Gas Dynamics
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Module - 15
Lecture - 33
Supersonic Jet, Numerical Examples

Hello everyone, welcome back, till now we have solved problems, where the back pressure outside the nozzle was brought down from P naught all the way down to third critical pressure, till yesterday we have done upto that point. That is, we also took care of over expanded flow, now we will go for remaining portion, only thing left is under expanded flow.

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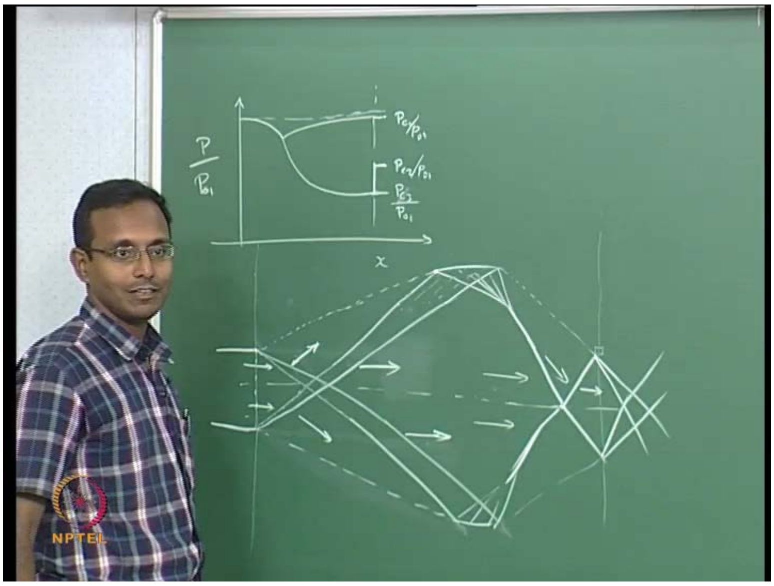

When we think about under expanded flow, what do you mean, we go back to the plot again. I am going to say that, P by P naught 1 if I plot then, I am going to get something like this, this is my P critical 1, there is a shock at the exit and that becomes P critical 2, this is my P critical 3, everything is divided by p 0 1. So now, we want to solve the cases where the pressure is below this P critical 3, initially we said the flow changes inside the nozzle itself upto P critical 2.

After that, we solved in yesterday last class, where we did this between these two cases and of course, we went upto point where we said, those oblique shocks that are forming at the exit, finally become mach waves. The weakest oblique shocks ever possible, we gone upto a point where, it becomes the weakest oblique shock ever possible and that just completes to become a mach wave. Now, we are going to start from that point and I am going to say, I have a nozzle exiting exactly parallel walls, that is just for simplification.

And then, I am going to say, I just barely have a mach wave there, this is the condition I have, this is wave mach wave and I am going to say, this pressures are equal at both side, outside and the exit which means, my jet will just go straight, it will be no difference, jet will just go straight. Now, if I decrease the pressure here, what should happen then, the P exit is more than the outside pressure which means, it has not yet expanded to that value.

So, that is why this is called the under expanded region, that pressure is the under expanded pressure region, which actually means by definition itself, I am going to say, it is under expanded means, it still needs to expand more, it has not yet expanded fully. What should happen, it should form an expansion fan around this corner, starting from the first point where there should be a change, which is that exit point, where it first meets the outside world, rather say different pressure.

So, at that point, it is going to send out some of set of expansion waves and what should be the very first wave, same as this particular wave we already draw. It should be the same wave, because the first wave and an expansion wave is your the same mach wave, it will all be the same. So, your expansion oblique shock that was inside, just came out slowly and then, became your mach wave. After that, if I keep decreasing my back pressure, it is going to continue from there and send out expansion waves that way.

Now, I will have set of expansion fans like this, when there is flow is expanding, velocity will be going outward, wait and I have to draw the arrows correctly. Anyway, I have to remove the jet boundary and I will change the arrow length to be little longer. So, this velocity has now increased a little bit, it accelerated along with change in pressure. So, if this is the new velocity vector then, the very last set of fluid elements that are coming here will also be going parallel to this.

So, that becomes your jet boundary, any fluid is that was coming in, will be inside this line under similar line this side. I have to remove my old jet boundary, so this will be my new jet boundary and the flow velocity is like this, this is what you get and the velocity initially like this. Now, again since we have discuss the shock problem, I can go a little faster in this time and I am going to say, it is going to turn like this and the fluid just keeps going like this till the other set of expansion waves cross this one and just go all the way to the end.

