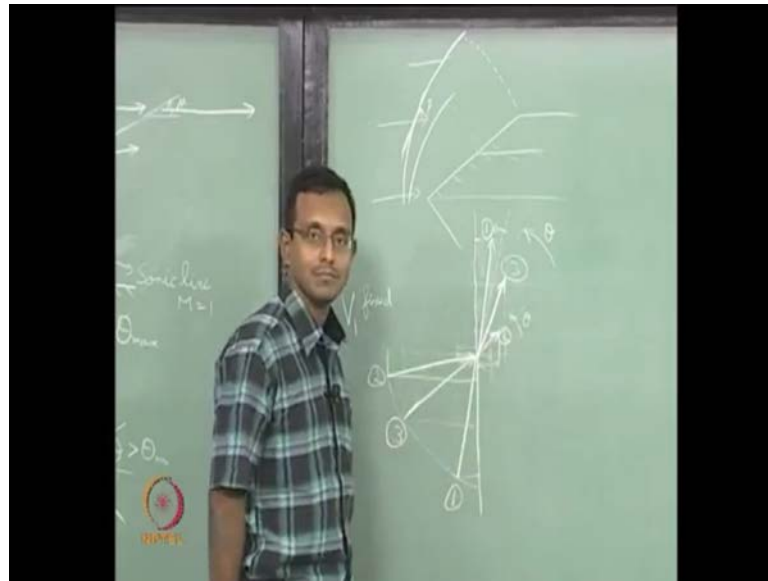


So, we are accelerating the flow here, that is why this dotted which line corresponding to the sonic line we call it as the sonic line, which is nothing but,  $M = 1$  there along that line. I will go to a point where  $M = 1$  sits here and after that, the flow is supersonic it. So, happens that typically, I will have to modify this picture such that, the  $M = 2$  line  $M = 1$  line will end on the shoulder of this body typically, that is where it stops, if it is the other way it will find it unstable. This is the only place where it will find easy to adjust. So, it will typically be sitting here, if it is a wedge problem something like this.

So, we have to think about what is happening inside here of course? I already gave you the answer streamlines, stream tubes are coming closer and closer after the shock, it becomes subsonic and it is coming closer and closer means, it is going to accelerate again go to supersonic condition. Once it goes supersonic here, there is something more that will happen here which we have not explained, we will come back to that. The flow has to turn back here along the wall will come back to that later in next class onwards we will deal with that.

So, when I go here, this streamline going is to accelerate to go there and there is something more which I have to talk about, let us say towards the last one-third of the class of course, where I will talk about, this region having a lot of vorticity in it is not a simple flow all this time we ignored any rotational flow. We said our flow is completely irrotational, we do not need to worry about any  $\text{del cross terms}$ , actually we just said  $\mu = 0$ , even if there was  $\text{del cross terms}$ , we wouldn't worry about it that was what we said. Now, we will just say that, this flow is very rotational behind this bow shock. We will not do anything more about that and I said that depending on how far the shoulder is from this starting point, the curve will shift, if I had a case I will draw there you know.

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Let us say I will draw only one side, I would not worry about the other side if I had a case like this, and then let us say my bow shock is like this and my  $M_2$  equal to 1 line is here. If I had another case, there it goes beyond and then turns, if I had had this case, it will most likely be looking like this, then there will be a higher stand out distance and this whole region will be filled with subsonic flow inside of course. The flow becomes supersonic after some time, even for this streamline. The streamline, that will comes here, will turn accelerate to supersonic at this point, it becomes sonic and then it becomes supersonic, just there you do not worry about that for the moment just leave it like this.

And one more thing, I just want to point out the streamline that is going here is processed by a different shock compared to the streamline, that is going here compared to the streamline, that is going here which means, each of them will have different entropy jumps across of course, we can easily say that  $M_1$  normal, the normal mach number is the highest. It is equal to  $M_1$  here and as it goes up, it is  $M_1$  normal equal to 1, it becomes as it goes out far which means the strength of the shock or a  $\Delta s$  the increase in entropy will be the highest here as it goes up, the entropy increase lesser and lesser, one more observation you would not do much with it. Currently we will come back to it after sometime. Now, I just wanted to tell you something extra. So that, you will have a little more physical feel for this.

