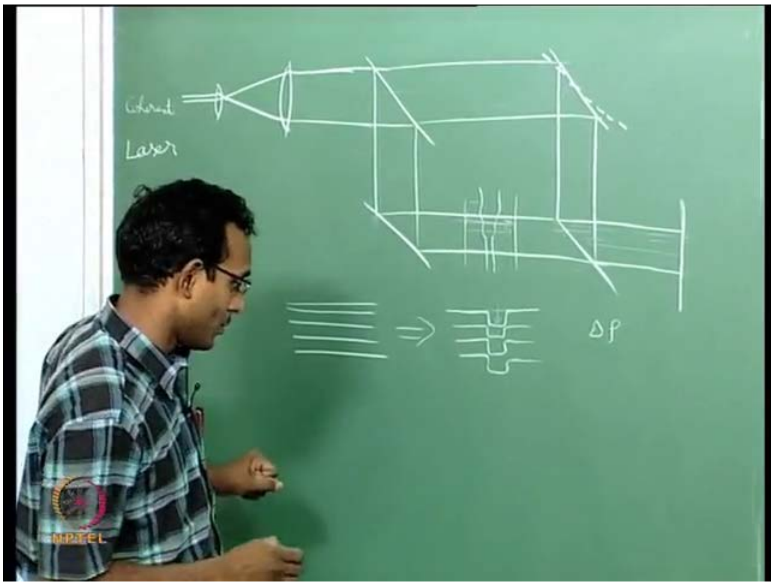
Gas Dynamics
Prof T. M. Muruganandam
Department of Aerospace Engineering
Indian Institute of Technology Madras

Module - 20
Lecture - 49
High Speed Flow Visualization
Method of Characteristics

Hello, everyone welcome back. We were, looking at interferometry for measurement of flow with density changes. I will just go back and look at the picture once of the flow field of the setup.

(Refer Slide Time: 00:29)



I made one small whatever slip which I want to correct right now, typically I did not tell you what the light source was that was a mistake I made. I am having a parallel beam of light coming I said I will put a mirror this is the beam splitter which splits 50 percent of light or whatever some fraction of light this way another mirror and the final output will be mixture of these two together again, this beam splitter which will recombine this beam and this beam together and at the end I will put the screen here I will be watching what happens on the other side from this side I will be watching.

If it is a screen I will just look from that side. So, in this overall ((Refer Time: 01:29)) I did not tell you I just slipped that this should be a Coherent light source. A Coherent light source is it is supposed to be something that has only some fixed phase that is it should be all the light wave that are coming in should be having the exact same phase why we are thinking about looking at the phase difference. So, phases are important for this.

So, typically we use Lasers for this Laser must be the source for this and after that we expand the beam to whatever size we want from there we will split it into two pieces. I will put a setup my flow facility somewhere here say a Jet or something they have a Jet in their then that Jet will deflect not really deflect the light in this case it will just slow down the flow of light through this.

So, the light when it goes through this if my Jet is say a Helium Jet then I am lighter than air. So, if density is lesser which means it will go faster. So, this light going through here will go faster while the one going through the other path will go slower because, of that there will be a phase difference at this point after that they are mixed together when they come here because of the phase difference there will be interference change.

I said change because initially in the alignment if we adjust everything such that every point on the mirror wherever the light comes in every light ray if I think about each of these rays if they come and meet exactly at the same phase here. Then there will be no interference at all everything will be exactly same intensity typically what we do is we adjust it slightly off either this is rotated this way or this way or rotated this way.

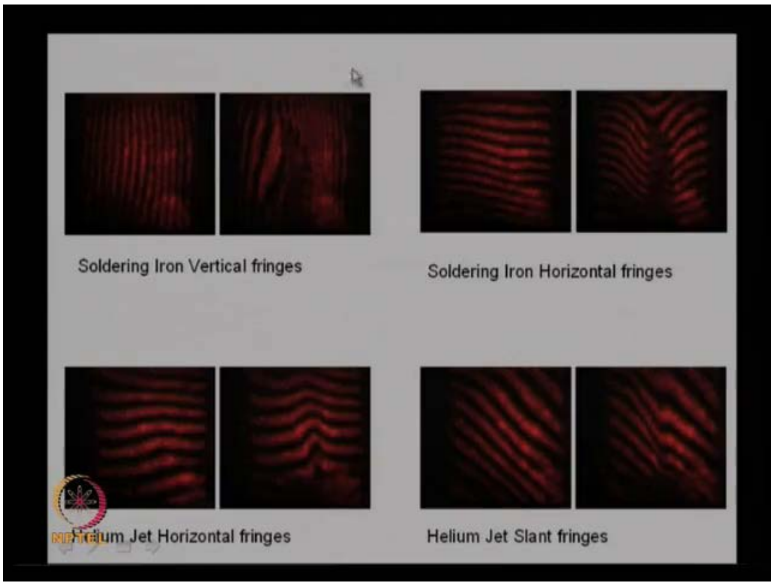
If, I do these I can get some fringes directly on this because if I made it go I will put a dotted line here if I make it to go like this top ray is taking a shorter path while the bottom ray is taking the longer path I am, imposing and known interference here. Typically I will create a set of fringes if this is the case there will be interference fringes this way perpendicular to this board there will be lines like this on the screen that is what you will see from here if we have this way.

