Gas Dynamics
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Module - 12
Lecture - 24
Expansion Fan, Prandtl-Meyer Angle, Smooth Compressions, Prandtl Meyer Function

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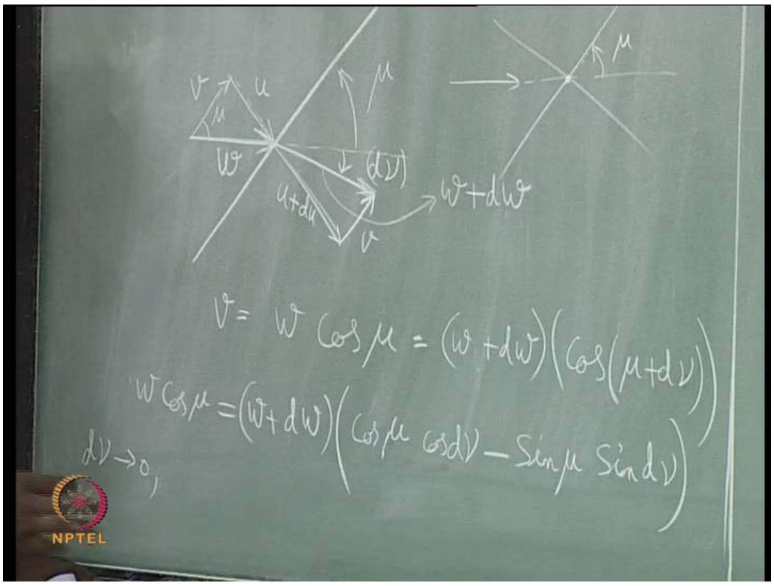

Hello everyone, welcome back, we looked at simple one dimensional waves, last class for expansion process and we said that we have to look at it as, if I have a flow and there is a wave and the wave is going like this, the induced flow will be the induced flow part alone will be like this, if the flow is already having this peak velocity. The induced flow should be added to it and it will be slightly lesser only this much will be the final velocity, this is what we saw last time. If, it is an expansion wave, if it is a compression wave, then the induced velocity will be in the same direction. I gave some statements like the wave will talk to the fluid and it will tell that there is empty space there go that side and it will keep going this side, that is what I said last class right.

Similarly, if it is a compression wave it will tell there is less space there come this side and they will go along with the flow, the fluid will go along with the wave, if it is a compression. If it is expansion, it will go against the wave flow direction. Now, we want to derive this in two dimensions, we have gone to take the similar approach like we did for oblique shocks derivation.

Where, we said if there is a wave. I am going to look at the normal component and how it changes; I am drawing the oblique shock picture. Here, the arrow lengths are changing that the actual velocity is this and we have a observer moving along the shock this way that with respect to that observer the flow field looks like this. Now, if I combine that with the actual flow along a normal shock, which is this velocity and that decreases to this velocity, with respect of that observer was moving along this way with velocity v.

He will see that, the flow goes like this and then it is going to like this. This is what, he will be seeing, that is the idea right this is how we derived oblique shock relations, now we will go and do the same thing for expansion wave, where we will start with one wave. We, already said last time that the waves cannot get together to form a strong wave last class. So, now, I am going to consider only one wave, they do not ever get closer ok.

Now, we will consider a similar situation incoming flow is parallel to my ground reference and they got split into normal and tangential components with respect to this wave direction. Let us, call this is my w velocity; I want three different variables for velocity. I want to use u, v and w. I have u here, v here and w is the total, the net velocity will be this, these are the components, this is how I am defining it.

Now, I have to tell more about this wave, if a wave is travelling at speed of sound or almost equal to speed of sound in a flow, which has a velocity direction like this, what will be its angle with respect to this velocity vector .what should that be, it should be same as your mach angle. Why, this is how you defined your mach cone right, we had flow velocity this way and. So, I just extended that line for this point.

This is the direction along which, the sound waves will propagate at most or inside this it cannot go outside this. How, we got to this. Now, I am going to use the same kind of explanation here and I am going to say a wave is travelling this way with respect to the flow velocity, at velocity wave velocity equal to speed of sound almost equal to speed of sound. I told this last class also right its slightly less infinitesimally, we will take it as equal to speed of sound.

So, it is gone to have this angle. Now, once I draw this angle as that value immediately I can just translate that into here because, this is parallel to that line, this is also going to be easy to tell. Now, once I draw these now remaining things, I just have to continue from here and I am going to have a deflection, this is my final deflection. I am saying this ok. This is the final deflection and I am going to call this as d nu because, we would not integrate it later I could call it d theta, theta is already used for oblique shock I will keep it as nu, for this one it.