Of course, I cannot give you very good physical feel, because it involves math after sometime anyways we will try it a little bit. I wanted to give you a slight feel for why will there be two solutions, if I keep on increasing my beta value from say mach angle to very high angles. The theta will keep on increasing after sometimes and then it will start decreasing again, we want to see why that is the goal? It is easier done, if I go to my fixed normal shock which is vertical and an observer is moving with the velocity.

We are here, the very first thing we did with oblique shock class, we start at this and we will say that I am going to have a particular incoming mach number  $m_1$ , irrespective of off what beta I have for the shock which means, my  $v_1$  value is a fixed, value  $v_1$  is fixed. it is  $v_1$  by  $a_1$  is my mach number,  $m_1$  and I am saying incoming gas having a particular speed of sound  $a_1$  starting with that. Let us say we are considering the vector, that is going through that point, I will take this as the length of my vector  $v_1$  and if I want to change the beta, it is going to be beta is what is nothing but, this angle alright the angle that shock makes with respect to the incoming that was beta in here we said this angle is beta.

That was our beta locally a shock makes this angle with respect to the incoming stream is. So, here this is my incoming streamline shock is making this angle. So, I am having very low beta value of course, I cannot have very low beta value close to 0, minimum possible is  $\mu$ . We proved it, in last class below that it will go entropy less than 0. So, it is a minimum possible, let us say our close to minimum possible, it is slightly more than  $\mu$  we will keep it like this. The other possibilities will be the vector can be here. I have to draw the vector, this can be my vector and I have to make sure that the length of the same. So, I will draw a circle along this with that as centre. So, is this has to be this length not a good circle but, it is reasonable.

Something like this and I can have close to normal, that will be a shock like this, that will be streamline like this with respect to that shock, something like that we want to see what happens in these three cases ? let us pick this particular case, the lowest beta value what I have to do is just go and draw the normal components and tangential components for this small beta value you think, you find that  $u_1$  normal is very small, this is my  $u_1$  normal for that case, if this is the  $u_1$  normal, the change will not be very high, because the  $m_1$  normal is exactly the same for all the cases, the change is not going to be very high or even if there is a change the velocity vector.

Say I multiply by 0.5 then I am going to say this vector is going to become only this length. The normal velocity vector tangential is still going to be exact same length. So, I will erase this  $v$ , because it is confusing I have to draw roughly, the same length let us say this is my  $u_2$  normal, I just guessed the value something smaller than this  $u_1$  normal. Now, my final velocity vector I have curving, this will be my final velocity vector, if you look at it there is a very small change but, it is not perceptible, there is a small change, if I extend it has changed by this much, this vector if I extended it will go along this line and it has changed by a little bit that is a small  $\theta$  value. Let us go to the other extreme, this highest  $\beta$ .

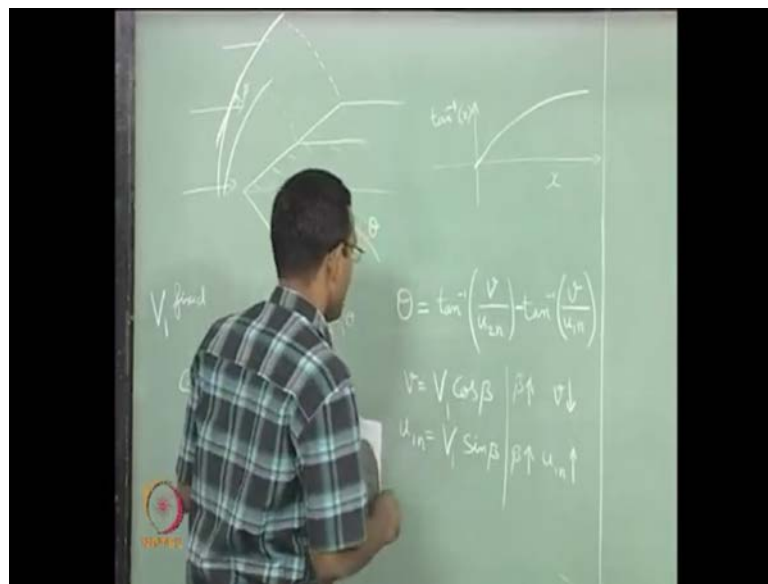
Currently we are considering here normal component is very high, this whole thing is normal component and this is my tangential component, in this case for this shock the incoming velocity is very high, which means it will drop very high, incoming flow it is going to drop very high unless I use the normal shock tables. I cannot exactly tell what the value will be I am going to guess again.

