Gas Dynamics
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Module - 21
Lecture - 50
Directions, Constitutive relations, Subroutines

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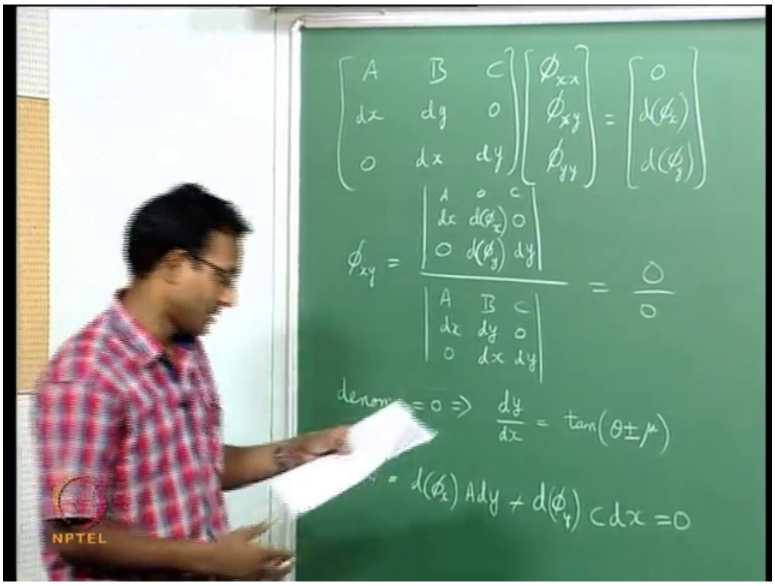

Hello every one welcome back we were somewhere in the middle of derivation of equations for method of characteristics we started with this matrix we went to mathematician they looked at this way basically they looked at the matrix and said if you have if you solve for phi x y and if it phi x y is solutions comes out to be indeterminate form then you can simplify the expression where this couple differential equation system of second order will now become independent first order differential equations, which is what we wanted to try and to solve for phi x y we use the Cramers rule and we got this from determinant of that matrix with the centre column replaced with that side and

The bottom will be just determinant of this matrix and for this to be in-determinant we should have denominator zero separately numerator zero separately and we looked at this part it should be of the form zero by zero we did this part separately already we did this and said when the denominator equal to zero this corresponds to we said characteristic directions. We said there are two directions theta plus mu and theta minus mu for which this will happen this is what was done up to last class from now on we will continue with the numerator getting to zero if you look at the numerator determinant and we want to make that zero if I

let us say I will pick the center line and do determinant based on that a determinant for numerator is equal to it is going to be d p x times A d y minus d of phi y d of phi y times this determinants which is minus c d x. So, that minus will just go away and becoming plus c d x and I am forcing it to be equal to zero and I am saying it has to be zero because I am looking at this phi x y in determinant form. So, I am getting this form now we will simplify this a little bit will rearrange it such that d phi x and d phi y terms are on one side and d y and d x terms are on the other side.

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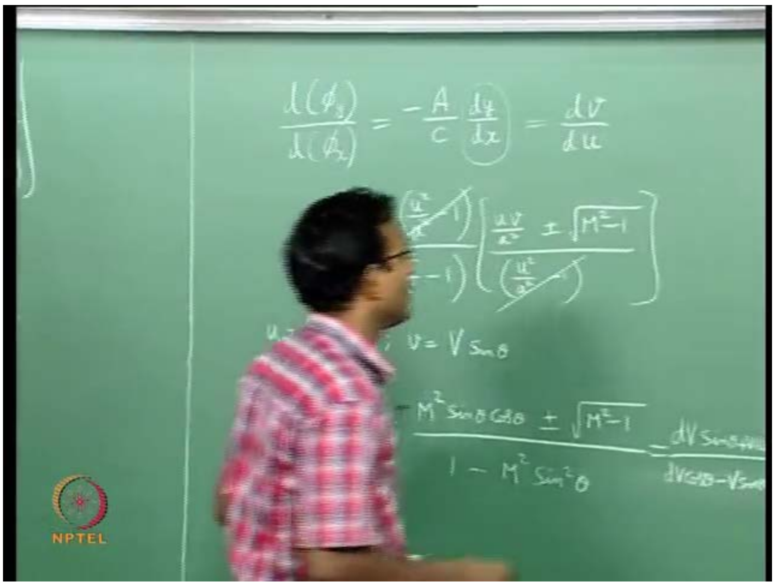

If we do that we will get to a form d phi y by d phi x is equal to minus A by c d y by d x now we remember our A by c A and c from our original differential equation we can go and substitute it back in terms of u and V, but before that I want to look at this d y by d x this d y by d x we already found that from the denominator being equal to zero I should have two solutions for this d y by d x only along that particular two directions the denominator is zero and if I satisfy this equation then the numerator is zero.