So, happened that planning mere functions use this mu, nu for several in several books, I will just keep that the same. And, in terms of perpendicular to the flow direction what should happen to this flow velocity, it should just increase to some other value, it should be more than that u value, u plus d u that is what it should become finally, and the tangential component should not change, the way I have drawn it looks like it as changed slightly, but let us say this v and this v are the same. I have done it slightly this is v slightly smaller; just assume that they are the same. Now, what we have to do is conservation of mass across this wave. I have to start conserving mass across this wave and I am going to look at the component directions ok.

Simplest thing, I can do is, I will go for v direction very easy to work with v direction. v direction, I am going to tell that ,whatever mass comes in this way, is gone the component of moment this way, is going to be conserved that direction the way. I am going to use it w times cos of this angle right. It is w 1 w cos mu, I want to use this, it is going to be equal to on the other side. What is this going to be w 1 plus d w, w is again velocity remember that, that is a total velocity, times what is the angle I have to consider this is the final w, this is supposed to be w plus d w.

I did not mark this, let us say this vector have a magnitude of w plus d w, this vector length is w plus d w. If that is a case, I want to find the component with respect to this line. So, that is going to be cos of this mu plus this d nu. We have this expression, now all I have to do is just expand this, because I know, I use this w 1 here just make it w, w1 or w are the same parameter, w times cos mu will come out to this side, which I can directly cancel with the left hand side of this equation ok.

So, I want expand it and cancel that particular term, if I write it again, w cos mu is equal to w plus d w times, cos of a plus b expansion, cos mu cos d nu minus sin mu sign of d nu, this is what I will have. I have this, now I have to try and rearrange the terms with a small limit. I am going to tell that this d nu is a very small deflection. Why, I have considered only one wave, one wave can effect only extremely small quantities right, that is what we said last two class.

One single expansion wave, which is almost isentropic can effect only a very small amount. So, the change cannot be very big it has to be extremely small value. So, I can now say that the d nu is very small, d w is very small, d u is very small d p, d t, everything across this is all very small, they are all of the order of d nu. If you want except for entropy, I am going to say it is approximately equal to zero. We will just force it to be equal to zero, because we are using isentropic implicitly.

When, I say mu is the angle here, we will keep it that. So, when I say limit of d nu tending to 0, what will happen this? cos of d nu is almost cos 0, cos 0 is 1and sin of d nu. I will just replace this sin of d nu as just d nu. I do not want to make it 0 ,if I make it 0, there is no d nu in my expression, the whole thing will just get cancelled and d w will become 0 .That is, just no use solution, we do not want that. So, I want to keep that d u in that ok.

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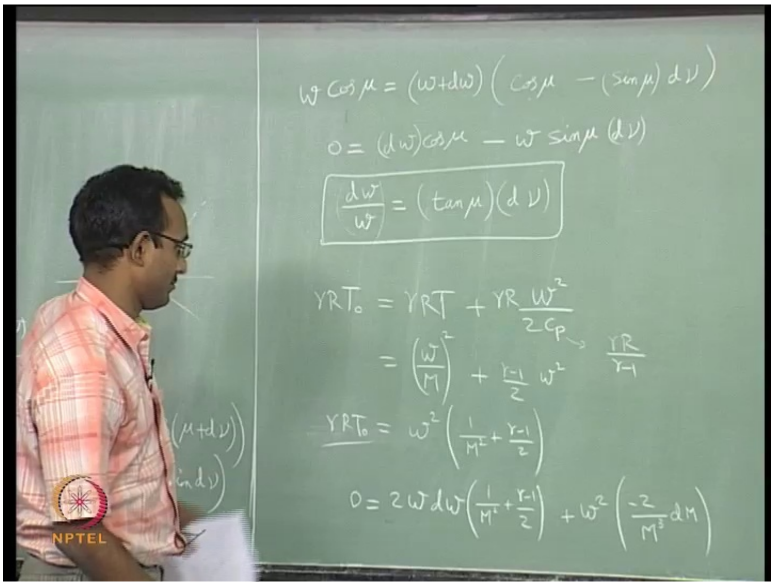

So, when I do that , w cos mu, w cos d w times, cos mu times, minus sin mu d nu, this is what, I just directly see that w cos mu, will just cancel with this w cos mu. So, I will put just 0, here equal to the remaining term will be d w cos mu I will put bracket. So, that we know what we are doing. Now, w and d w are multiplied with cos mu. Already now, I have to take this and multiply with this term.

So, I will have minus w sin mu d nu that is one term and then the other term is d w times d nu, which I am going to say is too small, multiplication of two very small quantities. I say that is very small. So, I have only this expression. Now, I will take one term to the other side and rearrange this, I will get d w by w is equal to, I have taken this term here and brought the w down here. So, it will become sin mu by cos mu, which is tan mu times, d nu this is one relation. What does this tell me, if my d nu is positive? I am going to say that, my d w is positive.