As I told before, they are just messengers, they are taking the message that the flow has a pressure outside. That information has to go all the way till somebody else says, do not change anymore, till that point it is just going to continuously send this information out all the way out there. Till where, will it be accepted as is, every fluid inside will accept whatever, outside atmosphere is very strong, it does not want to change the pressure. If I assume that to be correct perfect then, I am going to say, outside does not change at all.

So, when this expansion goes and meets this dotted line, this pictures become bigger, that is why I started with the smaller picture at the beginning. So, this line suppose to be one straight line, which just going out like this, till this expansion fan first wave goes and touches that. At that point, this expansion fan goes and tells that, your pressure has to decrease further and think about it, this expansion fan is left running expansion fan which means, it wants to accelerate the fluid to the right, it is going like this, it is accelerating like this.

While this is a right running expansion fan, it is going to the right which means, it wants to accelerate the fluid to the left. Remember that, if my expansion fan goes to the right, I will be told to move to the left, it goes to the opposite. So, currently the fluid experienced right running expansion wave and the fluid turn to the left and it is going like this. Now, it is feeling a left running expansion wave, now what will happen, since it is a left running expansion wave, that is going to go and tell fluid should be have more right velocity or it should have less of left side velocity, that is all it is going to tell.

So, at the end of it, it is going to go to a point, where it become straight and even longer velocity vector than that. Because, it is also expanding the flow further, it is accelerating the flow even more then, it is going to even higher velocity. Same thing will happen this side, I am talking about this fluid element now, it is first facing a left running expansion wave which means, the fluid is accelerating to the right.

And immediately after that, I keep going, I have to draw it is symmetric also, this expansion keeps going like this, looks like I am not drawing very symmetrically, I will try and match the top one, close enough. And after this point, this fluid element is going to see a right running expansion wave, what is that mean, it is going to get a velocity towards the left. So, it will turn less to the right and more to the left, so it is going to finally become this.

Why should it become this, that is because, the ((Refer Time: 09:01)) angle expected on this side is same as this side, that is the only reason. Ideally, if I have different pressure here than this then, this need not be very symmetric, some final flow will be some other direction, that can also happen. Currently, we are not having that crazy set up, we are having simple enough atmosphere, both sides are same pressure.

If I think about a very complex problem then, I can have this pressure different from this pressure and this expansions will not be symmetric which means, this will bended by some angle. This will not bended back by the same angle, it will bended by some other angle, in the final fluid, we will have some net direction and you can always expect that. Because, the jet will not go symmetric, it is going to be tilted to the down, if this pressure is more than this pressure why, there is a pressure difference across this fluid element.

Over all fluid element that is coming out is seeing that, this pressure is higher than this, it is going to accelerate down net, that is the one more way of looking that, it has to do that eventually, just think simple logic, it works most of the times. So, we would not pick that complicated case, we have going to assume symmetric problem, both pressures are exactly equal, I am not drawing it very symmetrically, but it is suppose to be symmetric, doing something like this.

So, let us say, I would not draw both sides and try to make it symmetric, I will draw only one half, it is the simplest way to get through in engineering style. So, I am going to say, I have this expansion wave coming here and now at this point, this expansion wave is just telling the outside atmospheric fluid that, it has to decrease pressure. But, the outside atmospheric fluid is going to say, no I will never change my pressure, that is why this becomes your boundary condition, it is telling I am not going to change, you have to change your pressure.

How does it impose it, it forces a compression wave on top of this expansion wave to negate it at that point, it will be exactly equal and opposite value, it will be a compression fan matching this exact expansion value, so that at that point, it is zero change in pressure. Similarly, it will happen for all those expansion waves inside this expansion fan, everyone of them will send out one such thing and a whole bunch of things together will eventually form one shock.

It is similar to what we did last time, almost the same concept, it is going to go through this whole thing. Similarly, there will be one more coming from this side and there will be a shock and I am going to say, these two shocks interact, ideally they should be symmetric. And after this we know what happens, this two shocks interacting, we just look at this point downstream and I am going to say, it is going to go crossing like this. What is happening here in the middle, the flow is just parallel, when there is a shock like this what happens, flow turns.

If it is a right running shock, flow turns to the right, compression wave is the wave which tells, there is too much crowd there, come along with me. Think that, it is a right running wave, so the flow will be turn to the right, going to be turning in like this. And then, this is a left running wave, so it will be turn back towards the left and it will go back to the parallel, does something like this, overall that is what is happening. I am also trying to draw the velocity vectors lesser and lesser as it goes, now if this is the flow vector, my dotted line should go like this.

