Rocket Propulsion

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Lecture No. 02

Motion in Space

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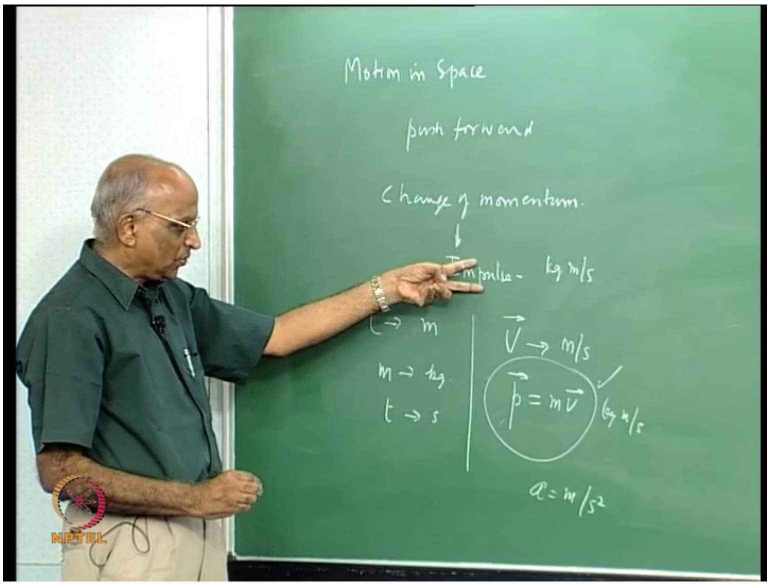


Good morning. In the last class we considered motion in space. For pushing forward we have to give or provide a change of momentum. Change of momentum is what we call as impulse and it has units of momentum namely kilogram meter per second. We also considered what are the parameters used to quantify motion. The length in meters is based on a standard, but ever since 1982 the standard is based on physical constant which is the velocity of light in vacuum. Mass in kilograms is, however, still based on the reference standard.

Currently, research effects are on to define a physical constant rather than a reference object by which we could define mass. It is not yet done. When we say time, we mean a period and that period is second. How do we define the direction of time? The direction of time comes from the second law thermodynamics which says that time progresses in the direction in which the entropy increases. May be we will try to take a look at it in the subsequent classes because thermodynamics forms the basis of the entire rocket propulsion.

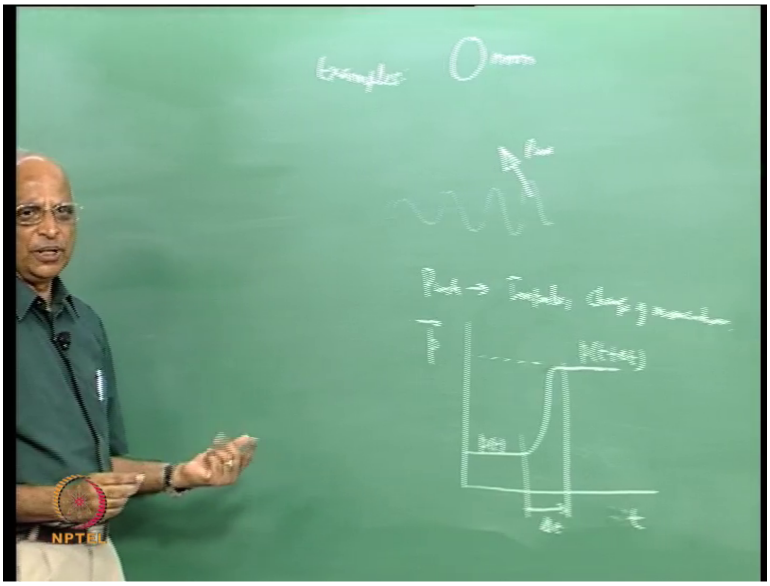
Having defined these three quantities mass, length and time we talked of change of momentum. Velocity is defined as meter per second. Since distance is a vector, velocity is also a vector. Momentum p equals to mass into velocity and is therefore also a vector.

To repeat, momentum p is also a vector being product of mass and velocity - kilogram meter per second. A question immediately arises why use the term momentum when I can use velocity. Momentum is a more fundamental quantity compared to the velocity. It can be seen from the following example. If I have an iron ball which travels at one meter per second and hits me and let us say I have a feather which travels at the same velocity and hits me; the feather does not produce any major sensation while, iron ball leads to significant effect. Therefore, in subjects dealing with motion of molecules or classical mechanics, we deal with the quantitative momentum.

The other quantity as we saw is acceleration, which is meter per second square (m/s^2).

To summarize, in order to be able to push and change the momentum we provide an impulse. While talking of a change of momentum and let us introduce the term Force.

Let us consider the following example.(Refer Slide Time: 03:43)



When people fancied in terms of going to the moon may be some 2000 years ago, a method suggested was if the sea became very rough and a boat tossing in the rough sea could be caught up in very rough and strong waves. The boat could be catapulted up by the waves and if the force of the push from the very rough sea was very intense; the boat could reach the moon. This was the first science fiction article on going from the Earth to the moon. The idea of being pushed up by storm in the sea namely when you have huge waves in the sea, that is huge tidal waves such as when a storm occurs over sea and a boat being pushed upwards towards the moon was proposed by Lucian. He was a Greek philosopher and satirist who live in the period around 40 BC. Therefore, we are talking of something being pushed with a large force. Can we quantify push in terms of impulse that is change of momentum? Or rather describe the push in terms of change of momentum itself.

We therefore look at the change of momentum which is a vector, may be as a function of time. Assume a body is traveling with constant momentum and after a short duration of time, we change the momentum to a slightly a larger value. How do we change it? May be in the example that we consider, we change it gradually from the steady value by increasing its velocity as shown. The final momentum again reaches a steady constant value. Initially the momentum is a constant value and it changes over a time period delta t; that means, in the plot the momentum is changing from p at time t to p + Δp at time t + Δt.