My flow will accelerate if my d nu, I chosen it which is here I chose it to be going below the horizontal if it goes like this then it will be positive. If, I chose it d nu to be negative now, I give it a negative value, what will happen it wills the flow vector will go the other side. If that is the case, automatically this expression gives me d w is negative, which means my velocity decreases, that is what I am getting here, which means that will correspond to a friential mayor compression wave ok.

A single compression wave we are talking about almost isentropic, but compression wave that is the answer we will get from that. Now, the next thing we want do is get rid of this d w by w and keep only mach number in there, for we do everything in terms of mach number to simplify the expression. Otherwise, I need to know the exact velocity there and work that is more compressive with a tool. So, we will start from our energy expression, t naught equal to t plus w square by 2c p this is how we stated and we this expression and of course, I could start from another place, I want you to get familiar right, this particular derivative.

So, I will keep deriving the several places multiply at gamma where at everywhere, one side do this, now I can rewrite this gamma r p as a square now that a I will replace it in terms of w, that will be w by m square, because m is w by a. I can write it like this, plus now my c p is gamma r by gamma minus 1and I already have gamma r, there which will get cancel gamma minus will go to the numerator, gamma minus 1by 2 and this is what I have ok.

Now, I could have derived it from somewhere else. I will keep it this just way of course, you guys know that directly this can be rearranged to the form t naught by t equal to

1plus gamma minus 1by 2 M square right, this is the same derivation we went through to get to here. Now, I want to keep things in terms of w's. So, I will not go that direction I will keep it this way. I did this specially, so that you will remember this particular formula. I wrote here, I will go gamma r t naught ,now I have to group them in such a way all the w are around 1by M square plus gamma minus 1by 2 .

I have this form here, I have to get here d w by w. If I take this expression and differentiate this with respect to some any variable, then I will get a d w from here. So, that is what I am going to differentiate from this expression and I know that it is almost isentropic problem. So, my gamma r t naught does not change. So, when I differentiate this side of this equation this is going to be 0. So, it is going to be 0 equal to, when I differentiate this term. I will keep this bracket constant.

So, that is going to be two w d w times this whole bracket 1 by M square plus gamma minus 1by 2, plus differentiation of this term, multiplied by w square, w square into one by M square differentiation. M power minus 2 will be minus 2 M power minus 3. So, it will be minus 2 by M power 3 d m and gamma does not come it at all, gamma is a constant any way. So, that does not come after different. So, this is your expression you have now. Now, I want to rearrange this a little bit. I am seeing that, I have, I want d w by w that is the goal, how will I get d w by w here, I will multiply and divide by w here. That will give a w square d w by w here. I have a w square already and I have d m by something, I will rearrange it such that, I have d m by m here and d w by w here, it is not very difficult to do. I told you already gas dynamics people like this d rho by rho, d u by u, d a by a, like that, it is d w by w and d m by m ok.

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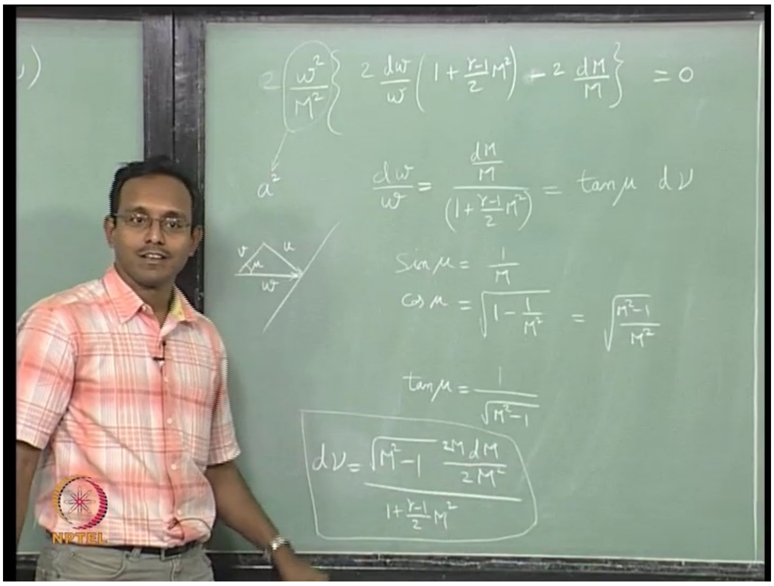

We will rewrite that, I will keep that 2 inside, I just pull out w square by M square from the whole expression. I am going to have 2 d w by w times, minus 2 d m by m, this whole thing equal to 0, this is what I will have. Just follow it, if I am making a mistake I am doing it out of memory and not from my notes here, I am having this expression here now I am going to say this is equal to a square and this is equal to gamma r t, which will never be 0. So, only this bracket thing most is 0.

So, now, I have expression for d w by w. I will rewrite this thing as; d w by w equal to d m by m may be 1 plus gamma minus one by two n square. I thing that two gets away from there. So, we derived one expression before this, I will write the other expression here, directly tan mu, d mu this we derived here, from this board. we derived this I am just putting that also equal to there.