And similarly, from the other side and then, it is going to go, at this point this shock goes and hits this corner, now what should happen. When the shock goes and hits to the corner, now this set of compression waves together have formed one shock and they are going and telling, you have to increase your pressure. Again the atmosphere is very adamant, that fluid element sitting there, it is going to say, I am not going to change. So, what it will do, it will put how many compression waves we have, so many expansion waves at that point negating each one of those compression waves.

So finally, there will be a whole bunch of expansion waves travelling from here and they start going like this. Similarly, from the other side, a whole bunch of expansion waves going like this. Now, they will start crossing and if I look at the problem from here, it is expansion wave crossing, which is the exact same problem, as what I started here. Now, all I have to tell you is, if I draw this line here and draw similar line here, take this picture between these two lines, take this whole thing, put it after that line and another after that line, again like this whole structure, just keeps repeating one after the other.

This is what is going to form your expansion compression cell structure, I have drawn one simple case, where this kind of the reflection at the axis happens. We also know that, when two oblique shocks interact, we need not have this simple reflection. Reflection at the axis or you are going to think about, they are just crossing each other simply, that need not be the case.

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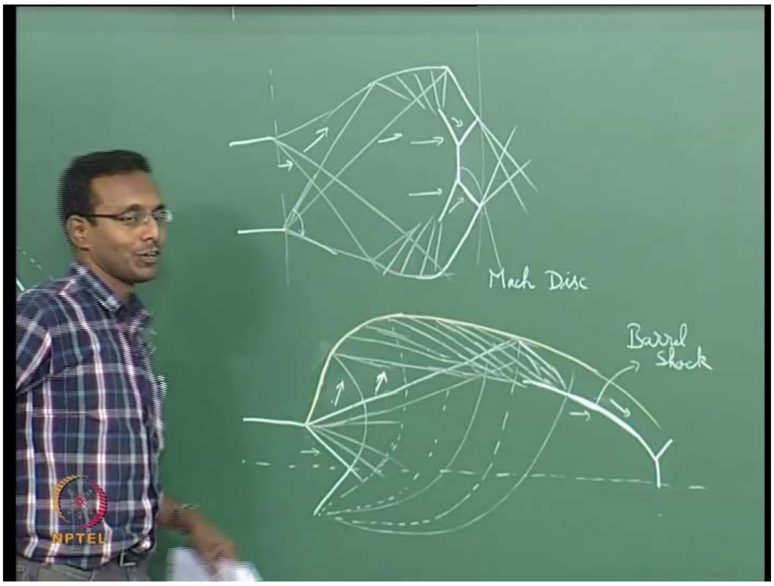

I may have a crazy situation., I will just start drawing from the expansion turning out the top and bottom, where I am having a stronger oblique shock. And now, if my fluid is coming like this and turning very sharply, it may not be having enough energy to turn it back straight parallel. There will could be a situation where, I may end up with a mark reflection at this point, in which case it becomes a shock and then, turns around like this. Now, it will be more difficult for me to draw velocity vectors correctly, because it will cross the region.

So, we would not draw velocity vectors right now, it is going to do something like this. Now, what is going to happen, the flow is going to have dotted line like this, this is my jet boundary now. And after that, again there is a shock coming and hitting this boundary which means, again there is this adamant fluid element sitting here, which will replace all the shock with full set of expansion fans. So, there will be a whole bunch of expansion waves going from here, same thing will happen from here, this whole thing repeats again, this can also happen.

When such a thing happen in a flow, where I have the expansion, I will just go back and reconstruct and i have my nozzle here. If I have my flow situation like this, if I have a case where I have an under expanded jet and it forms this kind of a shock. Typically if it is an anti symmetric nozzle then, what will be the shape of this normal shock, if I look from that side. It is a anti symmetric nozzle and it is like a tube, this will come out to be a circle, anti symmetric again.

If that is the case then, it looks like the disc and since Ernst Mach explained this kind of phenomena of shock reflection or shock crossing, this is called the mach disc. So, if you see books, which are talking about under expanded, just they typically tell there will be a mach disc. There could be a situation, where the number of expansion fans is not enough, number of expansion waves is not this small, it can be really high, can be as high as...