If, I have the other way tilt of the mirror then I will have the line that is going away from the board will have one phase while the one going on the board plane will have some other phase when it comes here now will have vertical lines here on the screen. Now, I can choose whichever setup I feel like or I can choose partially vertical partially Horizontal then it will get a slant lines on the screen anything can be possible with just adjusting one mirror. But, 1st what we will do is typically we will adjust such that everything is exactly parallel then we will adjust it slightly off the good alignment.

So, that we get reasonable fringe settings. Now if I put with Horizontal or vertical some such slant fringes already I put a density gradient here density variation here then there is going to be a change in the fringes now, the change in the fringes can be related to if my fringe pattern was like this say there are lines like this is my original fringe pattern from here if it moves to if it does something like this then, if we track this particular fringe it has shifted down now this shift can be exactly related to the density difference.

How much did this fringe shift from here? It went by 1 order down one fringe down if it went 2 fringes down then it may look something like this more deep depending on the number of fringes down I can tell that can be related to delta rho delta rho the particular ray has seen that is what we will be picking up that is the idea of this method this much of detail is enough I think for this we will just go and look at some images of interferometry which we did in our lab will go to the screen.

(Refer Slide Time: 06:10)



Here, we have this which is just no density gradient it is just aligned such that there are vertical fringes here. We are using a red Helium neon Laser. So, we are having red lines vertical bright and dark bright and dark etcetera. And in this image we have put a Soldering Iron this black shadow is the Soldering Iron of course, it is not full shadow because part of the beam has it the other part of beam will still go to without being blocked by the Soldering Iron.

So, one beam we have it and other beam we do not have it. Now this is a Soldering Iron which is getting hot slowly we turned it on and what weare seeing is this fringe patterns are no more the same see that there is gap in the fringe pattern changing that basically says that these fringes have shifted this way. In this particular case which means I am going to have some kind of density variation here you can see that far away the fringes are not changing much, but as I come closer there are getting expanded more.

So, there is some kind of changes happening in here you go to same Soldering Iron thing with Horizontal fringes this case we have adjusted the mirror such that we get roughly Horizontal fringe pattern it is not exactly parallel you can notice that means, there is a irregularity in my mirrors with this when we put Soldering Iron the same this is the shadow for the Soldering Iron this is sitting here where we are seeing that all the lines are being pulled down suddenly all the lines are being pulled down saying that the density inside is lesser suddenly in this particular case.

We have arranged it such that it will be lesser density in here can be arranged straight opposite also is depending on how you arrange it, but I know that in this case density is getting lesser and then going back to normal. Here it is same as this height if I follow that line instead of going and ending there it is going down coming back out and then ending there that says that there was some density variation in this region.

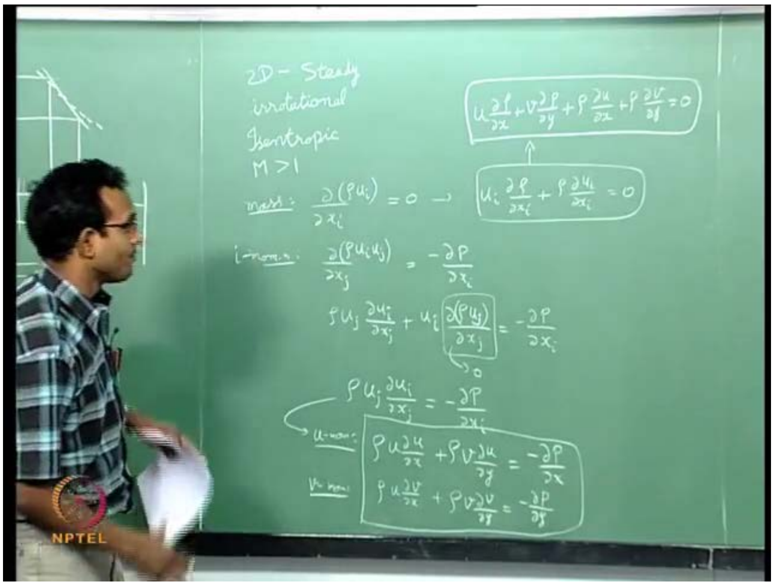
And similarly on this region everywhere around the hot metal rod youare going to have a difference. Now we did the experiment with Helium Jet I have adjusted this is not the same setting as the other Horizontal fringes I have adjusted such that it can be opposite now we are having fringes again Horizontal we readjusted it and then we put a small metal tube with a tiny hole where there is Helium Jet coming out of it you can see the Jet boundary like this here and here and everywhere inside the Jet boundary there is a jump in the fringe and that can also be used for telling density difference the same Helium Jet.

Now, we are doing with cross slant fringes depending on what you want to show in your images you can choose the fringe settings if you choose slant you can now see that I have realigned that now it is going down is density decrease saying that it is from here go down and then come back to the original line and go out. Which means this line is my Jet boundary here and this line is a Jet boundary here and this region is here metal block from which the Helium is coming out that is all you can see from here.

We already have seen other density based visualization methods alright we looked that shadow graph 1st then we looked at sleeve run and now this is interferometry and I think that completes pretty much.

Every kind of visualization method which we have for compressible flows which are simple enough to deal with there can be other methods which we would not deal with currently these are specific to density change based methods. Now, we will move onto computational methods used in compressible flows one of the most popular ones and they are simple enough method happens to be method of characteristics we will deal with only method of characteristics this time in this course actually.