Now, I have to go back and look at my mu function. I am going to look at it this way, I can tell you tan mu what is tan mu, sin mu by cos mu. So, what is it sin mu is what? mu is sin inverse 1by m. So, cos mu is square root of 1minus 1by m square, now you divide one by the other, what will I get tan mu. I will just rewrite this it is easier, M square minus 1m by M square, which will be 1by m outside, that will cancel with this. So, I will just have 1by square root of M square minus 1.

Now, I will substitute this tan mu here. I want to write expression for d nu is equal to, I will write it as that 1by square root of M square minus 1, I will take it to the numerator here, I have square root of M square minus 1, d m by m divided by 1plus gamma minus 1by 2 M square, this is some expression which is just nu is a function of mac number, of course, it is also a function of gamma it is mainly known as the function of mac number ok.

This is one important expression we have, but this is not easy to use as of now and of course, the actual thing you need to do to find, how much will be my mac number is the inverse problem. If, I deflect my stream line by so much d nu, how much will be my new mac number that depends on d m, m plus d m is my final mac number and that is not very easy to get from here.

So, of course, here integrate this then I will get nu as a function of m here that is why, I will end up with. So, how will I do that, I want to actually do the integration I will just go to mathematicians, but just to make it look like, I am doing something here. I can just go for now this looks like, I will put a 2 and 2 here I multiplied and divided by 2 M. Now, this is your d of M square, now every other function is just m square. Now, I can substitute x equal to M square and integrate this, I would not do integration, I will just give you the final expression.

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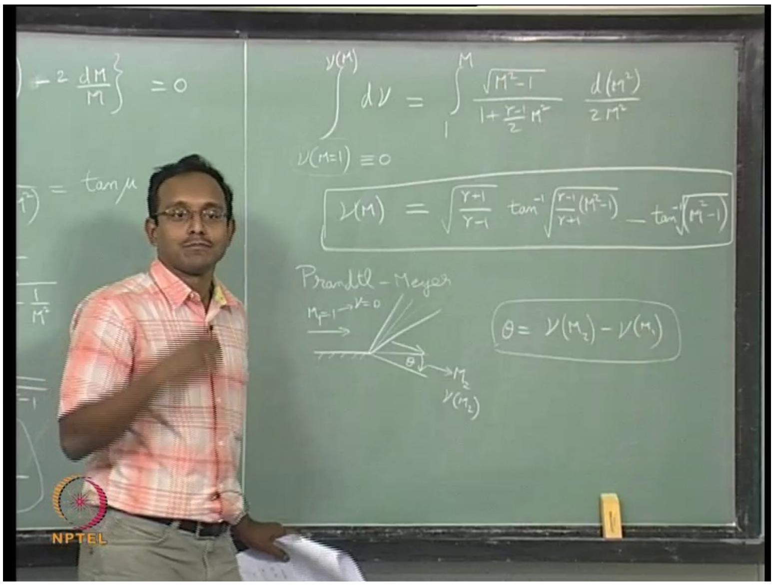

Now, before I integrate I have to do one more thing, when I put d nu integral, I want to put limits for it, what limit should I use? For this d nu, I have to give a lower limit and an upper limit, where am I starting from is the basic question? Now, I have to find out where I am starting from, not an easy thing to do, I will go back here. We know that all the expressions here will work only for supersonic, why? Somewhere along the line I said that, I have square root of M square minus1.

It cannot go complex this whole derivation assumed that my mac number is more than 1; otherwise, I cannot have a mu angle itself. Sin inverse 1by m is not valid for m less than1. So, I am assuming that it is supersonic already, which being said what is the least point, where this function will work m equal to1. So, I will pick that as my reference currently. So I am going to say, mu of m equal to 1, is one side and the other side will be nu, whatever value of m.

Now, this the way I will integrate a nu, I do not know the values yet we will figure out what to do, other side I am going to integrate from 1to m, square root of M square minus 1divided by 1plus gamma minus 1by 2 M square, d of M square divided by 2 M square, this is the function I have. Now, let us say I know how to integrate this, you can just tell directly that 1plus x square, kind of form will give you tan inverse function. It is not just a simple thing, because it is also having other term.

So, if I go and ask the mathematicians they are going to tell me, that this integral with the limits substituted comes out to be square root. Now, I have to do one more thing on the left hand side after that, I will put a big box around this. One thing I have to do is, I am going to start with a reference and at the reference I am going to set my nu to be 0. I am going to say this is defined to be equal to 0, nu at m equal to 1is defined to be 0 for me, my prenatal Meyer angle at m equal to 1is defined to 0 ok.

Once I do that, now the remaining thing will be just nu, nu of m is equal to this. This is a function, which is purely a function of gamma and m only and I can tell you for every mach number, some value called the prandtl Meyer angle. I will write that at least once, prandtl Meyer angle. Now, how will I use this is the next question?