I will pick the tangential value to be somewhere here. It should be the same length here and then the normal. Let us say I will put somewhere here, I do not know where it should be I will consider this to be my  $u_2$  normal for this case. So, I will end with this and that value is somewhere here. Now, again I will have to find the deflection in the stream line. So, I have to extend the original stream line, this is my  $\theta$  for this case. The way I have drawn it looks like, this is slightly bigger than this. Now, we will go to the next case, next case is something in between and I am assuming this is going to be somewhere close to  $\theta_{max}$ . So, I will draw the normal and the tangential on this. Now, it is somewhere this length same tangential that will be here at this line.

Now, I have to look at the shock strength. It is going to be some  $u_2$  by  $u_1$  in between this case and this case here the  $u_2$  by  $u_1$ . We said was roughly half and the other case, it looks like it is that full amount discovered and 1, 2, 3, 4, one-fourth. So, let us pick one-third for this case  $u_2$  by  $u_1$  is one-third less in this case I pick that, then it is going to be that distance somewhere here. I am having so many confusing clots here but, that is the only way I can explain this, I missed it slightly it is, we will just take it like this, it is going to be this particular case I will number it 1.

We said this was our 2, these are three, no these are four what we are finding here is this. one extends somewhere here this is my theta here, what we are seeing here is when I have 1, I have a low theta when I have 2, I have somewhat less theta compared to 3, I am going from lowest angle possible to beta to 90 degrees as I go what we are finding is theta is increasing and decreasing. I am giving you complete physical feel not exactly. So, I will give you a better example, by just going to math, a little bit of math.

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I want to find this angle theta, how will I find? It is just tan inverse of this vector components divided by minus tan inverse of these vector components right. So, I will just write it directly tan inverse of V by u 2 normal minus tan inverse of same V divided by u 1 normal. This is the formula we are having for that. It is a simple enough formula of course, we know that the V is the same but, V depends on beta, how does V depend on beta V is equal to V 1 cos beta. Now, of course we can find it out, it is not that much difficult we wrote it for mach number before. Now, I will write u 1 normal is V 1 sin beta, this is how you got m 1 normal equal to m 1 sin beta. So, if I say my beta increases if my beta increases and beta is somewhere between 0 and 90 degrees, if beta increases cos beta decreases we said fixed we want right. So, v decreases here v 1 is fixed 0 to 90 degrees sin will keep on increasing.

So, if beta increases u 1 normal increases. Now, I will go look at this term v by u 1, v by u 1 normal if my beta increases v by u 1 normal will decrease, because denominator is

increasing and numerator is decreasing simple enough and tan inverse is a monotonic function I have to draw that also here. I will draw it here, this is some x and this is tan inverse x, this function has got to look something like this, it is close to 45, I think it is 45 at the 0.0. It is got to go like this is the function for tan inverse of x, where x is your angle x is here number this is your angle.

So, x can go anywhere from 0 to infinity. So, what we are seeing is if beta increases, this quantity decreases which means I am going on this way, on this line. What we will find is tan inverse, that quantity will decrease which means I have a tendency for increase in theta based on this, if I increase beta my theta will increase, that is 1 aspect. Now, we will go to the other one v is still the same v 1 cos beta good enough, we will keep it that way I can also say it is v 2 cos. Let's not worry about it v 2 cos beta minus theta in is not complex. I will write u 2. Now, what happens to u 2 normal, If u 1 normal increases that is the idea u 2 normal is actually we have a function called u 2 by u 1. We will just keep it like this, u 2 normal by u 1 normal is equal to gamma minus 1, m 1 normal square plus 2 divided by actually I am getting too cluttered here. I will write it in the next place, next board.