This is the starting angle, but the ending angle could be somewhere here, I can have an expansion fan all through upto here, can also happen. Let us pick such a case, I will draw only the top half and put a central line, because I would not have space otherwise. I am going to draw expansion, starting will be same as original mach number mach angle that you do not have a choice. But, the pressure outside is extremely low, that my delta theta will be very high, because my delta nu is going to be pretty high.

We know you can have if it is mach 1.5 here and I have very low pressure here then, I know, I can have all the way upto around 120, 130 degrees turn. We said, if it is mach 1 then, 130 degrees is the best turning angle you can get for infinite mach number expansion. Pressure should be 0 for that, which is not really possible we also said that, when we are dealing with Prandtl Meyer flow long back. If I think about that case, I can tell that, my angle can be more than 90 degrees also, even if my mach number is 2, I can have more than 90 degrees angle change, if my pressure is extremely low.

So, let us pick such a case, where my expansion is this wide, this full region is my expansion wave. This is all my expansion fan, everything inside is my expansion fan, if that is the case and my flow is turning like this, it is going to turn finely out at some angle like this, that is also a possibility. It is going to come like this and is turning, turning, turning and is turning out, I may be put too high an angle, I decrease it a little bit, something like this.

I want to make sure that, this to this angle delta theta is same as delta nu, should be, now it need not be same as delta mu though, so I am probably ok with it nearest. So, I am going to have something like this and it is going out like this, you have to think about other expansion fan coming from the other side, that is also going to go through. And now remember, it is a left running wave, if I think about this particular wave, it is a left running wave.

What does that mean, it is running to the left of the fluid element local velocity vector which means, when this thing is going up like this, this wave comes here, this fluid element is going like this and this wave comes here, it has to run this wave with respect to that fluid local velocity vector. So, it can even turn this way, this particular expansion fan coming here can also turn that way then, you will have more complex phenomena happening here.

When this expansion goes and hits there and tells that, it has to become a compression wave. Typically, it is go and hit and come back as a compression wave from there, there will be whole bunch of compression waves coming. If I think about this as my jet boundary then, it goes and hits there and then, from here, there will be a compression wave coming back, till here it straight line, this is my first wave Ii said. Every other wave is also going to go and hit and come back, so my jet boundary become something like this, I will use some other color for jet boundary.

Let us say I use this whatever orange color for this, some such things, if I use this as my jet boundary, I can tell that, even before the top one expansion goes there, this expansion goes and hits there. And I am going to tell that, this whole set, I have to use white now, some other expansion that is coming here is also going to do the same thing, it is going to hit somewhere there. And some other expansion if an expansion wave is coming here, it is going to hit somewhere there, all these stats starts to happen.

And I am going to get this kind of jet boundary and a whole set of compression waves coming from all of these points, every point from here will send out the compression wave. When all these compression waves are coming, they are suppose to be right running with respect to that local velocity vector only which means, it need not be going downward, expansion. When the velocity vector is like this, it may still be running like this, that may be right running for that locally and have drawn such that, it make simpler for you to think about it.

What happens is, there is a whole bunch of these coming together along with these expansion fan, this one from this corner going and hitting the boundary, this is going to hit. At the same time, some wave from here has started turning and that is also going and hitting there, all of them going and hitting, they are all going to start turning there. That will be a little stronger, because two expansion waves go and hit, it will come back as slightly stronger compression wave.

Whole bunch of them coming together, they will all meet, compression waves you know come together and they going to meet and form a shock at some crazy angle like this, this also happens. When this kind of thing happens, this is going to experience the expansion fan from here also, that shock that is forming. Because of that, you are going to see the fluid turning that way similar to here, here I did not draw the velocity vector, I will go back and draw it.

This expansion is making sure that, the flow velocity vector turns back to axis, so here velocity vector like this, as it goes that way, the vector velocity is becoming parallel. And this shock is suppose to be right running for that local velocity it mean, that the angle slowly start turning like this, as it goes. So, we will go to a point where the shock looks something like this and I have drawn it as if the shock starts here, it could even start from somewhere here, anywhere here depending on how many compression waves come together, it can be even frankly, that does not even matter.

And I am going to have the jet boundary, this is my orange color or whatever color that is, it is going to go like this. Now, the flow as it is coming along this region, is going to slowly turn back to parallel before the shock. After the shock what should happen, it will turn back inward, similar to this, but it is just more curved shock. Similar to here after the shock, it is going to turn back inward, that same thing is going to happen in this region and your jet boundary will also start turning like this.