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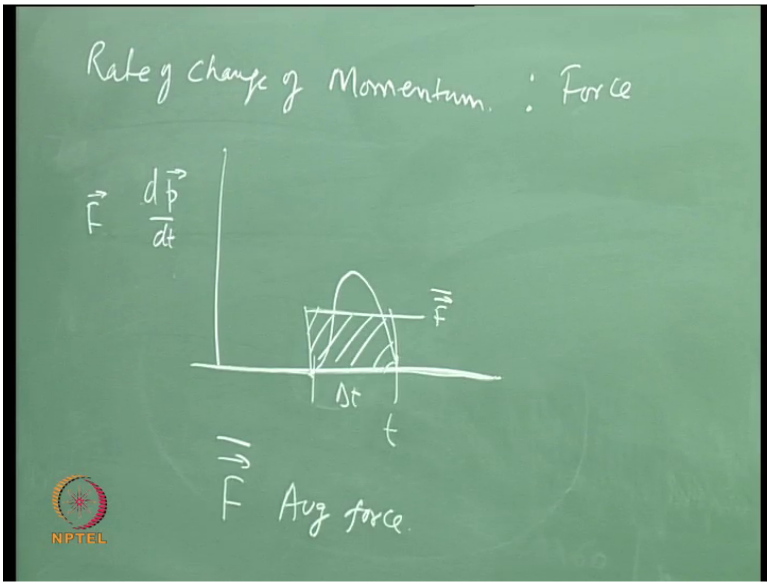

Therefore what is the impulse associated with the change? The impulse is equal to the value of momentum p +Δ p at time t +Δ t minus p at time t so much kilogram into meter per second.

Therefore, the definition of impulse is just a change of momentum that you give to a body. If we were to ask what is the rate of change of momentum? I plot the rate of the momentum change for the momentum change shown earlier. How will the rate look like? We find that the momentum remains at p over time t right from zero onwards to this particular time at t. In other words, dp/dt is zero up to this particular point in time t. Thereafter it increases reaches a maximum and then goes back to zero at time t + Δt. In other words over a period of Δt it increases, reaches maximum (at the inflexion point on the momentum curve) and comes back to zero. The curve is the type of signature that we would get for dp/dt over the duration delta t.

If now we state that the rate of change of momentum is force, the force due to the change of momentum is not something, which is a constant over the duration Δt, but keeps varying. However, whenever I change the momentum and I ask myself what is a force? It is difficult for me to specify the force since this force is going to vary with time. Therefore, what we say is that the force is continually varying during the duration of the change of momentum and it would be better for me to take an average value and say this is my average value of force for the momentum change over the time duration.

Force is vector like momentum and is an averaged out value. When we talk of force, we imply this average value.

Force is a derived unit. It is not something fundamental like momentum and we must keep in mind that the force during a particular change of momentum continually varies.

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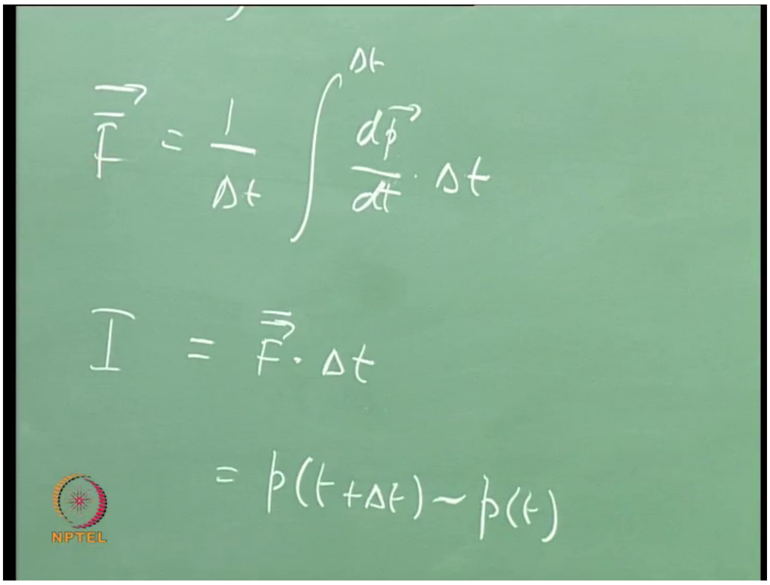

But I take the average value and what will this average value of be? Let us put it down as $\bar{F}$ and we say it is a vector. It is equal to the average value of dp/dt over the duration Δt. The momentary value of dp/dt, averaged over the duration Δt by integrating it over the time Δt and dividing it by Δt This must be the averaged value of the rate change of momentum, i.e., averaged force over a period of delta t.

Therefore, you see that force is something which is not that that good a unit compared to momentum change or impulse as some averaging are involved. If we have to write an expression for Force in terms of impulse; how could we do it? We can write Force into Δt as change in impulse over the duration. Denoting the change in impulse by I we find it is equal to Force into Δt. Because we find force is equal to this impulse divided by Δt or equivalently Force into Δt is the impulse or the change of momentum. This is the connection between impulse, force and change of momentum.

If we are very clear about the above relationships, it is about time to get into the universal law of gravitation. What did we discuss about planets orbiting about the Sun in the last class?

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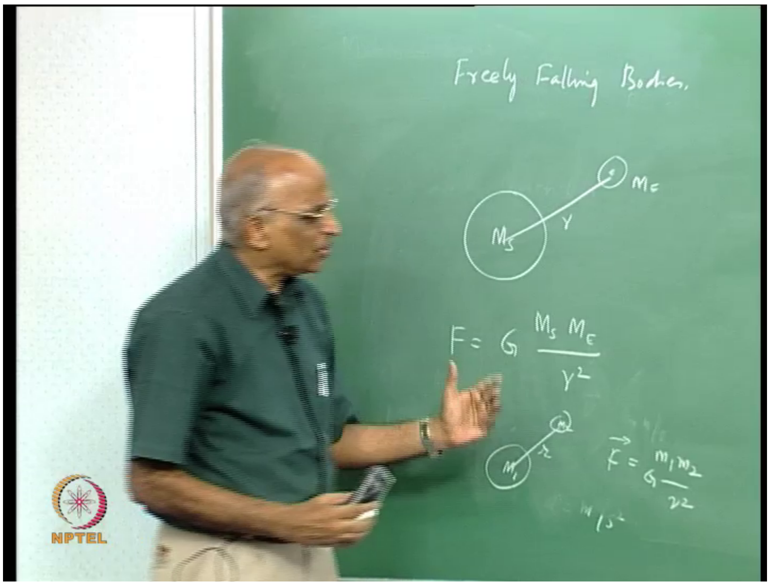

We told that all planets as they go around the sun are all freely falling bodies. A planet such as the Earth is continually falling on to the sun. So, also an apple is falling down and this is the observation which Newton had and he says a heavy body like the Earth attracts a body which is up in space in the same way that the much heavier Sun is attracting the Earth. And if we say this is the mass of the Sun and this is mass of the Earth; he said the force with which it attracts is given by a constant multiplied by the mass of the attracting body, the mass of the body which is attracted divided by the square of the distance between them. In other words it need not be only the Sun and the Earth. It could be any heavenly body or otherwise. Let us say mass m_1 attracts a small body of mass m_2. Let us say that r is the distance between them. Therefore, the force is equal to the product of m_1 and m_2 divided by r^2 into a constant.

There is therefore a field of attraction around a body of mass m and it is the field from the heavier mass that attracts the lighter mass. And this relation becomes the universal law for gravitation. The constant G becomes the universal gravitational constant. You know this law is important as we shall apply it in moment or two.

Let us also remember that force is a vector.

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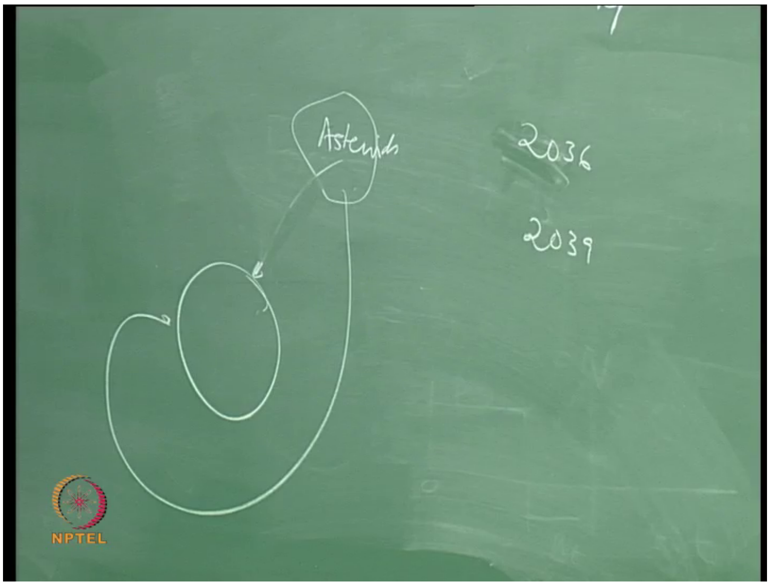

Therefore, I should have really written this as the force is equal to may be a body one being attracted towards body two or equivalently let us say the body two attracted towards one. Therefore, I have the body two which is a light body being attracted by a heavy body m goes as a gravitational constant into r^2. Therefore, I put it as r is a vector to the power 3 mod of this vector into r bar; that means, m_1 into m_2 by r^2 into G and it is being attracted. Therefore, we have a negative sign for the attraction.

This is the universal law for gravitational. G is the gravitational constant and therefore, the unit for G should be what? It should have the units of force divided by kilogram square multiplied by radius square or meter square. What is the unit for force then? We should be clear about it. We told that force is the rate of change of momentum; force is equal to dp/dt. Therefore, force is equal to 1 over time multiplied by kilogram meter per second which is equal to kilogram meter per second square (kgm/s^2). And this particular unit kilogram meter per second square is what we call as Newton.

Therefore, the force has units of Newton, which is kilogram meter per second square ($kg\ m/s^2$). It comes from rate of change of momentum and therefore, we have the units of G as Newton meter square by kilogram square and the value is something like 6.671 into ten to the power minus eleven (6.671×10^{-11}) Nm^2/kg^2. This is the constant in the universal law for gravitation.

In our solar system we had the eight planets going around the Sun. You also have some loose objects like asteroids, which are also going around, but they do not have a well-defined path as the planets have.

One such asteroid is likely to hit the planet Earth may be in the year 2036 and if if does not hit the Earth it is likely to come back and again hit it in 2039. What is it we are talking of? May be some of these asteroids may come and collide with the atmosphere above the Earth. The question is how to prevent an asteroid from hitting the Earth? What is the type of propulsion system that we could design such that we prevent the collision? Can we think of it from the universal law for gravitational forces? People talk of difference strategies how to how to prevent some of these things happening and let us take some time off and let us try to solve this problem if it helps us in applying the universal law for gravitation.

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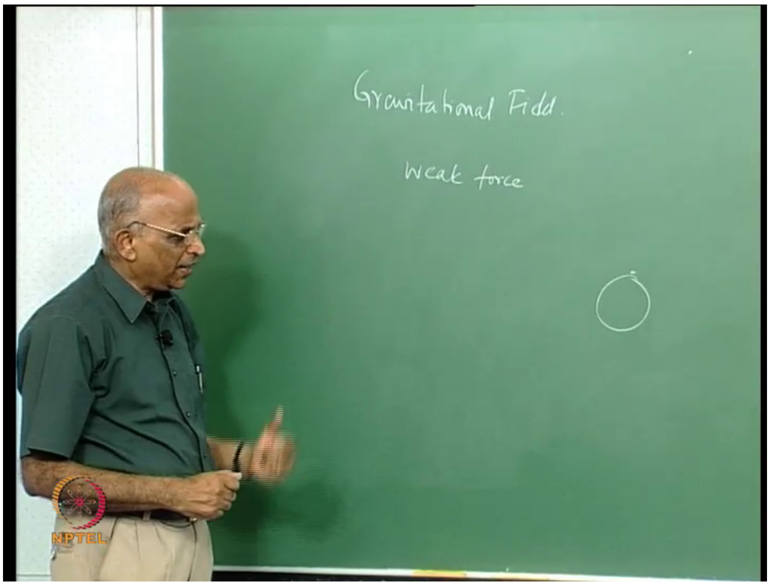

Let us say one of the thinking is that we launch a rocket on to space and we keep on accumulating satellite over here; I make a heavy mass over here and when the asteroid comes over here; this mass being heavy compared to the mass of the asteroid, will attract it towards the heavy mass and instead of the asteroid going in a given particular direction deflect it. In this way we change the direction and it will miss the earth. This is known as a gravity tractor. That means, we put a mass in space and make sure that this mass is large and its distance from the asteroid small, it gets attracted towards it and the asteroid instead of coming like this can get deflected away.

You know these are all possible. That means, you know the law is not only doing problems in mechanics; but can be applied for changing the trajectories and changing the trajectories is as good as giving some propulsion element to it. The gravitational force what we are talking of or rather the gravitational field is a weak force but it persists over a very long distance. Like let us say I have the Earth here. May be a mass near the surface of the Earth is attracted with a higher force than something which is very far away because the field decreases as the distance from the Earth increases. The attractive field from the Earth decreases as we move away from the earth.

Let us do one problem to be able to assess the gravitational field and I take a model problem again of an asteroid and let us calculate what is the force exerted by this asteroid on something which is moving near it. This would help us understand the magnitude of the gravitational field for some space related problems.