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The whiteboard contains the following handwritten content:

$$\frac{u_{2n}}{u_{1n}} = \frac{(\gamma-1)M_{1n}^2 + 2}{(\gamma+1)M_{1n}^2} \rightarrow A + \frac{B}{M_{1n}^2}$$

$$u_{2n} = u_{1n} \left( A + \frac{B}{M_{1n}^2} \right) = Au_{1n} + \frac{C}{u_{1n}}$$

Below the equations, there are notes:

$\beta \uparrow, u_{1n} \uparrow$   $u_{2n} \uparrow$  if  $u_{1n}$  high  
 $u_{2n} \downarrow$  if  $u_{1n}$  low

U 2 normal by u 1 normal equal to this is same as normal shock formula, m 1 normal square plus 2 divided by gamma plus 1 m 1 normal square. I can regroup this, such that it will become something like a plus b by m 1 normal square, it will just become that I will

divide numerator denominator by  $m - 1$  normal square. It will just become something like this where that will become  $\gamma - 1$  by  $\gamma + 1$  and  $b$  will become  $2$  by  $\gamma + 1$  and you will get to this  $1$ . Now, I want to write it as  $u^2$  normal equal to  $u - 1$  normal into. Let us say I want to write  $m - 1$  in terms of  $u - 1$  normal of course, it will become  $a$  prime plus  $b$  prime by  $u - 1$  normal square. So, this looks like  $a$  prime  $u - 1$  normal plus  $c$  by  $u - 1$  normal something like this is not a simple function in  $u - 1$  normal that is what is got problem currently. Now, if I say my  $u - 1$  normal increases, why am I saying  $u - 1$  normal increases, beta increase? We go back to the previous board.

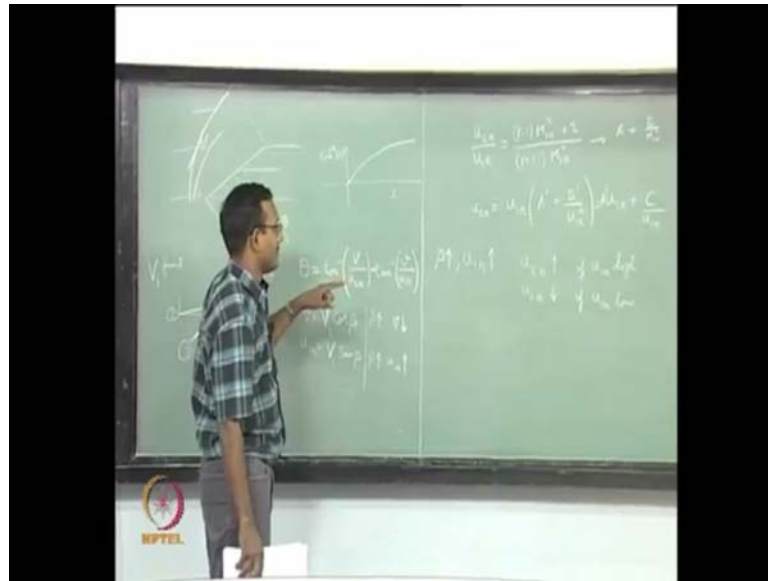
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Where, we said that if beta increases  $u - 1$  normal increases from here. I am going to use this.

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Beta increases  $u - 1$  normal increases and because when  $u - 1$  normal increase. I can probably say that  $u^2$  normal increases if this term is dominant, if linear term is dominant or if  $u - 1$  is high. I will tell that way, if  $u - 1$  normal high, if this is very high then  $1$  by  $u - 1$  normal will be very small then, this term will become dominant that is easy where to look at if not  $u^2$  normal decreases if  $u - 1$  normal is low, if it is low then this ratio will be very high and this will be very low, then it will go the other way. Now, this is not a simple thing and this is what is causing all the reversal in theta, we go back to our theta formula actually I want to show this whole thing, hopefully we will show the whole board this and up to here.

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Now, if I have, we already showed that, if I increase beta the second term will always decrease that is monotonic, if that decreases theta should increase. Now, I will go and look at the other 1. I am going to say if beta increases, if my  $u_1$  normal is high, if my  $u_1$  is high.

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If my  $u_1$  normal is high means my beta is high if beta is high, then  $u_2$  normal increases v anyway drops v always drops that. We have it here v drops  $u_2$  normal increases which means, this term decreases if this deceases and this decreases. Now, I do not know where it will go but, based on numbers  $u_2$  normal is less than  $u_1$  normal. this is more than this but, the difference is decreasing. So, I am going to end up with a case were theta will decrease. So, like that its little more complexes, I would not get you anymore physical feel than this is the closest I can get to after this it is all math you can probably you have to go and draw the functions out by yourself once and then only figure out. The physical intuition from here it is not very easy to get the full physical feel that is what I told, when I started also that being said I want to give you a feel for what happens, when I change gamma. Let us, not discuss anymore about that, this is the best I can give you that. We will go and change gamma and we will see what happens let us go to the screen it is easier to finish off with the screen then, you come back here.