Now, I will go extend my axis, at the axis for this much of expansion, if I form all those expansions come together forming a shock, that shock is typically very strong near the axis. So, when it crosses the axis, it goes through a mach disc, I am drawing only one half of the thing, otherwise its suppose to be a disc fully, it forms a mach disc. And this also happens, this typically a very under expanded jet, it has to expand a lot after it comes out, that is the case for this.

If this happen then, you have one more extra feature, which has the special name, typically this happens when you are having rockets go out in space. That kind of situation when that chamber pressure is so high, exit pressure is going to be matched by the chamber pressure, not matched, it will be proportional to that and compared to that, the outside pressure is almost vacuum. So, you will have this huge expansion coming and you will end up with this particular feature called a barrel shock why, it looks roughly like a barrel.

It is not complete barrel, but it is start somewhere here and it just goes like a barrel, barrel for storing liquids, that kind of barrel, just looks like that and at the end, it has the mach disc like the top lid of your barrel or something, going to look something like this. So, that is the barrel shock and this is also one more feature in this flow, when this happens. Typically this problem becomes much more difficult to explain after this point, because there is a expansion waves from both sides crossing.

While they are crossing, there is compression waves crossing and become much more difficult to explain this problem. At this point, if you think about it, there is an expansion wave coming from here, there is an expansion wave coming from here and there is compression wave coming from here, all that are meeting here. What will be the final result of the flow velocity vector turning, I cannot tell very easily from here, but if I use method of characteristics for solving this, most slightly I can still get the correct answer out of it.

If I assume, I know how to use method of characteristic really well, but that is one way of looking it, I have not given you the method of characteristic yet, we will go for it after some time. So, this is one more feature that happens in this kind of flows, otherwise it is similar to what you have seen before, there will be the shock structure, the expansion shocks cells will be there.

I have already told you, this dotted line, this line to this line, take this and copy it to that side and copy it to one more and copy it one more and it just keeps on going like this forever. If that happens then, you will see your expansion compression cells, just continuing to happen.

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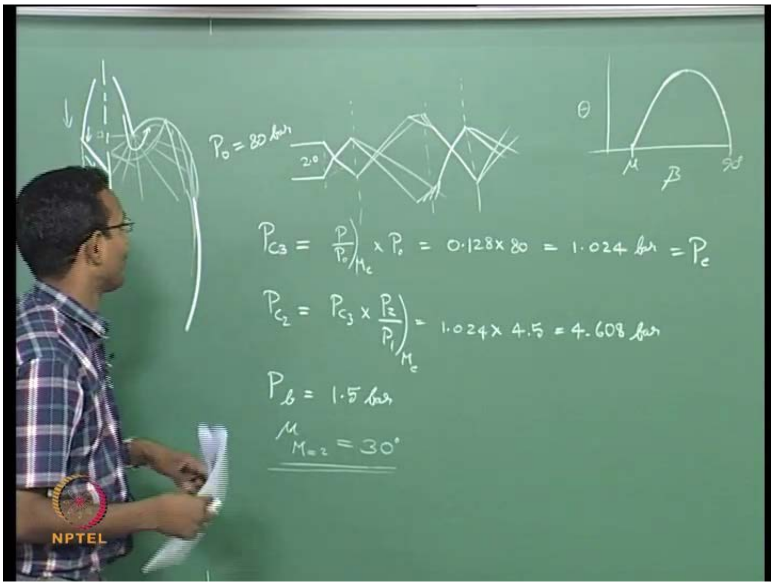

If we recall what we did last class, we said that, we will have a jet with first thing will be shocks and after that, we will just have expansion compression cells. If you look at it, the only thing that repeats will be from here to here, this whole structure can we put one more level here and one more level here and one more like this. It is always the expansion compression structure that is going to be repeated several times. Only difference in a over expanded jet is, it will have this initial portion of some two shocks crossing, after that it forms the other structure and it just goes on like this forever.

I will go back here and I will just label these number like last time 1 2 3, this region must be 4 5 and then, I can go for a 6 and 7 and it just keeps on going like this. If I think about the pressure at 1, this is a under expanded jet which means, it is more than atmospheric pressure, under expanded jet, it has to still expand to atmosphere. And region 2, this and this both are region 2, symmetry of course, region 2 is exactly same as atmospheric pressure.