Now I have this and I will tell you that this particular function it is not very easy to calculate with the calculator every time. So, this is tabulated it is also available in graphical form, that is what is more popular long back, now tabulated is easy now for us to use. Now, how will I use this function to calculate anything in my flow for that I have to get a practical field for what this means ok.

So, let us consider one particular specific problem, where I am having flow parallel to a wall and it is M equal to 1here and it is bending by some angle and its final mach number happens to be M, where I will let us say, this is M 1 and I will call this is M 2, it is easier to understand. Now, I will get a new value for this, as 0 and what will I get for this value, I will get a nu of M 2, now since we integrated from this point all the way to there and we already said that d nu is the small change in the stream line direction caused by that one individual wave.

When I integrate, what I am doing is, I am integrating over a whole bunch of waves, we said expansion fan contains a lot of waves we typically represent it by first wave and the last wave and probably a middle wave. That is all we do, but it is actually a whole bunch of waves inside, infinite number of waves and if I put that in here that is going to be somewhere inside ok.

This is a whole bunch of waves this is your expansions fan and now if I tell that, I am integrating over all the deflections of stream line. What am I going to say, I am going to tell finally, that this difference between this nu values will be equal to my total deflection of stream lines right. So, now, I will mark this as theta, the total deflection. I am going to say, theta equal to mu of M 2 minus mu of M 1,this is a very useful formula, I will keep it this form and then, now I will use this for the specific case, nu of M 1is 0 that is how we defined it.

So, theta becomes equal to nu of M 2 for this special case of this problem and the flow will go like this finally, this is what happens here. There are some set of expansion waves that are going from this corner all the way out like this, what if my M 1 is not1, but some other value how will I think about this problem, for that I will draw a different figure.

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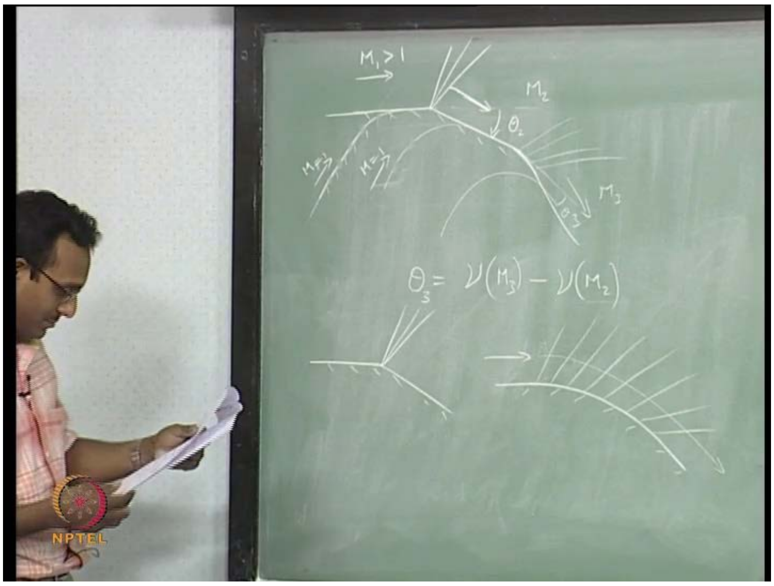

I will just draw the same thing, similar to what I have drawn before this is having a deflection of theta and this is having some M 1 greater than 1. But, if you have a field for this problem after that, you can apply it anywhere. So, I am just trying to give you the field for the problem and it is of course, going to have some set of expansion fans and then the final vector direction will be this and the velocity magnitude will be longer of course, velocity vector should be longer. It is accelerating the flow and this is what you should have, to solve this problem I need to find nu of M 1nu of M 2 and then the difference must be equal to theta.

I gave away the formula already, but to think about physical field I am going to imagine this problem of, I have some other direction here. I am going to imagine a wall like this and I will put dashed line for this wall because it is my imaginary wall. It is not really there in my flow field the actual wall is something like this. Let us say, I am going to imagine this wall, starting from M equal to 1 going to a case, where it is becoming M 1 equal to that particular value. Let us say it is 1.5, 1.4 whatever number it is going to that value. I am starting from M equal to 1and it is turning to go to that particular value and after that it is again turning at this point to go to this value ok.

So, if you imagine this in any problem, then you will never make a mistake in this. Prandtl Meyer angle corresponds to from here how much should I turn the stream line to go to that particular mach number. Let us call this M 2 again, from here. I have to turn by this whole amount to go to this mach number, I will turn a little less only up to here to go to this mach number, that is all it is right and by the way the prandtl meyer function is a monotonic functions.

It is not going to go up and down, it just keeps on increasing that is one more important thing you should know, but as it goes to very high mach numbers the number does not change, it is going to asymptote to a particular value. But, this is how you have to think about it, if I am thinking only about this problem and I have another change here say there is another change and there is some set of waves here, set of expansion waves again the flow is going like this let us say I did not do this problem already.