(Refer Slide Time: 10:23)



Method of characteristic is for a problem which is 2dimensional Steady irrotational Isentropic supersonic these are the standard conditions of course, other than this I will say that the flow is invisid nobody forces no friction or inviscid are the same no heat release those kind of assumptions also will include in this. So, I am just thinking about the standards set with these extra conditions in here this is the only case where I will be solving my flow field currently only this type of flow field I will be solving the think about it I will start with the equations of motion mass conservation it is dou by dou x i rho u i equal to 0 .

Or you can rewrite this as this is from fluid mechanics of course, which you can rewrite as ui dou rho by dou xi plus rho dou ui by dou xi equal to 0 this is one expression that is mass. Next we will write momentum equation momentum equation again I will write in this index notation later we will switch to this is 2 D right we can always switch to u v coordinate system we will go back to it after sometime 1st we will write the general thing anyway will switch in 2 3 minutes probably we will just simplify it a little bit dou by dou xj of rho ui uj equal to minus dou P by dou xi this is your i momentum equation if you think about it, I can take 1 2 3 values that is how it comes out which along with this expression can be simplified a little bit.

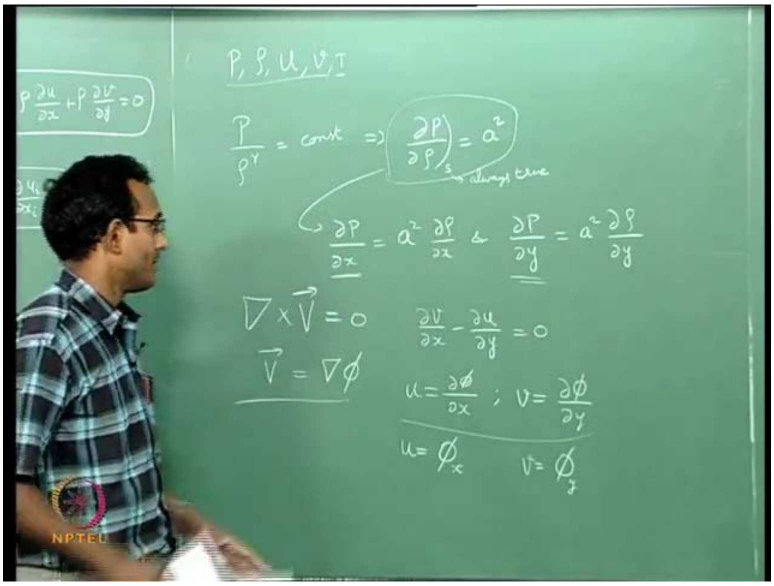
So, I am going to keep this rho ui together uj and multiplied by rho ui I want to do differentiation by parts if I do that I will get rho ui dou uj by dou xj this is one term I have plus uj dou rho ui by dou xj this is the other term I have. Now, I think I made a mistake somewhere rho ui hear dou uj ui rho uj should be kept together fine that is a mistake I made I am pulling out rho uj and instead of rho ui otherwise yes of course, it will just confuse you a little bit.

But, this one will be easier to think about. So, I will keep it as rho uj it can be done the other way also it just does not mean anything because i and j can be switched around, but this is rho uj is one unit and ui as the other unit we are going to do this differentiation by parts and this is equal to minus dou P by dou xi look at this now, look at just this unit this is dou by dou xj of rho uj if I look at this and this they are exactly the same except for i is replaced by j that is all we would not worry about this i it might as will be k and this can also be k if that is the case then I am going to say this unit is 0 this unit is 0. So, this is 0.

So, I will get my other expression this is actually a part of fluid mechanics derivation, but I like starting from here that is all this is your momentum equation. If I write it in 2D form these expressions mass equation will now become in 2D that is in terms of u and v coordinates I will substitute for i equal to 1 and i equal to 2. So, we will get u dou by dou x of rho plus v dou by dou y of rho plus rho dou u by dou x.

Plus rho dou v by rho y equal to 0 this is my 2D mass equation for steady flows steady compressible flows. Now, similarly I can write momentum equation, but of course, here I will get two equations it is I momentum equation. So, we will get two equations u momentum equation will be rho uj can be any value i is equal to one right now rho u dou by dou x of u plus rho v dou by dou y of u equal to minus dou p by dou x this is one equation. And the other one v momentum going to be rho u dou by dou x of v plus rho v dou by dou y of v equal to minus dou P by dou x. These are my momentum equation is two of them here. How many variables do I have my for my flow field? I have to now start counting variables.

(Refer Slide Time: 16:47)



I have pressure density u v so I have any other variables probably temperature these are my variables I have to think about , but if I do not think about temperature right now even now I have only 3 equations and I have 4 variables even without thinking about temperature. Temperature also is a part of the problem temperature I know I can use energy equation let us just think about the remaining things.

Now, I have to link any two of them independent of whatever I already linked to it we have one condition in here Isentropic it is isentropic then I can say P by rho power gamma is constant this is one way of looking at it or another way of looking at it this is equivalently given as I will put.

Dou P by dou rho at constant entropy equal to a square I have where a is my speed of sound I will say that this is true for our case this is always true why we have isentropic flow.

So, I am going to say for us dou P by dou rho is equal to a square I do not need to say only at constant entropy our case is fully isentropic. So, it is always constant entropy.

So, I have 4 equations what are the 4 equations? Mass two momentum equations and one isentropic relation I will use this relation. Now how will I use this relation in my momentum equation I want to remove my dou P variable, but I have dou P by dou x there.

