Gas Dynamics
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Module - 16
Lecture - 40
Rayleigh flow- Relations, Plots and discussion

Hello every one, welcome back. We were half way through the derivation of ratio of post heating in the tube verses before heating, ratio of 2 by 1 for various quantities. Today we will pick up temperature ratio.

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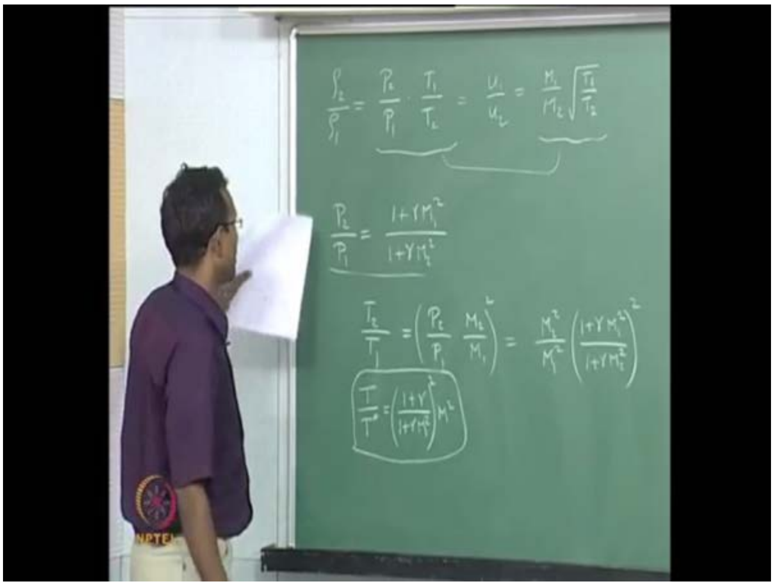

So, will start with this expression rho 2 by rho 1, this is of course P equal to from P equal to rho R T. I can write rho 2 P 2 by T 2, it comes out to be this relation. Now, we already had an expression for P 2 by P 1. But we want an expression for T 2 by T 1. I do not known rho 2 by rho 1 yet. So, I want to rewrite this rho 2 by rho1 in term of u 2 by u1 which will be we know from mass equation, it will be u 1 by u 2. Now, this can be written in term of T 2 and T 1 and M 1and M 2. Wait, I will just this as M 1 by M 2 square root of T 1 by T 2, mach number times speed of sound is here velocity.

So, now I look at this expression and this expression. I already had an expression for P 2 by P 1, which is same as your shock relations, this relation we already had. So, I have P 2 by P 1 in terms of M 1 and M 2 and the remaining things if I look at it. If I connect this 2 only T 2 by T1 and M 2 and M 1 are present. So, I can rewrite this and get to an expression for T 2 by T 1 and that will be P 2 by P 1multiplied by M 2 by M1whole square, the square comes from this square root and this reciprocal. You get the square root anyone. So, that expression comes out to be M 2 square by M 1 square plus gamma M 1 square by 1 plus gamma M 2 square. This is an expression for T 2 by T 1.

Now, I will again use this expression and substitute one of them to be M equal to 1 and I will get an expression for T by T star, this happens to be 1 plus gamma divided by 1 plus gamma M square, whole square times M square, another relation which is typically listed in tables. Of course, you know that these already listed in tables, already talk about this in terms of P by P star it will be listed. So, of course from here immediately I can go back and use this relation rho 2 by rho 1 in terms of either, I can use this or I can use directly this M 1 by M 2 times square root of T 1 by T 2, from here and I will get to rho 2 by rho 1.

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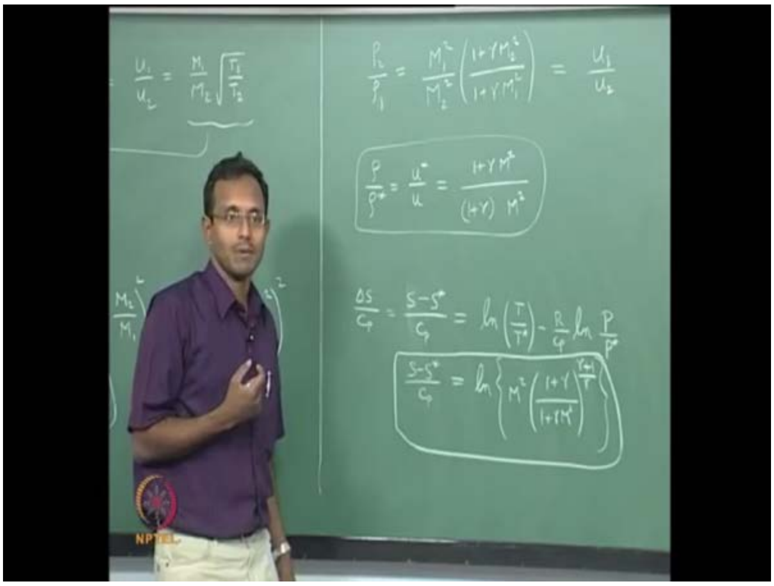

I will just write final expression for that, that simple enough for you to figure off rho 2 by rho 1. This is what you will get and of course, you can write rho by rho star and by the way, this is also equal to u 2 by u 1, the other way u 1 by u 2. So, it will be now I can write my rho by rho star which is also equal to u star by u is given by 1 plus gamma M square divided by 1 plus gamma times M square. This will be a relation, another quantity which is listed in your tables. Typically, they will list one of these two, they never list both of this useless like you know they are equal there are 1 is reciprocal of the other.

So, the next quantity we want to do is delta s, delta s by C p this is equal to log of 1 or I have to be a little more careful, I will write this more specially as s minus s star divided by C p. I want to defined what is state 1, what is state 2 so that, now when I write my expression, it will easier to think about this is equal to log of T by T star minus R by C p times log of P by P star, this is just coming from delta as equal to C p log T 2 by T 1 minus R log P 2 P 1 that is why you get. I just divide it by C p. So, I just put it as divided by C p of course, R by C p I can write it as gamma minus 1 by gamma and you can substitute for P by P star and T by T star in terms of mach number and you will get to final relation, log of M square 1 plus gamma divided by 1 plus gamma M square to be the power gamma plus 1 by gamma, that will just come out of your expression because there is half plus gamma minus1 by 2, gamma minus 1 by gamma, you will get s minus s star divided by C p, this is other relation you have.

Now, we know that you cannot have infinite amount of heat added to my system. How do I know that? If I start from a mach number, any M 1 does matter subsonic or supersonic I can utmost go to M equal to 1, finally if I heat it. So, I want to find how much is the maximum amount of heat that can be added to the system? So, if I want to find that, how will I find that it is not very difficult to find at all?

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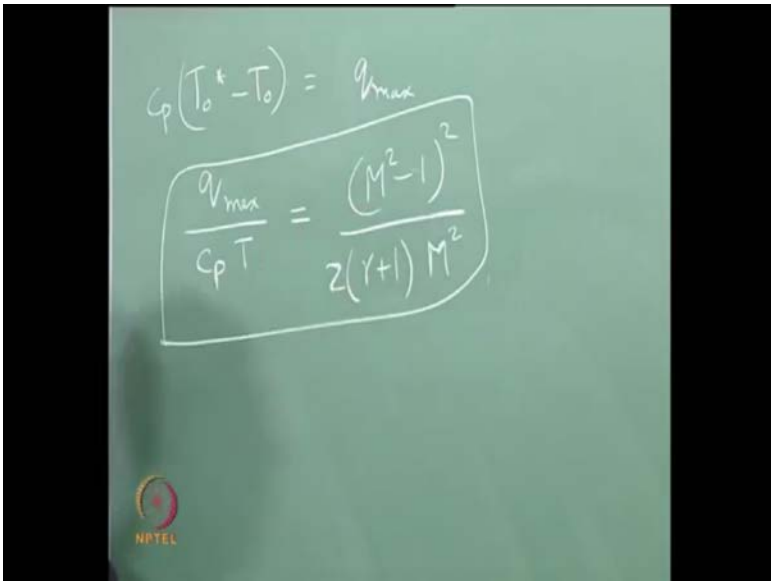

All I have do is T naught star minus T naught this times C p. This will be my maximum heat right this are basic heat equation we started with, this is going to be equal to your Q max. Actually I used small q I believe, I will stick to that convention as of now. q max per unit mass bases. Now, I will write this expression in terms of q max normalized with C p T and that expression in terms of mach number comes out to be. Yes of course, this is very simple to derive. So, I did not do it. We already have expression for T naught star by T naught and from there you can get T naught by T multiplied with that and you will get to this relation, not very difficult to derive I just do it. Now, once we have all this, we have to now understand the properties of these functions. So, we will go to the screen there.

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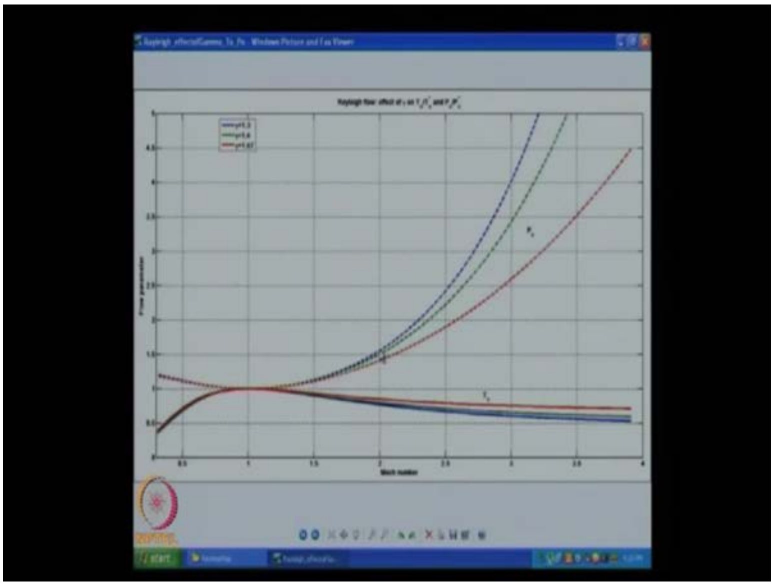

First I am plotting T naught by T naught star, you should understand here that T naught star is always greater than T naught for any mach number, utmost it is equal to 1 T naught equal to T naught star for M equal to 1. So, what I am plotting here T naught star is finally, think that is possible if I heated and T naught is the actually value. If I looked at these solid lines, those are T naught divided by T naught star, if look at it we know that if I am heating the gas, I am going towards M equal to 1. So, I am going to start from where to go towards M equal to 1. If I think about the small change in mach number from 1on either side, we can see that the slope is much deeper on the subsonic side than on the supersonic side, the way to think about it.

Let us just pick one of the curves here we do not want to look at all the gamma values. Let us just say thick red because that is very prominent here. Currently, I will just pick the red one, red happens to be gamma equal to 1.67. I will just pick the red one and I am thinking about, if I go from 1 to 0.5, I am dropping a lot in my T naught which actually means that, if I am adding heat I can add a lot of heat or my delta T naught will be very high form M equal to 0.5 to 1, then from M equal to 1.5 to 1. This is in a way similar to what we expected in our friction flows where we said, if I had subsonic flow it can handle lot more heating, lot more frication in this case. It can handle a lot more heating, it is taking the similar approach. This is the reason why the supersonic flow, I will suddenly take a subsonic part by going through a shock and then it can handle a little more heat and it can still try and manage to get M equal to 1 at the exit without worrying about changing the flow completely, that is a possibility.

