Rocket Propulsion

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Lecture No. # 30

Efficiencies due to Mixture Ratio Distribution and Incomplete Vaporization

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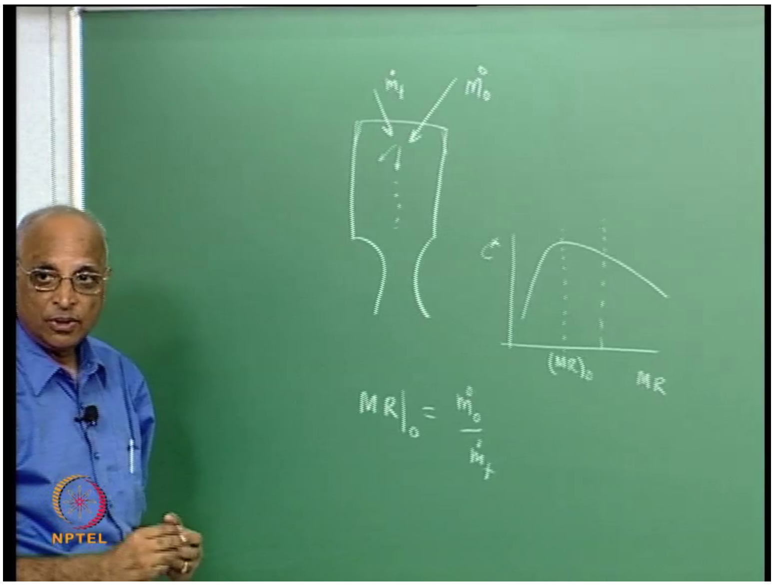

In the last class, you will recall I told you in a thrust chamber of a liquid propellant rocket, the mixture ratio is not distributed uniformly. Why was this? We said maybe fuel is used for cooling or you could have barrier cooling. We said in some injectors like shower head, the droplets travel straight into the chamber from the injector; whereas, in some cases the droplets are given an orientation. Therefore, it is really difficult to determine the performance of a chamber since the mixture ratio is varying in it. In the earlier classes, we found the value of C* as a function of mixture ratio and we wanted to operate it in the fuel rich region. Let us say this is stoichiometric mixture ratio. We have maximum value of C* in the fuel rich region. We would choose an injection mixture ratio to be in the slightly fuel rich region.

That means, we inject some mass flow rate of fuel m°f in kilograms per second and some mass flow rate of oxidizer m°o into the combustion chamber, and the mixture ratio that we get at injection, let us call it as nominal or injection mixture ratio ‘o’. It is Ro = m°o/m°f. We would like the C star to be a maximum. Therefore, maybe we adopt this as the mixture ratio and we call this as mixture ratio corresponding to injection. But what is it we found? We found that the mixture ratio at the walls would be different; mixture ratio in the core would be different; and it would be different at the different places. Now how do you compute the C*? We want to calculate the C* for this chamber and since C* × the thrust coefficient of the nozzle = Isp, well the Isp and C* are going to be different from the value corresponding to the mixture ratio value at injection.

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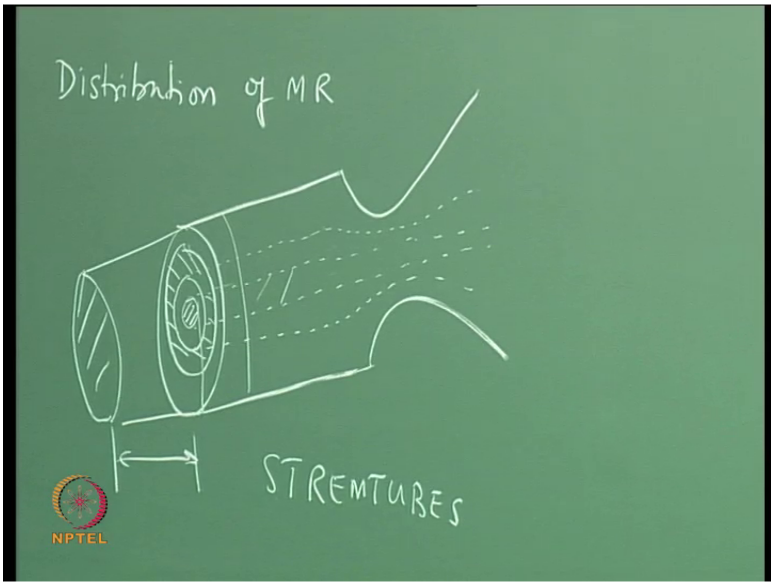

Therefore, we must find out what is the effect of variation of mixture ratio in the chamber? I think that is the one thing which we have to do without it we will not be calculating the parameters like Isp and C* correctly. Therefore, first I would like to say is that we need something like a distribution parameter distribution of mixture ratio in the chamber. Some injectors generate such fine droplets and the fans are so well distributed, whereas, in some cases it may not be that well distributed. When we have film cooling, we have fuel rich zone near the wall and maybe the core might be different, therefore, we need to have some method of characterizing the distribution of mixture ratio. How can we do it?