So, I will rewrite this as dou P by dou x is equal to a square dou rho by rho x and dou P by dou y equal to a square dou rho by dou y this is the way I will rewrite my expression. What I have done really its calculus really i have taken this any derivative can be represented as delta P by delta rho limit delta rho tends to 0 I will take that I will remove the limit currently and then I will go back and say I will take this delta rho multiplied that side I have this expression without this delta x at the bottom.

Now, I will divide each side by delta x now I will put the limit delta x tends to 0 again then I will get to this expression this calculus rearrangement the same thing done for delta y you get to this expression that is one way of looking at it whatever i have done that is a simple way of looking at it.

So now, I will go and substitute for my dou P by dou x and dou P by dou y in my momentum equation. So, I will write it as I go back here what I will do is I will write my a square dou rho by dou x here then, put that is equal to minus of this whole expression in here that is what I need to do. But, I want to do one more extra thing we want to put this condition irrotationality which we did not use yet.

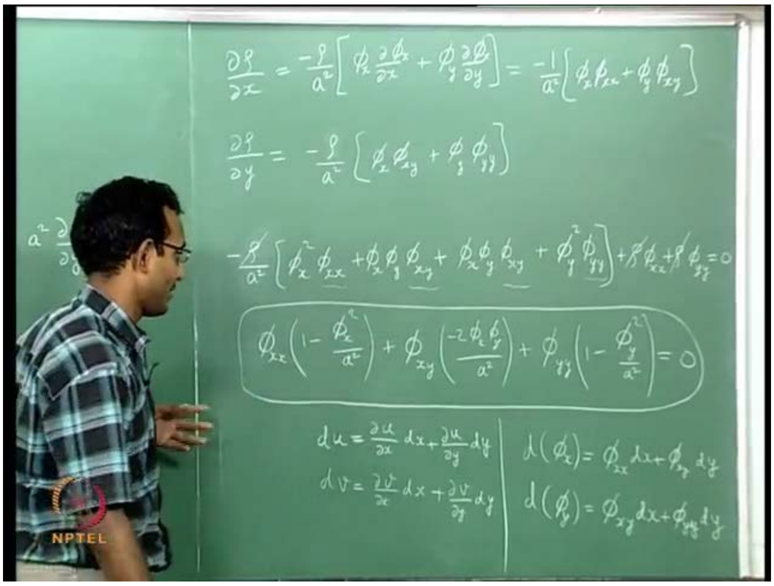
So, till now my flow all this equations if I solve will be satisfying mass momentum equation isentropic conditions that is what it will not do anything more than that now we want to say I want it to be irrotational flow also. How will I force this? I have to tell that I have del cross v equal to 0 del cross v equal to 0 in 2D will just come out to be it will just come out to be this relation.

Now if you are in fluid mechanics you will immediately tell that if there is a irrotational flow now my velocity field can be expressed as gradient of a scalar field. So, now I can rewrite my velocity field v can be expressed as gradient of some scalar field phi. Phi does not have a direction this is just a distribution like pressure is a scalar field like that phi is some other scalar field.

So, if this is a case if I know the phi value everywhere in my flow field that is all the x y locations then I can get a velocity by doing just the gradient operation on the phi. So, my u velocity will become dou phi by dou x and v will become dou phi by dou y. Now, I justified in doing this let us, do a quick check will just substitute this u inside here and this v inside here what will happen v derivative with respect to x will give me dou square phi by dou x dou y and here, It will be dou square phi by dou x dou y again. 1minus the other I will just abstract and it becomes 0 this is assuming phi is continuous in x y which is reasonable to assume for our kind of flow fields.

So, we were getting this correct. So, we are justified in putting this definition which we will use like this. Now, I will do one more thing for simplification of our expressions I will put, you equal to phi subscript x what this means is exactly same thing as this derivative with respect to x is now written as subscript x just for simplicity. And v is given as phi y subscript y if I have this now we will start using this in our expressions I already told you I wanted to substitute dP by dx in momentum equation with this expression will start doing all of them together.

(Refer Slide Time: 22:50)



So, what I will get will be I will take this dou rho by dou x is equal to 1 by a square dou p by dou x alright. I will go here dou rho by dou x equal to 1 by a square with a minus sign put whatever is in the left hand side of my momentum equation I will write that expression here.

What will that expression become? It is u dou u by dou x that is I am thinking dou p by dou x substitution right it is x momentum equation u is my phi x dou by dou x of phi x plus phi y dou by dou y of phi x this is what, I am having which now, I will introduce more convention if I put one more subscript there I will make this phi x x which means I am taking derivative with respect to x twice and here, I am taking I will make this phi x y I am taking derivative once with x once with y.

So, I will write this as minus 1 by a square phi x phi xx plus phi y phi xy. So, that the expression looks simpler it looks simple enough to handle we are going to get into very complex expression. So, we would not simpler notations like this. Next one I will write dou rho by dou y in similar fashion and I will skip one step I will just directly write the answer minus a square phi x phi xy plus phi y phi yy this is what you will get. I skipped this kind of step here this is jumped one step.

Now, I will use these two in my mass equation next mass equation has dou rho by dou x and dou rho by dou y what was my mass equation? I have dou rho by dou x oh I will just go back here I will just tell you what I am doing here and then go back there immediately. I will take this we have expressions for dou rho by dou x I have expressions for dou rho by dou y in terms of phi and of course, u can be written as phi x v can be written as phi y and here rho will keep as of now dou u by dou x.