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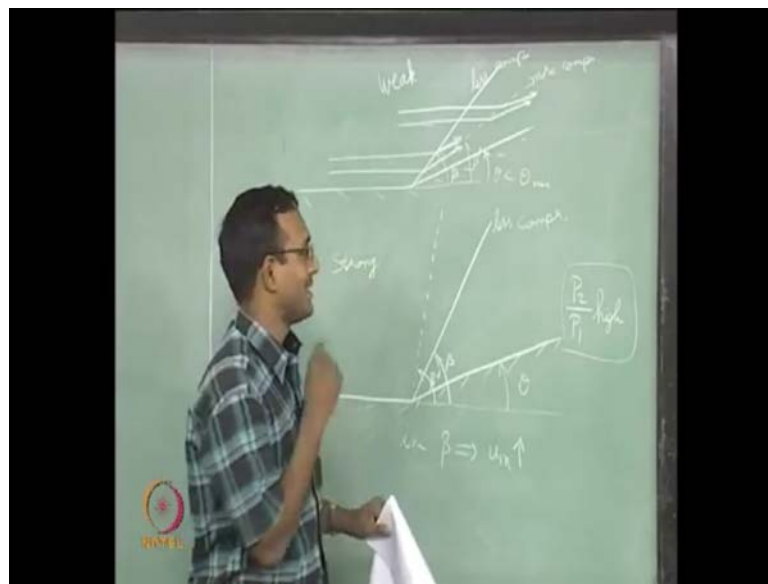
What I have plotted is deflection angle versus beta same as theta versus beta curve and I have drawn it for 2 different mach numbers 1.5 and 3 for each for mach number. I have drawn it for three different gamma values 1.3 on the top 1.4 in the middle and 1.6 in the bottom of course, all of them reach at 90 degrees in the corner and when I look at each individual plot it looks like whatever we have been seeing all this time. I just want to see the difference between, then currently if I go from 1.4 to a higher gamma that is a less compressible gas what we are finding is the beta is higher first of the ratio if the gas is less compressible beta increases for weak shock solution. I go to the strong shock solution it is the opposite.

If i become more compressible beta increases previously, it was when it became less compressible beta increased for weak shock in strong shock. If I become more compressible that is go from higher gamma to lower gamma. I am going to increase my beta, it is going opposite we have different trends and one more important thing the theta critical theta critical is higher for more compressible gas three different observations we want to explain and of course, I do not need to explain weak shock, strong shock difference currently in this course. It is not really important all you need to know is that the course shifts of by itself as gamma decreases that are enough for this course.

I just want to give a little more, in case you want to learn a little more and the same trend happens for every mach number, it is the same thing I do not need to show it again but,

anyway I will just show it once more for mach number three for weak shock solution. Let us say I will take theta deflection of 20 degrees, what I am seeing here is as I increase gamma my beta increases. If I go to strong shock solution as I decrease gamma from 1.6 to 1.4 to 1.3 my beta increases, these are trend currently. Now, we will try and explain each of these three points of course, the third point was theta critical increases the first one is easy to explain the weak shock.