After that region 3, it has expanded from atmospheric pressure across a expansion fan, this pressure is lower and then, I go more, is a shock and it goes to higher than atmospheric pressure. How do I know that, it is higher than atmospheric pressure, if I look at this region, these compression waves are turning to this direction and this region will be close to atmospheric pressure. Because, that it is what is forced by these compression waves, these compressions are going to negate the effect of this expansion on this region.

So, at the end, after all that compression waves, I will get back to atmospheric pressure in this region. And then, I will go for the shock after this and that is going to be your higher than atmospheric pressure condition. So, the whole thing together has started from higher than atmospheric pressure, went to atmospheric, went lower than atmospheric and then, went to atmospheric and then, went to higher than atmospheric. If I think about an non central line stream line, if I think about the stream line that goes like this, it goes like this, goes there, turns down and then, comes back parallel, if I pick such a stream line then, it is doing that.

If I pick the centre line stream line higher than the atmospheric pressure, during the expansion processes inside here, it will cross expansion somewhere atmospheric pressure. Then, it will go to lower atmospheric pressure, high, drops to low and then, stays low, suddenly jumps upto high and then, just stays high and then, it drops to low through this exit of expansion then, it goes low, like that it just alternates up and down. If you actually do the calculations, you will find that, 1 and 5 will have the same pressures and 3 and 7 will have the same pressures and it just goes like this.

And of course, you know 2 and 4 are supposed to be close to atmospheric, that is what you will get, 6 is also atmospheric, all the even numbers are atmospheric. And odd numbers 1 plus 4, 1 plus 4 n will be something and 3 plus 4 n will be something, this is the overall structure. This same thing you can see even in the over expanded jet case what we did last case, even there you will see the same kind of system. Now, I have to talk about one more thing, which I forgot at the beginning that is, when I think about fluid slab that is coming here, like what we did last time for over expanded.

Fluid slab is coming here, what it is happening to it, it is increasing in size, is that logical, yes it is logical. Because, it is going through an expansion process why, because the pressure outside is much lower, it is trying to occupy all the volume there, because there is nobody occupying it, it is much lesser pressure. It is expanding and occupying the space, so it is going up and in the process, it has inertia, I want to think about it has inertia if you think about.

It keeps on increasing so much that, the pressure at the center drops, when this happens then, they want to implode, come back in, that is this process. It is coming back in and decreasing in size and again it went pass due to inertia, over shot and it became very high pressure and then, it is again expanding and contracting, expanding and contracting. In this whole mechanism, there is no process that applies breaks, there is nothing that says, slow down you will be reaching there anywhere, that kind of thing is not there.

But, in real life, there is something like this that, something happens to be viscosity, which we are neglecting in our gas dynamics currently. In our gas dynamics, I am going to say, there is fluid going like this and the next fluid element here is just sitting idle not moving, that is not allowed in real life. In real life, that momentum from this last fluid is, part of it is taken and given to this fluid which means, this is also moving which means, energy of this fluid element is slowly decade out in both sides, it leaked out in both sides.

Eventually energy is so much spread that the jet is expanding out, the moving fluid if I call as a jet, there is more moving fluid on the outside now, that is going to change the system also, this whole sequence of things that happen. When i think about this case, this is very very under expanded jet, why do I call it very very under expanded jet, we go back to our pressure picture, I am going to talk about the case, where it is like pressure is somewhere very low.

So, it is going to have a huge jump from there to here kind of pressure, if I jump from some pressure, this is my exit pressure and I jump from there to all the way to here at outside pressure, say I am almost vacuum here. When I am jumping so much, now I have to think about the fluid element that is coming out from here, is going to expand from here suddenly to really huge size and it is going to expand to something really big like this, going to expand to huge size.

There will be times, if you see some rocket flows, you will see a nozzle of the rocket like this and the ploom will go like this, that also happens. If you think about what is happening there, whole set of expansion fans going all the way there, that the fluid that is coming here is turning around there. It is turning all the way out there and after that, this set of expansion fans that are coming here are turning around, that fluid back to this direction, that whole set of things are happening out there.

And if you have some such case, you will start having compression waves from here itself, it will form a much bigger barrel, start having a barrel shock like this, at that point onwards. This happens typically in upper atmosphere, you will see barrel shock with a mach disc kind of shape in rockets, when it is flying way up. And you can see pictures of that in several websites, ISRO website also has pictures of this, if not NASA website has it, you just go and look at those pictures, nice to see these, these are also nice things to absorb.