We know as phi x x rho dou v by dou y is phi y, y. You can write these things like this. So, now, we will go and write that kind of a formulation. So, that will skip one step in here again minus rho by a square times phi x square phi xx plus phi x phi y phi xy plus another phi x phi y phi xy plus phi y square phi yy plus rho phi x xplus rho phi yy equal to 0. These are the last two terms rho dou u by dou x rho dou u by dou y are these two terms the remaining terms if I look at these two they are coming from dou rho by dou x phi x phi xx is there multiplied by we had a u here in the mass equation yes.

Student: (Refer Time: 26:29)

dou rho by dou x in here u momentum equation I missed a rho term there dou rho by dou x I missed a rho term somewhere I have it right here I missed it on the board. this should be rho yes, it is not just 1 there should be a rho in here. That is why it came up here. So, you got it good. So, this rho is supposed to be there in here I missed it I put it as 1 these two places it should be rho.

Now, using that you get to this expression the 1st two terms come from this substituted into mass equation these two terms come from this substituted into mass equation and the last two terms are directly in mass equation in the last two terms. Now, if I look at this expression density is everywhere in this terms I know that density cannot be 0. So, I can divide by density if density 0 there is no gas.

So, we will never have that case. So, I will remove my density from my equation it is non0. So, now, I have an expression which is just a 2ndorder differential equation nonlinear 2nd order differential equation in phi alone except for I need to know the value of a at every point it may be changing from point to point that is also there .

So, now if I think about it if, I want to solve this problem it is not easy. So, I will just go and ask some mathematicians and when I go to mathematicians they ask these to be expressed in a particular form they want to group things together as 2ndorder terms as the main terms and remaining things are coefficients to that 2ndorder terms that is the way they want it. So, if I look at these two terms they are exactly the same. I can just put two times this these two are exactly the same terms. So, now, I will go on rewrite the expression.

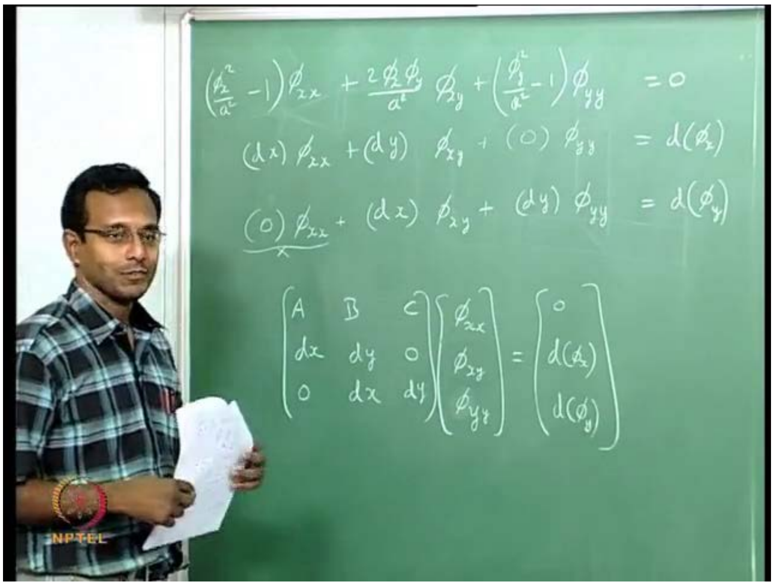
So, that it looks like something mathematicians will like phi x x 1 minus phi x square by a square plus phi x y minus 2 phi x phi y by a square plus phi y y 1 minus phi y square by a square this is equal to 0. This is some form which mathematicians like if I look at this they will directly tell it is a 2ndorder equation in nonlinear coupling great there just look at this and say that. They know how to solve this equation all we need to do is go and ask the mathematicians. How do I solve this equation?

Before we solve the equation we have to tell them some more things which we take for granted we have to tell the mathematicians very clearly. So, we will write two more terms two more expressions d u is defined as dou u by dou x dx plus dou u by dou y dy. We are basically giving the relation between du dx and dy that is the idea here, I am giving you a relation between this things which is probably its very silly for us in engineering world mathematicians want to define these things very clearly because, there can be some nonlinear coupling between this and this which we will ignore typically I can have other terms here tell a series expansion kind of thing.

I can have a dou u by dou x whole square or dou square u by dou x square kind of terms coming we will ignore all that right now. Similarly dv is given as dv by dx dx plus dou v by dou y dy these are the relation between dv and dx and dy. So, basically I am giving the connection between the velocity field and the spatial field similarly here v velocity field and the spatial field. And we have some other relation which is connecting phi and this thing I will rewrite these expressions in terms of phi because mathematicians weare going to show only phi to them. So, write that then it is d of phi x d of phi x is equal to phi xx vx plus phi xy dy and d of phi y equal to phi xy dx plus phi yy dy.

So, when we go to mathematicians we take these three equations and then we will tell these are the expressions I have this is a system of a linear equations the partial differential equations that are nonlinear why is it nonlinear? Because, I have phi terms here and here and it is already a phi square term there it is multiplying those things.

(Refer Slide Time: 32:28)



So, it is not going to be just phi derivative multiplied by phi derivative square that is in nonlinear kind of term that is why you are having it. So, weare going to take this three equations and go to mathematicians and when we take it to mathematicians they will rearrange this expression in a slightly nicer form. I will just show it to you. This is a same expression as before I just multiplied the whole equation by minus 1 the next expression they will write will be we will leave that gap for a reason you will see that immediately.