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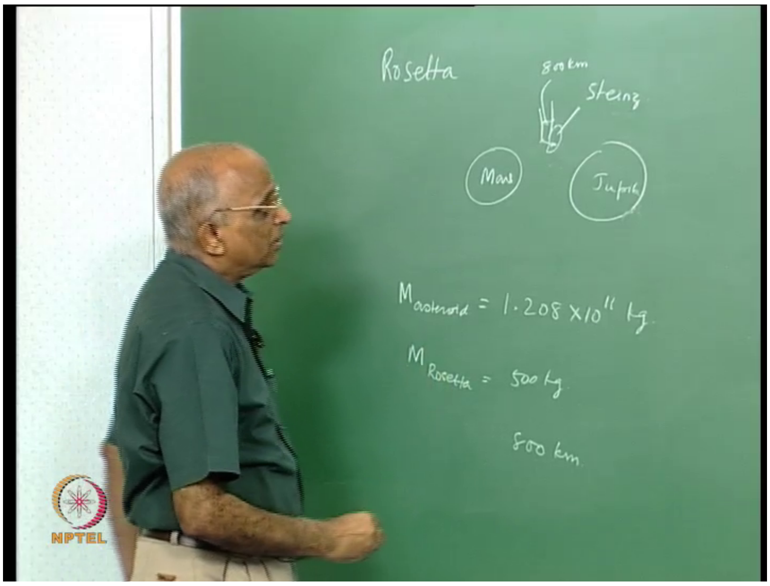

Let us do the problem of a space probe by name Rosetta that was launched to study the asteroids in the space between let us say Mars and Jupiter. This particular space capsule Rosetta was used to study a particular asteroid by name Steinz.

Now, Steinz, let us assume a mass around $1.208x10^{11}$ kg. You know the asteroids are somewhat composed of loose material and they do not have a particular fixed path in space. The space capsule Rosetta cannot be very heavy and let us assume the mass to be 500 kg. To get any meaningful attraction, it must be brought as near to Steinz as possible and the nearest distance it came near to this asteroid Steinz was something like 800 kilometers. We would like to know when this space capsule Rosetta is 800 kilometers from the asteroid Steinz, what is the attractive force exerted by this asteroid on this space capsule Rosetta? The nearest distance between the space capsule and the asteroid is 800 kilometers.

Therefore, what is the force which the asteroid exerts or pulls the space capsule? We say force is equal to G multiplied by the mass of the asteroid into mass of this space capsule Rosetta divided by r square. G is 6.67×10^{-11}. Mass of the space capsule is 500 kg and this is divided by the distance square. The distance between the two is let us say nearest position is 800 kilometers and the diameter of the asteroid can be assumed to be 1200 kilometers.

We can neglect the diameter of the space capsule. The total distance from the center of the space capsule to the center of the asteroid to the center of the space capsule is 600 plus 800 kilometers into 10^3 squared in meter square and therefore, we are getting a force of the order of $6.67\times 10^{-11} \times 1.208 \times 10^{11} \times 500$ divided by $(1400 \times 1000)^2$. Let us take look at the units. It is Newton meter square by kilogram square into kilogram into again kilogram i.e., kilogram square. Denominator is meter square and therefore, we have so much Newton as attractive force.

When we look at it, the type of force that we get is of the order of 10 to the power minus 6 of a Newton which is something like a micro Newton. That means, the attractive force exerted by this asteroid on this space capsule is something like a micro Newton; which is negligibly small. However, in space even small forces are of interest and therefore we find that weak gravitational forces attract the space capsule. However, if we had a very massive satellite like the gravity tractor then we could get more gravitational pull.

What is the gravitational field and how is it expressed? You say acceleration due to gravity. What is acceleration due to gravity? How would you define the field? What do you mean by gravitational field and what is acceleration due to gravity? How to define it? We say the force experienced by a mass m in a gravitational field g is equal to m g. What is the unit of g?

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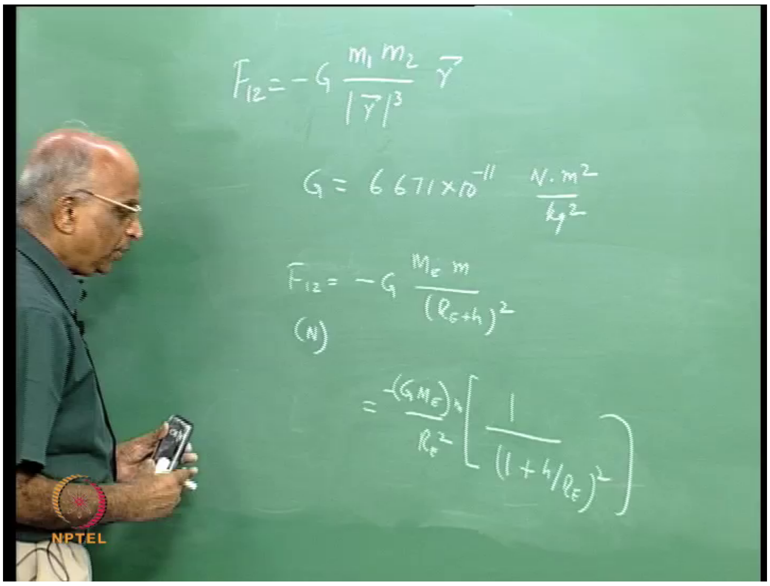

Meter per second square we call it as acceleration, but, how can Earth give something like an acceleration? Acceleration is rate of change of velocity. You know we go back to the universal law for gravitational and then we write the force F_{12} is equal to the mass of a body attracted by the mass of Earth of M_E. Let the mass of this body be m. Therefore, the force by which this body is pulled towards the Earth as per the universal law for gravitational forces is $- G \times m \times M_E \div r^2$ where r is now the radius of the earth R_E plus the height of the body above the surface of the Earth h. The force is so much Newton. Now I want to simplify this expression. Therefore, I write this is as equal to minus G M_E by R_E square and then I write within the bracket $(1+h/R_E)$. I expand out the term $(1+h/R_E)^2$ for height h above the earth to be is very much smaller than R_E.

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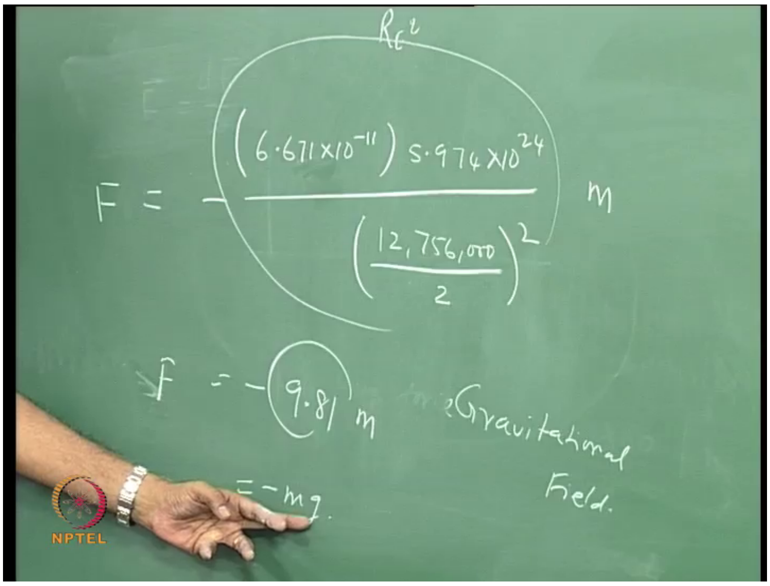

And therefore, I can write this expression as force is equal to $- G \times M_E \div R_E^2$ into mass of the object $\times$ $(1 - 2\ h/R_E)$ and the subsequent higher orders of h/R_E. Anyway h is smaller than R_E and the higher order terms could be neglected.

And therefore, I get the value of force is equal to minus $G \times M_E \div R_E^2$ into the mass. We have the mass of the earth as 5.974×10^{24} kg and the value of G was equal to 6.671× into 10^{-11} and the value of the radius of the earth was equal to its diameter 12756 kilometers divided by 2. This is multiplied by the mass m of the body.

Now, we simplify this. I find out that this will come out to be $- 9.81 \times m$ which is equal to force F. This value of 9.81 is a constant since the mass of the earth is a constant, G is a constant, radius of the earth is a constant and this is why we denote it by g and we say that $F = - mg$. Therefore, we are really not telling acceleration due to gravity we just tell ourselves as per the universal law of gravitation whenever there is a heavy mass it attracts a lighter mass and it is a field.

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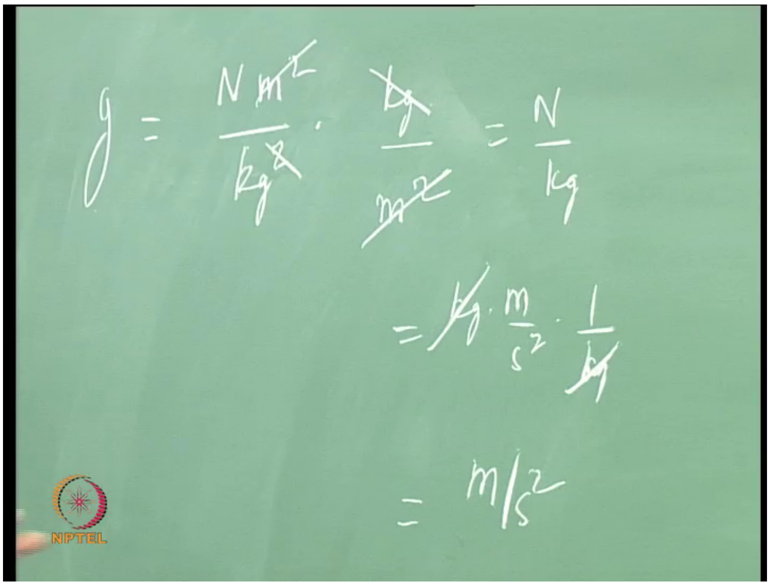

What is the unit of g? We put the expression down. The unit of G was Newton meter square by kilogram square. This is multiplied by mass of Earth in kilogram and divided by the square of the radius of Earth in m. Meter square, meter square gets cancelled. Kilogram comes here, you have Newton per kilogram. Newton is equal to kilogram meter per second square divided by one over kilogram this is equal to meter per second square. Therefore, the unit of the gravitational field comes out to have units of acceleration and therefore, many people refer it as acceleration due to gravity whereas, it is just a field attracting or interacting with a particular mass.

We had said that Newton formulated the law based on observation and since it is based on observation the law should not be extended beyond the realm of observations. Since it is based on phenomena it is something like a phenomenological law. When we talk of bodies in space such as Quasars that travel at a speed near to velocity of light, the law breaks down. The universal gravitational law is no longer valid. Therefore, whenever we base anything on observations; the conditions of observations must be related to the particular law and it has to be applied with caution for conditions outside the realm of observations.

Though we have seen the law for gravitational field, we have not really pondered over the question of why a heavy mass should attract a lighter mass. How can you justify it? Can we do an experiment to show why?

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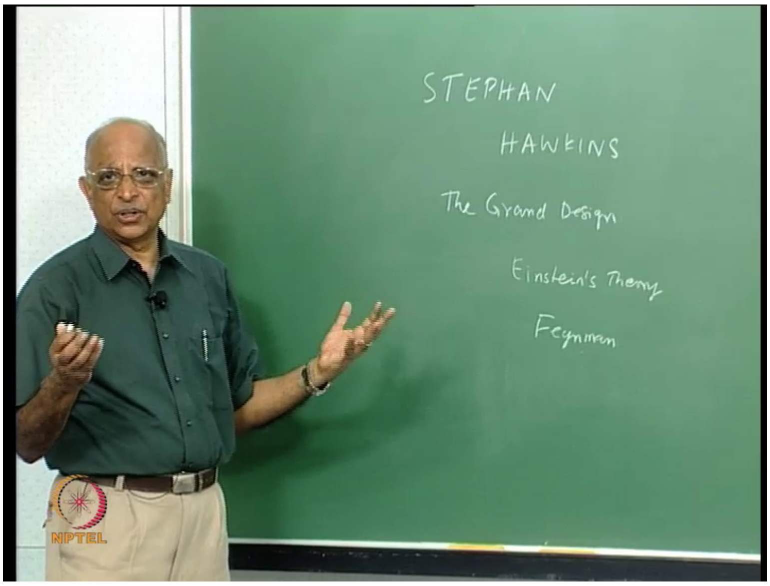

We have scientists, some very famous scientist like Stephen Hawking. He recently published a wonderful book known as the Grand Design. He talks in terms of a unified model for explaining the laws of nature. In fact, in this particular book The Grand Design, he talks about the phenomenological theories of Newton. Then he goes ahead to Einstein's theory and the pioneering work of Feynman. It will be nice to read through. But, all what I want it to say is Stephen Hawking among other great scientists has looked at the forces in nature but the reason why such forces exist is still not firmly clear.

Einstein gave an explanation for us to understand why such attractive forces should exist in the region of a body and the attraction of a lighter body by a heavier body.

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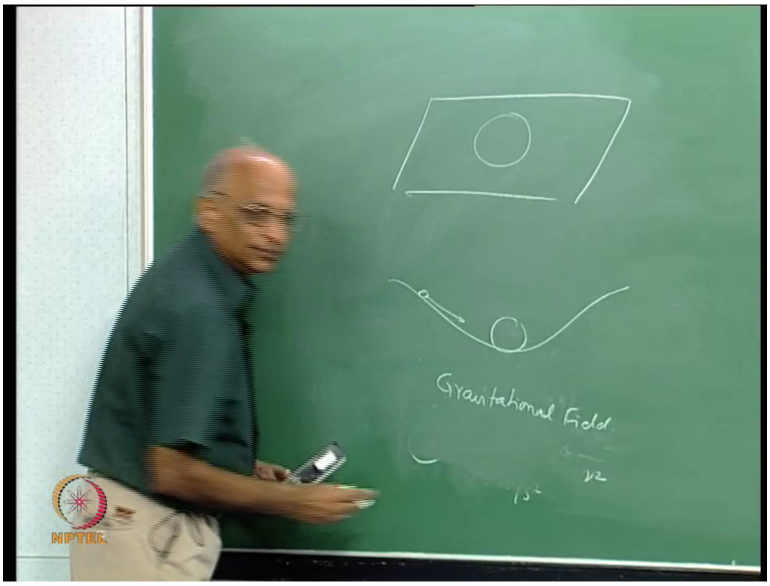

The reasoning was like this. Supposing I hold something like a towel or rubber sheet tight like is shown. In the center of the rubber sheet I put an iron ball. What would happen? The sheet would immediately sag as the heavy body pulls down the sheet. Now, if we were to place a small ball adjacent to the heavy ball. It will roll towards the heavy ball; that means, this heavy ball creates a field, which attracts the lighter ball. This helps in visualizing why a gravitational field should exist.

But, precise reason for gravitational field is still not understood. We can only understand it through an example like given above. May be when I have a heavy mass I have something like a gradient and that gradient attracts a smaller mass.

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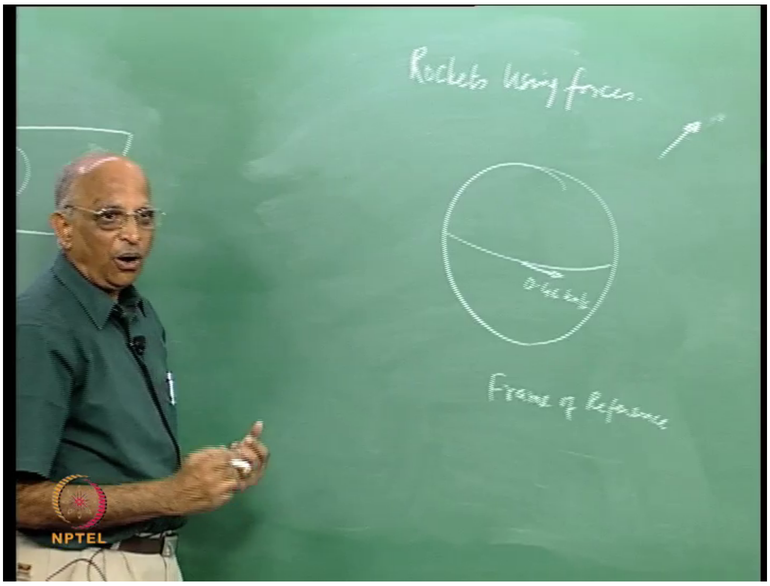

There is one problem, which we have not yet thought of regarding velocities. The problem is when we see an object traveling at a particular velocity v, how do we define its absolute velocity? We are on the Earth, the Earth is rotating and when Earth is rotating I am also rotating along with the Earth. The Earth is also revolving around the Sun. That means, we are also moving as we measure the velocity of the object in space. Our velocity is something like 0.46 kilometers per second. This is the speed with which we are moving because the Earth rotates once in 24 hours.

Now, if we are traveling at 0.46 kilometer per second and as I am rotating this body is moving. I am only able to relate the velocity of this body with respect to me? It is going to be difficult to even determine its absolute velocity. And therefore, I need to have something like a frame of reference in mind so that somebody else does not contradict my findings. How do I define a velocity? Is it relative velocity or absolute velocity?

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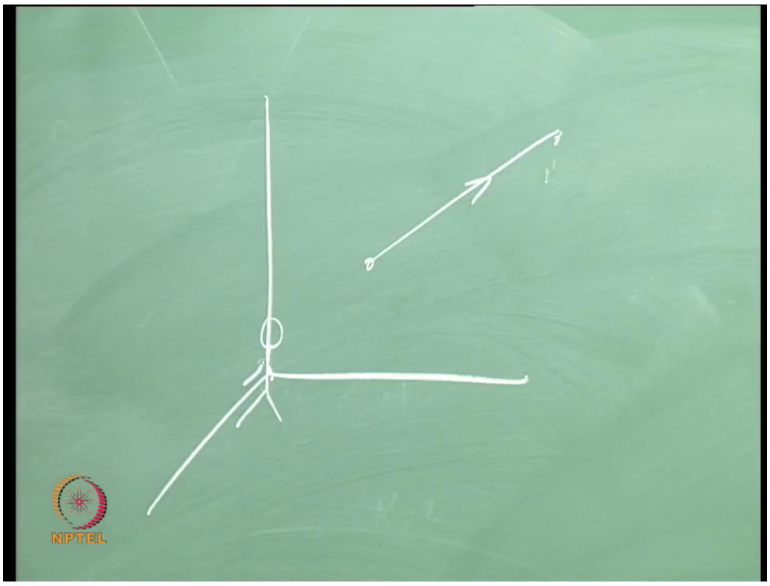

You will immediately tell me well, if I have if I am absolutely stationary like for instance I am standing here. This is my coordinate system. May be I stand over here I am absolutely stationary and watch a body that is moving; then I can say that distance travelled by the body divided by time gives me the velocity.

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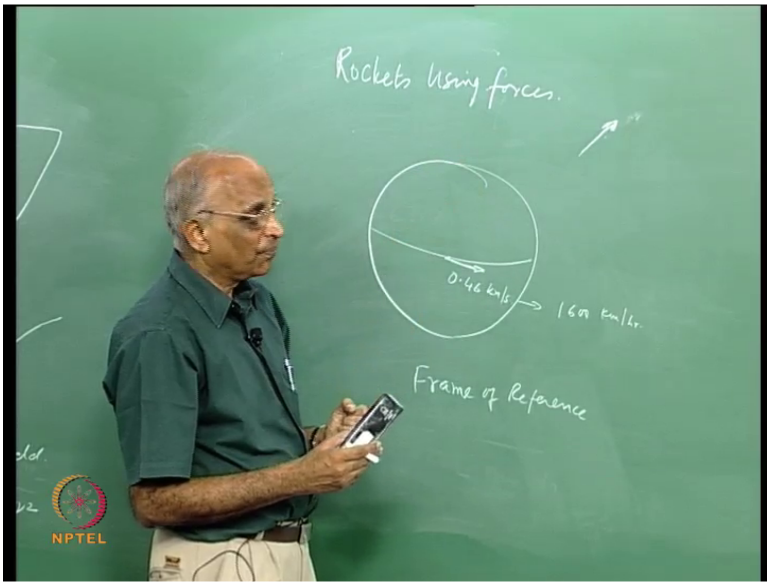

But if I am on Earth like all of us are, what is my speed? Mind you 0.46 is kilometer per second will translate into something like 1600 kilometers per hour which is going to be faster than the fastest car that I can imagine. I am also moving on the surface of the

Earth as an object is moving in space and my appreciation of the distance travelled by the body depends on my movement.

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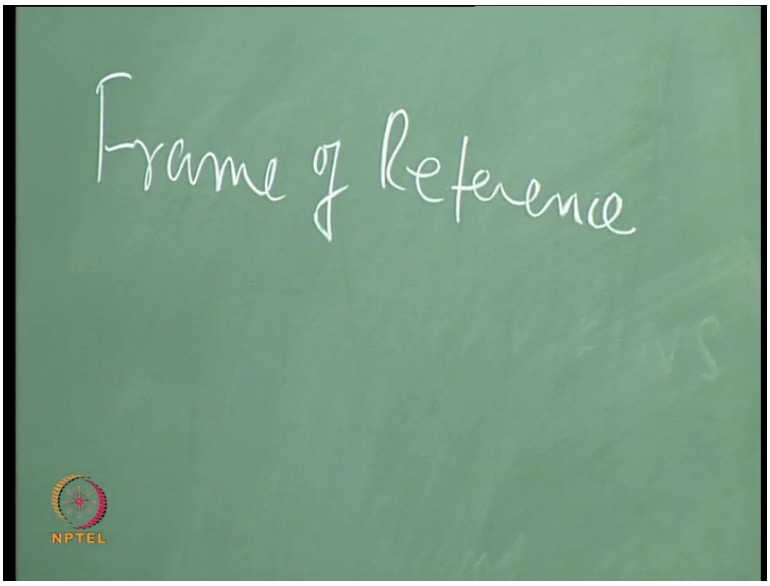

Therefore, we find everything is relative and we need a frame of reference to be able to describe the motion of bodies in space.

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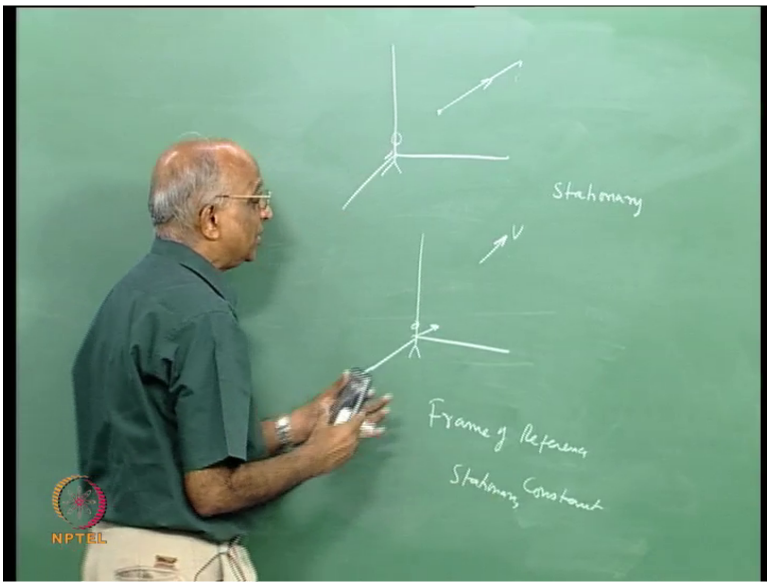

Therefore, in order to be able to do I must either be totally stationary which is difficult or else I must also tell myself if I am not stationary, if I am moving at a particular constant velocity, I can record changes in velocity of bodies clearly.

Supposing, I were to move at some constant (linear) velocity and I am observing a body at moving at a different velocity; the change in velocity of this with respect to my constant velocity will always give me the same change. Therefore, my frame of reference, whether I consider to be stationary or to move at constant velocity will provide me the change in the velocity of this body. Otherwise, if I am here on this Earth and it is rotating and all that my translation velocity is continually changing, I cannot really monitor the change in momentum or change of velocity of the body. Let us put this down.

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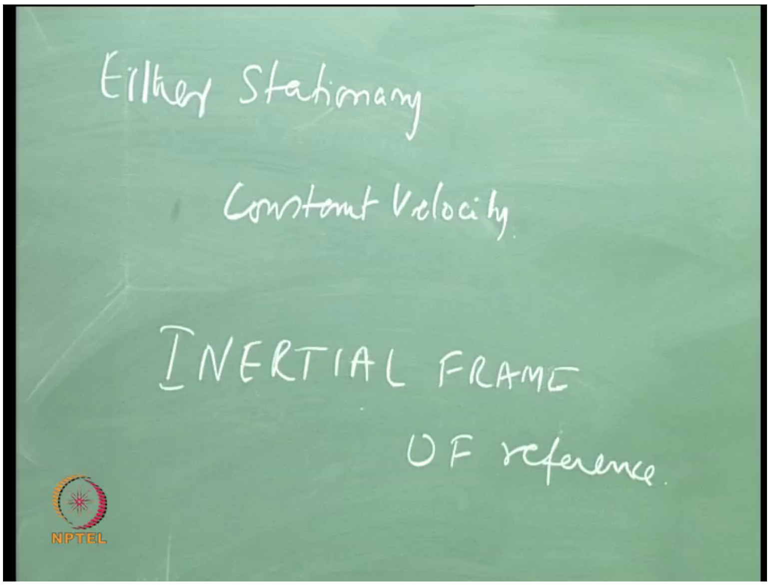

A frame of reference which is either stationary or is moving at constant linear velocity is known as the inertial frame of reference. This would provide the velocity changes and momentum changes which would be independent of the frame of reference.

Many of you are studying the subject of combustion. I will ask you a question?

What is the difference between flame speed and burning velocity? Are they the same? No Answers? Now, frame of reference is extremely important in any engineering problem. That could be a clue.

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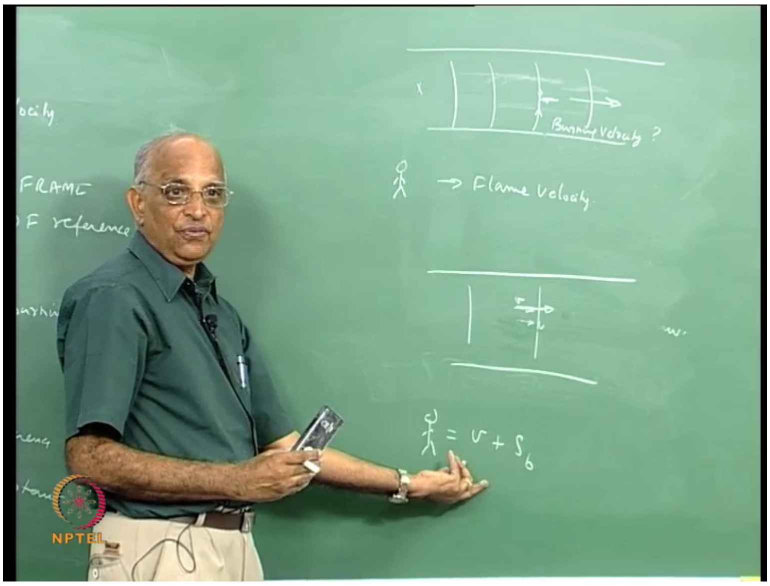

Let us say I have tube filled with a combustible gases and that is how we measure the flame speed in this tube after igniting the gases. When is it that I measure the flame speed or burning velocity? What gives me the flame velocity?

Suppose I am standing here i.e., I am stationary and the apparatus is also not moving. I measure the flame speed. I observe it from my frame of reference. I call this as flame velocity. But, if I were to sit over on the flame, I will find the gases coming towards me with a velocity. In the frame of reference of the flame, I determine the velocity with which the gases are coming towards the flame. This is what is called as burning velocity.

What is the difference? What is it that I have done? Should it still be the same or should it be different? Now, I pose this question to you; I have the tube, I have the burnt gases and the flame which are pushing the unburned gases ahead of it. The flame now moves in a region in which the gases also move. When I look at the flame from outside I am looking at the flame as it were moving in the tube. That means, I am looking at the speed of this flame as it moves along with the unburned gas. This will be higher than what I would measure sitting on the flame. Therefore, we have to be clear about the frame of reference with respect to which measurements are done.

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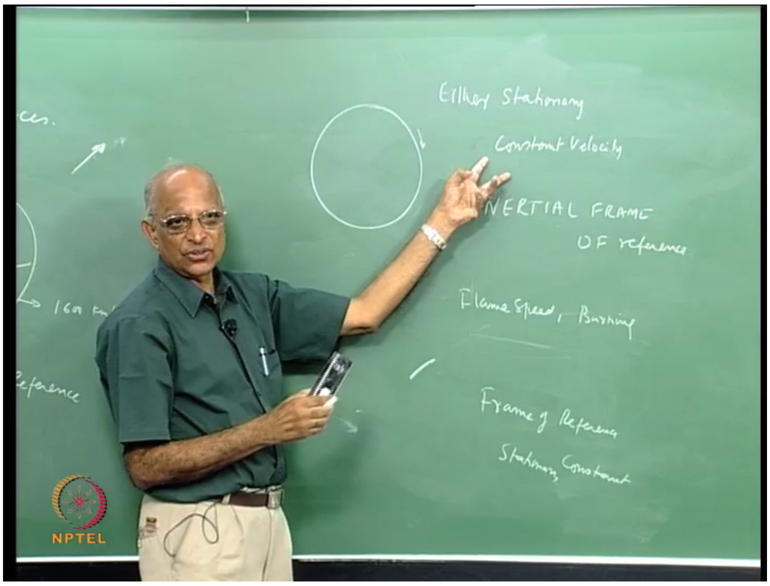

The inertial frame of reference is either a stationary frame of reference or a frame of reference which moves at constant linear velocity. If we are in a rotational frame of reference, we need to be more careful to describe the motion correctly. We will look at this in the next class.

To summaries then; what is it we have done so far?

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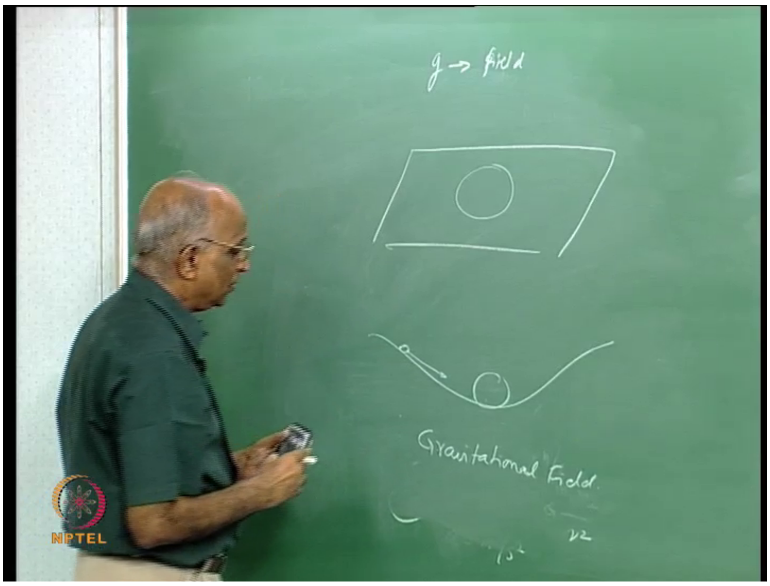

We looked at the parameters used for describing motion in space. We looked at the constituents of space. We talked in terms of planets moving round the Sun in the Solar system. We found that the planets are in a state of continuous fall on to the Sun and the planets are all freely falling towards the Sun. And Newton saw the commonalty between a freely falling planet and a body like an apple falling to the ground and formulated the universal law for gravitation. We looked at the value of g which we said is the gravitational field in m/s^2. Towards the end we looked at the frames of reference for describing motion in space and we said that to be able to measure momentum or momentum change; it is necessary to have an inertial frame of reference.

I continue with this in the next class. We will get into a rotating frame of reference, what are the corrections required to describe the motion correctly in the rotating frame of reference and that will help us to define orbital velocities and also the requirements of rockets.