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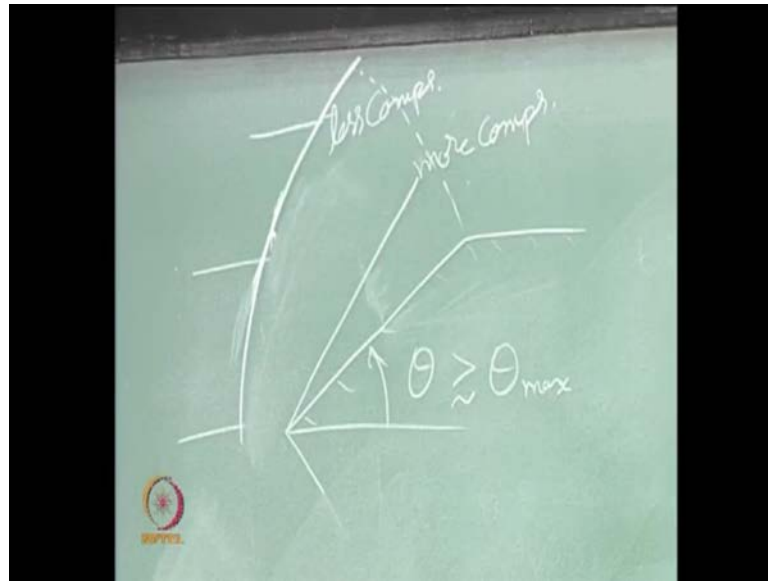
So, I am taking a particular case and we have theta less than theta max or theta critical whichever way people call it differently. I am going to have a shock, this is my original beta for one particular gamma value, and this is our theta. Now, I am saying the gamma changes to more compressible gas which is gamma decreases if I pick gamma decreases. Let us call this one less compressible, if it is more compressible then I am going to have lesser gamma in one way of thinking about it. My gamma is lower speed of sound will be lower, I think about in that way. The wave from here does not go this far into the flow against the flow, it will go lesser and it will end up with this angle theta will drop, that is one explanation I can give beta prime more compressible gas. Another way of looking at in the flow incoming is. So, fast that if the gas is more compressible it will compress whatever is behind the flow behind the shock to a smaller volume or if I put streamlines like the way we explained things before previously the streamlines were like this and the gap between streamlines decreased by this much.

That is getting compressed by. So, much fraction, if I draw this particular shock streamlines the other one does not exist for this shock. I will draw the same width at the beginning. Now, you find that from this gap to this gap it is much smaller, that is more compressed the fluid element is more compressed, if it is coming from between this gap same width as this it is much more compressed here than in this case, that is more compressible. So, it is getting compressed, it is easier to think about it this is very simple to explain these are weak shock solution is I will write weak here, strong shock solution is completely counter intuitive can be explained only, if you understand molecular gas dynamics what the molecule does with the energy given to it? We will try to explain without that as much as possible but, I am going to use one statement from there I have to keep the same theta, roughly same theta.

And now, I am going to say it is a strong solution in a strong solution we already observed that, the kinetic energy of the fluid is converted to pressure energy and temperature energy or enthalpy much higher than in the case of weak oblique shock. So, we are going to use that statement mainly, I am going to say this is my original less compressible situation and we already know the answer. That is going to be higher, we are suppose to explain that, if I just make one statement and you just believe me sure that if it is a strong. That is forming we are interested in making  $p_2$  by  $p_1$  very high compared to the weak shock, that is very high, that is what we observed last time also, anyway it is supposed to be very high to make it very high. That is I have to form pressure energy a lot where is energy coming from it should be from fluid.

So, it is coming from the bulk kinetic energy of the fluid element. So, there should be a lot more kinetic energy to be processed, if I decrease my beta my  $u_1$  normal decreases, which means I would not have enough kinetic energy incoming across a shock. So, I have to increase my beta. So, that increased beta which will give me  $u_1$  normal increasing, which means I have lot of kinetic energy, that I can process form the shock and produce the high pressure. That is all dependent on one statement I said. I have to increase  $p_2$  by  $p_1$  to very high value, you would not explain that part currently here, we will just believe that that is. So, if I assume that then things are all fine. So, we will keep it in that way and move on I will go to the next case, which is in the front the theta max is changing. So, I will keep parts of this board and continue.