If, I think about this gap regions it is actually plus 0 it is equivalent to that 0 times phi xx that is what this is it is actually 0. So, it is really not a term alright similarly here we can add 0 times phi yy. So, it looks like it is actually a matrix multiplied by a vector that is the idea that is what mathematicians like. So, this can be decoupled into a form where it looks like I have A B C dx dy 0. 0 dx dy this matrix multiplied by phi xx phi xy phi yy equal to 0 d of phi x d of phi y.

This is what it is if I multiply this with this I will get to that form that is what we have where Acorresponds to this coefficient here and B corresponds to that coefficient C corresponds to that coefficient just remember that. So, if I go and show this I have to actually go to linear algebra people in mathematics and.

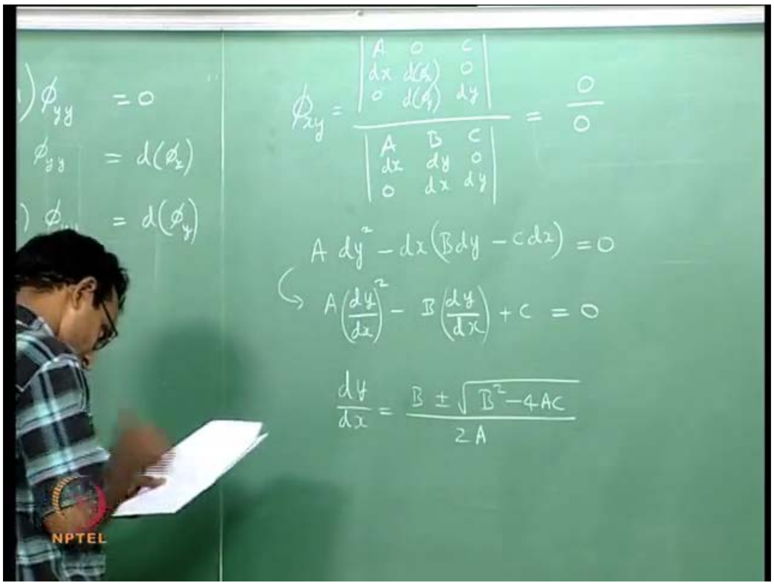
They will tell me that directly I can solve this problem with Kramer's a rule. If we look at this can be solved for phi xx phi xy phi yy from Kramer's rule that is what they do is they will take this column substitute say if I want to solve for phi xx they will take this column substitute with the 1st column find the determinant of that matrix divided by determinant of the original this matrix that will be the value for phi xx.

Similarly, if I want phi xy I will take this substitute the 2nd column and then do the determinant that will be in the numerator divided by this original matrix determinant in the denominator that will be my phi xy like that you can solve for all these three variables this is our Kramer's rule.

Now, mathematicians turn around and say if this is a differential equation of this form it may. So, happen that I can decoupled this coupled nonlinear equation into an ode in each of these dimensions x and y separately or not really ode is an equations in directions x and y but, some other directions special directions that, can happen if my when I solve for phi xy if my solutions becomes indeterminate that is it is of the form 0 by 0 it should become of the forms 0 by 0 if it happens then I can say that yes I am having a situation where I can decoupled expression.

So, we will look for that for now and then we will see if it is working before erase this there is one condition in this which I did not use still now I used 2D I used Steady I used irrotational I used Isentropic I did not used m greater than will come back to it after sometime. And of course, remember I did not use energy equation that is also waiting will come back to it when it needed by the way already told you something extra it says energy equation is not needed will get back to you later.

(Refer Slide Time: 37:40)



So, I am looking for phi xy solution which is A dx 0. 0 d of phi x d of phi y c 0 dy divided by that is a determinant symbol divided by A B C dx dy 0 0 dx dy. Now, I have to show that it may. So, happen that it is 0 by 0 form if that happens then my equations can be decoupled mathematicians told this. So, let us go and try this if this happens actually before that I will get carried away by that.

So, we will go back one step in here if I go and say that my A tends to infinity in this expression if my A tends to infinity in this expression what will I have this term goes to 0 this part of this term goes to 0 this part goes to 0. So, I will have phi xx less phi yy equal to 0 which is my Laplace equation which is my incompressible flow equation on phi.

So, A tending to infinity means what I am going to incompressible that is my waves are travelling extremely fast if I make a change immediately flow at infinity knows that I made a change that is the idea. So, that is also expressed mathematically in this form automatically. If, my A is not infinity, but anything lesser I will get to this expression finally, that is the one more thing I wanted to talk about.

Now, because of we wanted to get phi xy solution to be in determinant which means I want to show that numerators are 0 denominator is a 0. Now I am going to get two separate equations for this when I take the denominator alone we will take the denominator 1st we will go to numerator after sometime. If, I take the denominator alone I will find the determinant of this which will be A times this minus 0. A dy square dx will be minus dx times determinant of this and this B dy minus C dx this must be equal to 0 we said.