So, there is one more thing, I just have to tell you about this under expanded verses over expanded, which is when they talk about rockets. When they are taking off from the ground and going all the way up, the rocket is actually experiencing high pressure at the bottom, as it goes up, it is experiencing low pressure. What pressure am I talking about, the back pressure, for the nozzle jet. So, if you think about it, the P b is decreasing continuously which means, in my problem, I am looking at that in my pressure plot, I am starting from up and I am going down continuously.

I may not be really starting at P b equal to p naught, I will be starting somewhere down there. But, from that point on, I am continuously going down, that is what is really the case which means, I will typically start with a over expanded jet to an under expanded jet. This whole sequence happens when the rocket is being launched, which you will actually see, if you look at the ploom size, as you go when you are beginning know, this same rocket, I will erase one part of it.

Let us say, this is the high altitude jet ploom like this, the low altitude thing will be looking more like this, that will be your rocket ploom when it is taking off initially. It will not be looking very big compared to the nozzle size, it will just be looking in fact, it will contract slowly inward at the beginning, which says that my fluid coming here is turning inward. This is my shock and of course remember that, my plot is different across my centre line, this is one picture, that is another picture.

Initially my jet is going to be very small thing and when the pressure outside is decreasing, the jet expands to fill that space and that is what you will see out here, this whole thing start happening. Now, when you talk to people they will say that, the pressure is decreasing, why are you talking about going from over expanded to under expanded, should it be the other way, it is just a confusion. But, I told you last class also, again I am repeating this, over expanded or under expanded is with respect to the fluid element at the exit of the nozzle.

Is that fluid element already expanded to the outside pressure, that is what we are asking, that is the question we are asking. When the pressure decreases, we are going from over expanded to under expanded, it is confusing, that is what the typical confusion to actually only the under graded time. If you think about it, what we are talking about is pressure outside is decreasing which means, initially pressure was very high compare to this pressure which means, I have expanded too much this thing, this fluid, so it is over expanded.

Later, this pressure decreases and so, I end up with this picture then, I have expanded too little, I have to expand much more after I go out, so it becomes under expanded at that point, that is what happens out there. How many such expansion compression cells can I have really, how many such expansion compression cells this repeatable packets, how many will I have in a jet. Ideally it should be infinity, if you think about it, it should be how many ever number, it should just be going oscillating up and down, up and down, up and down, forever.

I said ideally, because we have neglected viscosity, if there is viscous effects then, depending on the velocities, if the velocities are very high, viscous effects will be much stronger, the velocities are very tiny, the velocities need not be dissipated so well. And so, depending on how the velocity effect is in this case and whether the fluid here is already moving or not, that kind of will change this answer, it is so happens that in a rocket, the fluid outside is moving already.

Because, the rocket wants to go up, the fluid is going down with respect to the rocket and the jet is also going down with respect to the rocket. We need to fix some reference, I will fix the reference to be on the nozzle, so everything is going down with respect to it. So, the velocity difference need not be very high, the typically there will be some difference. So, there will be some difference in velocity, the jet will be higher velocity than this, so there will be some effect of shear along the edge of the jet, that also happen.

We have not talked about viscosity affecting the gas dynamics of the problem yet, but we will try and avoid the gas dynamics part of it and we will try and explain how a shear layer has the effect on the flow itself. Say, one more class after or may be next class itself a little bit we will talk about it, just wanted to give a glimpse of how a supersonic shear layer looks like, just it is fun to explain something like that, it is nice to explain, that is the only reason I will go for that.

The next thing I need to start is solve a numerical example of a problem like this, not really this type, I believe I picked a problem, where we are in this kind of situation. We are picking a case, where it is over expanded case, so we will have a shock at the beginning, over expanded case. I am picking a nozzle and I am going to go through the solution of this whole thing, till where can I solve this problem, can I solve this problem all the way, looks like I can keep on solving this problem all the way.

It so happens that, it becomes complex when I have to think about, there is a set of compression waves and they start forming a shock. At that region, the problem becomes not simple to solve and we would not solve after this point, we will solve till here, we would not solve anymore after that. We will try and solve the problem upto this point, we will see how well we do this, what we have to do is, I erase. Let us just go through this, I do not want to talk too much, I am having a P naught of 80 bar, something close to what typical rockets had some time back.

And I am having exit mach number is 2.0, I am picking such a nozzle, exit mach number is 2.0. So, I want to find a pressure where it is over expanded, that pressure I want to decide, I am choosing my rocket to fly at different altitudes, I do not know what atmosphere it is, I am just going to pick some case. So, I am going to say, let us find the value below P critical 2 and above P critical 3, for that I need to know P critical 2 and 3. So, P critical 2 how will I find that value, to find P critical 2, I need P critical 3, because you need to know the supersonic pressure and then, multiplied with P 2 by P 1 at that shock.

So, P critical 3 will find first, that number is related to P by P naught at that exit mach number into P naught currently, that will be the pressure at the exit, if it is fully supersonic. That is your critical pressure 3, that value for assuming this gas to be air, gamma is 7.4, we get to one number 1.024 bar, this is a number we have. This happens to be your P exit for this condition is that right, because mach number is 2, there is no shock inside my nozzle till that point, that also happens to be equal to my P exit for my nozzle.

Now, I will go for P critical 2, now I will take P critical 3 multiplied by P 2 by P 1, where shock mach number is equal to exit mach number, this is the only change I need to do and that happens to be 1.024 into 4.5, which is 4.608 bar. These are the numbers we have, now I want to choose the problem, am going to pick some back pressure, I have to pick something in between these two for me to solve a over expanded jet problem. So, I am going to choose 1.5, which is close to almost expanded kind of condition, I did not want to have a mach reflection here, if there is a mach reflection, it is difficult to solve.

So, I choose a case where it, I know is closer to P critical 3, so I am picking 1.5, so I am going to say, I am going to solve a problem P b equal to 1.5 bar, this is a problem I want to solve. So, how do I start, the first thing I need to do is, find the beta of the shock such that, the pressure downstream is equal to 1.5 bar, that is the idea of this shock. It is suppose to have an angle such that, it corrects the pressure of the fluid to outside pressure, that is the job of this shock.

Previously, we used to do this flow angle turning as the boundary condition, that is the simple thing. I told you gas dynamics has two things, one boundary condition is pressure, other boundary condition is flow velocity vector direction. In this case, it is not velocity vector direction, it is the pressure, so I want to match the pressure. So, how do I do this, I have to first guess a beta value and then, I will go through the calculations and then, I will find what is the pressure here.

Then, if the pressure is lower then, I will increase the beta so that, it is more normal, pressure will be higher or if the pressure is too high, I will decrease the beta so that, it is less normal, so the pressure goes down, that is a logic. What should I start with, what beta will I start with, these are the range of betas which I can use or can I use anything from 0 to 90, what is that.

Student : ((Refer Time: 48:07))

14 degrees what is it, 40 degrees, why 40 degrees, is there any reason to choose that?

Student: ((Refer Time: 48:26))

Is there any limit on the beta value I can use, can I have all beta values for any particular case, what is the problem, where is the problem. You have to remember your oblique shocks now suddenly, so what I have to think about is this, I will draw this picture, may be suddenly you will know the answer. What will be my beta values, weak part of this is one thing, but what beta values can I choose, is 0 to 90, what is it.

Student: ((Refer Time: 49:22))

mu to 90 degrees, that is all I can choose, it may so happen that, I may have a solution in the strong side also, but currently will say that, it is only weak solution that happens. So, ideally I have to guess somewhere between here and the weak, let us say I will choose something close to this weak, the mu value so that, things will not be very wrong. What is the mu value for my mach number, mu for M equal to 2 is what, that you should know, 30 degrees, sign inverse 1 by 2, you should just know the answer to that.

So now, I have to go through this iterative process which, I do not have the time for today, iterative process goes for like 5 columns and so many rows, we will do it next time. But, I have to guess something close to this, because I know my final answer is suppose to be close to mach wave, instead of oblique shock. If it is a mach wave then, my pressure will be 1.024 bar, that is my P critical 3, while I choose 1.5 bar as my back pressure.

So, I want to choose something close, let us say I will choose 40 degrees, 50 degrees, that kind of number, start from there and I know the answer should be somewhere between say 50 degrees and 30 degrees. We want to work through that with iteration, we get to the answer, I know answer is definitely more than 30 degrees. So, you have to see what should be the answer for this, that is not enough. Immediately after that, I have to also find, what is my jet boundary line, the picture I have drawn is not correct here.

If we look at it, the jet boundary goes and the shock goes faster, that is not right, but anyways we will draw better picture tomorrow or next class and then, we will figure out what to do, this is the idea. Now, you just have to go through guessing betas, getting the pressure after the shock and try and match it with the back pressure, that procedure will go through next class. Then, we will see, if we can go to supersonic shear layers also next class, will stop with this for today, see you people next time.