So, ideally I have to say use that d y by d x here those two values and I will get a particular relation here what is this relation really this is equal to d V by d u right because phi y is my V phi x is my u small u. So, now, the next thing we want to do is go and substitute that particular d y by d x here, but I want to not use that tan of theta plus minus mu value there I will use one step before in terms of u and V.

So, I will write d V by d u equal to this A and c I will write here these are my A and c values now inside I will write the expression we wrote for d y by d x in terms of u and V. So, that things are getting simpler that b square minus four A c terms we got it to be m square minus one we will keep it that way writing it in this form, so that I will get rid of this term. So, only thing remaining we will now again substitute u equal to capital V cost theta and V equal to capital V sin theta I will use this substitution again and we will go back and write this every V and u there like that V sin theta d of V sine theta its d V divided by d of V Cos theta this is equal to you know how it comes out that is going to be capital V square coming out from here capital V square by a square is my math number square and remaining will be Cos theta sine theta. So, I will write that whole thing together it will be m square sin theta Cos theta plus minus.

Square root of m square minus 1 divided by I have sin square m square sin square minus one and there is a minus sign here. So, I will put that inside here that will become 1 minus m square sin square theta I got it to this form now I still have this d of V sin theta now V can vary theta can vary. So, I will expand this again further and rearrange terms again how will I do this d V sine theta d v cos theta I will just put it here it will be d V times sine theta plus Cos theta V Cos t theta d theta divided by similarly I have to do.

Differentiation by parts again on this one d V Cos theta minus V sin theta d theta this is how it should be now I will cross multiply this denominator with that and this denominator with this numerator and then rearrange terms such that all d V terms are one side and all d theta terms are on the other side if we rewrite it that way.

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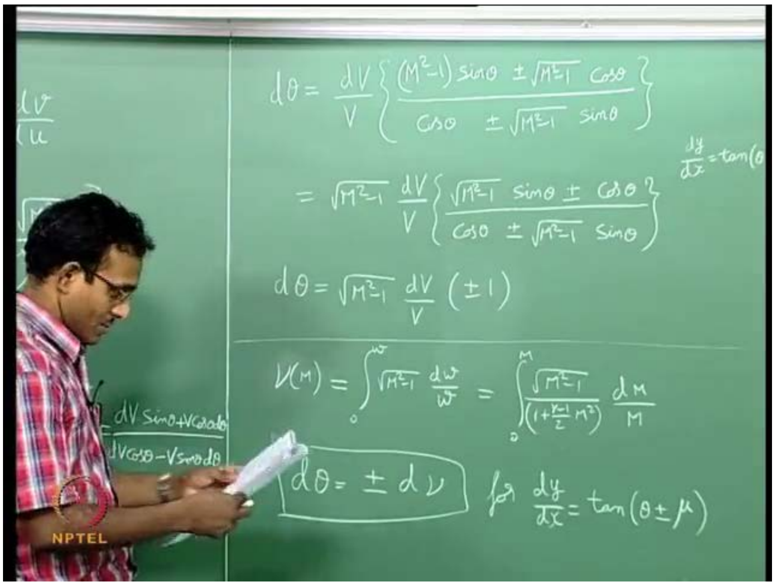

I will finally, end up with d theta equal to d V by V times m square minus 1 sine theta plus minus m square root of m square minus 1 multiplied by Cos theta square root is not for Cos theta there divided by Cos theta plus minus square root m square minus 1 sine theta this is what I will end up with.

And if I look at it I am having plus minus in the numerator and in the denominator how did this come up I am having a plus minus here and there is one term which will have d V with plus minus other term will have d theta with plus minus and remember when i put a minus d theta multiplied with this it should become minus plus, plus minus will become minus plus, but when I rearrange it to the other side of the equal to sign it will again become plus minus that is how it got to be plus minus in the numerator and denominator out there finally, now this actually means that when I have one particular solution.

d y by d x there are two solution we said that one solution was actually I will write both the solutions together we had this two solutions if it is theta plus mu d y by d x is equal to theta plus mu then I have to use the plus here and the plus here if I have theta minus mu here then I should use the minus here and minus here I cannot have four combinations of this plus minus they are coupled if there is plus there this should be plus if there is minus here this should be minus that is the only way you can use it. So, because of that what we are ending up is with two different differential equations in terms of d theta and d V.

