Introduction to Computational Fluid Dynamics
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Lecture – 32
Accelerating Convergence - Preconditioning, Dual Time Stepping

Okay, so what we will do is; in the last class we saw a demo of the one dimensional Euler equations, right and we were looking at various behaviour, there are certain things about the demo that I just want to recollect here, one we already know that there are 3 different propagation speeds right, so in the demo I took a constant time step everywhere, it did not look like right, I took delta t/ delta x, I kept it a constant which basically means that I was taking delta t to be a constant.

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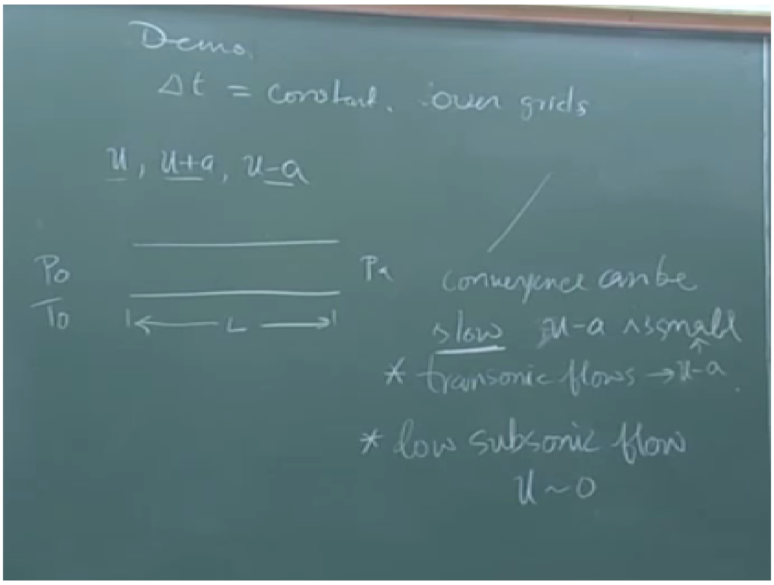

In the demo, I took delta t equals constant, so in the sense when I say delta t equals constant, I mean delta t equals constant, it should say delta t is in a sense homogenous that is delta t is a constant, this may not make sense to you, delta t is a constant across grids, right just all over grids. The other thing that we saw was that these waves were propagating, we saw that there were compression waves not necessarily corresponding to u + a or propagating at u + a.

The compression waves possibly corresponding to u + a propagating at higher speeds one way and rarefaction waves coming in the opposite direction, right which were taking more time because they are propagating something like u - a so, because the characteristics are u, u + a and u – a, those the characteristics are u, u + a, u – a, it is very clear that as we get close to convergence, one of these may become very small, right.

So, just to recollect what we did in the demo, we had a simple pipe and we had a pressure here P0 and initially because the flow and the pipe which is of some length L and you have K ambient on this side, I will indicate T0 here because we have the boundary condition but I am really interested in the pressure coming to equilibrium in this case; in the demo, so if you recollect the demo you had a compression wave that propagated left to right.

Of course, if you have a discontinuity you know that you have to take the left conditions and right conditions and right, to figure out the speed, there is a specific way by which we can do it but we will just say that it corresponds to u + a, right and those contact surface that was essentially traveling at the speed of u and when this T0 was communicated from this end to this end because it is a compression wave bordering on a shock, it was traveling quite fast.

It propagated through this and when the P ambient condition which were in enforcing here caused an expansion fan to travel upstream, you understand what I am saying, so there is an expansion fan traveling upstream which is propagating at u - a and as u gets larger and larger u - a is going to get smaller and smaller, so it is going to take more and more time to propagate upstream.

Now, it is clear that when you get convergence, if you are looking for a steady state solution these static pressure here and the static pressure there have to come to terms in a sense, they have to come from some kind of an equilibrium in this; the reason why I took a constant area duct is that they have to be the same, the P0; P in fact will be a constant throughout that is the solution.

So, you are going to see this reflection back and forth constantly, right and if the back part is going to take more and more time as time progresses, convergence will take more time, so one of the observations that I have is; I have taken delta T constant and convergence can be slow; convergence can be slow, so if u - a is small, this happens when near transonic flows, right, so near transonic flows, u - a is small.

The other is very low subsonic flows; low subsonic flow, so that transonic flows; low subsonic flows, u is approximately 0, u is very small, so any time these propagation speeds are small, we are going to have difficulty, right and I ended the conversation in the last class by just mentioning the idea of stiff differential equations, right, I mean you use the word stiff, which is a structures kind of a term.

So, you apply, you imagine that if you have a stiff beam, you apply an extremely large force and you get a small deflection, it is very stiff right, so there is equilibrium involving very large entities and very small entities that is the idea, right so it is very stiff and those problems like that can sometimes be very difficult or very often can be very difficult to solve and in this case, I am pointing out only a very simple scenario.

Sometimes, what happens is; as in the case of u approximately 0, you could even get spurious answers, you can even converge to answers that are not the right answer, right based on what is the algorithm that you choose okay. So, there is an issue of slow convergence; is an issue of slow convergence, there are 2 things that I want to take from this demo; take forward from this demo, one is how to speed up convergence, right.

The last time we talked about speeding up convergence was when we are talking about SOR okay that was the last time we talked about speeding up converging, the other we will look at now; we look at some mechanism, we have actually looked at what do you call it okay, this fast and slow we will talk about it, we actually looked at other schemes where we tried to speed up convergence.

The other thing that I want to look at is getting actually, a time accurate solution, so far we have been talking about steady state solutions, is it possible that I can actually calculate the transient okay, right, given that we kept this delta t constant, so we were sort of evolving an idea Runge Kutta method, which is a higher order scheme and time right, so we will see; we will say; we will try to say something about getting time accurate schemes.

Something about what I mean by slow convergence, so it is not enough see, you can do the math, you can look at algorithms, you can say oh, this is an order and squared algorithm meaning that if you have something like a matrix which is of size and right, that the time that it is going to take to compute is going to be n squared, it is possibility, right, you have all of these order of magnitude calculations.

You have; you can be given any number of guarantees by the algorithm, by the programming language, by the machine that you have people can make all sorts of claims but finally what counts to you is what I would call the wall clock time. I sit down, it is 9 o'clock, I start running my program when do I get my answer back, you understand. We can stand here waving hands, talking about floating point; number of floating point operations, gigaflops, mega flops all of this kind.

So, it does not help, those numbers do not help, what really matters are there is a clock, you have a question, you start your program running, when will you get your answer back, okay and if it is going to take a long time, if it is going to take a day that may be considered slow, if it is going to be taken a minute that would be considered very fast, right, it depends on what your application, depends on the size of your problem.

For a given problem right, obviously something that takes few days is slower than something that takes a day; wall clock time, am I making sense okay, so in that context when we did approximate factorization, we are looking at an acceleration scheme. If you are looking at something that takes less effort and you get the answer back quickly, we were actually looking at when we did approximate factorization for implicit schemes.

We are trying to look at ways by which we could make our program run faster from the point of view of wall clock time, when I say faster, I only mean from the point of view of wall clock time, so you have looked at 2; SOR and some approximate factorization schemes and so on. So, we will look at another; we will look at another class of schemes, we will look at one now and the little later I will introduce other ideas to talk about how to accelerate convergence and so on okay.

So, we will try to do this first, I will set this up and then I will just say something about unsteady flows, fine. Since, I have talked about approximate factorization, we recollect that basically, we had some term if you recollect if we had some term going to 0, we said what is the big deal, what happens to the coefficient, we are going to use the same idea here. What did we say we have very disparate wave numbers, we have very disparate wave speeds, I am sorry.

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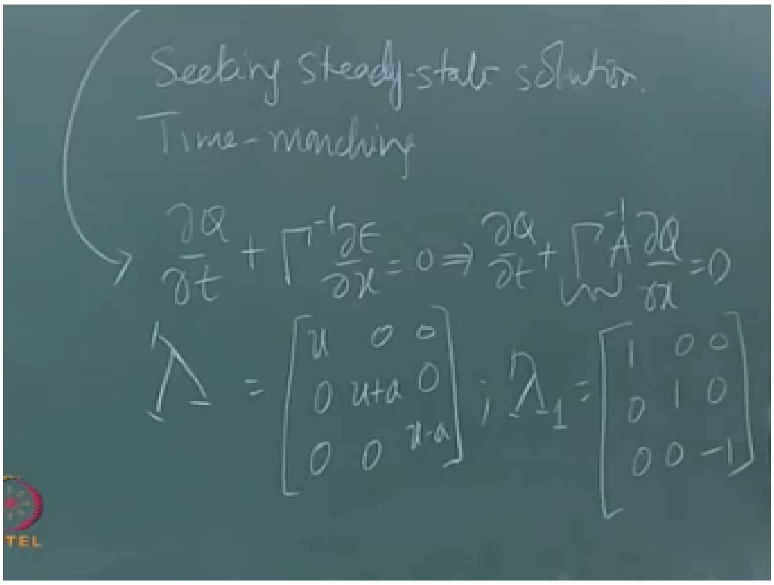

We have very disparate wave speeds, so if you have dou Q dot t + dou E dou x equals 0, one dimensional flow, you have propagation speeds that correspond to u, u + a, u – a, where a is the acoustic speed okay, if we are looking only for the steady state right, only for the steady state solution seeking steady state solution, if you are seeking only steady state solution, what is going to happen? Dou Q dou t will go to 0, okay.

When I say I am seeking steady state solution that is I am proposing to use a time matching scheme; I am proposing to use a time matching scheme to match these equations in time to steady state that is the objective and if I were to match these equations to steady state, dou Q dou t would go to 0, okay that is the opportunity for us. The minute we see dou Q dou t will go to 0 that gives us the opportunity.

So, I say using literature; the notation used in literature out there, why not multiply by a; pre multiply that term by a matrix gamma, okay pre multiply that I say pre multiply this term right by matrix gamma because I know that this quantity is going to go to 0, right so how does it matter, I just pre multiplied by gamma, fine. So, as a consequence, this equation having done this, so I propose now to solve this instead the original Euler equations.

If I pre multiply this by gamma inverse, what do I get? I get dou Q dou t + gamma inverse dou E dou x equal 0, this tells me that you have dou Q dou t + gamma inverse A dou Q dou x equals 0, fine and what we want to do is; we want to choose the gamma, so that these eigenvalues are nice, we want to choose the gamma, so that the eigenvalues of that matrix are nice, is that fine okay.

I want to choose the gamma, so that the eigenvalues of this matrix are nice and keep life easy but the eigenvectors are the same. Eigen vectors are same as that of A, fine, is that okay. So, what I am saying is; so what would be nice Eigen values, they are all says the same order 1, okay so the nice eigenvalues would be; the eigenvalues that we have right now are u, 0, 0, 0, u + a, 0, this is the matrix.

This is the diagonal matrix that we get, what I propose is that you; we build something that is I will put a 1 there, which is they are all the same magnitude, am I making sense, I propose that we take something like this, fine and having the same eigenvector.

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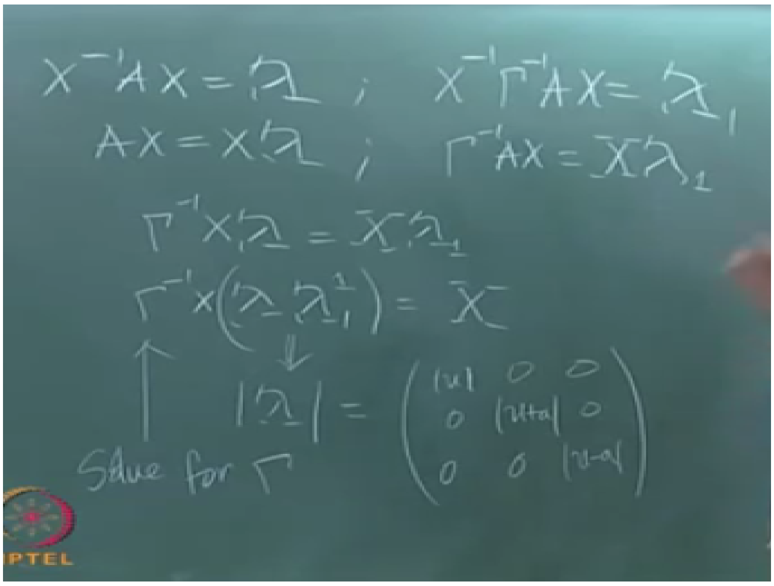

So, what I am saying is that X inverse AX equals lambda, what we normally write is AX equals X lambda that is what you normally write or what is the other possibility that I suggested now, X inverse gamma inverse AX equals lambda 1, okay or gamma inverse AX equals X lambda 1, is that fine, okay. What do I get now? Maybe I can substitute from the AX here, so I get gamma inverse AX X lambda equals X lambda 1.

Yeah, maybe I can multiply through by lambda 1 inverse; gamma inverse X lambda lambda 1 inverse, what is this product; equals X. So, in our case right now, this looks like mod lambda, this looks like that you can just check this out which is mod u, 0, 0, 0, mod u + a, 0, 0, 0, mod u

+ a, so is it possible for us to find gamma from here, how do we find gamma from here? Excuse me, so this may not be the greatest way to find gamma.

There are different ways by which you can do it, I just wanted to give you a clue, so you can work this through, it is actually possible for us to solve for gamma now and once you have solved for gamma, you come back you can solve that equation, am I making sense in the propagation speeds will be almost equal okay, so there is something that you can try out, so it is very easy that you; this is as I said this is a very naive method.

This has been tried now for almost 25 years, people have been fiddling around trying to figure out various values of gamma that will give you, right extremely good convergence, this process by which in any stiff equation, whether it is a linear system as in algebraic equations, right or differential equations, any stiff equations where you have very disparate eigenvalues and you do something.

Typically, you pre multiply by some matrix or pre multiply by some operator right, this process where you do that, so that the eigenvalues become nice, it is called preconditioning, it is a big idea, so numerical analysis we use it quite a bit, it is called preconditioning, right.

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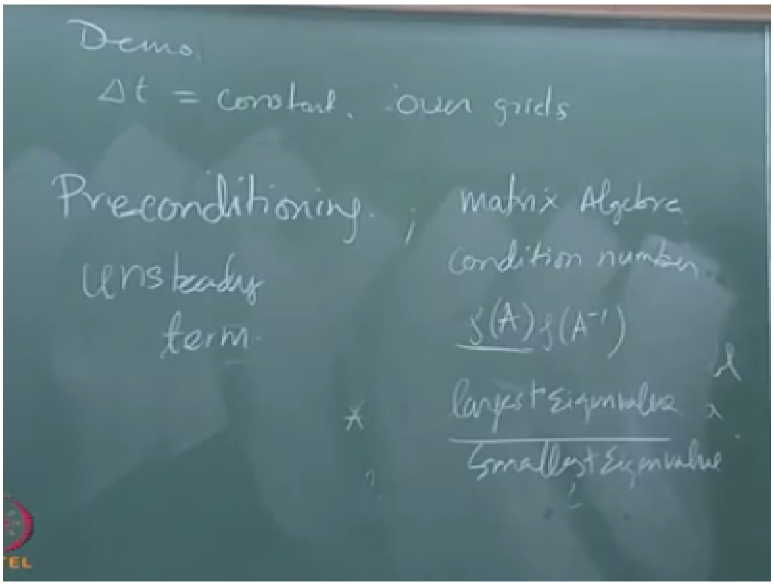

It comes from, in matrix algebra I do not know if you are aware of this, the ratio of the largest eigenvalue to the smallest eigenvalue is called the condition number, just to give you an idea as to where it comes from, in matrix algebra condition number is; I will just say rho A/ rho A inverse given that A is invertible, okay, so it gives you the ratio of rho A times rho A inverse, anyway, rho A times rho A inverse; A inverse will have rho A times rho A inverse, is that right.

So, I am going to make it up real time I have to be careful okay, fine, given that A is invertible okay, largest Eigen value/ smallest Eigen value okay, is that fine and if the Eigen values are very disparate, this ratio will be very large, it is called the condition number, fine and we will see, we will keep on coming back to this, this is a big headache, we will keep on coming back to this, a lot of the acceleration schemes that the other set that I am going to look at will deal with the same idea, okay.

We will deal with the same ideas, in this case because we are multiplying the unsteady term it is called preconditioning the unsteady term okay, so in this case we are preconditioning the unsteady okay, is that fine, right now what I will do is; as I said there is another class of acceleration schemes that we are going to talk about, I will get to that later, since I am right now talking about fiddling around with the unsteady term.

And simultaneously, I said look I am going to tell you how to do time accurate calculations, we went to the effort of taking a constant delta t everywhere, I want to say something about the consequences of this okay, is that right okay. So, how do we calculate; how do we get time accurate computations, what are the issues involved, I am not going to spend a lot of time like as I mentioned, there are lot of these topics that I am just going to give you enough that you have a flavour for what the topic is about, right.

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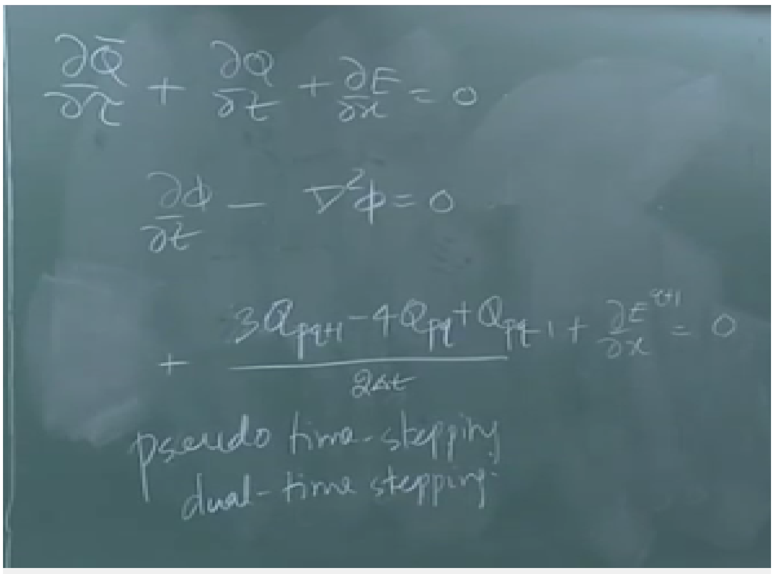

I am not going to spend a lot, there can be a whole course on unsteady aerodynamics and then consequently a computation of unsteady flows okay, so but what we do here is; if you have this equation, we have already seen there are 2 issues that are involved, so it is not enough that I use Runge Kutta method right, to integrate in time, it is not enough that I just use Runge Kutta method to integrate in time.

Sure, that gives me a higher order accuracy right, the truncation error is much smaller, the order of the truncation error is smaller, convergence is better all of that is nice but if you still have dissipation and dispersion, you still have dissipation and dispersion, what you may be; what the accuracy with which you may be calculating something that thing may not; may itself not be that accurate.

The other sources of errors that you have to be careful right, so when you say unsteady; computing unsteady flows, I want you to realize first that what I am giving you, this is a disclaimer basically and saying I am only giving you a tiny introduction meaning that there are a lot of serious issues involved that which you have to struggle if you want to get into doing actual unsteady flow.

It is not just that oh, I have a high resolution scheme, I am going to take; I am going to resolve timescales, I am going to resolve length scales, I am going to take right, Runge Kutta in time and I am going to take a 4 points centred or 4 point upwind or whatever in space and I am going to get great answers right, so you have to there is an issue here, right that you have to worry about dispersion, dissipation, there are other issues that you have to worry about that.

You have to pay attention to it, okay given that let me just go ahead in this naive fashion saying that yes, I used Runge Kutta in time that is one possibility, one way to do it, you can use Runge Kutta on time, right but then we have all this machinery that we have just developed right now, right whether I look at approximate factorization, whether I look at you know, there is a whole host of other schemes that I will talk about.

Or preconditioning the unsteady term, there are other machinery that we have done that the time that we are put into developing time matching schemes to get steady state solutions, am I making sense, we have spent efforts so far developing schemes, so that we can get steady state solutions to time matching schemes fine, see I am going to; I am obviously, you can see by the way and start going around here, I am going to do something a little fishy right.

So, I am trying to set you up for doing something fishy, the other observation that we made when we solve heat equation or if you want to solve Laplace equation is; if you have nabla squared phi equals 0, right you could either solve this using sweeping in space or you could actually add a term and look for a steady state solution matching in time okay, so you could either sweep in space or match in time, fine.

So, since we did this and you could add any term; unsteady term here, the question is why do not I add an unsteady term to this, why do not I just add an unsteady term to this right, they say why would you want to do it? Well, if I want to choose, if I; what was the time step that I took in the demo yesterday, what was the largest time step that I took in the demo yesterday, do you remember? 5 microseconds, you know what I am saying that is it.

I want to take larger time steps, I want to take something in milliseconds there, right I want the spatial resolution but I want to take time steps in milliseconds not in microseconds, so then I would have to go to an implicit scheme. So, if I went to an implicit scheme, I have to struggle and solve a system of equations, if I solve this using an implicit scheme right, I have if I did an approximate factorization, then I have lost the accuracy.

So, if I want to use, so if I want to use for example backward space right, so if I wanted to use backward time, so I could use; so, let me see how does it go; 3Q pq + 1 – 4Q pq; p is in space, q is in time, + Q pq – 1/ 2delta t, right. I could go to a higher order accuracy this way, am I making sense and do an implicit scheme plus dou E/ dou x at q + 1 equals 0, right, I have discretized, I have shown only this being discretized.

Am I making sense and I can do the same thing, chain rule I can have a flux Jacobian here, I can write a Delta form and so on, the only difference is that now I have higher order accuracy and in order to retain this higher order accuracy but still take that large time step, I will actually have to the system of equations okay. I actually have to solve the system of equations, am I making sense.

In this case because it is one dimensional flow, fortunately I still get a tridiagonal system, it is not that bad but the minute you go to 2 dimensions or 3 dimensions it becomes expensive, it is one thing, in the demo I took 1000 or 100 points, right so if you at it as a block matrix that is a 1000/ 1000, 999/ 999 but 1000/ 1000 block it is a 1000/ 1000 matrix or if you want to look at it component wise, that is a 3000/3000 matrix.

And I am only solving a one dimensional flow, am I making sense, it is an expensive process, so we have already said how matching is the same as sweeping, so we say why do not I add an extra time, why do not I add an unsteady term right, so the confusion comes the sort of physical look, everybody's face comes saying wait a minute, there is already a time, so I add one more time, I create a pseudo time, okay.

So, I add dou Q dou tau + that so, all I have to do is; I have to discretize this dou Q dou tau, in fact as I said this need not even be Q it just has to be something that depends on Q, so I will just say Q bar, is that fine okay. Now, if you are willing to squint a little and ignore the fact that this is time and treat this tau; this pseudo time okay, so terms that you will see is pseudo time or you sort of admit that there are 2 times or they are called dual times, right.

This is called either dual time stepping or pseudo time stepping, if you are going to go out and look at; look to see what are; if you are going to go out and search right, these are 2 possible search terms that you would use, either a pseudo time stepping or dual time step. Now, what I propose is; I have this equation as I said we keep squinting at this and treat that as though, it is not time, I am going to match, I am going to use a time matching scheme and tau.

And I am going to converge, go to convergence and tau and when I go to reach convergence in tau, dou Q dou tau will be 0 and I would have solved the resulting system of equations that is the plan okay, is that fine. So, I would have; I am going to match; I am going to do time matching in tau, I will use a higher order accuracy representation for dou Q dou t for the real time derivative.

This will evolve; it will evolve in the pseudo time and when it reaches a steady state in pseudo time, this will go away and I would have effectively ended up solving the equation that I want that I set out to solve, is that okay and because I have a new coordinate, so what is the price that I paid? I have a new coordinate originally, one spatial dimension, so we call it 1D but actually it is 2 dimensional because I have x and t, now it is become 3 dimensional that is the price that I paid.

What is the advantage that I get? I am thinking time matching, oh, I have done this before I can do this, I can handle this okay. So, as a consequence when I discretize this, I get a Q bar pq + 1 and then I get an r + 1, am I making sense, maybe I will write this out separately, why do not I write it out here separately so that you will get. Shall I first do it with a wave equation, will you be more comfortable if I do it with wave equation first.

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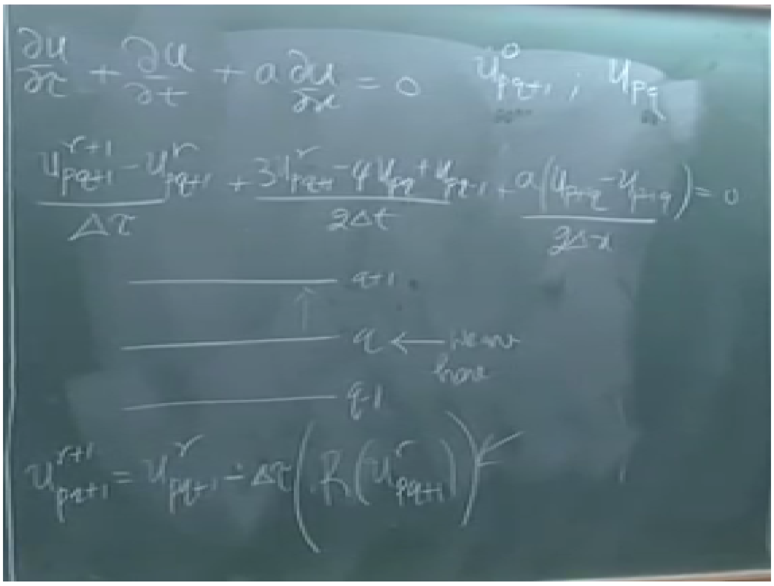

I think okay, maybe why do not I do it with the wave equation first, then you can, maybe I should have started that off okay. Dou u dou t + a dou u dou x equals 0, right, we will do it with the wave equation first and then apply it there if you want. so I am going to add a dou u dou tau, so if I were to discretize this, remember now I will have 3, right subscripts and it is a total of 3 subscripts and superscripts.

So, this becomes u pq + 1 r + 1 – u pq + 1r/ delta tau, that is a time derivative in tau plus, we have choices, I will make a particular choice and later on, I point out to you that I have made the choice, u pq + 1, so you have to decide, this is at q + 1, this is the time that we are going to do, so that is r, choices; I will make a choice r, there are 3 of them; - 4 u pq + u pq – 1/ 2 delta + a, how come they do not have a third subscript?

They do not have a third subscript, right I mean, I asked how come they do not have a third because they do not have a third subscript, there is no pseudo time, these are in real time, okay let me write it out and maybe I will explain that so, a up + 1q, we will do central differences; - up – 1q/ 2 delta x, so what we are doing is; I will write it here, q -1, 1, q + 1, we are here and we want to go to q + 1, you understand.

The tau is a coordinate that occurs between this and this everything is known below that q is known, q - 1 is known, q - 2 is known, they are all known when you converge, when r, qr becomes qr + 1, when this converges, when this goes to 0, you would have the q or u will become uq + 1, you understand, this is something that is happening in this gap okay, is that fine, these are known, these quantities are known, okay.

So, we are set, we can know this is sort of like it, this looks like very suspiciously like an explicit method, right it looks like everything at r is known, so you just take all of this; this equals 0, you take all of this to the right hand side, u pq + 1, r + 1 equals u pq + 1 r + delta tau times, the equation that you are actually trying to solve, am I making sense; plus, minus, minus; - delta times delta tau times the residue, right delta tau times the residue.

These are all known, these are all constants for all, these iterations in r, these are all constants, the only one that changes is this, is that fine, you keep on; so what would be a good first guess for this 0; what would be a first guess for that? Use some explicit scheme; use some standard explicit scheme you understand or u pq, you would be a good guess, this is a cheaper guess, right you could choose.

I will be honest, this is what we needed, right with the suggestion that everybody made is what we did the very first time that we did this, u pq would be the good guess then we got a little bolder and say hey, I can actually take whatever explicit thing that I was doing earlier and use that as the first time step right, it is an improvement, so you could use; you can use even with all this satisfying the stability condition and all of that stuff, it is still a little better.

You could use that as the initial guess, calculate the r, update the u, you keep repeating this process till it converges, okay I am not going to do it here but you can actually do the stability analysis for this just like we did for FTCS okay, there is a stability condition, we are using an explicit scheme here in r, we are doing forward time in r, you get r + 1 explicitly in terms of all the terms in r, it is an explicit scheme.

It turns out there is an associated stability condition, fine okay, are there any questions? So, how do you decide on delta tau and delta t that will actually come out of the stability condition, there is a stability condition on delta tau and delta t, see there are 2 issues, when you say how do you calculate it, it is like saying how do I decide on delta t in my computation? The delta t in your computation will depend on other parameters.

What is the accuracy you are looking for, how much you know there are; it depends on other parameter but there is a stability condition, there are constraints right, so question you should ask is what is the constraints; are there any constraints on delta tau? There are constraints on delta tau, if you ask the question, are there optimal values for delta tau, so that you get the steady state, yes there are optimal values for delta tau, right.

So, you have to figure out how to pick that every time you introduce a parameter, it is not obvious that just like SOR, we introduced an omega, so you can ask the question, how do I pick omega? Well, yeah that is an issue, if you get a good omega, it will converge fast right, it is just a parameter that we introduced, it is not part of the problem. So, one way to look at it is; look at it as a problem, it is a difficulty saying that why should I do this, why do I do this?

It is just a headache I have one more parameter to determine, the other way to look at it, it is an opportunity, you look around figure out, if you can get a delta tau or a delta tau/ delta t that ratio if you get it right, we will get very rapid convergence okay. So, our experience with this just as the initial transients; the initial first few time steps the number of delta tau time steps that you take is large so, if the order of 1000, 500, 1000 of that order.

But after that is like down to 4 or 5, okay convergence is very rapid after that it is only the initial; we have always found that initially, it takes some time for the and after that subsequent time steps, it is of the order of 4 or 5, okay convergence, so but you take some; you have to implement it try it out see what happens, fine. What is the other possibility; I said I made a decision, what is the other possibility?

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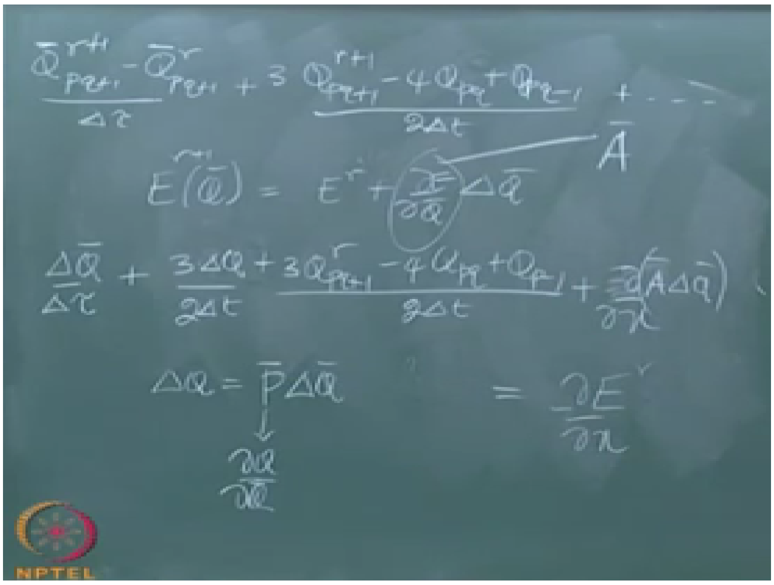

Maybe this I do using the full equations okay, what is the other possibility? So, I have Q pq + 1 r; r + 1 - Q that was Q bar, is not it? Delta tau + 3 Q pq + 1, so you could instead of choosing r, you could choose r + 1, then it sorts of looks implicit like; so it is implicit okay, - 4 Q pq r; no r + Q pq – 1/ 2 delta t. What do we do for E now? E which happens to be a function of Q is also a function of Q bar.

And if I represent E at r + 1 is E at r + dou E dou Q bar delta Q bar right, anyway I have used forward time here, I am not using such an accurate scheme there, so I am doing the same thing there okay, we have done this the only difference is that I am doing it with respect to Q bar, so this is dou E dou Q bar, so I should most probably call this A bar, right I am trying to write it in the delta form now that is basically what I am trying to do, okay.

So, this gives me delta Q; + dot, dot, dot; delta Q bar/ delta tau +; here, I want a delta Q bar, what am I going to do? I will add and subtract 3Q pq + 1r, I will add and subtract that so I will get a 3/2; 3 delta Q bar, you have to be bit careful with that maybe, I will do that a little; let me just leave that I will get that, this is Q, remember this is not Q bar, 3Q well, I can add and subtract, it does not matter.

3 delta Q + 3Q pq + 1 r - 4Q pq + Q p – 1/ 2 delta t + A bar dou Q; dou/ dou x A bar delta Q bar equals; what is on the right hand side, I take this Er to the right hand side, dou Er dou x -; is that fine, everyone okay and as we did before, we will use, we can relate delta Q to delta Q bar, right, so delta Q, we can write this, we can substitute delta Q, you can write that delta Q as some P bar times delta Q bar, right, where P bar would be dou Q dou Q bar, is that fine.

All I have done this, I am just using chain rule, I am just want to see I have some Q Bar for instance, this could be a rho UT right, these variables could be rho UT, this could be a rho, this is our standard conservative variables; rho, rho u, rho ut, right we may figure, we may find and actually it is a fact, we may find that using rho ut or rho up or something may be better for the pseudo time, okay.

So, for various reasons I will point out one possible reason for a various reason, so we choose to have this; pick this variable that is going to go to 0, the dependent variable we choose to pick it, right and if we pick it, then of course I have a delta Q, which I have to convert to a delta Q bar, I have no choice, so I do it through chain rule, I just basically perform a change of variables only for that single term.

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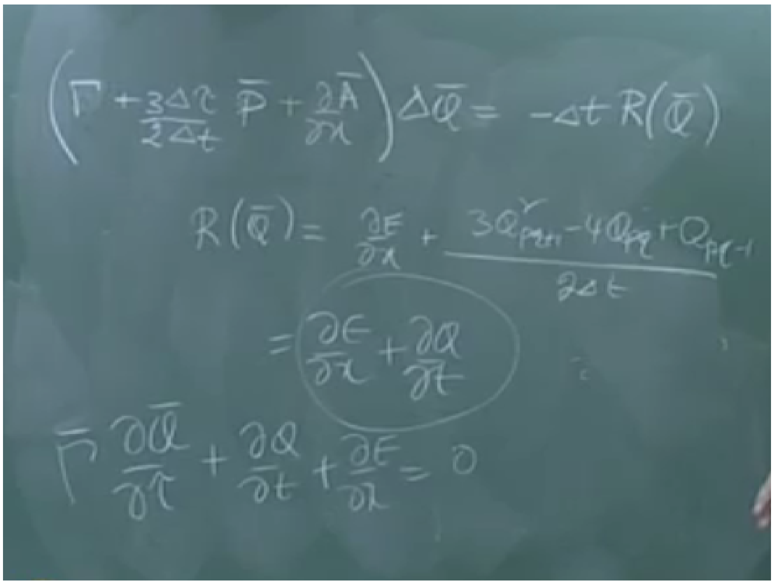

So, my delta form then turns out to be I + delta tau/ delta t that is that I specifically write it because we had a question about that multiplying P bar; 3/2 delta tau/ delta tT multiplying P bar + dou / dou x A bar acting on delta Q bar equals - delta * r of Q bar and what is r of Q bar? The residue; dou E dou x +; however, you discretize this plus that term 3Q pq + 1 or - 4Q pq + Q pq – 1, am I making sense?

In fact, r; as far as I am concerned I do not write it this way, I write it as dou E dou x + dou Q dou t that is the residue, so I look at this equation, it looks like a familiar equation, I have it in delta form, the residue decides when I am done and when my solvers done and when the residue is 0, I am solving this equation with dou Q dou t represented second order accurate or if you want to do something fancy, you know maybe you can step in and do it.

You understand what I am saying, is that fine okay, so now the little twist that we throw in; the little twist that we throw in is; we say wait a minute, we just looked at an acceleration scheme, pre conditioning the unsteady term, so you can precondition the pseudo unsteady term, so you basically take this and you can multiply that by a gamma or a gamma bar if you want, since we are saying Q bar fine, right.

This is as I said, this is a sampling but this gets you very quickly to where we are, so the kinds of things that one can do, you can add a pseudo term; the pseudo term is going to go to 0, when you are; you can pre multiply it by some gamma, if you pre multiply it by some gamma there will be a gamma that shows up there that is basically what happens, if you pre multiply this by gamma what will happen is; this will become gamma that is the only thing that will change.

The change is a small change, the change is seemingly a small change, the idea is that if you have a code that you developed, can I make a small change to the code to make it run faster rate, right to make my convergence better, so we can pre multiply this pseudo dual time stepping unsteady term by the gamma precondition that term, so that this converges faster to what looks like a steady state and tau but it is in fact a transient.

And when you have done that what you have done is; you would have gone from Q to Q + 1, it will take a one step, am I making sense then you shift; whole machinery shift, restart again, fine great. As I said here, please remember so I am only talking in terms of oh, I addressed only one issue how can I take a very large time step? I wish I could do implicit schemes without all that effort.

Well, it is implicit scheme, there is a certain amount of effort but the effort is what we have been doing so far, so hopefully it is not that difficult, we are just using the ideas that we had in our time matching schemes so far, so which means that you can now try doing approximate factorization here, you can do whatever you want in tau here because when this delta Q tilde goes to 0, r will be 0 and you would have solved your unsteady equation exactly and more accurately, right.

Does that make sense, so you can precondition, you can do approximate factorization all the games that we are playing earlier, you can do the same things here, everything that we have done so far, you can do the same things here knowing that you are going to solve the full unsteady equations ultimately, right but you do all of that stuff, you have taken one-time step that is the point to remember, do not lose sight of that.

You do all of that stuff, you have taken one-time step, then you have to repeat the process, the only question is; is this less expensive or more expensive than solving the full system of equations okay, fine and of course, if you are going to solve a full system of equations, there is whole bunch of machinery to allow you to help you solve systems of equations very efficiently, so the competition is there, the comparison right, the computation of ideas is there.

Because there are; it is not as though people have not tried to solve large systems of linear equations right, then there are algorithms there too do not get; do not let me, you do not allow me to give you the impression that oh, this is it and the other one is difficult to know it, there are other possibilities, I am just basically saying we have developed certain schemes, certain skills using to get steady state solutions using time matching schemes.

They can also be used to solve for the unsteady problem fine, okay. In the next class, what we will do is; we will look at some other acceleration schemes right, we will look at another class of schemes to increase the; improve the performance of your solvers, fine, thank you.