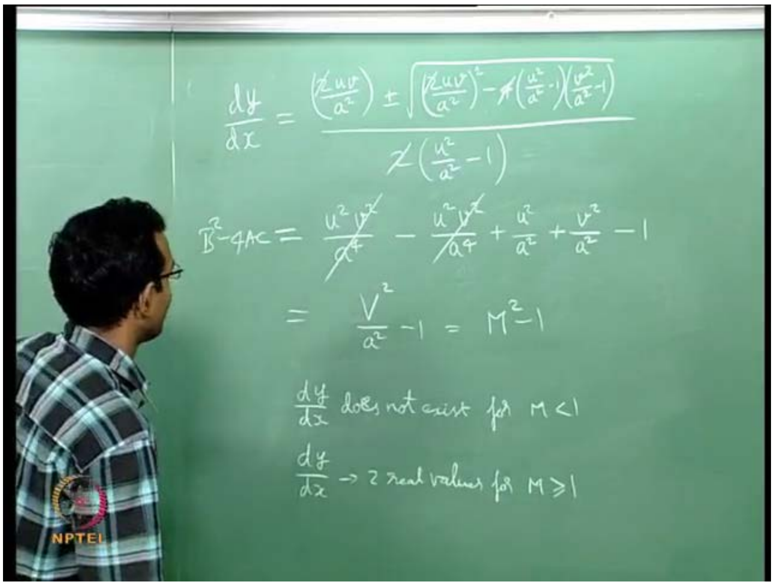
So, look for this now again I will do this divided by dx and then make it a derivative this can be rewritten as, A I am dividing by dy dx square and then taking the limit I will get this expression what is this is nothing, but a quadratic equation in dy.

By dx which means I can go to a point I will write the solution for this is easy is already having a minus sign should know that minus of B will become plus B. Now, B plus minus square root of B square remains the same minus 4 AC divided by 2A.

So, I am telling that for that phi xy to be indeterminate denominator should be 0 of course, numerator should also be 0 currently of denominator has to be 0. Then the relation between dx and dy must be this and we are finding that there are two solutions for that plus and minus two solutions are there. So, I am telling my phi xy may become indeterminate.

Only if I move along dy by dx given by this relation these are called the characteristic directions that is how you came to method of characteristics which is what we are dealing with for now. So, now I have to find these I have to substitute my original expression in terms of phi xx phi x square etcetera for B, A and C will do that.

(Refer Slide Time: 42:08)



If I put that in dy by dx equal to our B was 2 uv by a square I am rewriting it in terms of u and v where this, was actually phi x phi y by a square I putting it back in.

Terms of u and v plus minus square root of again B square which is 2 uv by a square whole square minus 4 u square by A square minus 1 into B square by a square minus 1. I am replacing all phi x by u and phi y by v I am doing that inside here. So, I am getting this particular relation divided by 2 A 2 into u square by a square minus 1.

Then, if I look at it I have 2 here 2 here 2 square inside a square root 4 here inside the square root I can cancel all of them. Now, only the remaining thing I need to think about. So, I will look at this is going to be uv by a square plus minus let us say we will look at it in a simpler way we will just look at what is inside the square bracket square root inside the square root it was B square minus 4 AC term this is equal to u square v square.

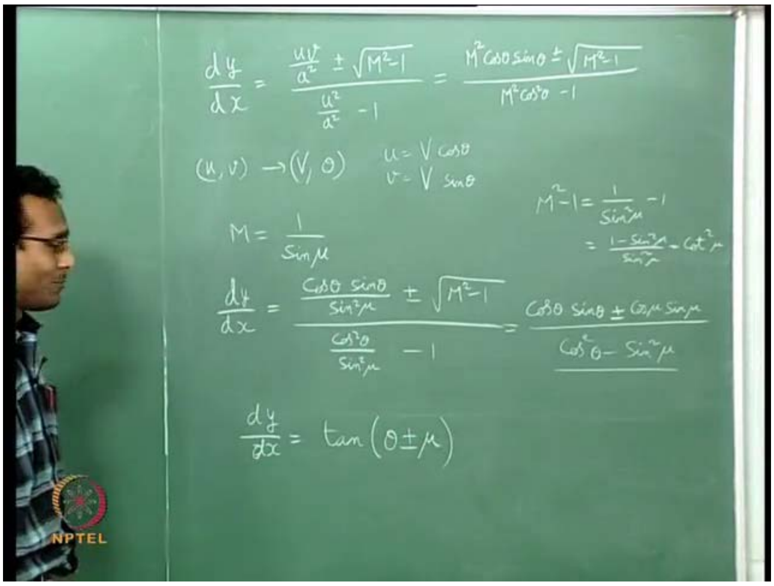
By a power 4 minus multiplied these two things which is again u square v square by a power 4 this into 1 that will be plus u square by a square and this into 1 that is again plus v square by a square and then this one into this one with the minus sign minus 1 these two get cancelled.

Now, we know it is in 2D problem u square plus v square is my total magnitude square. So, I can write this as capital V square by a square minus 1 which is our M square minus 1. I get to this form now I will go back and look at my dy by dx expression I am saying inside the square root I am going to have M square minus 1 finally, if this is M square minus 1 it can have a possibility of going negative right if number is less than 1 my square root goes negative. Which means I do not have really a real dy by dx direction it is going to be complex which means it is not real itis not true.

So, dy by dx does not exist for M less than 1, but if it is equal to 1 or greater than 1 dy by dx exists and if it is greater than 1 alone then, I have two separate values for dy by x if it is equal to 1 I have same value twice that is what I have if dy by dx two real values.