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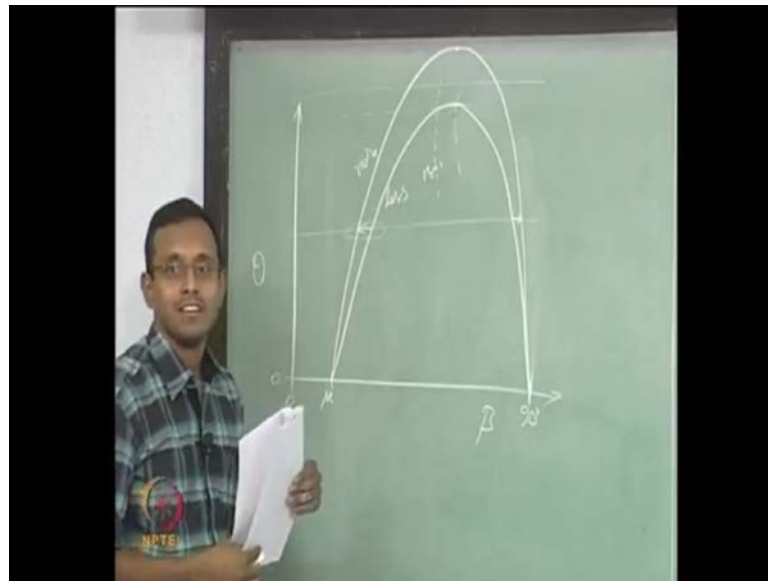
Let us say I have a body which has theta greater than theta, critical theta greater than theta max or theta critical people call it as theta max or theta critical get used to it. Now, let us consider a case where it is only slightly greater. This is not usually used in math but, I will just tell it is not a common sign I am just saying it is approximately equal but, slightly greater I will pick such a case and I have this particular condition. This is for less compressible gas and it has to go through a subsonic solution and then go adjust itself along the wall and get out all, we did already in today's class if the gas is more compressible, what should happen to this? Now, gap can be decreased, why similar to weak shock answer. The flow incoming can push the wave more back and keep the gas behind more compressed, keep the gas is behind more compressed, because it is more compressible it is possible let us compress then it is compressible.

So, when that will happen? flow can get attach just giving you a possibility, I cannot give you an exact answer because, nobody has proven it exactly this way, chest give you that it is a possibility of this skating compressed to a point where it gets attached and once it get attached, there is proof that weak shock solution is stable. So, dissolution is automatically going to from this curved shock to straight-line shock it will automatically go there.

We do not need to worry about that part, that proof came up only like two years back. It is very recent in research we are still fresh in solving this problem. So, it closes to this

situation. So, I am just using this one statement if the gas is more compressible, the shock will want to compress it more have been telling this statement from normal shock time onwards, let us keep on telling the statement again and again. So, that it is in your brain. So, overall what is happening might it is now attached shock for this to get detached. Now, I have to increase my angle much higher. So, that it will go and sit outside this is not critical angle for this particular mach number, if the gas is more compressible that is what we are seeing here.

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Now, we will go back to that and just properly we have to stop it to this. I will write like this, I would not worry about where the peak is shifting actually? The peak will shift slightly to the right, we will just ignore that currently I do not know the explanation for that. We will ignore all I can say, we explained that oblique shock. This is less compressible, this is more compressible of course, you have to draw it again from your notes but, I am just giving a less compressible, more compressible when I go to higher gamma. It is less compressible lower gamma it is more compressible I am having some curve like this. Of course, I am going to give whatever I am putting on the screen, on the website. So, that people who are watching you tube can avail it later they can print it out and keep it for themselves, if it is a weak shock solution. We find that beta decreases, we proved it, if it is a strong shock solution it will be proved that as, it has become more compressible it goes that way beta increases and you notice that there is no change at 0 deflection.

There is no change at 0 deflections of course, if it is  $\mu$  there is no shock, it is 0 strength shock. So, it just goes through straight there need not be any deflection. Normal shock again is again perpendicular change and there is no difference in beta really there cannot be any difference, beta is 90 degree normal is normal of course, there will be a difference in  $p_2$  by  $p_1$  values slightly you do not worry about that really no  $p_2$  by  $p_1$  formula has gamma. So, if I change gamma will have an effect on  $p_2$  by  $p_1$ . Let us, not worry about that part, also the main thing I want to show in this plot is theta critical has. Now, substantially increased, it is not very small number if I increase my mach number. This change will be really drastic you are going to see a huge change up here of course, when I go to very high mach numbers like more than 10 or 15 this change may not be very high, it may saturate to some particular value, that is hypersonic limit everything goes through crazy there. So, I am going to have something like this here.

If I had a case where my theta was this, if my gamma was higher it will have a detached shock, if the gamma is lower. I will have an attached shock, it is related to compressibility. So, do we have a solution for solving flow behind a normal bow shock yet not really we cannot solve it for our bow shock. We will just leave it like this, we still have a little bit of stuff left in oblique shocks, we would not deal with it today, and we will wait for the next class. That will be, how will I solve problems that is one aspect, other aspect is what if this shock needs something else, these are the two aspects we have to talk about. We will talk about it next class see you people next class.