Why not we compartmentalize the mixture ratios. Maybe we look at a zone here, maybe I look at a zone surrounding this, maybe we look at a zone still surrounding the outer zone and so on and we find the mixture ratio at the different zones and then have something like a distribution parameter.

Let me explain it through another figure. This is the chamber and we have a nozzle. We are looking at the distribution of mixture ratio; maybe in this zone of the core we have a particular mixture ratio; and let us say the surrounding core has another mixture ratio and then it evolves further in the same way. The same evolution extends into the nozzle since the length of the chamber is small. So on we have different regions of mixture ratios.

Maybe this is the third region of mixture ratios. We could have a central core at a given mixture ratio; I have an annulus of a different mixture ratio; I have different annuli of different mixture ratios. What would happen to C star with a distribution in Mixture ratio like the above?

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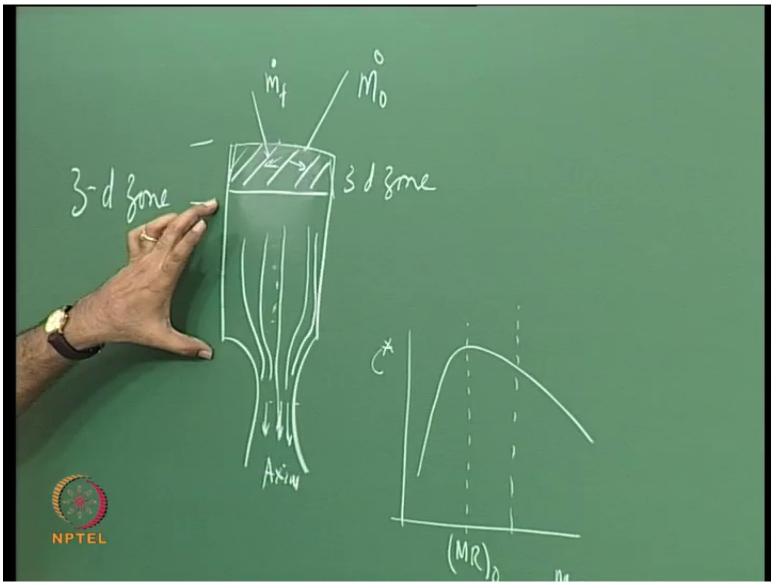

When we inject the fuel and oxidizer into the chamber, the droplets of fuel and oxidizer are initially travelling in all direction therefore we have a very strong three dimensional zone of varying parameters. And what do you mean by a three dimensional zone? Droplets are travelling both radially and axially and also in other directions and vaporization is taking place, and therefore, to quantify this zone is difficult. Therefore we say immediately downstream of injection because we inject in different directions we have let us a three dimensional zone and the process of evaporation and mixing would be three dimensional. But we also know when we come to the throat of the nozzle, the flow is sort of axial. If the flow is going to be axial at the throat somewhere upstream the flow should have become axial, in other words if the flow is along the axis at the throat maybe the flow should have come like this as streamlines.

In other words even though in the zone downstream of injection we have a highly turbulent three dimensional zone, somewhere along we should have started getting flow along the axis because at the throat it is anyway axial. Therefore we say initially we have a three dimensional zone, but subsequently we can think in terms of tubes or stream tubes which are progressing axially. Now I slightly extend this figure I say this is my injector over here, I have initially a zone wherein the three dimensional flow takes place, and thereafter since the flow at the throat is one dimensional I have something like tubes or something like stream tubes in which combustion takes place.

Now a rocket chamber is not that long after all and we do not have sufficient length for one stream tube to get effected by the neighboring stream tube, because it takes some time for fully developed turbulent to manifest itself. Therefore, we can say in each of these small stream tubes I can construct as many stream tubes as I like, in each of the stream tubes I could have something like a laminar combustion taking place and the stream tubes do not mix with each other. Why they do not mix? The chamber is short and we do not have sufficient time for turbulence to develop and intermixing between stream tubes to take place. Therefore we make these assumptions and these are quite valid because of the short length of the combustion chamber.