For M greater than 1 this is where I am going to use my M greater than 1 assumption which I did not used till now. I am going to say I want to solve for supersonic flow alone no subsonic flow in my flow field. Then I can go directly into this part then, I will say yes, I have this particular condition if M equal to 1 happens then that is not really bad. So, happens that the two values will be the same value. So, I can might as well add this to be M greater than equal to 1 in here. We still have not found the value for dy by dx we did only halfway we have just done what is inside the square root we have to do the remaining terms we will do the remaining also now.

(Refer Slide Time: 46.14)



So, we will go back and look at dy by dx it was uv by a square plus minus square root of M square minus 1 by u square by a square minus 1.

Now, I will say I will go back and convert uv coordinates to V theta coordinates it is magnitude of vector and direction of the vector. So, I can write u equal to V cos theta and V equal to V sign theta. I can write this.

Now, I will substitute this inside there. So, I am going to get and I know that v square by a square which will come from here will be my M square I will write that also in here. So, it will be M square cos theta sign theta plus minus square root of M square minus 1 divided by M square cos square theta minus 1.

Now, we want to look at that M square again we want to write M equal to 1 by sign of mark angle which is true right we defined sign of mark angle to be 1 by M. So, it is going to be this was I believe around the 4^{th} class or 5^{th} class of our course. Anyways we get this mach cone comes out to be this we will define it like this why we will go and see the reason later it simplifies the problem that is why we will do this now, my M square will be 1 by sign square mu.

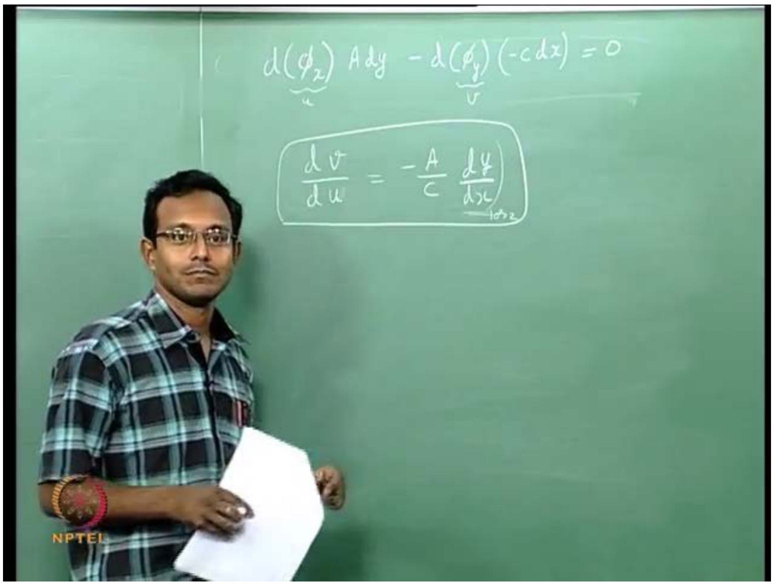
So, now I can write my dy by dx as cos theta sign theta by sign square mu plus minus square root of M square minus 1 divided by cos square theta by sign square mu minus 1 which of course, now I can rearrange this a little bit and by the way I can substitute this M equal to 1 by sign mu also in my M square here which will be M square minus 1 equal to 1 by sign square mu minus 1 which will be 1 minus sign square by sign square. Which is my cos square mu becomes cos square mu square root of that will just become cot mu and then I will multiply it with sin square mu in the numerator and denominator which will give me a final form.

Cos theta sign theta plus minus cos mu multiplied by sign square mu will be cos mu sign mu divided by the denominator also multiplied by sign square mu which will be cos square theta minus sign square mu yes it does not look any more simpler than this. So, after this we will go to mathematicians and say simplify.

This actually it is just trigonometry and algebra we can simplify it I will just skip that for this course it will probably take half a class to do that we will skip that, and we will just directly jump to. Now, you know why we wanted to substitute M in terms of mu finally, you will get to this form if we go through that ½ a class of derivation you will end up with this form dy by dx will give me two solutions tan theta tan of theta plus minus mu plus will be one solution minus will be another solution you get two solutions this is just, one part of the problem we go back to our determinant we still have to look at the other.

Part the numerator should be 0 now you have done if I give the relation between dy and dx to be that particular dy by dx equal to tan of mu plus theta or tan of theta plus mu and theta minus mu then I will get, two different directions along which my denominator is 0 now.

(Refer Slide Time: 51:12)



I have to make the numerator 0 we will look at that now and the numerator if, I look at it that gives me determinant of that is going to give me I am going along one of those directions multiplied by minus C dx equal to 0. You get to this particular expression this is the determinant of that particular thing if I rearrange this where I will again substitute this is my u this is my v. I will get to form where dv by du is equal to minus A by C dy by dx.

I get to this particular form of course; we already know from denominator determinant 0 that dy by dx can take two values. So, I have to actually think about dy by dx 1 or dy by dx 2 which means, again I am getting two relations for each of the dy by dx directions I have one equation like that I have two separate conditions between dv and d u I get that is the idea now.

Now, I am going to tell that if I get this expression and this is just a number. Now, for us right I can substitute this in terms of u and v I can substitute this in terms of u and v and simplify this which we will do next class of course. And when we do that you will find that the equation becomes ordinary differential equations which is the idea of this, we started with something very complex nonlinear coupled partial differential equation of 2nd order finally, it came down to simplified version for M greater than 1 alone not for M less than 1 and this is what we want to solve any way we want to solve supersonic flow field currently we find that this particular method if, I more along some characteristics direction things become simpler which is what we want to look at the remaining part.