That is what we are ending up with finally, now we will simplify this a little bit further if I look at it there is m square minus one here and square root of m square minus one here I can pull out a square root of M square minus 1 common we will do that I will get square root of M square minus 1 d V by V into square root of M square minus 1 sin theta plus minus Cos theta divided by Cos theta plus minus square root M square minus 1 sin theta this is what i end up with.

Now if I look at it numerator and denominator are almost the same except for lets split the terms now if I have a plus there and a plus here they are exactly that same terms. So, now, I can write my expression as d theta equal to square root of M square minus 1 d V by V into one solution is plus 1 right what about the other solution if I look at it if I put minus here and minus here numerator will have just a minus the square root sin theta minus Cos theta denominator will have Cos theta minus square root sin theta. So, this will give me a minus 1.

I will have a plus 1 or a minus 1 two solutions this we typically write instead of like this we will write it as multiply this with plus minus 1 where now we have to again think back d y by d x having tan of theta plus mu corresponds to this plus 1 here and d y by d x corresponding to tan of theta minus mu corresponds to this minus 1 here. So, those are specific relations we are getting for those two directions. So, now, there is one more thing I have to do this is from some class long back we did in Prandtl Meyer expansions there was a point where we wrote mu of M is equal to integral zero to w square root of M square minus 1 d w by w.

This w was our local velocity field vector value we wrote this and this in converted to math number came to our standard expression zero to M square root of M square minus 1 divided by 1 plus gamma minus 1 by 2 M square d of M by M and we integrated this to get our Prandtl Meyer function, but I want to use this expression now because this is exactly the same expression sitting here where we said w is our local velocity vector magnitude this is also the same thing capital V is a square root of u square plus V square that also local loss vector magnitude.

So, this is the expression that is sitting here which means derivative of this nu will be without this integral this whole thing alright. So, I am getting to a form where this expression which is exactly same as this expression is equal to d nu. So, now, my differential equation comes down to extremely simple differential equation d theta equal to plus minus d nu there are two differential equations here I have to write immediately for d y by d x equal to tan of theta plus minus mu. So, if I have a theta plus mu then I will use d theta equal to plus d nu if I have a theta minus mu I will write d theta equal to minus d nu this is what I will come up with.

So, what we are essentially done after this whole analysis this is end of our serious math sections what we have come up with finally, is two differential equations which are valid along some particular characteristic directions which are tan of theta plus mu and theta minus mu where along each of those directions there is one of these differential equations valid and the differential equation is extremely simple first order simple differential equation right it come brought it to a extremely simple form integral of this is very simple.

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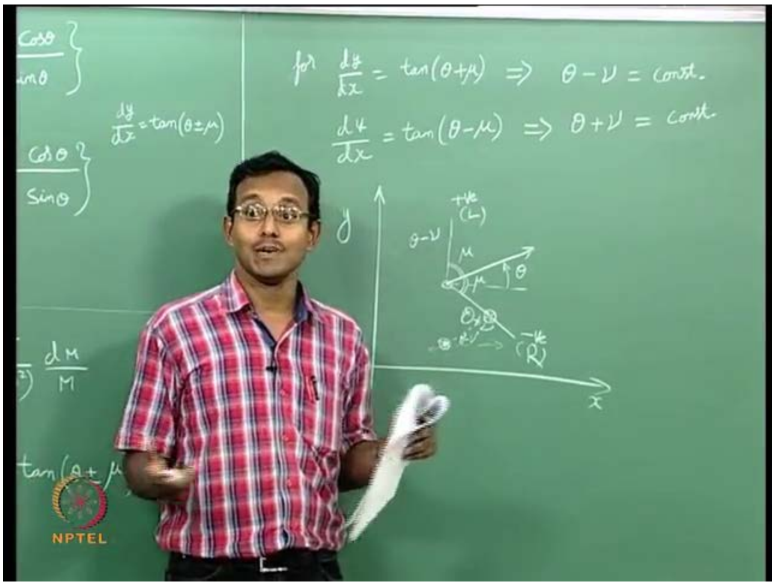


So, we will write the integral form finally, we will say for d y by d x equal to tan of theta plus mu I am going to say theta minus nu equal to constant and for d y by d x equal to tan of theta minus mu we have theta plus nu equal to constant these are the main results which we will use for two d method of characteristics whatever we have derived till now is two d steady isentropic all those I say irrational that kind of flow field and you get to two differential equations which are valid only along some specific directions this is what we will end up with finally.