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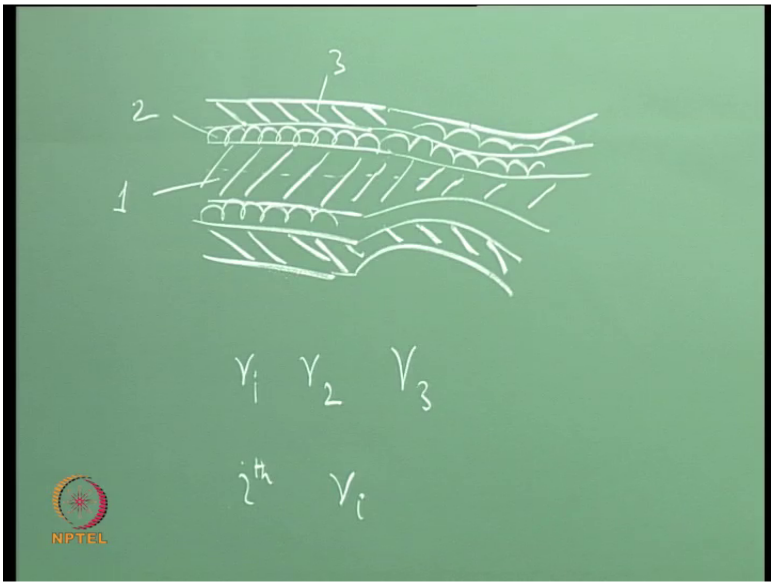

And now we can think in terms of the total combustion in the chamber proceeding in a series of stream tubes. We say this is the centerline of the chamber. We have initially one central core and let us say it has mixture ratio R_1, surrounding it we have another stream tube this is now an annular stream tube with mixture ratio R_2, surrounding it we have another stream tube with mixture ratio R_3 and so on. And when it comes to the nozzle the stream tubes converge and diverge out; maybe you have a series of stream tubes which burn, therefore, this is how it burns and gets exhausted through the nozzle. Now the core having a mixture ratio R_1, the annular stream tube surrounding it at a mixture ratio R_2, the next with a mixture ratio R_3, or rather the ith stream tube that we consider has a mixture ratio R_i which is the local mixture ratio in the ith stream tube.

Now, we want the net composite performance of a series of stream tubes. With reference to the figure, the core has a certain mixture ratio R_1, maybe the outer most near the wall has a mixture ratio let us say R_5. In between, we have the third stream tube with mixture ratio R_3. Therefore, now we want to put this in terms of a single parameter say a distribution parameter. The mixture ratio at injection it is MR_0, this is the value which we are injecting which is the ratio of mass of oxidizer to mass of fuel injected.

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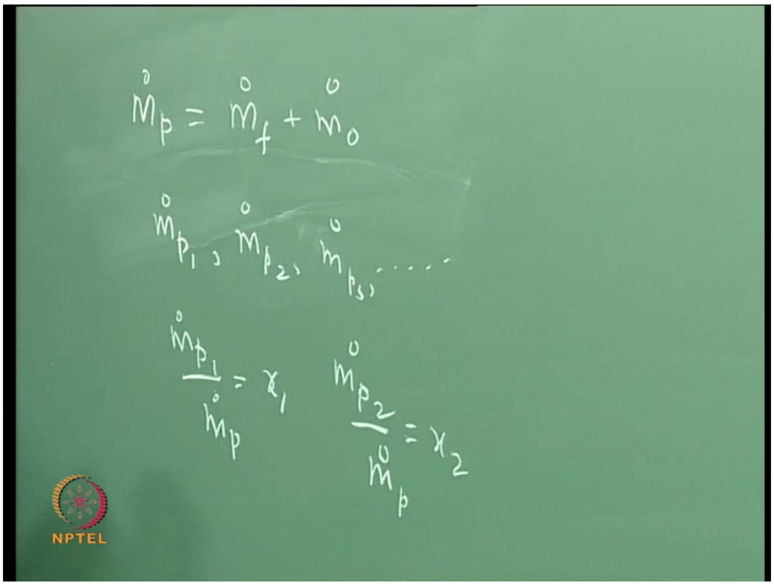

$$\overset{\circ}{m}_p = \overset{\circ}{m}_f + \overset{\circ}{m}_o$$

$$\overset{\circ}{m}_{p_1}, \overset{\circ}{m}_{p_2}, \overset{\circ}{m}_{p_3}, \ldots$$

$$\frac{\overset{\circ}{m}_{p_1}}{\overset{\circ}{m}_p} = x_1 \quad \frac{\overset{\circ}{m}_{p_2}}{\overset{\circ}{m}_p} = x_2$$