This is very similar to what we did in friction flows. So, I am just telling you directly that, there could be shock sitting in your flow and that can jump across to the other path and come from there to the subsonic solution. Now, we will look at the other curve P naught by P naught star, we know that P naught will always decreases because of adding heat. By P naught is related to my enter P. P naught is going drop and it is going to come down to P naught star and of course, I know that as I am heating, I am going to come down this way, this particular path. By the way if I am cooling it can go the other direction, always remember that. We are thinking about heat only as of now. If I cool everything was the opposite direction, I can remove a lot of heat from M equal to 1 flow, if I go subsonic path, if I am cooling any ways.

So, now we will just think about P naught curve. We find that, if it is supersonic flow is huge drop in P naught while subsonic flow, there is lesser drop in P naught and it can handle a lot of heating also, that is what we see in general. Now, we will look at the variation and gamma. I am saying blue is the most compressible, gamma is very low and red is the least compressible and gamma equal to 1.4 is our air sitting somewhere in the middle.

So, if I look at my temperatures, I see that the most compressible gas can be heated a little more than the least compressible gas. It is the same here also it not very clear here, red is on the top, green in the middle and blue at the bottom. The difference is to be very low, when it is subsonic curves. In supersonic, they seem to be clear difference that too only after 1.5. There is finding that, when the gas is more compressible it can handle little more heating. So, it will not reach choking as earlier as the less compressible gas. It can get a little more compressible try to squeeze through the gap available, that is the way to think about it pushed can squeeze through that is means it more compressible.

I just want you to get these physical fields so that, you will never forget this, that is why I talk about squeezing through the same area instead of. So, if it is more compressible, it is going to be able to handle a little higher heating levels but, they compromise for that is you are got to lose more on P naught. If it is more compressible, I am going to lose pressure energy a lot. It can handle a little more heating but, pressure energy is higher. I have to drop a lot more. For higher P naught, if it more compressible gas this is properties of P naught and T naught variation.

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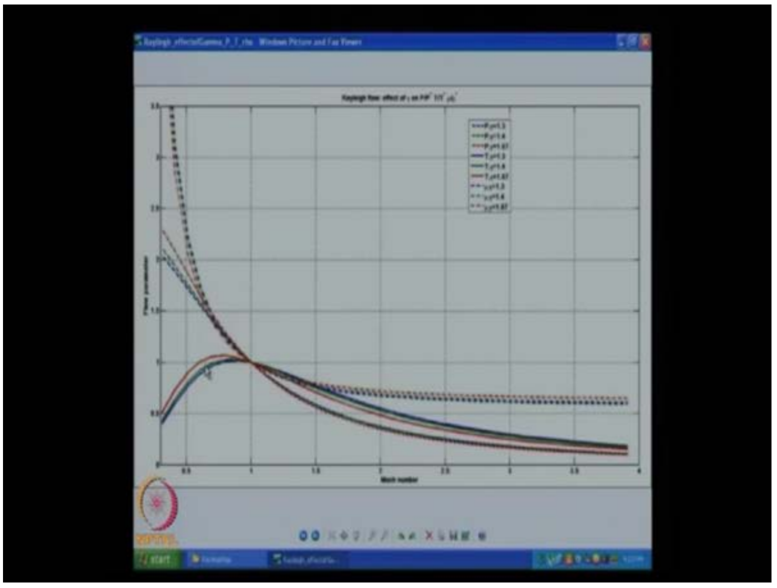

Now, we will go for pressure, temperature and density. Without even looking at legend, you should be able to tell for fan of flow, the only curve that goes up and comes down is temperature. And what is the peak point there? 1 by square root of gamma, that is what you have to look at and you can see that when gamma is decreasing that number is going closer to 1, that is what is happening here the peak shifting towards that direction, 1 by square of gamma.

Now, we will look at details. If I look at any one of these curve, let us say I pick pressure, the dashed line here. Pressure is coming very slowly changing towards M equal to1. And remember, that my flow always ends at M equal to 1, if I am heating the gas. If I am heating a subsonic gas, the pressure drops because it is accelerating towards M equal to 1. If I am heating a supersonic gas, the bottom one happens to be dashed line and this one happens to be for density. Supposed to be dashed dot line, the dots are not very clear on screen. Anyway this one your pressure curve and if I start with some higher mach number and I heat the gas, it is going to go towards M equals 1 and the pressure increases because mach number drops, it is easy way to remember it.

Now, I will look at temperature that is going to do the same thing here. Only in the subsonic side, there will be a small change where I will now use density as my explanation. I am going to say now, I will look at density and then I will come back to temperature. I will say this dash dot line for low mach numbers, the change in density is high when I heat the gas. As I go towards M equal to 1 that is not as high but, it is still changing at M equal to 1, it is not slope 0. The change in density is very high for very low subsonic and pressure is not changing so much.

So, what should happen now P equal to rho R T, temperature should compensate for this change that the density drops a lot, temperature has to increase so that, the pressure drop will be matched, that is why temperature is rising and beyond a point, they all look the same and the temperature seems to flatten out at that point. Temperature need not change, pressure is as same as density slope there. After some point it will just change to the other way, everything will go the way we expect things to go. Temperature should drop when mach number increases, that is happening around this region that is what we are seeing in here.

The remaining things gamma explanation, I believe you can explain, I gave this also as an exercise for fan of flows. I will just leave this. I believe you can explain this very easily for compressible gas. Just leave that part to you guys.

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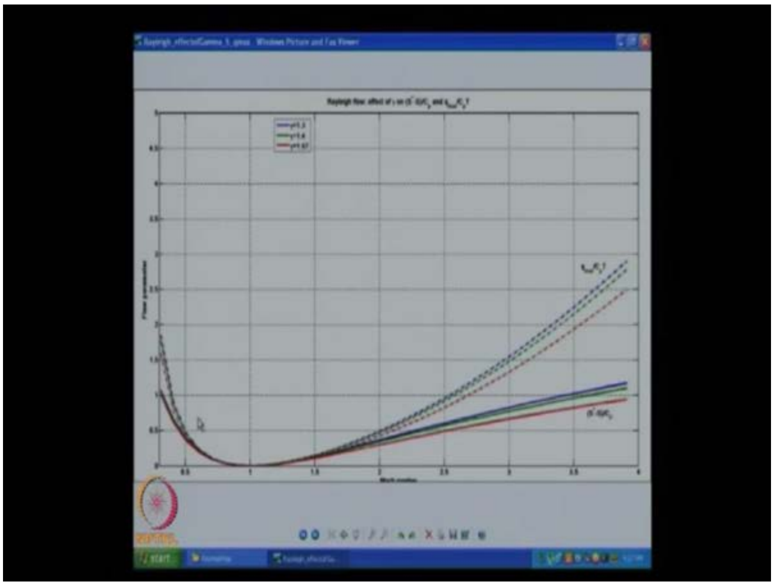

Now, I will go to the next one, more interesting curve. I am now plotting the opposite of what I given as expression in your notes, I am plotting here s star minus s, if I plot s minus s star that will go below its negative. Of course, if I am heating, I am thinking about this is entropy change, s star is going to be the highest value and s will be lesser, whatever expression I gave as s minus s star s star is higher than s. So, it will be negative always. So, I am plotting the negative of that so that, it fits in the same axis. I do not want put axis in negative also.

So, let us look at the solid lines. Those are s star minus s, what that says is, if I am starting somewhere subsonic, I am going to drop in s star minus s. If I drop s star minus s a lot, what does I mean, I am changing my entropy a lot in that region. When in here, supersonic condition that is not the case, s is almost equal to s star already, compared to the subsonic situation. Supersonic subsonic case, the entropy change is higher than the supersonic case.

Of course, now will again say subsonic case, it has only from1 to 0 while supersonic case has from 1 to infinity all the way. May be, that is why the curve is getting euclidean squeezed. You want worry about that any more here. But, if I think about the compressibility effect here, if the gas is more compressible and there is ideally you do not known answer to this really. If there is more possibility of rearranging energy inside the gas and so the entropy will increase more, that is the actually reason for this. This is out of scope of this course, if you want to deal with this. You want to go read high temperature gas dynamic or hypersonic close, we want to worry about that in this course.

Now, we will go the next one, q max. q max divided by normalized by C p temperature. Let us assume we start with the same temperature for the whole flow field case which I have plotted here, all same, any mach number, same temperature. I start with, let us say if I pick such a case and finding that subsonic case change a lot, q max is high when I go far away from 1, much more than the supersonic case for the same amount. I will just go for say 0.5 and 1.5. If I look at it, supersonic cannot handle as much heat as subsonic that is the main point I wanted to say again. This is the reason why I will say, if there is possibility of going through a shock, the flow can go from this point to a subsonic solution and still add a little more heat.

There is a possibility of this flow doing that and it may do that in depending on the settings. By the way, temperature is not the very important factor for determining the flow of conditions in gas dynamics, we know that already. It is pressure and velocity direction, those are the main boundary conditions, we will just keep this in mind. Now, we will go and do little more analysis. I want to now think about the fan of flow or not fan of flow, highly flow in terms of T s diagram again. Just talk about a little bit.

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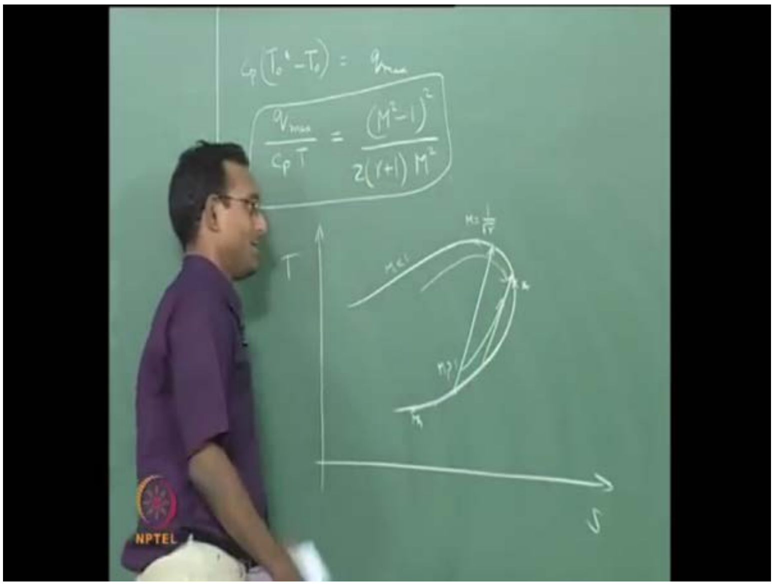

We already had this kind of curve for this T s diagram. We said that this particular point corresponds to M equal to 1 by square gamma. This is my star condition and this is my

M greater than 1, M 1 greater than 1 is my M 1 less than 1 and we said the define heating, I am going to go this ways. Heating will go towards M equal to 1 from both directions, cooling will go the opposite arrow that is what you found here.

Now, what will happen if I have shock somewhere sitting inside, if I have a shock sitting somewhere inside here in the supersonic section then, it is suddenly jumping from here on to some other point and of course, i do not know the exact shock characteristic here, I do not whether it is jumping to this point or this point. I am just generally drawing a line, here it will jump from some supersonic point to subsonic point, if it does that, after that it is going to go this way and reach M equal to 1. This is what will happen, just remember this that if I say the supersonic flow and I give some amount of heat I give. So, amount of heat it can either go this way and go there or it can go jump across the normal shock and then go along this line to reach that value somewhere, if I put T naught star then it will go and reach this point irrespective of whether it goes on subsonic path or the supersonic path, then when will the flow choose to have a shock are not have a shock.

Basically, I am telling if I give heat addition from this point, say I started somewhere here, this is my M 1. I am saying this is my M 1, supersonic M 1 I am having from here I am going to add exactly the q max value, which means the final point should be this. Now, I am saying I can either go straight along the supersonic path to this or I can go this way pick a normal shock in middle and then go this way. I know that normal shock does not change T naught, it does not change any heating value or T naught value anywhere.

So, it is going to have the same T naught whatever it was here, here also, from here it going to choose this solution. Now, which one will my flow choose and why? That is the thing we need to think about a little bit. Once I understand this, now I can go proceed further into P V diagram. I already gave a clue just before starting this discussion. What is the main difference because of normal shock? T naught remains the same. P naught changes stagnation pressure drops.

So, it may so happen that I am having some other downstream boundary condition in my duct which is adding heat to the gas, such that the pressure may not be matching the downstream pressure. If such a thing happens, then there will be a shock forming to manage the P naught value such that, it will meet the pressure value at exit. That is how this shock forms in the middle, it has nothing to do with, it is not similar to our fan of flow where we will tell the P naught drop should not match the exit pressure exactly. Here, again I am going to say this same thing P naught drop should the exit pressure but, our main problem is not P naught, it was T naught.

T naught does not change anything, it does not decide the shock location. It can be anywhere from here to here that I could have another path that will be this, it could be this path also. It starts from here and jumps to this point and from there it goes to subsonic condition, that might also be a possibility depending on whatever is the P naught required, it will choose whatever solution it wants and by the way if I have partial heating.

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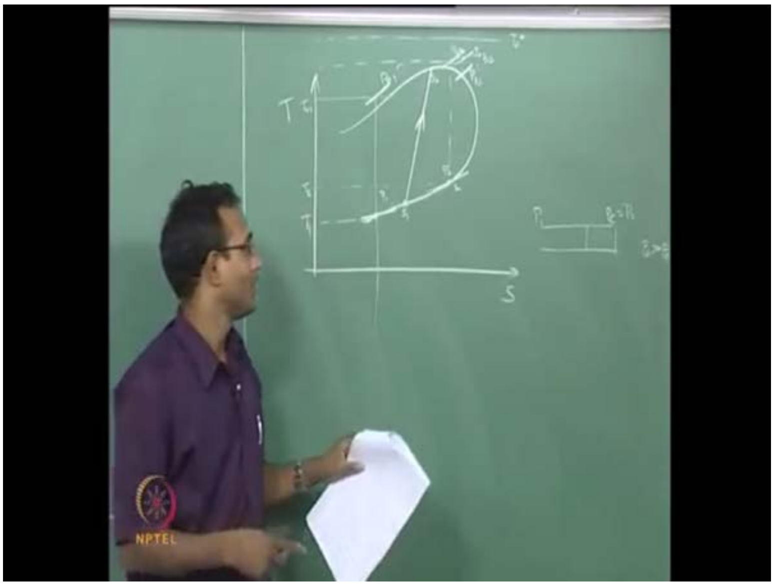

I do not want to erase this picture. I will draw one more and say I heated only up to this T naught value, that is p naught. Let us say this is my T naught star and currently I am sitting at this T naught, T naught 1 and let us say this is my s value, this is where I am currently, this is my T 1. Let us say that is my t 1, I am sitting in supersonic condition and I am here. If I supply heat such that it is going to go end up only here. Let us say this is my 2 T 2 and T naught 2 will be slightly higher than that, this will my P naught 2 while P naught 1, this is my P 1, P 2 curve and P naught 2 curve will be P naught 1 curve will be here. If I have some situation, I see that P naught 1 decreases to P naught 2 curve, curve has shifted to the right that means pressure decreases.

Now, yes I am going from here to here, this curve is not right, pressure should increase. I will draw the curve like this for pressure here and this curve will go up like this, pressure will increase a little bit to reach that point, because mach number decreases. It is going to go that way. May be I heated only up to this point, instead of going all the way to T naught star, I heat only upto this point. This will have a particular P naught 2 but, let us say I apply a back pressure that is more than this P 2, I give the same amount of heating but, I apply a higher back pressure than this.

What should happen similarly, nozzle flow problem or like friction duct, friction duck problem, everywhere there will be some external compression waves coming from exit because draw this. Let us say this P 1 P exit equal to the current P 2 and but, P b is greater than p exit. If I have such a case, what will happen there will be compression waves coming inward and as it comes inwards somewhere it will go, form a shock which will become stable. By the way I am assuming somewhere here that the heating uniform along the length, it need not be. I am assuming that the heating is slow and uniform along the length as it goes through length, heating is also increasing that is my assumption here, need be the case.

Let us assume that for now, if I think about that kind of slow heating system then, I am going say that there is going to be shock that is coming inside here and setting such that I am going to jump from here to some other value across a normal shock from some location here. Let us say this is s 1 that may just before the shock location and this is my s 2, just after shock location and from here it is still going to reach the same T naught. But, the P naught can change now. Of course, the P itself has changed, it can go, sit somewhere else. And the pressure will now be this, P 2 prime I will call it, P 2 which shock P 2 s, I will call this, same T naught and it will sit with this P naught 2 s. There is a extra P naught drop, you can see because of that, because of the shock sitting here and the pressure has now gone up to somewhere else.

But it still the same T naught line. I need not heated all the way up to T naught star. I need not have heated all way up to that. So, I can have 2 possibilities, when I have supersonic incoming flow I can go through just full supersonic path all the way with the heat the same amount of heat or it can have a normal shock such that it will match the pressure and sit somewhere else have their.

If I had some other pressure, let us say the pressure is such that it is lower pressure value, let say it is between this and this. What will happen, I need a curve it is closer to this line. So, my shock will shift this way. My shock will shift way so that, it will go and do that there that means I am basically saying that my mach number at which the shock happens will now decrease. You remember that here is high mach number, there it is M equal to 1. My mach number at which the shock happen decreases all this we can just get from just thinking through the curves here. Of course, I am going little fast but, just go and think through it, you will get familiar with this. We will come back it when we go back to P v diagram for the same discussion that again gives you a little more flavor to this problem. Now, we go start our P v discussion. Of course, P v is the set of variables which people use in thermodynamics and in gas dynamics we typically use P and 1 by rho, which is same as specific volume any way.

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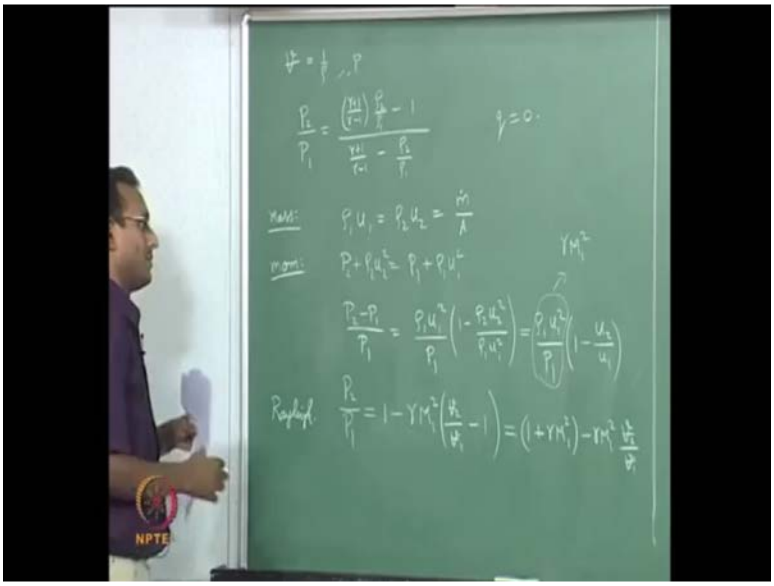

But, anyways I will use small v cut as my specific volume, this is equal to 1 by rho and P, these are my 2 variables. I am going to have for my plots but, before that I want to relate density and pressure so that, I will know what plot I am going to have. This is not new to you, you have seen this in shocks. Shock relation is going to be the same but, of course there will be some small difference, in shock there was no external heat added.

Now, there will be external heat added that will be the main difference. I will just recall that expression which we derived in shock, just show you get orientated with what I am going do and then you will find that this is not expression for our problem here. You may recall this expression, this particular expression is true for q equal to 0 case. This is the special case of what we are going do, derive now and you will put q equal to 0 in that new derivation and you will get to this expression exactly. Now, you want start from scratch, I am going to say mass rho 1 u 1 equal to rho 2 u 2 equal to M dot by A momentum. Again, I am doing the 1 d derivations not new to you. From here I can directly go from P 2 minus P 1 by P 1, I take it to this side and divide by P 1, that is going to give me rho 1 u 1 square by P 1 into 1 minus rho 2 u 2 square by rho 1 u 1 square. I jumped as the step in middle but, you can easily see that.

Now, I am going to say I will use mass equation inside this, till now this is only momentum equation rearranged. Now, I will use mass equation inside here rho 2 u 2 equal to rho 1 u 1. So, i will cut of rho 2 u 2 from here only one u 2 will remain, only one u 1 will remain. So, M 1 write this as rho 1 u 1 u square by P 1 into 1 minus u 2 by u 1. Now of course, we are been doing this a lot, this particular term will directly become gamma M 1 square, you will be doing this in the last few days, a lot of it. I can directly see P by rho is R T and then you multiply and divided by gamma you get that expression.

So, now u 2 by u 1 from mass equation, u 2 by u 1 is rho by R 2 but, from our v equal to 1 by rho, I can rewrite this as u 2 by u 1 is equal to V 2 by V 1 where V is my special volume. So, I will write my expression as P 2 by P 1equal to, I am taking this my take this minus 1 to the other side, 1 minus gamma M 1 square times V 2 by V 1 minus 1. I will end up with this particular expression and I will just rearrange the terms so that, I will directly talk about P 2 and V 2 relation, 1 plus gamma M 1 square minus gamma M 1 square times V 2 by V 1.

What we are seeing here is this expression, I got from mass equation and moment equation. By the way, this is the same thing as what I did for shocks. I just written it in different form. This is your rayleigh in your shock. I do not whether gave it as rayleigh but, now I call this the Rayleigh line. This is conserving mass and moment that is all. This particular expression is converging. What I am seeing here is for my flow we know mass and moment are conserved which means P 2 and V 2 are related in this way. They are linearly related and they are inversely related in a minus y not invisibility like minus V 2. P2 is propositional to minus V 2. So, I am going to have negative slope, if I draw P 2 and V 2 curve. So, if I draw my P v diagram, I will draw it as P 2 by P 1and V 2 V 1.

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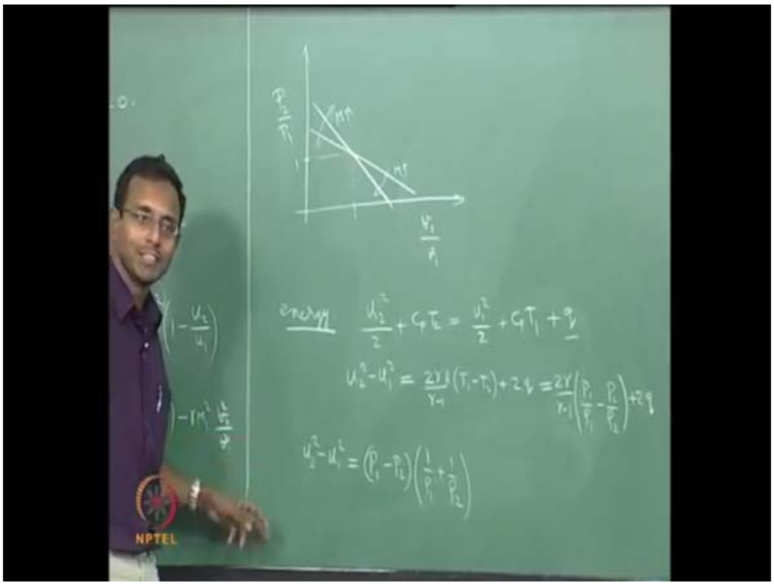

I am normalizing with some particular value, the initial pressure and let us say this is my 1 and 1 which is the starting point P 1 V 1 that particular line. From here I am going to go to some curve like this, that particular curve corresponds to one of this equation M 1 values. Some M 1values that is all it is going to be. We will end up with that and we know that it has to satisfy 1 and 1 always. Because if I substitute P 2 by P 1 equal to 1, then this term should cancel this term, that will make it V 2 by V 1 equal to 1. So, 1 and 1 are always satisfied even though it looks as if the intercept keeps the changing with M 1, it is changing such that it will match this particular point always.

If I have another mach number, this is going to be like this, which one is higher mach number I have to label them first. A and B, they are all straight lines. I have A and B, which one is higher mach number, the slope is what matter here. The slope magnitude is gamma M square, both are negative slope. The slope is gamma M square, if M is higher then it will have higher magnitude, it will be more vertical. So, I will replace this B will be higher mach number. So, I will replace this with curve this thing saying M increasing this way and here M increasing this way, when I go this way M is increasing it as if I am having a straight line and I am rotating it about this point, as M increases it is going no more like this. Just remember this, this is our Rayleigh line and how it works.

Now, still I have not used the ideal gas law of energy equation etcetera. Now, we will start from wherever we are currently and add energy equation to this. If I add energy equation, now it is going to be u 2 square by 2 plus C p T 2 is equal to u 1 square by 2 plus C p T 1 plus my q, where q is my new addition compared to my shock case. This is the only difference. Now, because of this of course, my expression is going to have q in every term.

Now, if I make q equal to 0, I should get back to my normal shocks as of now. We will keep it that way, we will substitute C p in terms of gamma and gamma R, gamma minus1 by gamma R gamma minus 1, that will be my C p and I want to rearrange this term u 2 square minus u 1 square. Of course, I could have A divided by 2, I will just take the other thing multiplied then by 2 V equal to 2 times C p's are together. So, I am going to take gamma by gamma minus 1 times R times, T 1 minus T 2 plus q, the q should also have a plus 2, 2 is multiplying all the terms. 2 is there all.

Now, of course this can be now rewritten, R T together R T 1 can be written as T 1 by rho 1 and R T 2 can be written as P 2 by rho 2. So, I can write this as 2 gamma by gamma minus 1 times P 1 by rho 1 minus P 2 by rho 2 plus 2 q. This is my u 1 square minus u 2 square. Now, I will just take one expression from our normal shock derivation. From mass equation and momentum equation it can be shown that u 2 square minus u 1 square is also equal to P 1 minus P 2 times 1 by rho plus 1 by rho 2. I am just taking this otherwise I have write three more lines. I will just wanted to take this from normal shock relation, we derived this exact equation when we were doing normal shock relations. So, I want to link this with this relation because both are u 2 square minus u 1square. I want to equate this 2 right hand side.

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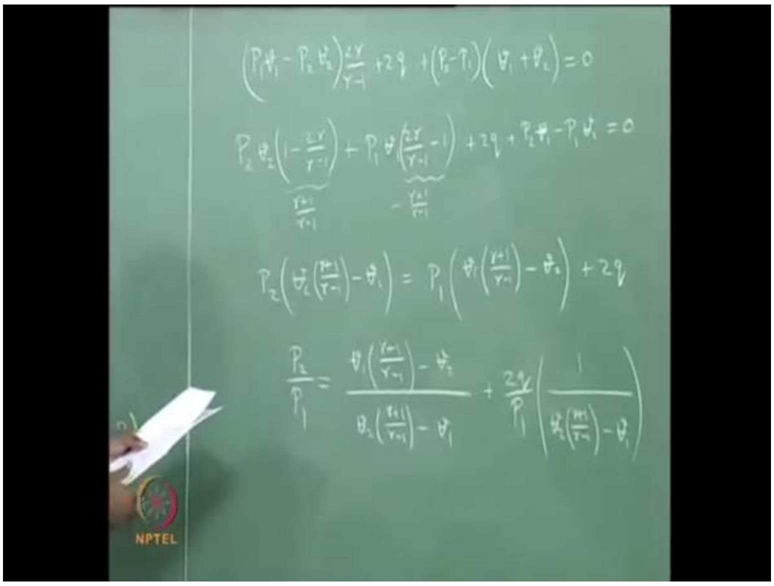

And I will write 1 by rho as my specific volume now. So, my expression becomes P 1 V 1 minus P 2 V 2 times 2 gamma by gamma minus 1 plus 2 q plus P 2 minus P 1 times P 1 plus V 2 this whole thing is equal to 0. I just rearrange these terms so that it looks like this. Now, from here I will group that terms such that, I will have P 2 V 2 terms together. I have P 2 V 2 here. Similarly, P 1 V 1 here, P 1 V 1 here and of course, I also have P 2 V 1 P 1 V 2 we like those have for. Now, we will group all the term in the nice way P 2 V 2 times 1 minus 2 gamma by gamma minus 1 plus P 1 V 1 times 2 gamma by gamma minus 1 minus 1 plus 2 q plus P 2 V 1 minus P 1 V 2 equal to 0.

Now of course, this will become gamma plus 1 by gamma minus 1, this will be minus of gamma plus 1 gamma minus 1 and the other terms where just simple. So this expression if you look at it, this is P 2 V 2, I am going to say P 1 V 1 are fixed numbers for my flow. I just want to see what are the relation of a exit P and V. P 2 V 2 product of these two and here it is just P 2 multiplied by constant, V 2 multiplied by constant. So, over all this is an expression for a hyperbola. Over all its an expression for hyperbola, if I do not have this, it is a rectangular hyperbola. If I have this it is a slightly tilted rectangular hyperbola and that is what you will get. I hope you know some amount of geometry to understand that. If not just go and draw the figure you will anyway see it.

I have the plot anyway I will show it to you after I finish these derivations. So, if I rearrange this, I just want go back to the original thing I wrote there. If I rearrange this, I can get it to. I will get to this expression from here. Now, I can get P 2 by P 1. I will just divide the whole expression by this value and P 1, this whole bracket and P 1 I am dividing so, I will get P 2 by P 1 has V 1 into gamma plus 1 by gamma minus 1 minus V 2 divided by this whole expression T 2 times gamma plus 1 gamma minus 1 minus V 1 plus 2 q divided by P 1. I will write this first, I will put it as 2 q by P 1 as a separate thing multiplied by this it comes to this expression. Now if you look at this I divide the numerator and the denominator by V 1 sorry V 2 so that, I will get back to the original expression I had out here for q equal to 0. I will get to this expression finally, because rho 2 by rho 1 is equal to V 1 by V 2, this will become V 1 by V 2 So, I just start to divide this expression here numerator, denominator by V 2, then I will get to that exact relation there for q equal to 0 case. So, this is the super set of the previous expression we had for normal shocks.

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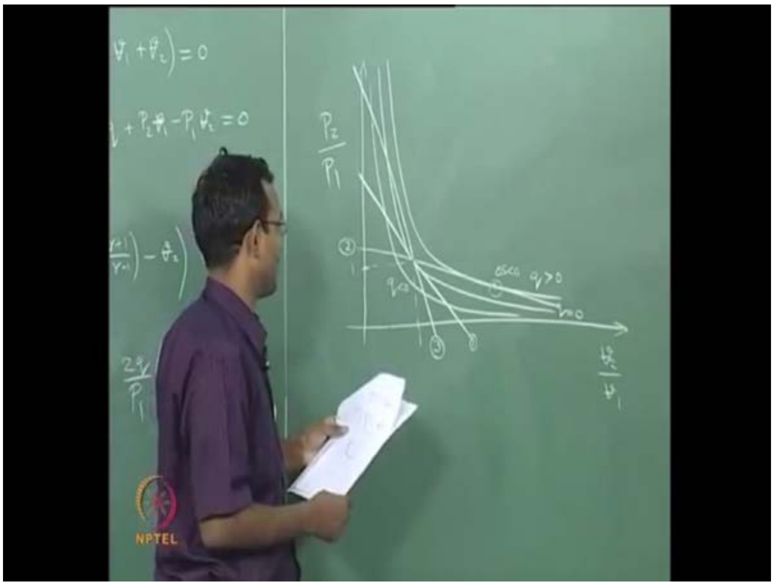

Now of course, the only thing remaining is have to know the functional form of this. Again I am going to say 1 and 1 is some starting point, first I will draw curve for q equal to 0 I said it is a rectangular hyperbola with axis tilted slightly, it is going to look something like this. This is for q equal to 0 that is no heat added. Now, if I add any amount of heat, if I have a q then from this expression is indirectly see that if q is positive, immediately this expression is going to have a higher value. It is going to have a higher value for the same V 2, P 2 will be higher for the same V 2. So, if I look that the curve is going to be above this curve q will be for q greater than 0 it is going to be like this and if q is less than 0, that is I am cooling that will be like this. These are your curves, this is how it will be.

Now, if I want to find what is my final state P 2 V 2, this particular curve set of curves will satisfy mass momentum energy state and the previous one will satisfy sorry a mass energy and state for this one and mass momentum energy for the Rayleigh line. From here I will draw the rayleigh also, say for a particular M 1. For a particular M 1, if I start with it is going to be some curve like this. So, my final solution has to be a intersection of these. I have drawn it such that its almost tangential to this particular curve.

So, it can only be cooled it says, heating does not give me solution that is what I have. If I draw another slope for my Rayleigh line, let us say I will pick very low slope that is mach number is very very small, then I will have something like this. I may be cutting it several points here. I find that if my mach number is very very small, then if I cool it I can have a solution there where pressure increases which you know is our going more subsonic, more subsonic pressure will increase that is this solution or I can heat it and go to a solution where my pressure decreases a little bit, that is I am having a higher mach number and it goes to this particular point, it is so happens that there are 2 solutions.

And I will just tell you directly here that, this particular solution delta s is less than 0. I can prove it tube at just ignore that point for now. I will prove with you some other time. delta s is less than 0, if I have to go there. So, it is going to tell me that I start from subsonic condition and it is going to end up supersonic just by heating that is not possible that is not allowed by entropy anywhere. So, it is going to be only this.

Now, if I consider a very high mach number that is supersonic mach number, this will give me this kind of curve I am drawing too many curves here. I will go back to plots with colors later. I call this curve as 1 this rayleigh as my 2 and this rayleigh as my 3. So, I have this and if I do not heat of course, I did not talk about do not heat at all. If I do not heat of course, there is a solution of P 2 e 2 equal to P 1 V 1, that is a trivial solution always, that is always a solution. And there is no other solution for one other than cooling that is a special case we will come back to that special case when I discuss it with the figure on the screen that currently we will just look at this. And if I go to high mach number case, if I cool the gas I am going to get lower pressure that is I am going to even higher mach number that is this solution and you can see that the volume increases that means my density dropped. I am expanding further to higher mach number that is this particular solution and I go the other side, this particular line if I go from here and heat it, I drew the q such that it never touched it.

Maybe I have to go my plots and I will do the plots thing next class because it looks like the time is not enough. You will find that there are 2 solutions probably possible or just one solution depending on the situation. That particular plot we will discuss next class because there is a lot of information hidden in there, after that we will go solve some numerical examples and we will close discussion on heat addition in supersonic duct. After that, I will just give you a brief idea of what if everything is present. Heat addition, mass addition, friction and area change, just probably one class after and then I will move on to some other sections. Experimental methods and method of characteristics those are left, we will go for that after. Any other questions? See you people next class.