Now what are its implications now I look at it from x y coordinate system we have d y d x here I have x y coordinate system and let us say I pick some point in the flow some random point x one y one and I am going to say it is having a flow velocity vector this direction which means it is having some theta value the flow velocity vector has a magnitude of capital V and the direction is theta from our reference axis x and now I am going to say there are two special directions which are having slope tan of theta minus mu and tan of theta plus mu this is theta minus mu let us say mu is different from theta that is going to go somewhere like this is one line and another line same angle the other side which will be going something like this both are mu angle with respect to the velocity vector direction and now I am going to say theta plus mu that is the top line is going to have theta minus nu constant and this one will have theta plus nu constant those are special information carried along this line.

That information does not change it just keeps going along like this similarly it is going to go along like this it is going to be this value here this is what is happening in here now we label these characteristic something typically the one with positive mu is called the positive characteristic this is called the positive characteristics and this one is called the negative characteristic that is one way of writing it, but this becomes confusing in problems which are not having all most flow parallel to the axis there will be problems where things get complicated. So, we have another convention people use both interchangeably depending on what you are discussing I can also have another case where I put myself on this fluid element sitting

Their locally and if the fluid element is moving this way to the right of the fluid element this characteristic runs. So, this is called our right running characteristic and the other one is called the left running characteristic that is another name given to it that is if I am the fluid element going like this there is a wave that is running along I did not give you the word wave yet there is a direction which goes to the right of me while I am moving straight like this and.

There is another to the left of me going this way if you are looking from the top that is the picture you will see here. So, that is what is written here this is our right running and left running characteristic. So, now, we have defined whatever has happening what all have I used here I have already told that it is m greater than or equal to one cannot be less than one if it is less than one I cannot define my d y by d x we found that square root will become complex.

So, it is not a real d y by d x there is no real direction along which I can decouple my equations, but if it is m greater than one definitely I can de couple my equations to two different equations which have this solution the integrated form of the differential equation you get to this particular form. So, what is the connection between this and our already existing understanding of gas dynamics if I think about it now I have to give this connection of math cone and this you have seen this picture before where we are saying there is a fluid element that is going along this direction there is a specific cone or a region inside which the flow will be having influence of this point and that happens to be this region exactly this region and if there is a point here this point does not

Have any influence on it or this point does not depend on this point that is what we are ending up with. So, now, math is coming up and telling you that our analysis matches with what is happening physically you been thinking physically for some time at the beginning of the course you were just telling math cone based on whatever we feel physically from acoustic wave propagation etcetera now we are going and solving it equation wise and it is telling you exactly the same thing.

So, our analysis seems to be matching in multiple ways, but if you think about it in olden days when people did not understand this very well they followed the math first then from there they got this physical understanding which I gave you at the beginning you should know that it is a cycle keeps going up and down anyways math got to got us to this point anyway thinking about it now we have to think about one more thing supersonic flow we said depends only on upstream conditions and not on downstream conditions if I think about one particular stream tube

The flow happening downstream stream depends on the upstream and not depend on downstream it may depend on the side of the stream it does not depend on downstream conditions that is all I am going to say is that shown here it is not very clear yet we will wait for one more discussion and then we will come back to this then it will become more clear.

so will think about currently whatever information is here typically I am going to think about at this point I know all flow properties how many flow properties do I really need to know for this problem ideally in gas dynamics typically it is two, but we got this thing where are those two constants two numbers that specifically talk about this point I can think about theta and nu probably at this point that is one way of looking at it typically people use theta and math number because once I get mach number I can get this mu easily of course, I have nu from there I can get math number and get nu mu also typically people store theta and mach number at every point.

That is how typically people work with in method of characteristics now if I want to find another new point say I want to find a point on this say this point I want to find how will I find the value there I need to find a theta and math number there at that new location how will I get that I know one relation between the theta and math number there what is that theta plus nu should be the same value as theta plus nu at this point, but that is not enough I need one more where am I getting the other one now you go back and think about the zone of dependence right this point depends on the upstream cone in this direction right these two lines. So, I am going to say I need one more point given on this.

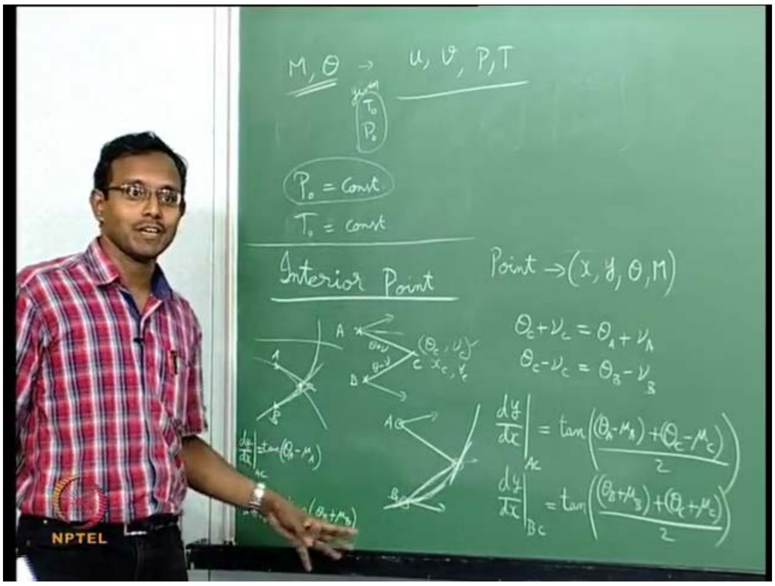
Maybe I will use a star here one more point given on this line of dependence where this is going to tell me another theta this is what characteristic typically if the velocity vector is like this and this characteristic is coming from this side it is a left running characteristic left running characteristic will have theta minus nu as a constant, but remember this theta and nu or this theta and math number need not be same as this theta and math number.

So I will put a theta prime and nu prime there. So, theta prime and nu prime and I am going to say this is going to set a condition on the difference between theta and nu on this point x while this is going to set the summation of theta and nu at this point summation and difference will now be fixing the exact value of theta and nu at that point that is how we can calculate another point on this flow field now we will go back and look at our previous statement which we were waiting for which is we said supersonic flow depends only on.

The upstream conditions have I used that here yes this point was found from two upstream points how do I know its upstream points we already assumed the velocity vectors to be like this one velocity vector like this one velocity vector like this based on these two points I got this point. So, I am thinking about upstream point calculated upstream points used to calculate the downstream point that is how I am getting to the new point if I want to find another point on this I may need another point from here sending a characteristic this way right running characteristic and that will give me another point out there that will become my downstream point. So, if you are clear with all this the next step is for me to talk to you about how to write a set of algorithms which you will need to write any method of characteristic solutions problems; however, you want to.

You want to say calculate flow around a body with method of characteristics or you want to calculate flow through a nozzle those kind of problems if you want to solve those things you need to have some set of standard routines subroutines which you need to have.

So, let us look at one by one by the way there is one more thing which I forgot to talk to you about in this whole set of differential equations we used we did not use the energy equation we I told you that time that I will talk about it later right there were two things I wanted to talk to you about long time back at the beginning of the course we said if I use isentropic relation mass momentum I do not need to use energy equation hopefully you remember that somewhere I said energy equation becomes redundant somewhere. So, somehow I have used it inside, but if I want to think about it again in our case if I want to think about to find temperature how will I find temperature if I know mach number at that point I still need to calculate some more things if I know velocity vector value and magnitude a velocity vector value and the theta or I know my m and theta I said right(Refer Slide Time: 28:23)



If I know M and theta I actually can find out from here u and V given temperature, but temperature depends on math number. So, I should be given T naught at that location for me to find based on that particular local math number what will be the u and V values can I find pressure yet from here no I still need to be given P naught at that local point. So, that I can now use this M to find the local pressure and once I know these variables I can find every other variable present. So, now, I can find density if I want entropy if I want all those can be obtained after this point.

So that is why we listed all the primary variables and then we went in a particular direction where we ignored one equation and that equation comes up finally, here how will I use how will I know P naught in my flow field we assume something about the flow we assume the flow to be isentropic and because of that we know that P naught is a constant right isentropic flow we had this derived long back delta s by r is log of p naught 2 by p naught 1 right and if delta s is 0 p naught 2 is equal to p naught 1 that is p naught is a constant this was already assuming that t naught by t naught t naught 2 by t naught 1 was 1

If that is not the case of course, you are going to have entropy rise anyway we had a full expression in terms of t naught 2 by t naught 1 and p naught 2 by p naught 1 hopefully I remember that it went something like delta s by r is equal to log of t naught 2 by t naught 1 to the power gamma minus 1 by gamma r something multiplied by p naught 2 by p naught 1 some such expression we had and if I now say my problem is isentropic

Then now I can link that t naught 2 by t naught 1 as 1 and p naught 2 by p naught 1 as 1 for every point across from one point to other always if I say that then now I am going to say t naught is constant also given these conditions I can now find every point in the flow as long as I can find theta and m at every point in the flow. So, typically method of characteristics people when they solve they will just solve for theta and M at every point in the flow and then they just do post processing of this data to get all these values if they want that is what they do typically in case there is a shock T naught does not change, but P naught may change right we know how to take care of that p naught jump, but remember

I cannot have subsonic flow to be solved by method of characteristics. So, I cannot have a normal shock in my flow and I want to still solve it with method of characteristics that is not possible, but if there is a oblique shock and the flow downstream is still supersonic I can use this as long as I take care of this P naught drop after that line I will change my P naught to be some other value and continue then I can still solve this that is the only way I can take care of this now we will go and look at individual ways of doing this the first subroutine we want to talk about is interior point sub routine.

If you are writing a program you need to write these sub functions keep it ready the interior point sub routine is basically whatever we just discussed I am given point A and point B and when I say I know points A and B I have to write here point actually means I know the x location the y location the theta at that location and the M at that location I know those four properties if I tell I know a point then only I fully know the point now from this given A and B; that means, I know a x y theta and M at each of these points now I want to find another point c.

This is the most used sub function in method of characteristic this is what fills your whole interior of the flow that is why it is a interior point this is all these points are inside in the middle of the flow not near a wall or near a jet boundary extra it is inside of the flow. So, we are given these and we want to find this point we are given x y values of this theta M values of this

So, what should I do let us say velocity vector is like this velocity vector is like this I do not know what values it will be some values arbitrarily now I am going to say I have drawn a right running characteristic or a negative characteristic from a which means I will have theta plus nu is a constant on this line and on this line it will be theta minus nu is a constant on this line positive characteristic will have theta minus nu constant i like to think about it as positive and negative it makes sense

When I want to use the opposite sign in theta plus nu versus theta minus nu if I say it is a negative characteristic it will have theta plus nu constant if it is positive characteristic then it will have theta minus nu constant that is why I prefer to use positive and negative, but it will get confusing after sometime when angles are not going to remain positive or negative there it will help if you have left running versus right running anyways now how will i solve for this simple thing I already told how to solve for this theta c and nu C those are your unknowns for this right I do not know theta c and nu c I also do not know x at C and y at C first we will look at theta C and nu C easy to look at we know theta A and nu A we know theta B and nu B. So, I can just go and write

Theta C plus nu C is equal to theta A plus nu A and from line B C I can get that theta C minus nu C equal to theta B minus nu B we know the values on right hand side we do not know the left hand side average adding these two and dividing by 2 will give me theta C first one minus the second one and dividing by 2 will give me nu C. So, I can easily get theta C and nu C. So, I have solved the flow properties at that point C now you want the exact location of the point C

Lets think about that part again I will draw fresh drawings and this velocity vector will give something else now i know that this characteristic line need not be a straight line always if I think about a bigger flow field if it is a bigger flow field depending on the flow direction locally that particular theta and mu tan of theta plus math angle that will decide my left running characteristic and at this point if the math number and theta are such that it is going to change the characteristic may still turn it may keep turning like this similarly the other characteristic may turn like this turn like this.

Whatever it may do different things if I think about it from this perspective I have one point here and I want to find this point this is the other point if I just use the slope from here and extend it I may reach somewhere else similarly if I use the slope from here i may reach here instead of this point, but with the given information how close can i get to the actual point that is what we want to think about. So, the simplest approximation if I do not worry about too much errors then I will just do the silly thing I will just assume let us say this is my A and B points right A B points I am going to assume that from here my characteristic is running as a straight line up to this point here it is a straight line.

So, I can just go ahead and write tan of theta A minus mu A is my d y by d x of A C and similarly d y by d x of B C is equal to tan of this is a positive characteristic theta B plus mu B I can write like this. This will have some errors if the flow is not very uniform if the flow is very uniform this is enough if you already know beforehand that the flow is not going to have So, much of change in characteristics then I can just go ahead and use this is enough, but if there is too much change.

Then I am putting the value that is supposed to be here that is the theta and nu values which are supposed to be here now I am putting it there it means I am already off on where the data should be written to correct for that with the data available the only thing we can do is we will take an average of this slope here on the curve as a slope here i need to draw it again I will draw it here I am taking one slope here and one slope here for the same positive characteristic and I am going to say I will take an average slope between this value and this value and put that as the slope for this line if you go do numerical methods you will find that I am approximating this curve to be a parabola at that point instead of it being a straight line which is this I am assuming it to be a parabola at this point I am interpolating in slopes which means it will become parabola that is the idea it will basically I am using the slope here at the midpoint.

As the slope of this line that is what I am doing really I am using the slope of this point here if I use this, this will give me a closer approximation it will not be very bad that is the best we can do because that is all the information we have in this. So, what we do is instead of this version of d y by d x we will write it different d y by d x of A C A C is my right running characteristic. So, its theta minus mu I will write tan of theta A minus mu A plus theta C minus mu C divided by 2 I will write it like this.

I am taking an average of the angles and then putting a tan of that do I actually know the value of theta C and mu C yes I can know it because I have already have this equation which can be independently solved of it. So, i know theta and nu. So, I can use theta and mu here easily similarly I will write d y by d x at B C tan of theta B plus mu B plus theta C plus mu C whole divided by 2 I will write it like this of course, you will see different books writing it differently some books will write theta B plus theta C by 2 plus mu B plus mu C by 2 it is the same thing anyway you can look at it whichever way it is just different groupings.

So, if I have a sub routine for an interior point I am given x y theta M for two points A and B I need to get x y theta M for C how will I do it I will start with solving for theta C and nu C immediately after that I will use these two expressions I will draw straight lines from A and B these two points.

And now I have to write another sub routine for two lines intersecting with some particular slopes and particular points solve for the point of intersection have that code also ready we have that silly subroutine you will get the x y locations of this point C. So, that is all I need to do I need to have a intersection code intersection simple subroutine and then I need to have a interior point sub routine which we will call that thing after this calculation once I have this I can calculate most of the points in my m o c in my flow field except for or few. So, we will need some more we will give some more, so what if my flow field is near a wall now I am looking at some point near a wall.

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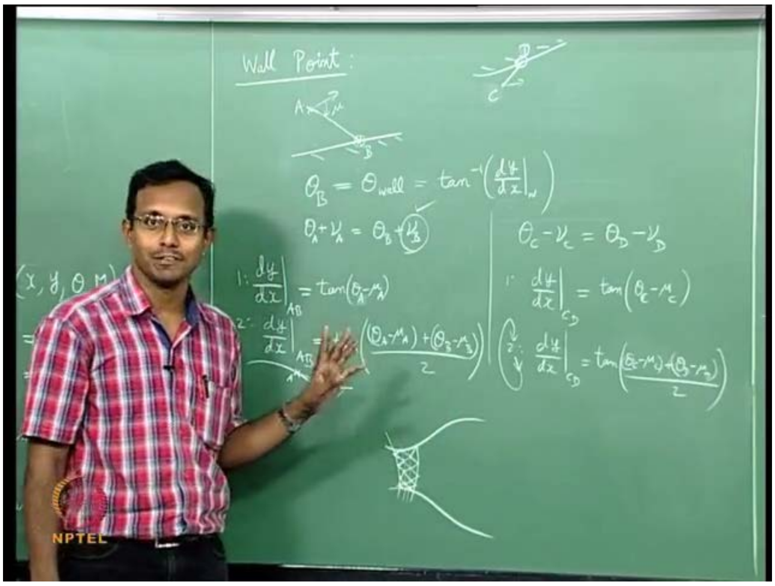

So, we will write a wall point I will tell you how to use all of them together probably next class wall point that is I am given a point a and there is a wall somewhere nearby and there is no other point here no more interior points I have this. So, flow is going like this and I am going to say there is a right running characteristic for this reaching here to this wall I need to find properties at this point B this could be one situation or an equivalent situation will be there is a wall on the top side and there is a point here A and now I need to find this point B both these situations can arise if you are thinking about flow

Through nozzle both of this can arise even flow around bodies both of these can arise in such situation we want to find how to solve this how to solve for the point B. When I say solve for the point I mean we need to find the location and theta and nu for that point theta and math number for that point. So, how will I solve it immediate thing you should know is wall enforces velocity parallel to it which means at this point B immediately theta B is known theta B is equal to theta wall what is this equal to tan inverse of d y by d x at that wall at that point, but of course, I do not know the point yet this may be a curve and the slope may change from point to point I do not know the exact point, but I can find slope at any point

So, I can find theta at any point the next thing you have to think about if I know this if it is a right running characteristic theta plus nu is a constant this is one case or i could have the other case two cases right the other case could be parallely I will write for the other case here probably I will just to be very clear I will use C and C here whether they are two separate cases I can also write parallely theta C minus nu C equal to theta D minus nu D I can write this also right if it is a right turning characteristic or a negative characteristic then theta plus nu is a constant if it is a positive characteristic then theta minus nu is a constant depending on what hits the wall your sub routine should adjust that slightly. So, your sub routine can have two parts or you can write two separate sub routines wall for negative characteristic

Wall for positive characteristic something like that. So, now, what is unknown here only unknown is this. So, I can find it directly everything else is given assuming you already know the location on the wall it is not really known now we will go back and look at how to find this point we can think about simple iterative procedure which is I am going to think about from their first I will just send a straight line with d y by d x of A B is equal to tan of theta minus mu here theta B theta A minus mu A I can just send this is I am taking into account only theta and mu from this point not an average between these two points I will just take the first one alone and from there I will just go and write this expression here that is one way of doing it

If I do this if the wall is curved a lot I will have a lot of problems I will give you an example case my wall is curved like this and my characteristic is curved like this and currently my point a is here if I have such a situation if I just use the slope from here I am going to meet this point which is actually having a some other slope here. So, this will have more errors. So, what will I do first I will do this for the next iteration I will just go through this iteration till it converges to some reasonable accuracy what I will do is first iteration I do this second iteration this is first iteration second iteration I am going to write d y by d x of A B is equal to tan of theta A minus mu A plus theta B minus mu B divided by 2 I am taking an average now and I will keep on doing this step two till I am getting d y by d x exactly the single value how will I get to all these points I am assuming a lot of things probably in this routine the way they solve it first I have to go and use this is the first thing i need to do in my routine step one I will use this d y by d x.

Now, I need to have a routine which gives me wall curvature versus this line intersection function of the wall and function of this line intersecting point i need to find I will find the point where this line intersects the wall I need to have a routine for that after I get that once I know that point exactly I can now tell d y d x at that point from there I get theta B I will put that in here I get nu B now I know theta and nu or theta and math number at location B once I know that I can go and use it here theta B and math number will give me math angle theta B and mu B I can use once I know this I can get a new value for d y by d x previous value for d y by d x is

Now, discarded we will get a new value go through the same thing I will get another nu value this time it will be closer to the previous one I keep on doing this till it converge automatically will come to some value within some accuracy will stop let us say if it does not change more than one percent I will stop something like that will keep on doing this similar thing for that it will be all this theta minus mu will become plus mu there that is the only change I will write step one d y by d x for c d is equal to tan of theta C minus mu C and for step two d y by d x of C D equal to tan of theta C minus mu C plus theta D minus mu D all divided by 2 an average of those two and this one keeps cycling across we keep doing this several times step two is just done till you converge to slope does not change more than one percent kind of condition.

And finally, that gives out your point B or point D depending on whether you are having right running characteristic hitting the wall or left running characteristic hitting the wall and this gives you the next point now the other one's are this with this most of the problems can be solved if you are thinking about simple flow problems like say a flow through a doubt I still did not talk about one particular thing somehow you know some array of points which I have to talk about next class if I know a lot of points here now all I have to do is take this point the top most point find the wall point C D based wall point from here I will send A B here based wall point in here all the other characteristics interior point.

So, I have a basically advanced my data by one step forward that will form my new line here from here this last point is already on the wall. So, there is no more line going to the wall from there it is only interior points that forms one more level of data and like this I can keep on going the next time it will go to the wall here also it goes to the wall now I have data for one more layer and like this I can do this for ever. So, I can solve flow through a nozzle using this itself except for you need to know this very first line of data somehow it has to be given to you we will talk more about this next class