Therefore now, we can say that the mean mixture ratio at injection is equal to MR_0 whereas each of the stream tubes has a value R_i, with ‘i’ depending on the number of the stream tube. The total propellant, which is injected is equal to mp° so many kilograms per second is equal to m° of fuel + m° of oxidizer. Now the total quantity of propellant, which is injected is shared between the different stream tubes. Let us say the stream tube 1 has a mass of propellant flow rate mp°_1, the stream tube two has a propellant flow rate mp°_2, mp°_3 is the propellant flow rate in the third stream tube and so on. We can divide the flow rates in the individual stream tubes by the total propellant flow rate mp° and call these values of the fraction; that means, x_1 is mp°_1/mp°, x_2 is mass flow rate of propellant mp°_2/mp° and so on. We therefore have fractions x_1, x_2, x_3 that is the fraction of propellant flowing in stream tube 1, 2, 3 and so on.

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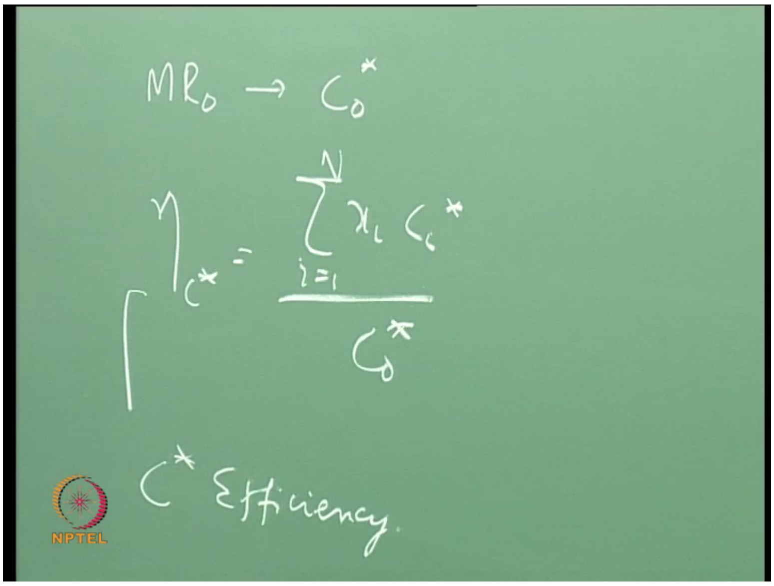

$MR_0 \rightarrow C_0^*$

$$\eta_{C^*} = \frac{\sum_{i=1}^{N} x_i C_i^*}{C_0^*}$$

C^* Efficiency.

What would the summation of x be? Summation of the mass fraction of mass flow rate going from let us say stream tube 1 to the nth stream tube when we have ‘n’ streamtubes is one. This is my first equation.

And my second equation concerns the mixture ratio and C*. We have mixture ratio over here corresponding to a lower mixture ratio or corresponding to a different values of mixture ratio in each stream tube. Let us say denote the value of C* at a mixture ratio R_i as Ci*. It is not going to be the optimum value, but it is going to be a different value for each of the stream tubes. The mass weighted C* for the chamber would therefore be and the summation of the fractional mass flow rate in the individual stream tubes multiplied by the relevant C* ($C^* = \Sigma x_i C_i^*$). The fractional mass x_i in the ‘i’th stream tube into the value of C* at this value of mixture ratio R_i for this stream tube which is Ci star, summed over all i’s as ‘i’ goes from 1 to n gives the net mass weighted value of C*.

Now, this is going the value of C* instead to of the value of C*, corresponding to MR_0 that is the mean mixture ratio at injection. Let us say the value at MR_0 is C_0*. Then the value of efficiency what we get is equal to x_i into C_i star as ‘i’ goes from 1 to n divided by the value of C_0*. This is due to the mal distribution or distribution of mixture ratio. What is it we do? We calculate the performance of each stream tube as if it were an individual rocket; we calculate the fraction of the propellant in the stream tube and the summation gives us the mass weighted C*. The mass weighted or averaged out value of C* divided by the conditions at injection is what gives us the value of C* efficiency.

I think this concept is useful and we tried it around 10 to 12 years back and found it to be useful. We can derive an idea on how to distribute mass of fuel and oxidizer such that we can get the optimum value of C*. This is because we have to distribute the propellants in the chamber and when you distribute it you get variable mixture ratio, and the assumption is individual stream tubes do not mix with each other, and why they do not mix? Because the length of the combustor is short, the turbulence still does not develop and therefore, mixing between stream tubes is not important. And now how we distribute the propellant mass and the mixture ratio? Can we characterize a distribution parameter?

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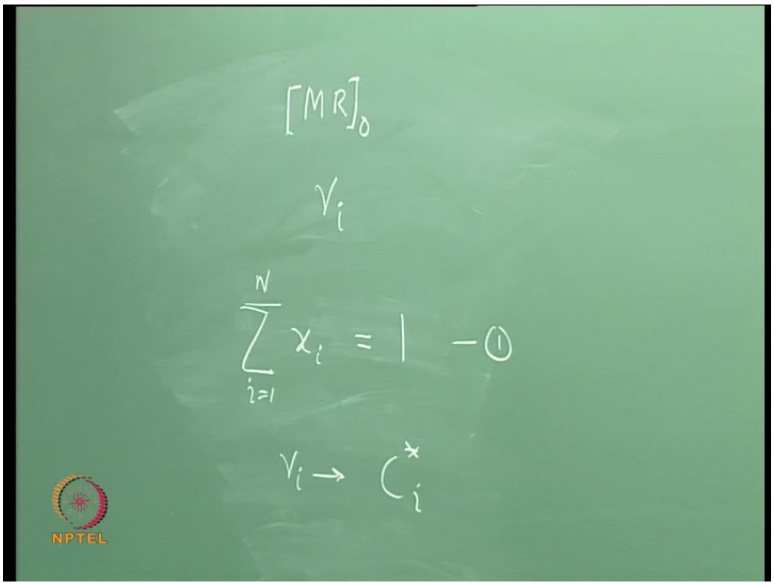

$$[MR]_0$$

$$r_i$$

$$\sum_{i=1}^{N} x_i = 1 \quad -①$$

$$r_i \rightarrow C^*_i$$

What should the distribution parameter consist of? Let us erase this out and put the distribution parameter.

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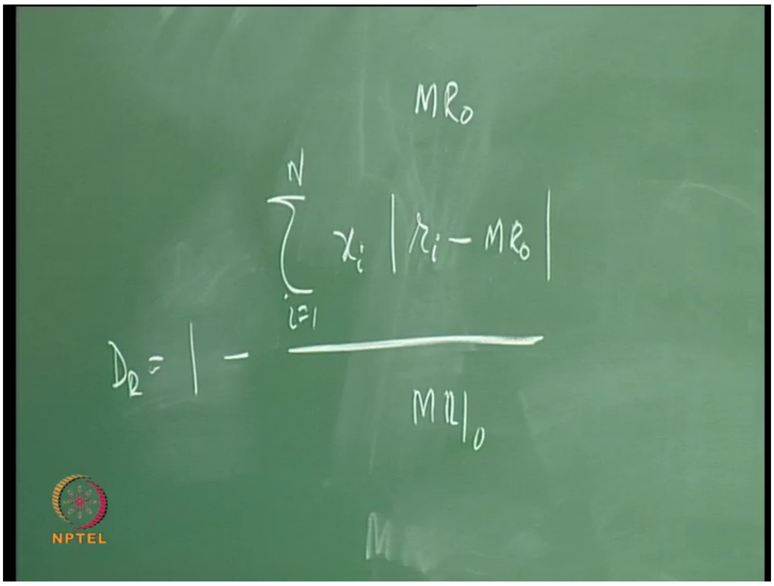

In the stream tube i, the value of the mixture ratio is let us say R_i, the overall mixture ratio is let us say MR_0 and the change from the mean is equal to R_i minus MR_0. This is the change in mixture ratio in the 'i'th tube from the mean value at injection. It could either be positive or negative value depending on whether the local mixture ratio in the 'i'th tube is less than or greater than MR_0. The change from the mean is the modulus of this value is the value. The fraction of the propellant in the stream tube is x_i. We can represent a distribution parameter in mixture ratio as D_R as equal to the fraction of the propellant in the particular stream tube x_i into the modulus of the deviation in this stream tube summed over all the stream tube going from 1 to n, and we divide it by the mean value of the mixture ratio MR_0. In other words, we get the distribution parameter D_R as equal to the dispersion multiplied by the fraction divided by the mean mixture ratio.

If we have a perfectly homogeneous mixture ratio distribution everywhere, we get the same value, well my value comes out to be 0. Therefore, I have to make some changes in the definition. That means, I am looking at a distribution of mixture ratios. Therefore, D_R should be equal to one minus this value. If everywhere we have the same mixture ratio well my distribution parameter is unity. If we have at different places different mixture ratios, the deviation from unity will tell me what is the distribution index, which could be termed as mal distribution or distribution parameter.

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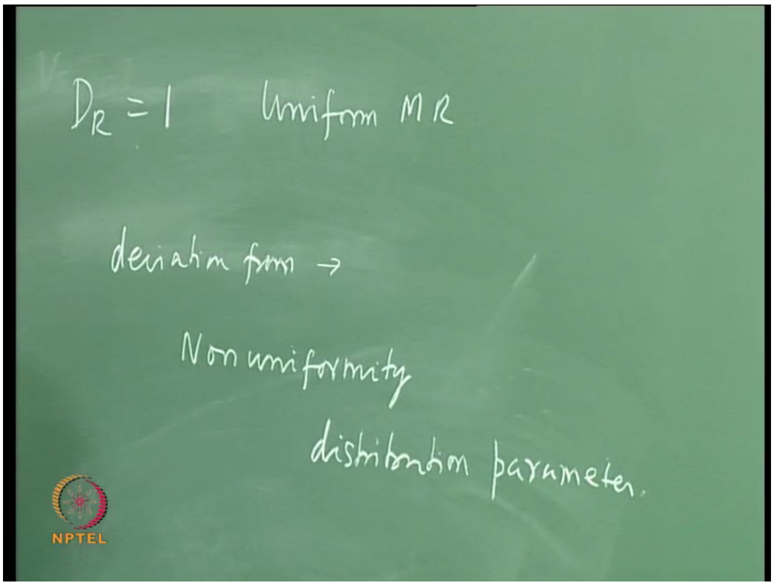

$D_R = 1$ Uniform MR

deviation from →

Non uniformity

distribution parameter.

The distribution parameter D_R; if it is 0, it tells us that we have uniform mixture ratio, whereas, the deviation is > 0, and it is always going to be less than 1, tells that it is not uniformly distributed. The amount by which it is less than 1 tells us how much it is deviating from a uniform distribution. And therefore, we say D_R is a non uniformity parameter for mixture ratios.

Now, we would like to again revisit the problem of C star efficiency, We found it to be given by η_{C*} is equal to sum of the x_i's into C_i*'s divided by C_0*. We expect it to be less than one. There is no way it can be greater unless something wrongly specified and you have a lower value for the mean. How does η_{C*} depend on D_R? Well, let us plot it out.

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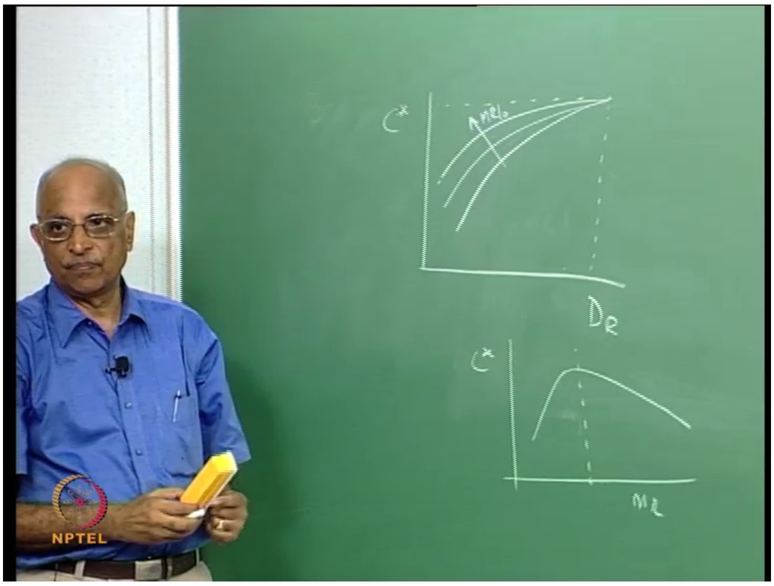

We have the value of C star as a function of D_R. When the distribution parameter is unity we get the value corresponding to the maximum. As the distribution parameter decreases the C* value falls down, but when we looked at this particular figure which I keep erasing out each time; what was the shape like? It is like this. This is the nominal value of let us say MR_0 which I choose. I find that the slope of this in the fuel rich region is somewhat steeper than in the oxidizer rich region. This suggest that as MR_0 increases, the change in C* from the maximum will be smaller and the line will be little less steep. Therefore, if we increase the value of the nominal mixture ratio at injection MR_0, we will get curves do not change very much from the nominal value as the non uniformity of distribution increases. The shift is a function of the mixture ratio.

All what I am saying is if we were to have the nominal here in the oxidizer rich region, then the change in C* due to mal-distribution of mixture ratio is little bit less, whereas, if we choose a value here in the fuel rich region for which the gradient in C* is higher, the value of C* is more adversely affected by the mal-distribution of mixture ratios. The mal-distribution causes a penalty in C*.

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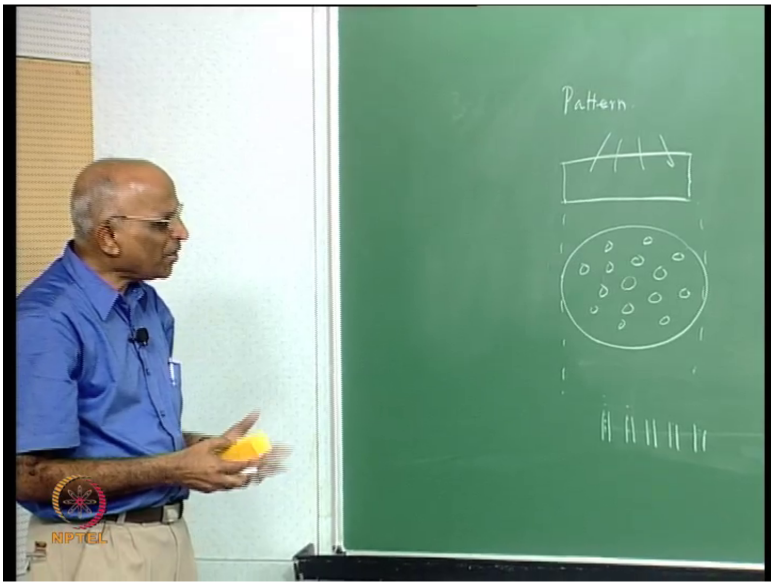

The mal distribution or distribution of mixture ratio in the combustion chamber or thrust chamber is of cause of concern and leads to a penalty. How do you qualify this penalty? In terms of efficiency which is the real value of C* due to distribution divided by the value of C* at the nominal injection conditions. And how do you get this? Let us denote the actual value as C*net or C* equivalent that you calculate from the C* corresponding to the individual stream tubes and the fraction of mass of propellant flowing through the stream tubes. I hope this is clear. You may not find this in text books, but it is something which we must understand.

Can we restate your question; you want to know how to determine the distribution of mixture ratio? Well you know the injector type that you use, you know how it is forming droplets, you know the distribution of droplets at the head, you know what type of droplets of oxidizer and fuel are coming out of the injector. But how do we estimate the value of let us say x_i for each of the stream tubes and the value of C_i^*. You know how C star varies with mixture ratio. The only thing you need is what is the value of x_i? You know you can think in terms of a simple experiment, let us think of it, and such experiments are known as patternation; that means, you would like to study the pattern from the injector.

What you do is you have an injector set up above a table. It could be any form of injector with either coaxial elements, impinging jet or whatever be the configuration. We simulate the oxidizer by a particular fluid, simulate the fuel by another fluid and now we do a cold test by passing the simulated fluids through this injector may be at ambient conditions or hold the chamber under pressurized condition. We put a series of test tubes here under the injector at different axial locations. We put a series of test tubes all along a little bit away in the three dimensional zone, and collect the mass of the fuel and mass of the oxidizer at the different places. We can study the pattern formed by the injector, and this is what is used to calculate the x_i. Once you know x_i at the different locations we can find out the non uniformity parameter and the efficiency due to the non uniformity.

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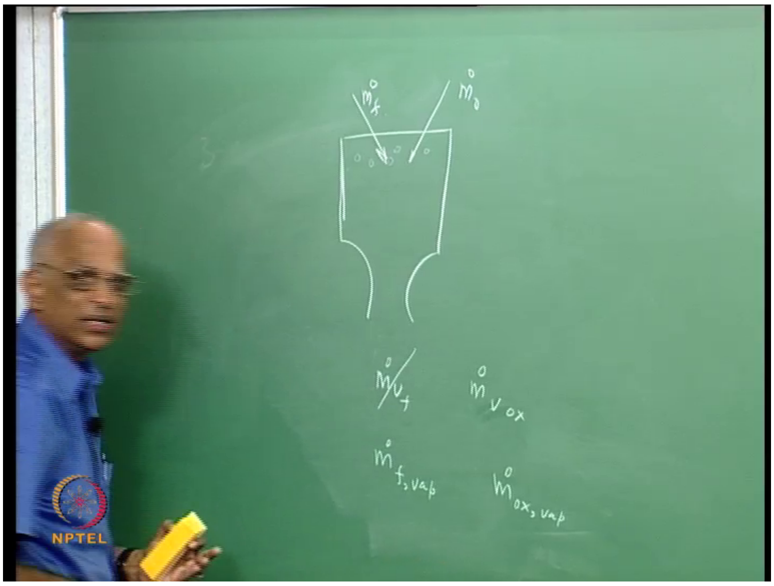

Are there any other questions?

The next problem that we consider is somewhat a little more challenging. What is it? We have the thrust chamber and we inject let us say a mass of fuel at a particular flow rate like kg per second, mass of oxidizer at a certain rate into it, and these are both liquid propellants that we are injecting. But what really reacts is not liquid with liquid; we form droplets the droplets have to evaporate, the vapor has to mix and burn. Therefore, what will react is mass of the vapor formed from the liquid fuel let us say m°vf and m° vapor of the oxidizer.

Let us make this even clearer. Instead of using this symbol we say out of liquid fuel which is sent to the chamber, not everything evaporates, and the unevaporated part of the liquid drains out. The entire liquid must evaporate if it is used for generating thrust. Of m°f the liquid fuel which is injected, only a certain portion evaporates and this mixes with the oxidizer and releases energy. Similarly, of the liquid oxidizer injected, only a certain fraction evaporates, because of the limited length of the chamber. We cannot keep on increasing the chamber to infinity to make sure everything evaporates and everything burns.

Therefore, it is quite possible that not all the fuel which is injected vaporizes and burns and not all the oxidizer which is injected, burns. After this we will discuss what is the time taken for the liquid fuel and liquid oxidizer droplets to evaporate. If all the liquid fuel and all the liquid oxidizer, which I inject into the chamber, does not evaporate and mix and burn we wish to find out the efficiency.

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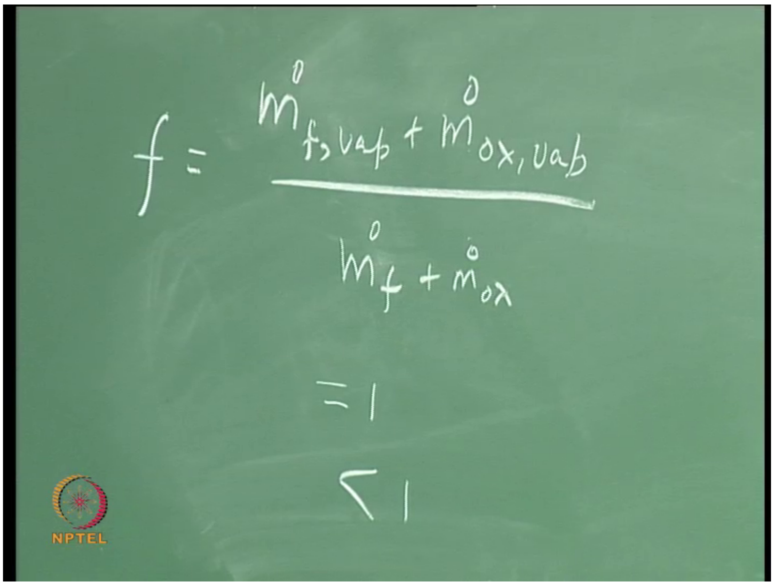

$$f = \frac{\overset{0}{m}_{f,vap} + \overset{0}{m}_{ox,vab}}{\overset{0}{m}_{f} + \overset{0}{m}_{ox}}$$

$$= 1$$

$$< 1$$

Of the total propellant which is m°f + m°o, the part which gets converted to vapor is equal to m dot of fuel which vaporized + m dot of the oxidizer which vaporizes viz., m°f,vap + m°o,vap. Therefore the fraction of the propellant which really contributes to combustion is m°fuel which is vaporized, plus m° the oxidizer which is vaporized divided by the total liquid which gets injected over here. Is it all right? That means, now we have to say this fraction, the maximum of it will be equal to 1 if the entire propellant vaporizes. If not it will be less than 1. