Rocket Propulsion

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Lecture 11

Area Ratio of Nozzles: Under Expansion and Over Expansion

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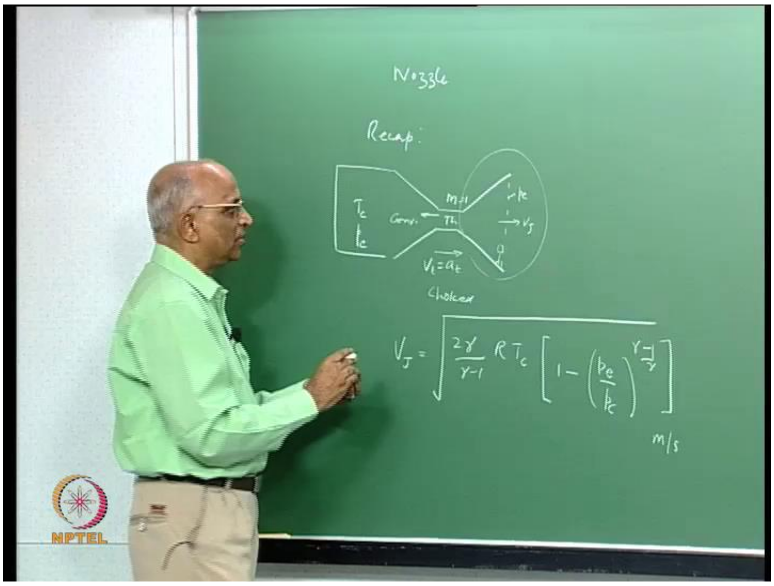

Good morning. We will continue with the portion on nozzles. We will find out what is the effect of area ratio and define area ratio of a nozzle. We will also find out how the nozzle operates at different attitudes and look at some typical results. But, before getting into it let us quickly recap where we so that we can connect it with what we are going do today.

We said in the last class that if we need to have a high jet velocity; we need to have a convergent, it should have a throat for which the Mach number is equal to one and then we should have a divergent. We were very clear about the throat and we said it is the place where the velocity is sonic; that means the gas flow velocity at the throat is the sonic velocity: $V_t = a_t$.

We also found that any disturbance generated downstream of the throat; suppose I stand on nozzle here and make a loud noise or we make some disturbance, this disturbance cannot enter the convergent and therefore, the chamber is isolated. The reason is that the velocity at the throat is equal to the velocity of sound.

Disturbances generated downstream cannot travel upstream. This is because the disturbances travel at the sound speed. Any disturbance downstream of the throat cannot enter the chamber.

And therefore, the sonic throat essentially decouples the convergent and the chamber from the downstream portion. Second point was that the throat is choked. In other words, we told that for the given mass flow rate we can have a maximum velocity, which corresponds to sound speed at the throat and if we want to have higher pressure in the chamber or if the gases are sucked it at lower pressure we cannot exceed this condition of sonic velocity. This means that the throat always will have Mach number equal to 1 or the velocity here should be the sound speed. I think these findings are important.

We also derived an expression for the jet velocity V_J at the exit, which we found V_J as equal to $\sqrt{(2\gamma/\gamma-1))RTc[1-(pe/pc)^{(\gamma-1)/\gamma}]}$. We did not consider the convergent divergent shape while deriving this equation; we just said that if the chamber pressure is Pc and if the exit pressure is Pe then you have the pressure ratio alone which is important. The temperature in the chamber was Tc and this is how we derived the expression for V_J; so many meters per second. Is it alright?

And what we started was with a vent, we derived the velocity and then looked at the shape of the vent; it was necessarily for us to have a convergent followed by the divergent such that if we were to plot it we have a convergent divergent nozzle.

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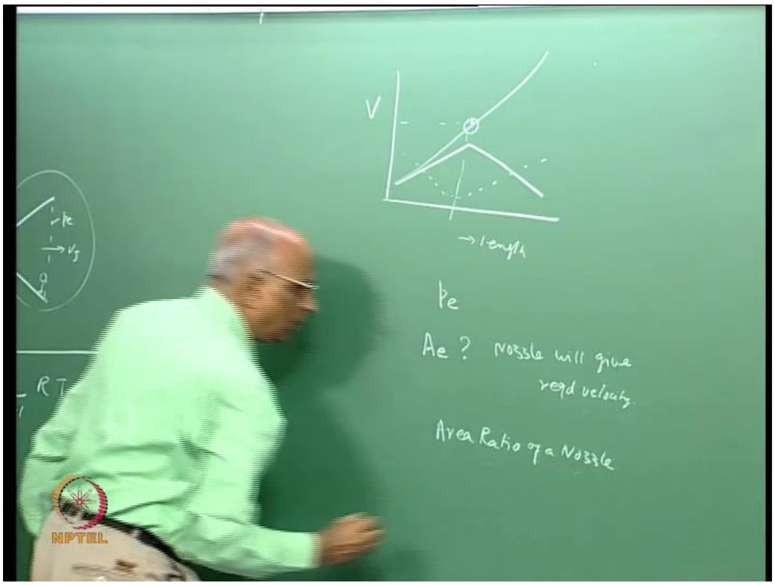

The velocity of the gases as a function of the length of the nozzle mind you increases along the length of the nozzle. Initially we have a converging shape then I have a throat then we have the diverging shape. I get the sonic velocity at the throat and therefore, the velocity will keep increasing along the nozzle in the divergent.

The moment we have at the throat the velocity less than sonic velocity, the velocity thereafter the drops. Therefore the necessity to have Mach number one at the throat was essential. I think this we must remember. Having said that, in today's class we will try to see instead of mentioning that the exit pressure is Pe can I put it in terms of the area ratio and area at the exit Ae. What must be the area Ae such that the nozzle will give me the required velocity and that is we want to do today.

Let me repeat again; see when we realize a hardware we do not know the value of Pe; all what we know is that we must have a configuration like this. We must have a diameter over here of a given size, I must also have a given the diameter at the exit or rather the exit area ratio. Therefore it becomes essential for me to define something like area ratio of a nozzle to be able to give me the value Pe such that I can get the jet velocity or rather I want to know the configuration of a nozzle which will give me the required velocity.

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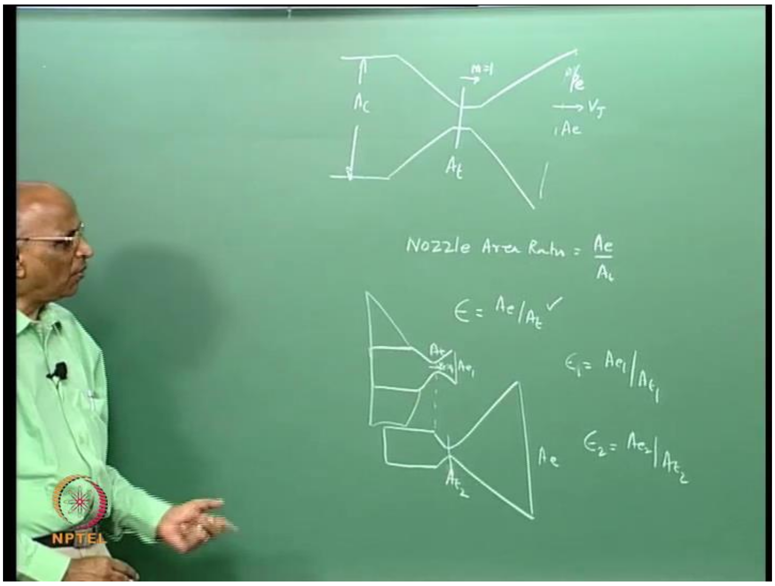

Let us put it this way: I have convergent, I have the throat, I have the divergent. I want know what will be my exit area here such that I can get the V_J what we want. We want to find out the expression for the jet velocity in terms of a diameter or an area ratio rather than put it in terms of the value of the pressure at the exit, which we called as Pe. To be able to do that we are looking at the exit area, I would like to define the area at the throat, because I know that at the throat the Mach number is always equal to 1, therefore I can define it as a critical or an unique particular area for reference and I call the nozzle area ratio as equal to the exit area divided by the area at the throat.

Well. If I have to define something like an area ratio maybe I should think in terms of the area of the chamber over here and relate it to the area at the exit. But whatever be the area here the reference is the throat because that is where the velocity is always equal to the sound speed or Mach number is one. And it is related to the area of the chamber. The gas accelerates from Mach one at the throat in the divergent to high velocities. Therefore we define the nozzle area ratio as the exit area divided by the throat area and it is denoted by Epsilon (ε). E = Ae/At. Is it okay? Having said that area ratio of a nozzle is Ae /At, we want to derive an expression how the area ratio will affect the jet velocity or how should the performance of a nozzle be linked to the exit area ratio.

Let me repeat the problem such that it becomes further clear. Supposing, I have a small rocket, I have the throat Mach number as one. I could also have a small area ratio or a large value of area ratio. I could also have a same rocket in which now I again draw this nozzle over here, I could have a very large area ratio and how do I find out and compare the performance of a nozzle with a nozzle of exit area Ae_1 with a nozzle of exit area Ae_2 and if the throat area is the same in the two cases; I have area ratio in one case which is equal to Ae_1 by at At. In the second case, I have area ratio is equal to Ae_2 by At. I want to compare which one gives me higher velocity, I want to compare these two nozzles and therefore, we define area ratio as exit area divided by the throat area.

I could also have had a larger chamber something like this with much larger chamber, but still even for a larger chamber the throat area would describe the same flow condition namely V_t is equal to the sound velocity a_t. The area ratio would be Ae/At. Area ratio is always defined with respect to the throat that is exit area divided by the throat area.

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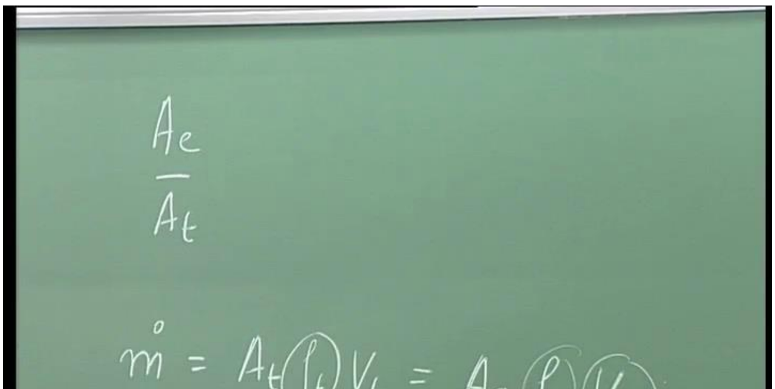

We want to determine V_J and therefore the ratio of Pe/Pt as a function of the value of Ae/At for the nozzle. We just look at the continuity of flow; we look at the mass which is entering the nozzle, mass which is passing through the throat, mass which is passing through the exit and I write m° is equal to the mass which is passing through throat area × density at the throat ρ_t × the velocity at the throat is equal to the area at the exit × ρ at the exit × V at the exit.

Therefore, we find that we need the condition at the throat namely the ρ_t at the throat. I need to be able to find out ρ_e. I do not know Ve; but Ve is the velocity with which the gas is exiting the nozzle. It will be equal to the V_J, which I have already derived. Therefore, I need to find the conditions at the throat. Therefore, let us first spend a couple of minutes on deriving the expression for the conditions at the throat, which are critical to a nozzle.

The condition at the throat will specify the mass flow rate, because it is choked here and I will clarify this later on. Let us first find out what are the density, pressure and temperature at the throat.

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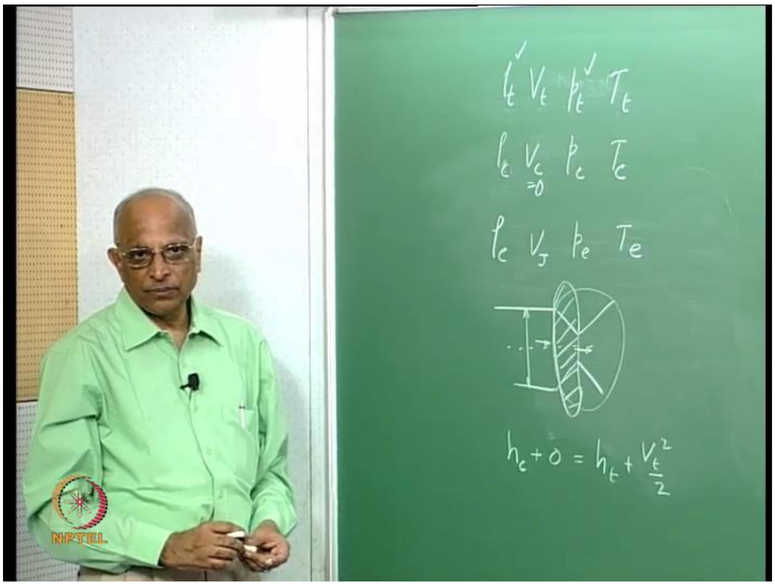

We define the density at the throat as ρ_t, velocity by V_t, pressure at the throat as p_t, and temperature at the throat is T_t. This means that subscript t denotes the throat condition. And how did we define the chamber conditions? The density is ρ_C, velocity in the chamber V_C, which we said was equal to 0 pressure in the chamber p_c and temperature in the chamber Tc. At the exit ρ_e, exit velocity Ve , which is equal to the jet velocity V_J at the exit, pe is the pressure at the exit; well these are all the variables what we have.

We want us to find out the value of ρ_t as a function of ρ_c, may be p_t as a function of p_c and T_t as a function of Tc. Therefore, we again just look at the flow conditions. We are interested in the condition at the throat, the conditions are given by subscript t over here for ρ and V. We treat this convergent as a control volume or we are considering our attention only in this small region, which I show hatched over here. Gas enters at a pressure p_c at velocity 0 at a temperature Tc at it leaves at the throat with a condition of ρ_t at a velocity equal to the sound velocity. The pressure is p_t and the temperature is T_t.

Let us write the expression for this control volume. Let us again assume adiabatic condition and therefore we can write the enthalpy entering is hc plus kinetic energy 0 is equal to h at the throat + we have $V_t^2/2$ which is kinetic energy per unit mass. We must be able to write this steady flow energy equation; same mass flow is here, enthalpy in the chamber corresponding to this initial kinetic energy of 0 while at the throat the enthalpy is h_t and $V_t^2/2$ is the kinetic energy.

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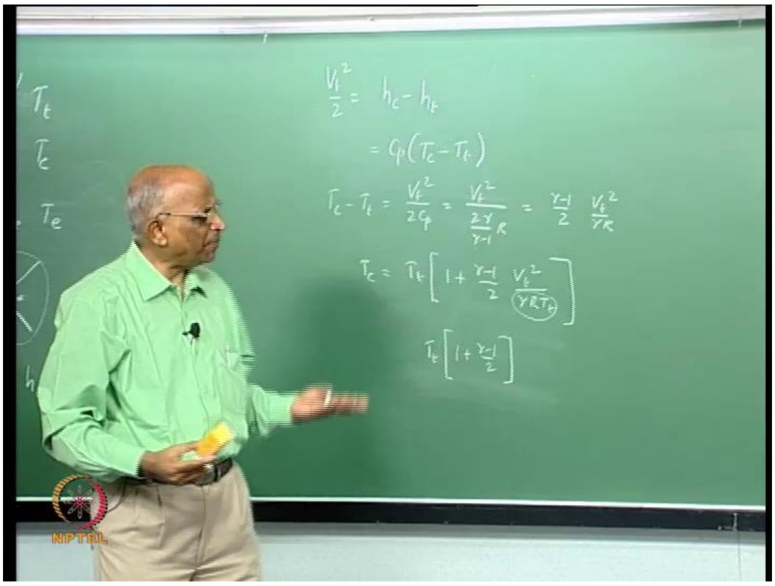

Now let us simplify this equation. We get from this equation $V_t^2/2$ is equal to hc minus h_t. And what is the difference in this enthalpy? It is equal to: Cp $\times$ (Tc – T_t). Therefore what is Tc minus T_t; tt is equal to $V_t^2/2$ Cp. What is the value of C p in terms of gamma: $\gamma R/(\gamma-1)$. How did this come? We had derived in the earlier class: Cp minus Cv is R, Cp by Cv is gamma and therefore, Cp is equal to $\gamma R/(\gamma-1)$. Therefore, we can write this expression as equal to $(\gamma-1)/2 \times V_t^2/ \gamma R$.

And therefore, we can now write the value of Tc: We take T_t on the other side to give $T_t \times \{1 + (\gamma - 1)/2 \times V_t^2 / \gamma R T_t\}$. What did we do? We have taken T_t at the denominator, and therefore, I have gamma $\gamma-1$ into $V_t^2/ \gamma RT_t$. We know that γRT is the sound speed square or $\gamma R \times Tt$ is a sound speed at the throat square. V_t is also equal to the sound speed at the throat and therefore this is the Mach number of one square and we get $1 + (\gamma-1)/2 \times$ Mach number square. Mach number is one and therefore I get the value of $T_t \times (1 + (\gamma-1)/2)$. This gives us the value of the temperature at the throat as a function of the chamber temperature Tc. $Tc/T_t = (\gamma+1)/2$ or rather $T_t/T_c = 2/(\gamma+1)$.

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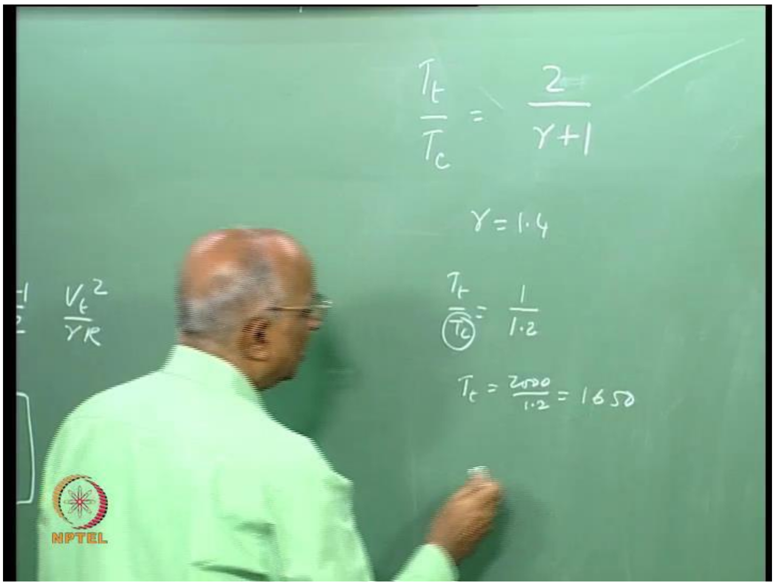

Therefore, for gamma of 1.4, we find that the temperature at the throat is approximately 1 over 1.2 times that in the chamber. If the chamber temperature is 2000 K then it will be something near to 1650 degrees; in other words, if gamma is equal to 1.4 the value of T_t by Tc is equal to 1 over 1.2. Therefore, the temperature falls at the throat and it is less than the value in the chamber.

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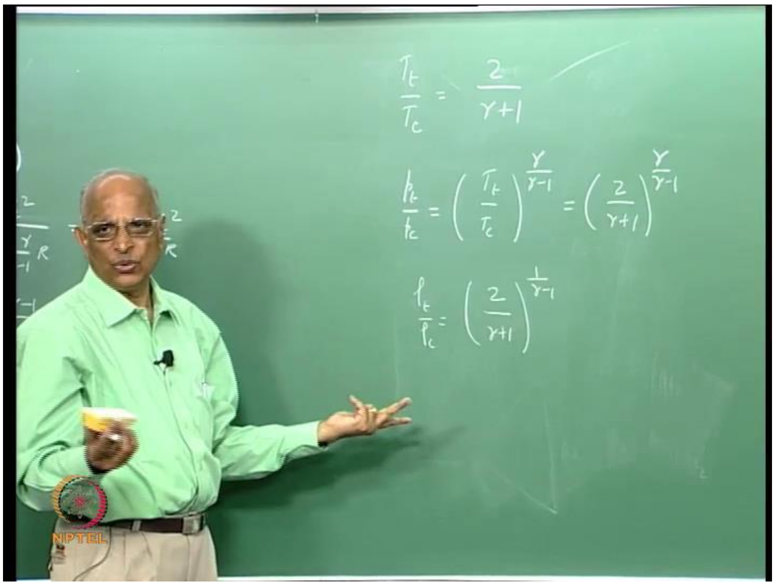

Now what will be the value of p_t by p_c? We have derived the expression in the last but one class. It is equal to $(T_t/Tc)^{\gamma-1/\gamma}$. Please look back in your notes. Let us see how we got this value. We had p/ρ^γ is a constant for an isentropic flow and p by ρ×T is a constant for an ideal gas from the equation of state. Solving for this we got this particular expressions. In fact, you will remember in the expression for V_J, we had the expression $1 - (Pe/Pc)^{\gamma-1/\gamma}$ and how did it come, this was essentially Te by Tc and we expressed it in terms of the pressure ratio. We therefore have p_t /pc = $[2/(\gamma+1)]^{\gamma/(\gamma-1)}$.

What is the value of ρ_t/ρ_C? This equals $[2/(\gamma+1)]^{1/(\gamma-1)}$. This comes from the ientropic relation p/ρ^γ is a constant.

We have derived the conditions at the throat namely the value of the temperature at the throat, the value of pressure at the throat and the value of density at the throat as a function of the conditions in the chamber pressure and which is known to us. The chamber conditions are given to us.

I want us to go back and apply these three relations since we know the density at the throat may be we have to find out the density at the exit. And then find out the value of the exit area ratio as a function of the exit pressure or alternatively the exit pressure as a function of the area ratio that we are interested in.

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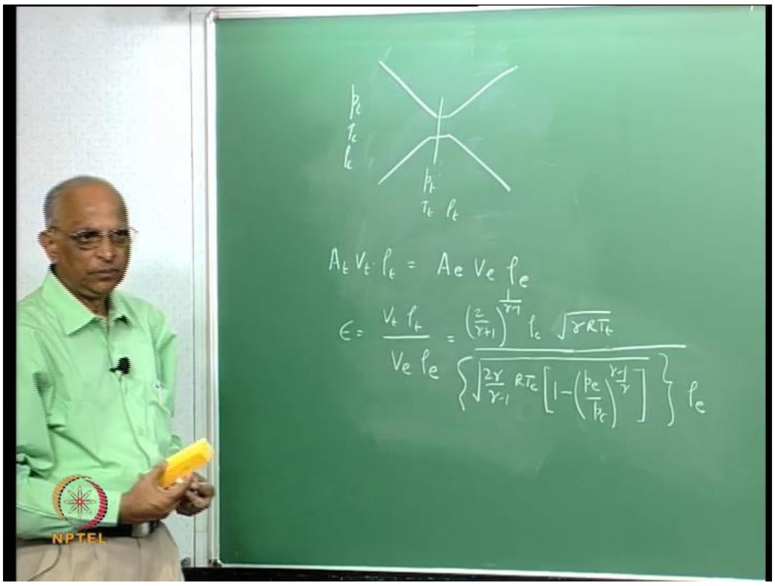

Therefore, I hope that by now we know how to evaluate the conditions at the throat of a nozzle. We have the throat conditions from a chamber pressure p_c and temperature Tc and density ρ_c. We know how to find out the conditions of p_t, T_t and ρ_t .

Now, let us go back and solve the continuity equation; we said area at the throat× velocity at the throat × the density at the throat is equal to area at the exit ×velocity at the exit × density at the exit or rather from this I get the area ratio ε = Ae/At = $V_t \times \rho_t \div (Ve \times \rho_e)$.

I want to substitute the values. We know the value ρ_t in terms of ρ_c; it is equal to $[2/(\gamma+1)]^{1/(\gamma-1)}$. Now we have the value of ρ_t/ρ_c into V_t. We know that Ve is equal to V_J. What is the value of V_J? $V_J = \sqrt{}2\gamma RT_C/(\gamma-1)\times\{1-(p_e/p_C)^{(\gamma-1)/\gamma}\}$.

Now, we would like to somehow get rid of ρ_e and also V_t. We can write V_t as the sound speed and this is equal to a_t and therefore we can write it as $\sqrt{}\gamma RT_t$ viz., equal to under root gamma into specific gas constant R into temperature at the throat.

Please be careful since these are all simple algebraic expressions and we are substituting one into the other and in the process we are also learning how the properties are varying.

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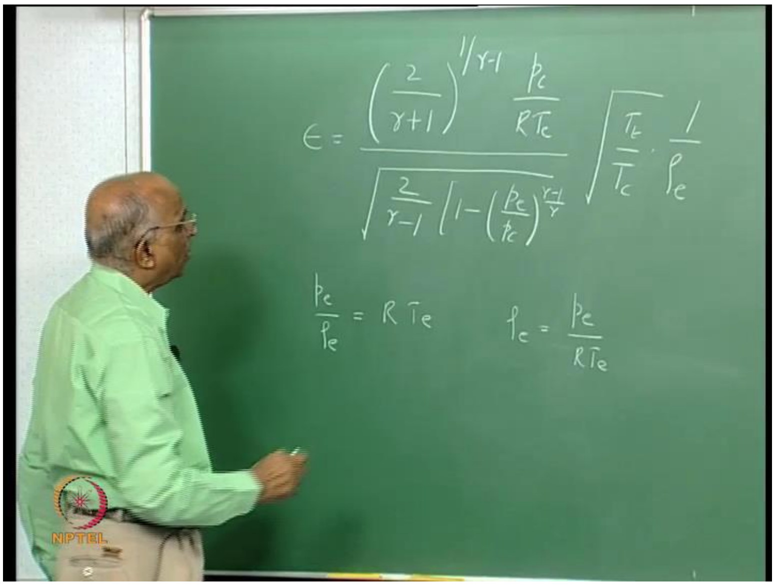

Let solve the equation for the area ratio ε: It is equal to $\rho_t/\rho e \times V_t/V_e$ and equals $[2/(\gamma+1)]^{1/(\gamma-1)} \times \rho_c$; ρ_c can be written as p_c/ RTc from the ideal gas equation $p = \rho RT$.

And now, we have γR and let us strike of some of the numbers in numerator and denominator; √γ and √R go. We will take Pc outside; $\sqrt{2/\gamma-1} \times [1- (p_e/p_c)^{\gamma-1/\gamma}$. We have $\sqrt{T_t} /\sqrt{T_c}$. This is divided by ρ_e.

We now have an expression for area ratio as given by the above expression viz., $\varepsilon = [2/(\gamma+1)]^{1/(\gamma-1)} \times p_c/(RT_c) \times\sqrt{T_t/T_C} \times 1/\rho_e \div \sqrt{2/(\gamma+1)}[1-(p_e/p_C)^{(\gamma-1)/\gamma}$. We would like to simplify the expression by expressing it as ratio of pressures so that we can write it as a function of the pc/p_e alone. For this purpose, let us take a look at ρ_e. We can write the value of ρ_e in terms of the pressure at the exit. The pressure at the exit divided by density is equal to specific gas constant into the temperature at the exit. I simplify this expression to give me $\rho_e = p_e/(RT_e)$. Substituting this value of ρ_e, we get an expression in terms of pc/p_e. We can also make some changes for the value of T_t/T_c that is the temperature at the throat and chamber.

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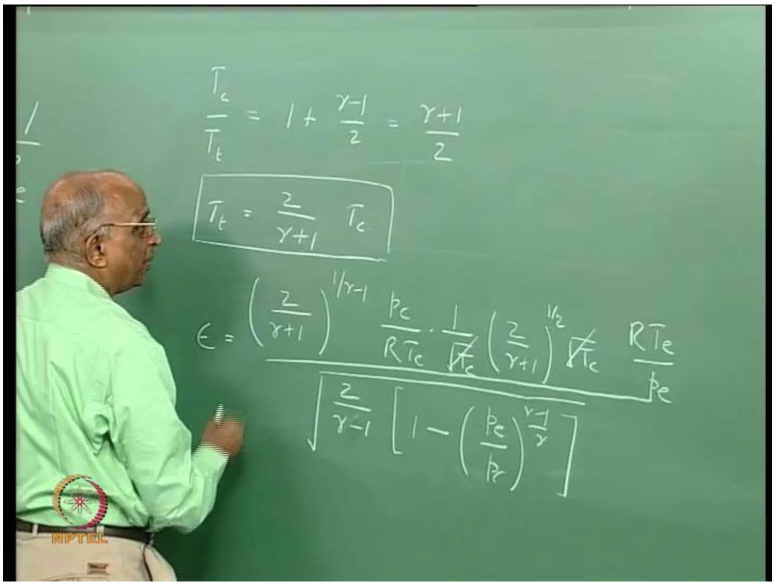

And as we had seen earlier that the temperature at the chamber divided by the temperature at the throat is equal to 1 + $(\gamma-1)/2$ into Mach number squared and the Mach at the throat is one. The value of $T_c/T_t = (\gamma+1)/2$.

Now, these two equations namely the value of temperature at the throat in terms of the chamber temperature and the exit gas density in terms of Pe by RTe are substituted in this particular expression for ε. We therefore get the area ratio $\varepsilon = 2/(\gamma+1)^{1/(\gamma-1)} \times p_c/RTc \times \sqrt{T_t} /T_c$. We can also write the value $\sqrt{Tt/Tc}$, as $[2/(\gamma+1)]^{1/2}$. We had got from ρ_e which was equal to RT_e/p_e and this is divided by the same value $\sqrt{2}/ (\gamma-1) \times(1 - (pe/pc)^{\gamma-1/}\gamma)$. Now, let us simplify this: R and R gets cancelled and we get p_c/p_e and Te/Tc. If we were to put it in terms of Te by Tc in terms of pc by p_e

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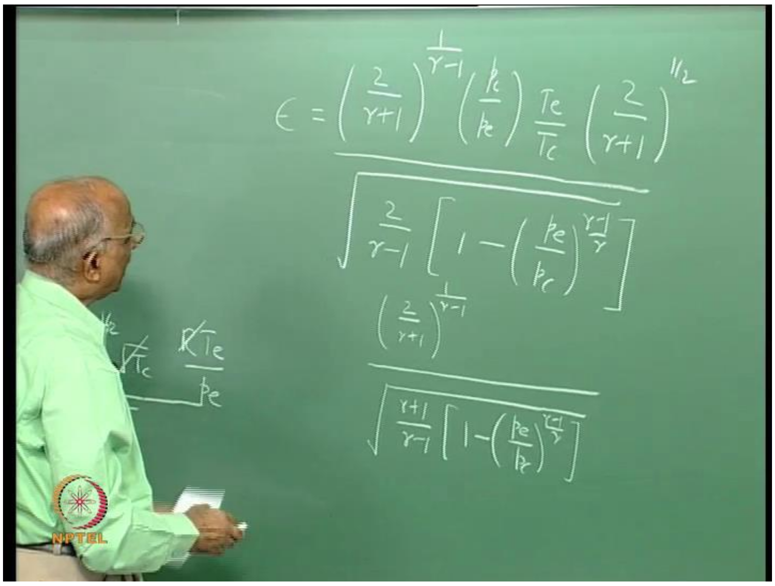

we get an expression for epsilon $\varepsilon = \{2/(\gamma+1)\}^{1/(\gamma-1)} \times (pc/pe)$ and R got cancelled out. Therefore, now we are left with Te, that is the exit temperature and there is nothing else left. Let us write the value of Te over here and we have taken Pe inside and here we have Tc. And this $\times\ 2/((\gamma+1)^{1/\gamma-1}$ and this $[2/(\gamma+1)]^{1/2}$ ÷ under root of the denominator. This comes out as $\sqrt{}2/(\gamma-1)[1-(p_e/p_c)^{(\gamma-1)/\gamma}]$.

Now immediately we see that $2/(\gamma+1)$, 2 gets cancelled and $\gamma+1$ comes on top in the denominator and therefore now I can write the denominator as equal to $\sqrt{}(\gamma+1) \div(\gamma-1) \times [1 - (p_e/p_c)^{(\gamma-1)/\gamma}\}$.

Let us now simplify the numerator; we have $[2/\gamma+1]^{1/(\gamma-1)}$.

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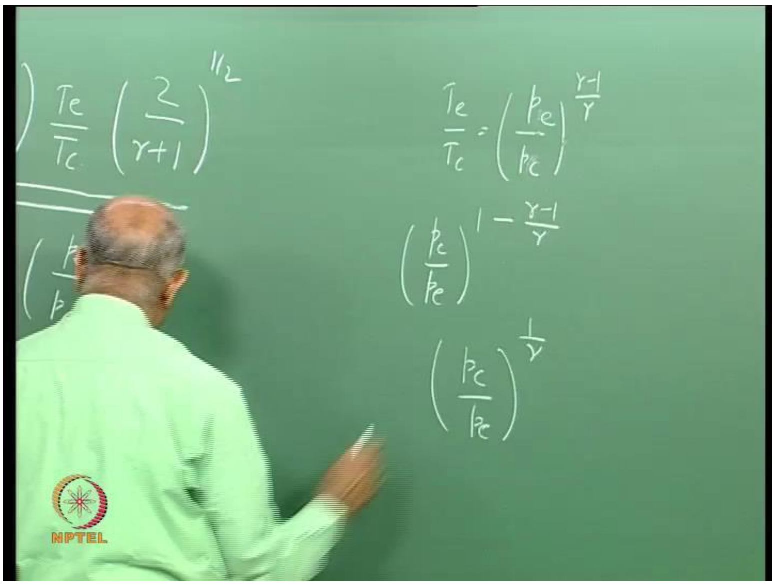

And now we can express these terms: express Te/Tc in terms of p_c/p_e. Please observe that we have been doing this by setting Te by Tc using the isentropic expansion process as $(p_e/p_C)^{\gamma-1/\gamma}$. You will recall we have done this several times and therefore, if now we say p_C/p_e and $(p_e/p_C)^{(\gamma-1)/\gamma}$ (multiply this together), we will get $(p_C/p_e)^{1-(\gamma-1)/\gamma}$, which is equal to $(p_C/p_e)^{1/\gamma}$ because $1 - (\gamma - 1)/\gamma$ gives $1/\gamma$.

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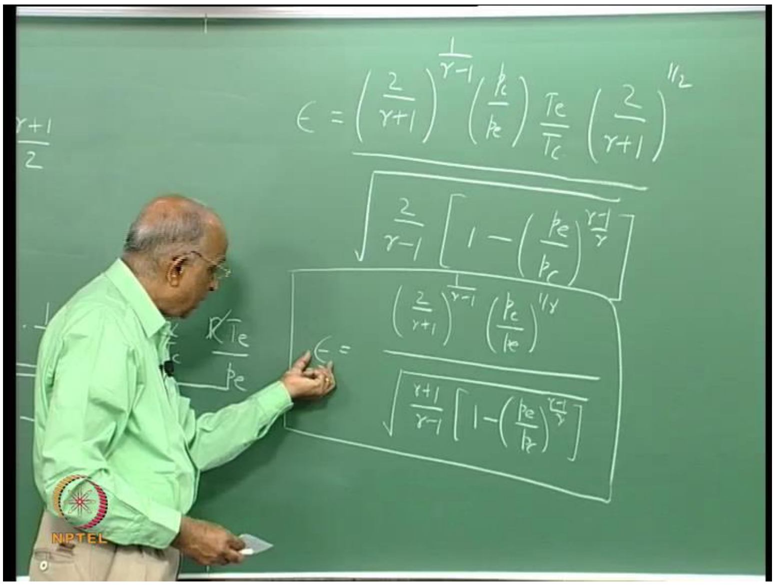

And therefore, we write the area ratio $\varepsilon = (p_C/p_e)^{1/\gamma} \times (2/(\gamma+1)^{1/(\gamma-1)} \div \sqrt{(\gamma+1)/(\gamma-1)}[1-(p_e/p_c)^{(\gamma-1)/\gamma}]$.

What does this expression tell us? This expression tells us that the area ratio of a nozzle increases as the chamber pressure increases or rather as the ratio of p_C by p_e increases. The increase in Pc/Pe can come about either by increasing the chamber pressure or by decreasing the exit pressure. If we have a very low value of exit pressure my pressure ratio is larger and we require a larger area ratio nozzle. Of course, gamma also plays a role, but only a secondary role. The main aspect of area ratio comes from the change from the variations in the value of the chamber pressure to the exit pressure.

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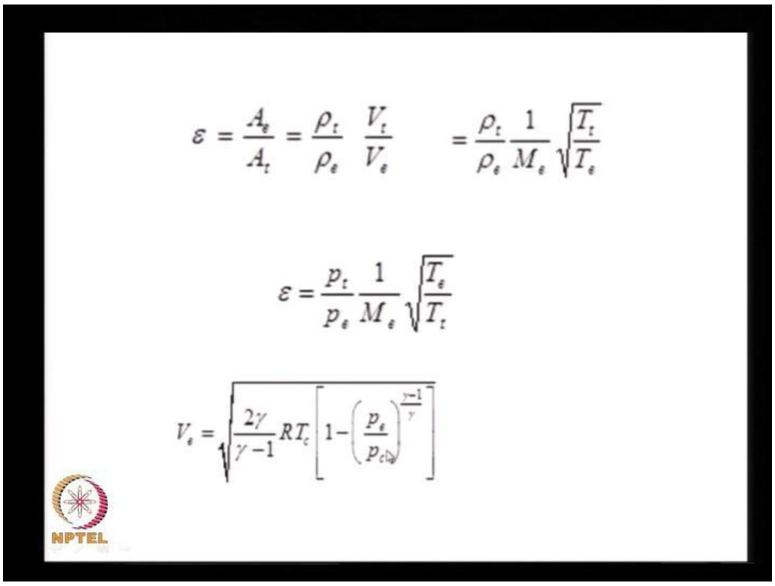

Let us take a look at some of the values which I have plotted in the slides which will be now presented. First, we see that area ratio is defined by the value of Ae by At. The expression we had got we had the jet velocity V_J or Ve given by $\sqrt{2\gamma/(\gamma-1) \times RT_C[1-(p_e/p_C)^{(\gamma-1)/\gamma}}$.

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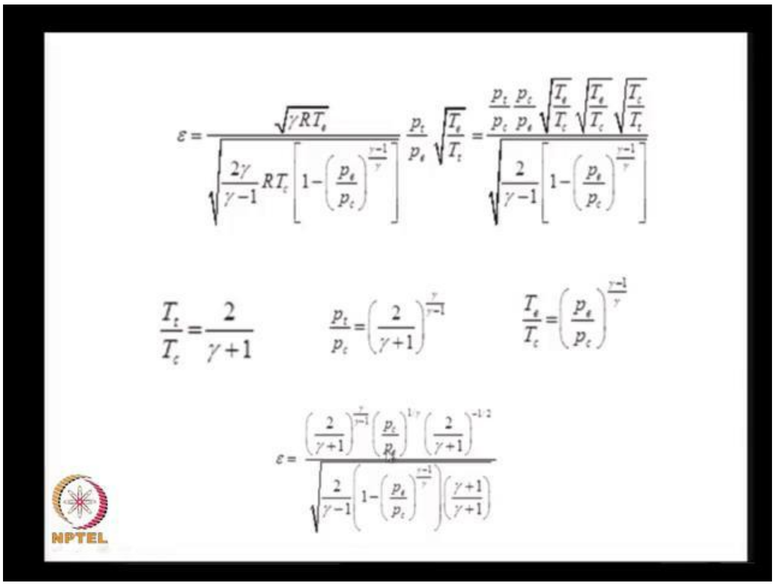

And thereafter we wrote the area ratio in terms of these parameters and got this particular value, which worked out to be $\{2/(\gamma+1)\}^{1/(\gamma-1)}$ and a whole series of gamma terms.

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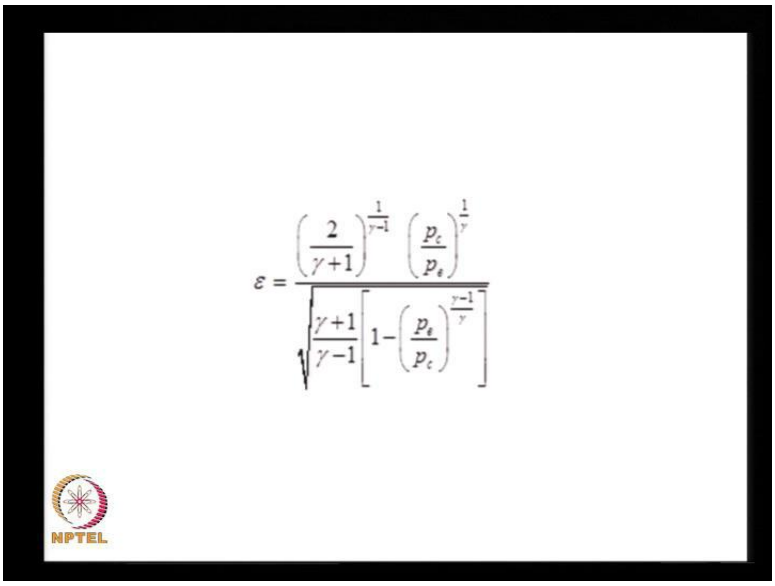

We also had the expression in the denominator as $\sqrt{(\gamma+1)/(\gamma-1)\times[1-(p_e/p_C)^{\gamma-1/\gamma}]}$. When we plot the expression for ε, we get:

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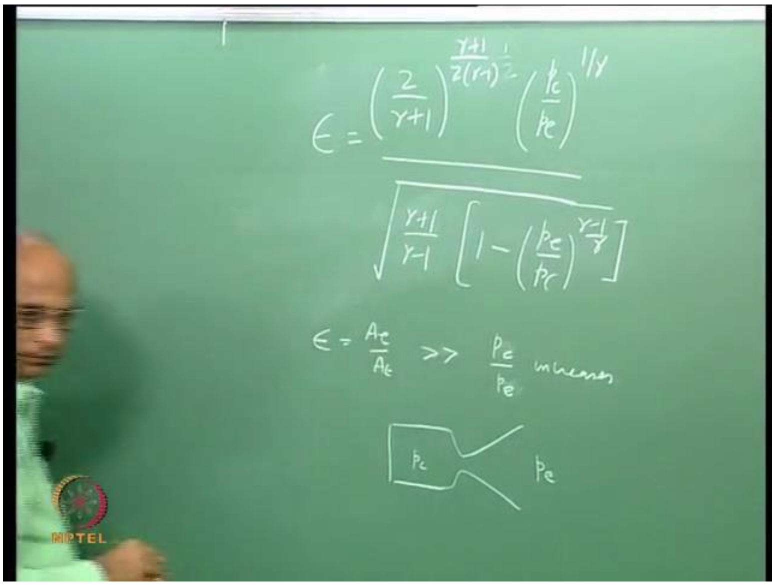

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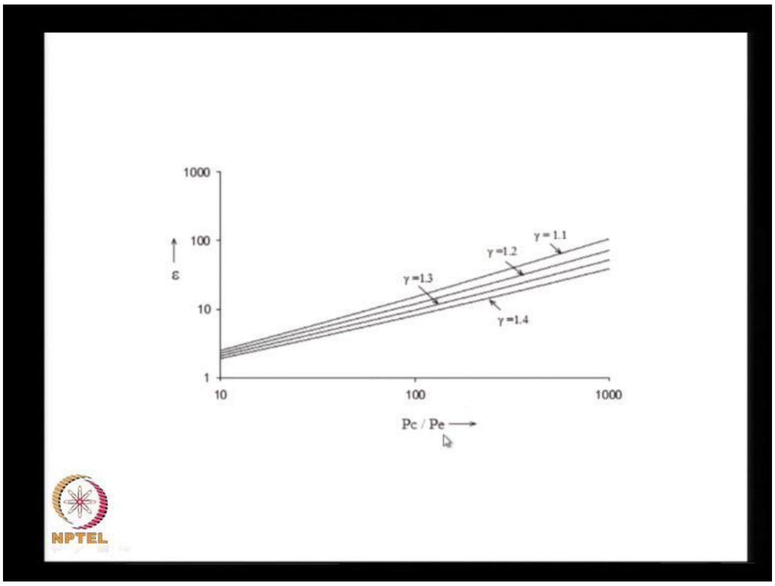

the area ratio as a function of p_C by p_e. As p_c/p_e increases, the area ratio increases. Further, as the value of γ decreases from gamma of 1.4 to 1.1, we find a larger area ratio is required to give the same value of p_C by p_e.

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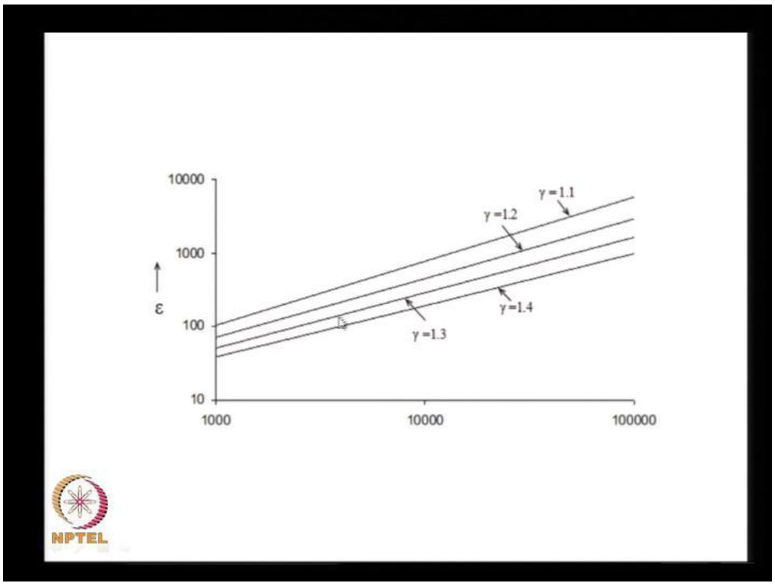

This slide gives area ratios for larger value of pressure ratio. Again, the value p_C/p_e is expressed on the X axis while the area ratio ε is shown on the Y axis. You find that as γ decreases we need a larger value of the area ratio for the same pressure ratio. In other words what it tells me is if the gases have a smaller value of γ then I need a larger area ratio to give me the same value of pressure ratio. This is all about area ratio.

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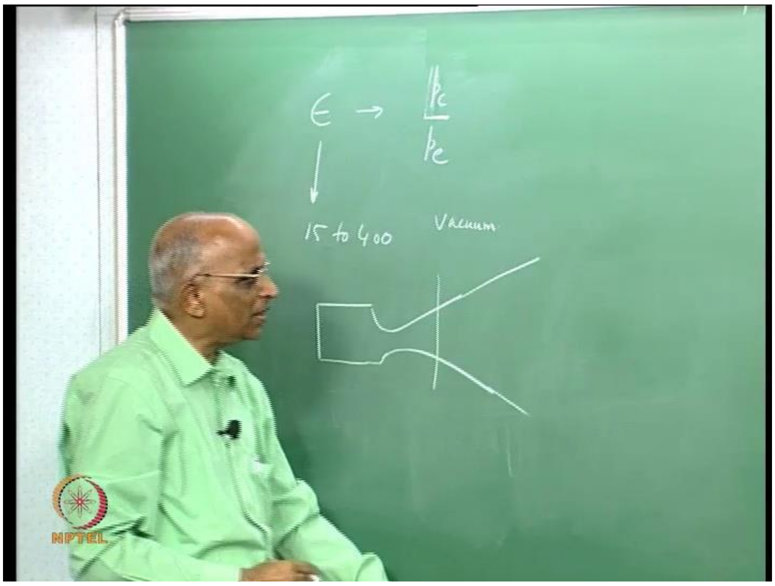

What is it we have done? We found out the value of the area ratio and related it to the value of the chamber pressure divided by the exit pressure. In general area ratio of most of the nozzles are between 15 to 400.

If we were to look at the expression for area ratio you find that when p_e becomes zero, we need area ratio, which is something like infinity. We cannot have infinity, because we cannot construct a rocket, which gives me a very large value of area ratio going to infinity. We cannot keep on extending because the mass of my rocket will keep on increasing; therefore, the general practice is to have area ratios between 15 and 400, 15 for those rockets, which operate within the atmosphere or which operate near to the Earth and 400 or values around this for rockets, which operate in the vacuum regions. Therefore, the question which now crops up is if I have a rocket whose nozzle whose area ratio is either too small or large, how does it perform?

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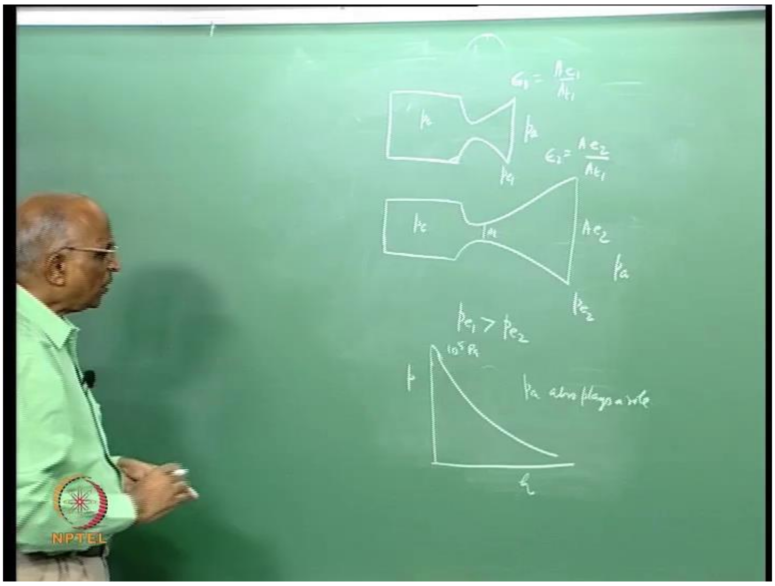

Suppose, I have a rocket nozzle, which let us say has a small value of Ae by At: this is the value ε_1 which is equal to Ae_1 divided by At_1. For the same condition of the throat, I also have another rocket, which has a larger area ratio, let us say Ae_2 for the same value of let us say At_1. The latter is $\varepsilon_2 = Ae_2/ At_1$. Now suppose the chamber pressure is the same in both the cases. What we going to get is a smaller value of p_{e2} as compared to pe_1 since the area ratio ε_2 is more. Stated in the reverse, $p_{e1} > p_{e2}$. The smaller nozzle expands to a higher value of pressure; if area ratio increases as in the case of nozzle with area ratio ε2, the value of p_{e2} is less than p_{e1}.

We also know that the ambient pressure decreases as the altitude above the surface of the Earth increases. At sea level, the ambient pressure is one atmosphere i.e., 10^5

Pascal. As the altitude increases, the pressure decreases till at an altitude corresponding to one at which at the edge of the atmosphere let say around 50 or 60 kilometers altitude, the pressure will go down to a very small value and when we go to geosynchronous altitudes its almost perfect vacuum. The ambient pressure with respect to the exit pressure of the nozzle is expected to play a role. Let us assume that in the specific case p_{e1} for the nozzle with area ratio ε_1, the ambient pressure is Pa.

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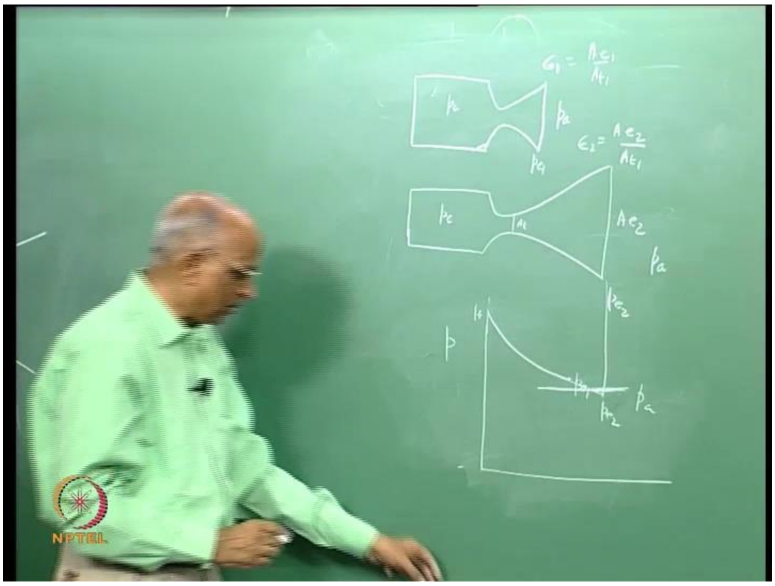

Let us show the values or pressures in the figure. In the figure here, we show the pressure variation along the nozzle. In the first case with ε_1 area ratio, the pressure in the nozzle continually decreases. It starts with the value of p_c comes down to a value of p_{e1}. In the second case, for the same chamber pressure p_c, it starts from p_c, but continues further till we get a much lower value of p_{e2} at the exit.

Let us consider a situation where in the ambient pressure is equal to Pa. I show Pa to be somewhat less than the value p_{e1} this figure.

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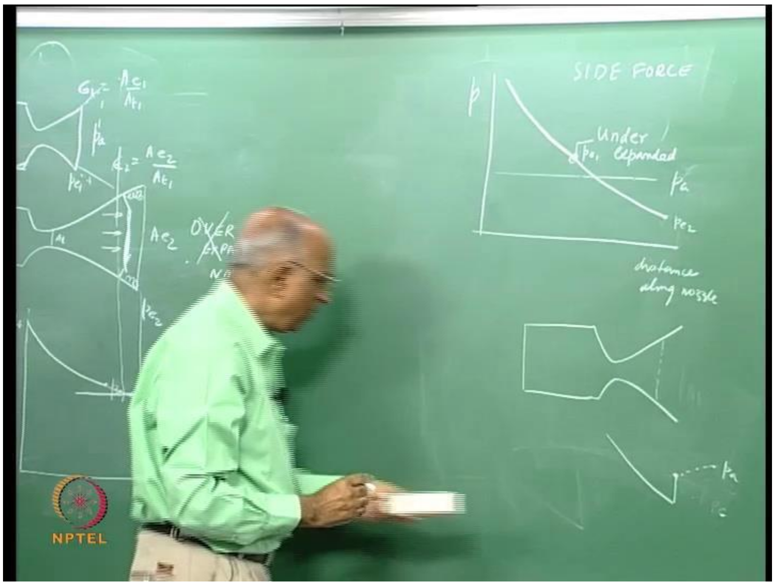

The pressure is varying in the nozzle along its length. The ambient value of pressure Pa in the first case of the small rocket nozzle is less than p_{e1}. In the second case, the rocket nozzle is bigger and therefore the gases expand further with the exit pressure being less than the ambient pressure Pa.