How will I think about it? I am going to imagine that wall like thing here, that dashed wall I am imagining this and I am going to say this is M equal to 1.From here, it is turning by so much, how much equal to nu of M 2 and it came to here. Now, I am telling it has to expand more, how much more by that small angle. If I think about, what should be the mach number here, I am going to say, I start from M equal to 1.

I am turning the whole amount here; this turning is more than this turning. When that is happening, I can tell that this will have higher mach number, because I know nu of M is a monotonically increasing function. So, and what will be the difference, I stop here, I continue to move over here. So, I can tell that theta equal to, I will call this M 3. Now, I named it theta. I call this is theta 2, I call this is theta 3. I will put theta 3 equal to nu of M 3 minus nu of M 2, something like this.

Basically, it is just coming down to the deflection is equal to the difference, my prandtl meyer functions for those mach numbers only one caution in this whole thing, the mach number can never be less than1ok. It is less than 1; I cannot use this whole analysis, because during the analysis, I assumed that I have supersonic flow.

So, I cannot use that analysis. Now, I will give you different problems to think about all this time I have been thinking about, corner I said this is a corner, which I have typically called the sharp corner. It is a sharp corner expansion. If this is the case, then I will have expansion waves coming from that point, the corner point, all concentrated from that point its coming out if that is not the case and it is more like this say the same angle deflection.

Then, I can imagine this wall as a whole sequence of small deflections for every tiny deflection, there will be one expansion wave something like this, and now the flow is going like this, it is going to slowly turn to finally, become that in both these problems, I can use the same function the delta theta is equal to delta nu. I can use that, now the only other thing I have to think about how I relate this and this.

If, I go really zoom into this region and just see what is happening at that corner alone there will be a point where, it will become a radius right. Mechanical machining whatever, it will always be this form if I zoom into it. So, much it will finally, end up here and if I zoom out of this problem this radius will become very small change and eventually it will go look like this. This is what, they call as smooth corner, the change is low and this is the shaft corner or a sudden corner, sudden change in angle these are all expansions. How did we define expansion the wall moves away? So, that there is more space for the streams lines, this is an expansion. We already did compression and there the wall will move into the stream line path. So, that will become a compression wave.

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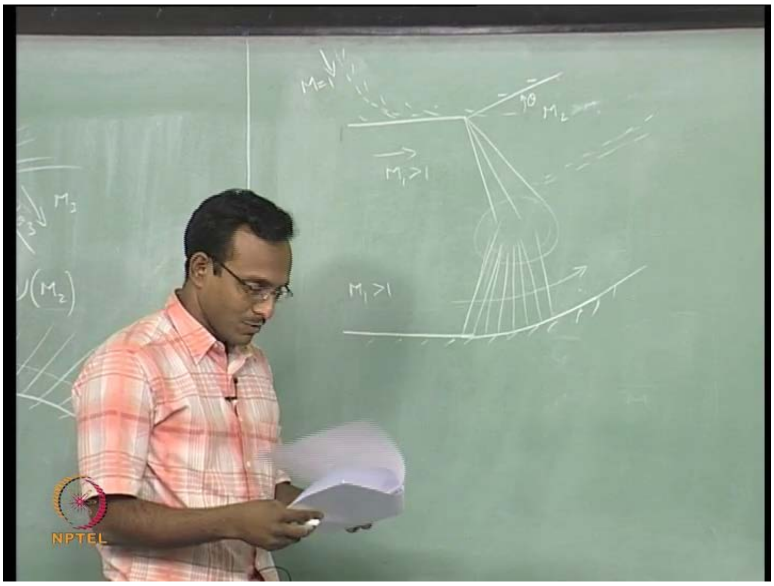

Now, I just have to give you one more picture, this particular picture will be useful, when you are thinking about solving problems. You will always never have a case, where it will be always wall at the bottom you may have wall at the top. So, how will I imagine that, I just have to flip this picture upside down ok?

I will draw a wall like this and now I am going to say this is M greater than and its becoming M 2, which will typically be greater than M 1 right. There is no other case, I do not need to write it all, you should know this and there is gone to be expansions this way. How will I imagine that, M equal to 1 point here? M equal to 1 point, I have to imagine it is going to come from the top this side, the dashed line wall.

I am going to draw from here will be coming from here, M equal to 1. It is going to turn deflect this way and come to M 1, which is greater than 1.Let us say, it is M 2 and then if I expand more it may go to 3,the total expansion from M equal to one direction to here will be some value and then from here. It is going to expand more the difference in the nu will be equal to that deflection here totally; this will help you understand the problem. So, you will never make mistakes in this case. And of course, you can always make it a smooth corner or a rough sharp corner with depending on the problem. It should not change anything, now we will just go for the animation there.

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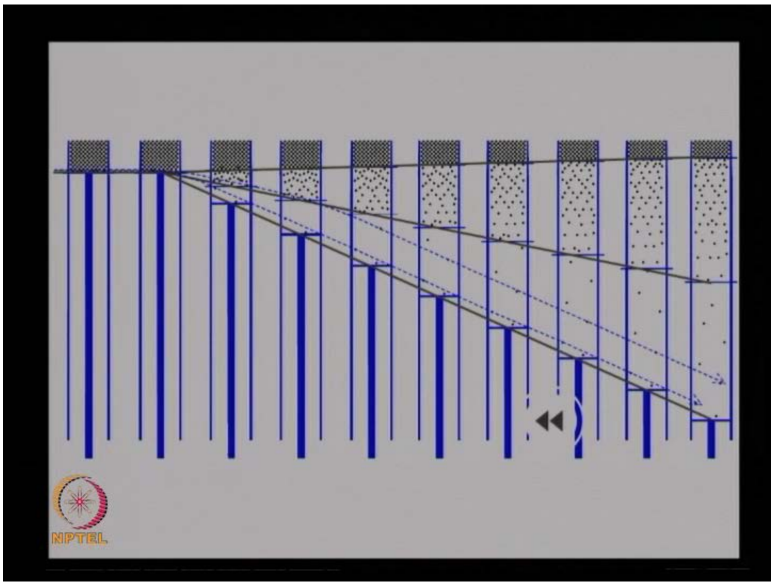

The piston analogy problem we are showing here, we are having a piston similar to your compression wave piston analogy we did before, I am having a piston and it is expanding out at a constant rate and when it is expanding, the first expansion wave is going in there. This is the first expansion wave, while this is the final expansion wave, which tells that after that the velocity is constant. Nothing, more changing there and this is your high pressure gas, which is slowly expanding, because this wave is going and telling there is lesser pressure here. In fact, I have drawn only two lines here. Ideally, I should draw infinite number of lines here I have. So, many waves here this is the first expansion wave and this is the last expansion wave and we already did the pressure plot of what this cross section will look like pressure variation in last class.

So, this is this case this is 1 d problem, but we did all 2d expansion fan, we will go look at that next. Now, similar to our piston analogy for 2d from 1d to 2d, if I have a piston cylinder arrangement and it is moving across a horizontally in a constant velocity. I am seeing what is happening to the particles inside, the piston starts moving somewhere here, somewhere this point onwards I will run it again.

The same way as it goes, whatever is happening inside is same as what we saw before and this is your first expansion wave going that way. That time, trace of the first expansion wave going that way and this is the time trace of the last expansion wave that is going here. And then, this is your time trace of the piston, which is equivalent to our wall in this case. So, I am going to have an expansion fan this is the region where there are so many expansions waves and the expansions waves are more and more apart as time goes. So, this whole region is expanding out like a fan, also shown here it is the stream line path for these two particles here.

In fact, this is like exact particular tracking thing, we just picked the two particles here and we are just tracking what it looks like in time and what you are saying is the one particular particle, this particular particle we are tracking is here and that case and it is now here and it is now here it like that, if you are tracing that particular particle that is the lower stream line here, its coming here it is starting to turn somewhere around here.

Where, the first expansion line touches that, after that it changes and when it comes to a constant rate after that it is becoming parallel. Its moving parallel to the wall and they are also tracking another particle here, some other particle some other stream lines slightly higher stream right, starting almost close to that 1 and its coming here and its getting accelerated and it is slightly higher.

So, it is experiencing the expansion for a slightly longer time. Its going through the expansion like this, its stretching inside and then it the gap is increasing here. And finally, we come out from here and you can again see a particle here. It is just moving at a constant velocity parallel to the wall. If you imagine the 2d case, it is just a constant distance from the piston, which means the relative velocity is 0. It is moving same velocity as piston in this reason, in here the velocity is 0 the top region and inside here the velocity is increasing from 0, at this point to velocity of piston at this line. After that, it is all velocity of piston in all over here.

That is, what is actually happening inside, this is just two different ways of looking at things? Once this is given, I will go to another problem. I will go back to the board, I will tell another problem. Where, I have a mach number greater than 1flow and it is having a smooth compression corner. If it is having a smooth compression corner, then of course, the same thing like expansion will happen for every small change, there will be one compression wave.

There will be one compression wave coming for every small change. Now, I am trying to pull you out of this thinking of there will always be a shock. I am going to say that for very small changes the wave that is produced is very weak, you can consider it as a very weak oblique shock or I will just consider it as a compression wave, which is almost a mach wave I will think about it that way.

Now, I am going to say this is a mach wave and there is next mach wave, which is telling that there is a small change, some other increase in angle. So, now, I am going from here continuously changing the other direction. How will I know that my now my flow will turn this way, it is a compression my d nu is now negative by our convention going this way is positive, going up is negative and my d nu is negative, my d m is negative from that formula, we wrote the formula is still here this one ok.

If my d nu is negative, m is positive always. So, my d m will be negative. So, I can tell that, if my wall is going in, then my mach number will decrease and that is your compression phenomena. It is going to have a whole bunch of compression waves coming together and of course, we would not worry about. What will happen, when they come close to each other? They eventually collapse to form a shock oblique.

Finally, we would not worry about that part, right now, we will deal with after some days and it is it is going to go through this. This particular problem is very close to the wall of this particular smooth compression. I can still use prindtl Meyer angle to solve this problem. I will get the mach number here exactly that, but when these things interact similar to what we did last time when oblique shocks come and interact they may form a slip stream kind of stuff.

Here also, there will be some slip stream forming here. There will be a slip forming not just one slip stream probably there are a whole bunch of slip reason might be form of there we will just ignore all that for now. I will show you pictures of that, and so this I can now start solving problems like, I can just tell what will be the mach number. I can start predicting mach number values. Now, just to have feel for these numbers, I will tell you the limits for these numbers. We just have few more minutes.

So, what will happen I have the expression here? If I set my mach number tends to infinity then, I can easily tell this term is easy to work with this is going to go to infinity and I am going to tell tan inverse infinity is pi by2, which will become 90 degrees. If I convert radians to degrees, but this term this is going to have some number. Still tan infinity, that I do not need worry about that, is going to be pi by2, but square root of gamma plus 1by gamma minus 1for gamma equal to 1.4 this is 6.

I will write this long back, just use to get this number. Now, I have square root of that. So, it is going to come to some particular number it will be 2.236 multiplied by pi by 2, now that I have to convert to 90 degrees. It is going to go to that, what will that number be ok.

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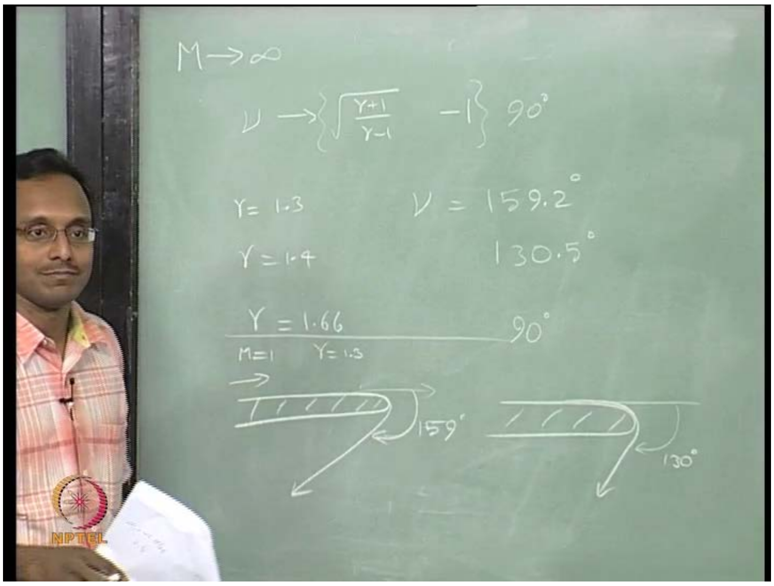

Let us say, I am going to write that, M tending to infinity case, nu tends to square root of gamma plus 1 by gamma minus , minus 1, remember that there is a minus gamma by 2 there this whole thing multiplied by 90 degrees. I have converted it from radians to degrees. Now, let us look at this number for three different cases, gamma equal to 1.3, this number is 159.2 degrees. Gamma equal to 1.4, this number is 130.5 degrees. M is equal to 1.6. Oh sorry, I put nu here it should be gamma here and that is nu and this is gamma 1.66, this value is 90 degrees this is what we have ok.

What does this mean, even if I accelerate my flow, say my flow is coming with M equal to 1 here and I turn it straight back to 180degrees, my flow has to turn around fully the flow does not follow the wall any more. That is, what it says, if my gamma is 1.3 case, if I have a case of gamma equal to 1.3, then it is going to turn by that 159 degrees alone. That will be something like this, my last stream line near the wall will separate from the wall and just gets out here, and this is what will happen? this angle is not there sorry, the angle of deflection for the stream line. So, this is stream line and it turn like this.

This is your 159 degrees, what you will get? If, I have the same thing for gamma is equal to 1.4. I will draw the another picture, then this is going to go for 130 degrees, here this deflection is 130 degrees and inside here what will it be, there is nothing vacuum as far as our simple gas dynamics works this region this is vacuum there is nothing, which breaks down our assumption of continuum breaks down at boundary. So, at M equal to infinity, we cannot use these analyses; just know that this gives you these numbers. But, these numbers are not valid, when I go from M equal to infinity, we will pick up some specific numerical examples next class onwards, see you people next time.