Therefore in the first case the pressure at the nozzle exit is greater than the ambient pressure? In the second case, the pressure at the exit p_{e2} is less than the ambient pressure Pa. In the first case the expansion is not completed; therefore, we call this nozzle as being an under-expanded nozzle. In the second case, we expand it over and above the ambient pressure; therefore, we call this particular nozzle as an over expanded nozzle.

Are there any problems with these two nozzles? What we have done when the nozzle area ratio is small, that is, the area ratio is to a lower pressure than what is possible, the expansion is lower than what could have been possible. Therefore we are not able to get a high jet velocity because the exit pressure has still not been able to match the ambient value. The expansion is incomplete. We could have got much more jet velocity had we really expanded it a little bit more come till the ambient pressure. We are losing some velocity.

What is the problem with over expanded nozzle? Well the pressure here itself within the nozzle is equal to the ambient pressure. At the exit, the pressure is going to be even lower. However, at the exit, the ambient pressure is higher. Mind you, the flow in the divergent is supersonic and does not know the conditions existing ahead of it. All of a sudden the supersonic flow finds a higher pressure because it has already been expanded to a lower pressure and this is clearly not possible. Therefore, it is necessary that something like a shock stands over here within the nozzle; that means, I have supersonic flow it is not able to see anything before it, but all of sudden when the flow reaches it sees a higher pressure and therefore something like a shock is required to match the exit pressure.

The situation is like the following: I have a nozzle here and now the pressure has come to the ambient over here itself and therefore, if I were to plot the pressure, the pressure is going to decrease further. I need to have a shockwave and the flow downs stream of the shockwave is subsonic. The divergent nozzle considering the subsonic flow will act as a diffuser instead of a nozzle and the pressure will increase till it reaches the ambient value at the exit. That means there is going to be a shock and the adverse pressure because of the shock would cause flow to separate at the walls of the nozzle. Since we have a higher pressure at the nozzle wall, the performance of this nozzle may be even better than had the flow not separated. But, normally this flow separation does never happen symmetrically, and it's leads to something like side forces, and therefore over expansion is never preferred at all. I will get back to this point a little later; this point may not be clear at this point in time.

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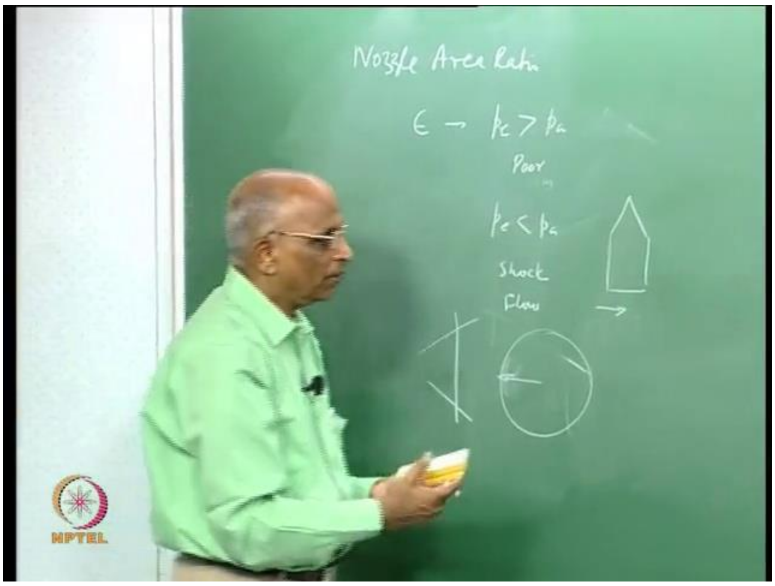

What we are discussing is that if the nozzle area ratio ε is not properly tailored and we get the pressure at the exit of the nozzle p_e to be greater than the ambient pressure Pa, we have under expanded nozzle. In this case the nozzle performance is rather poor; we do not get the high value of jet velocity which is possible by further expansion to the ambient pressure. But, in case the nozzle exit pressure p_e is less than the ambient pressure Pa, we will have something like a shock. The increase in pressure at the shock and in the subsonic flow subsequently will lead to flow separation at the walls of the nozzle. And the flow gets separated from the walls due to the adverse pressure gradient. Flow separation does not take place symmetrically along the circumference, with result that in some regions we have higher pressure, where flow separation takes place. In regions where the flow is not separating, the pressure is lower. The low and high pressure distributions along the circumference which give rise to side forces and this is not desirable.

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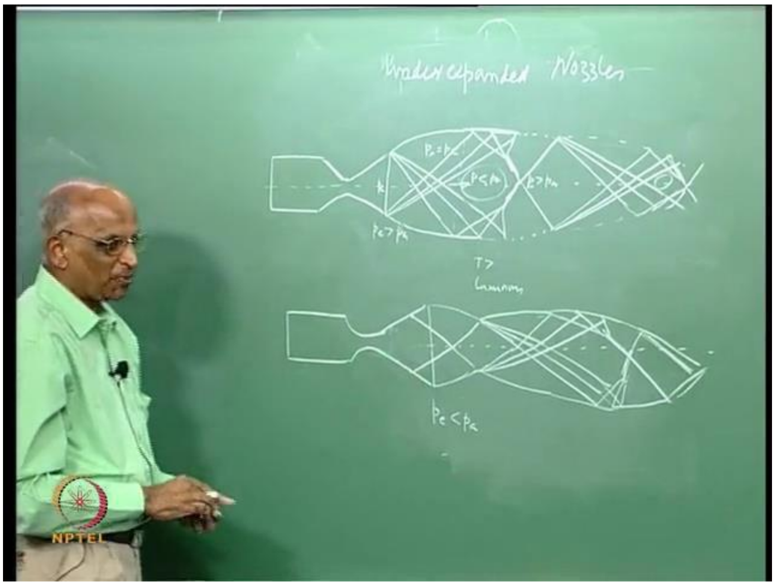

We will come back to this point after seeing a few pictures on flow separation in nozzles. What do we really mean? Let us go back and make a plot of pressure distribution with flow separation or let us say an under expanded nozzle. How does the flow behave? Let us consider this diagram. A chamber, a nozzle and the center line of the nozzle. We said that for the under expanded nozzle p_e is greater than Pa.

Therefore the flow comes here, it meets lower pressure; therefore I have something like flow is going to expand out; as if it bellows out. And how does the flow expand out; we have rarefaction fans or expansion waves which are generated at the nozzle exit. And similarly over here that means the plume comes here, this is the low-pressure region; this is higher value of pressure and when the flow expands out as a series of expansion fans. So we have the expansion waves being shed at the nozzle exit while the plume spills out. I have here the back-pressure, which is the ambient pressure surrounding the expanded plume. The same expansion fans are shown for a two dimensional geometry..

After the expansion waves, pressure in the plume matches with the lower pressure ambient. At the center the flow velocity is still higher. Therefore, at the center line, we have expansion, therefore here the pressure is going to be less than the value of Pa; that is the nozzle with under-expansion forms expansion fans that meets the expanded boundary of the plume. The expansion waves are reflected back from the plume surface as compression waves, the compression waves converge to form shock waves as shown. The pressure behind the shock waves increases more than the ambient pressure and the shock waves intersect as shown. We have compression region after the shocks. The compression waves subsequently hit the plume as shown. They are reflected back as expansion waves. And therefore now, the pressure decreases from the expansion waves. In this way a series of zones of pressures more than the ambient pressure and less than the ambient pressure are formed in the plume from the nozzle. The formation of these zones is due to the interaction of the rarefaction fans and shocks with the boundary of the plume.

In regions wherein pressure is high the temperature is also high. If the temperature is higher, the plume becomes luminous and you can see the pattern with alternate bright and dark zones.

We find that because of under expansion, there is further sudden expansion that means there is an expansion fans and this expansion fans impinges on the plume surfaces. And when the expansion fans impinge on the plume surface, the expansion waves are reflected as a compression that means as weak oblique shocks. These oblique shocks further compress the medium. The interaction of the compression waves with the plume surface forms expansion waves and the process of compression and expansion continues.

If we were to have an over-expanded nozzle, we will have different flow pattern in the plume. A shock is formed within the nozzle and this compresses the initially expanded flow. This is because p_e is less than Pa. The flow being supersonic, we need a shock which will match the higher value of pressure. Therefore, what is going to happen is the plume boundary will come down like this since the ambient pressure is higher. The shock waves interact with the plume boundary and are reflected back as expansion waves. The plumes expand following the expansion or rarefaction fans. The expansion fans are reflected from the plume as compression waves and the pressure in the plume thereafter increases. And so the processes of compression and expansion continue along the plume.

In other words, in the case of overexpansion, we get a higher-pressure region little bit away from the nozzle exist. In the case of the under-expanded nozzle, we get a high-pressure region just at the nozzle exist, that means over here, I get a high-pressure region following the oblique shock waves in the case of under-expanded nozzle.

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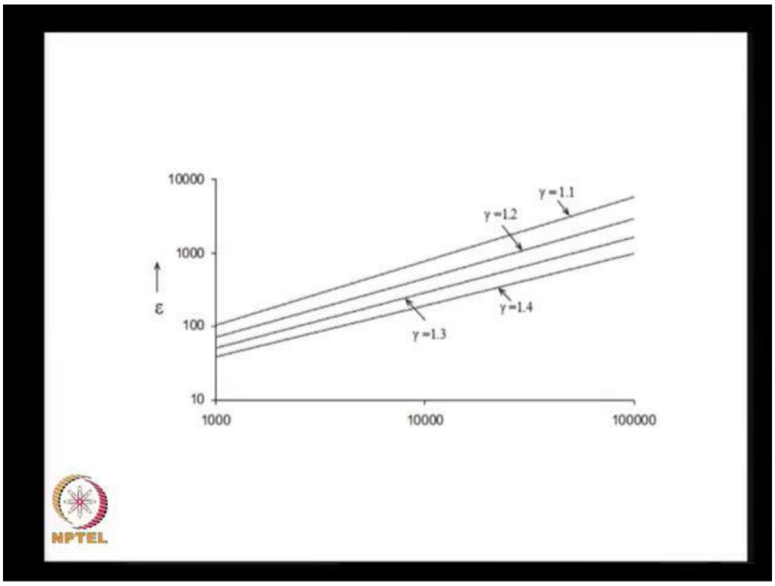

To be able to appreciate this point, I show some slides of the nozzle plume and this will become clear to you now.

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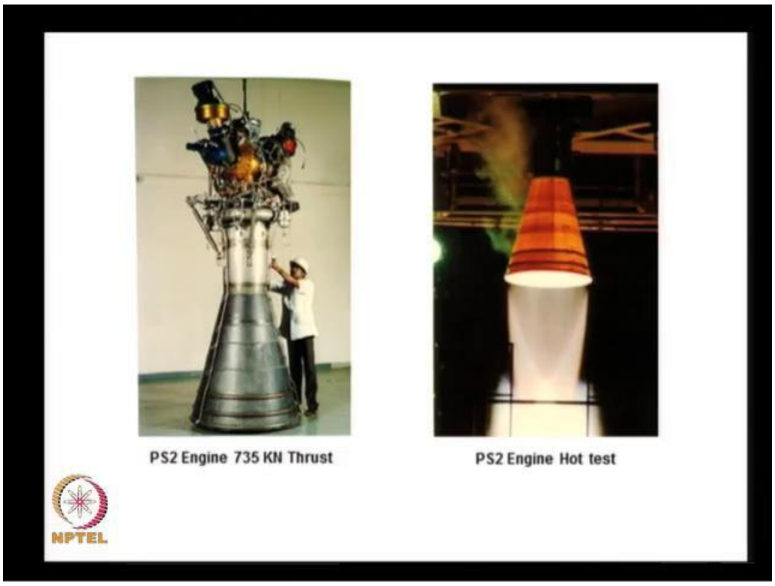

This shows a particular second stage rocket of PSLV, and here you see this is the divergence portion of the nozzle. This is the combustion chamber. And if we take the inside configuration of the nozzle, it will have a throat and will come back with a convergent shape like this to the chamber. Therefore, we are looking at the outer portion of the nozzle here.

When the rocket fires for some time, the nozzle runs hot and become red hot. We are looking at this hot diverging nozzle. It becomes at red hot as time progresses, because it is heated by the hot gases. And then hot gases are converging towards the center after leaving the nozzle like in an over expanded nozzle. You see the plume becoming luminous after the shock wave. The downstream is not clear, because water is sprayed to cool the plume.

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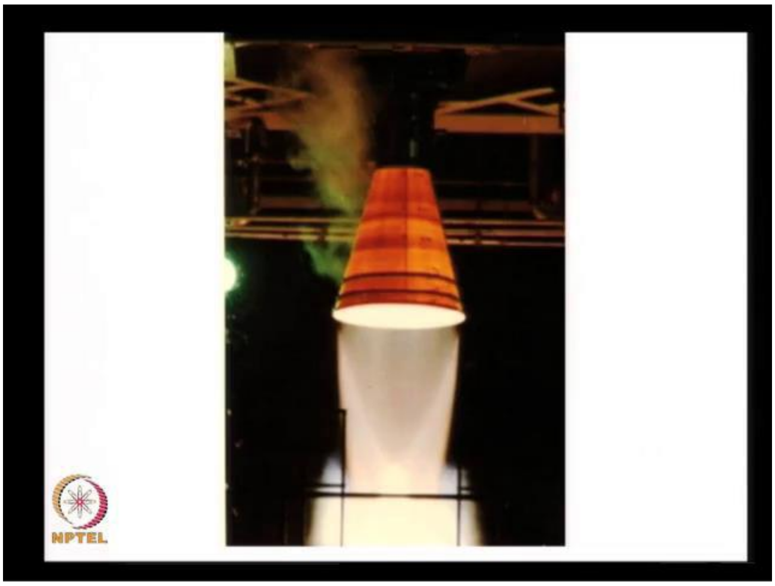

Let me go to the next one, I show the same nozzle again. It is red hot. The white part is the luminous zone after the oblique shock waves. The oblique shocks are seen and they interact along the base.

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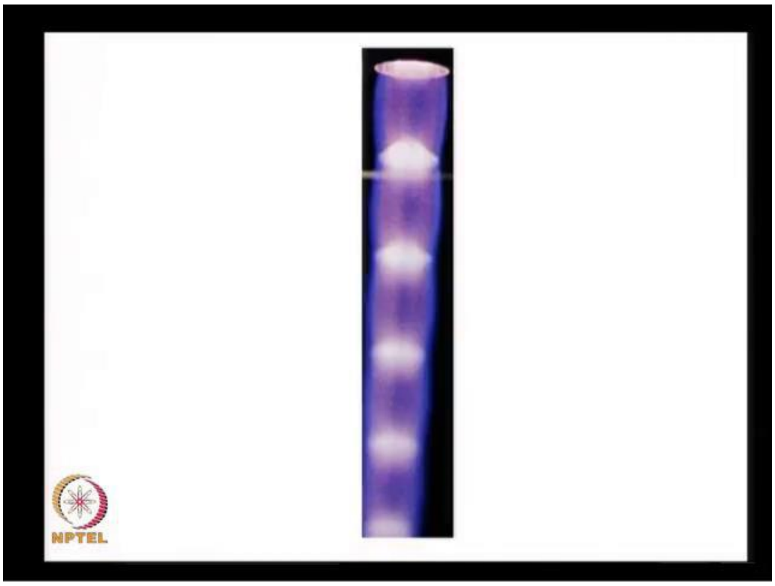

Let us go to some other experimental firing; this shows the engine test wherein at the exit of the nozzle the flow is probably over expanded. And therefore, you have something like shock waves, which increase the pressure and temperature in certain regions of the plume. These regions become luminous. And what happens is the high pressure region over here, gives you a higher pressure and higher temperature. If I have oblique shocks like this, which give us a high temperature region. It looks like a shock diamond you know, I have a high pressure region which is luminous. Afterwards, the oblique shocks comes here, I have rare fractions fans coming another oblique shocks coming; I have another white patch over here. Again the process, I have something like a series of shock diamonds from the shocked high temperature gases..

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In continuation, this slide shows a space plane SR-71 We see the shock diamonds in the plume in this particular case. And we continue with this, this is a test of an engine. And here you find, there is a shock here and something like this. This is because the exit condition is over expanded.

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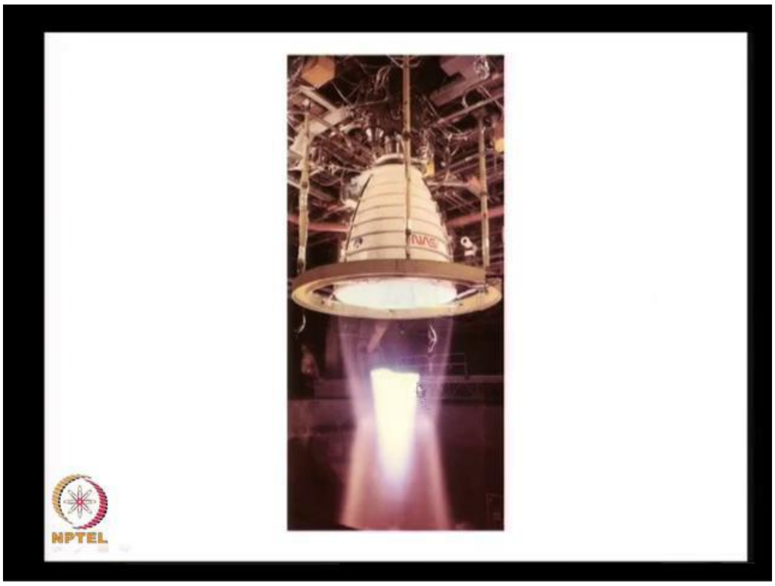

We will continue with nozzles in the next class. We will review overexpansion and under-expansion and then proceed further.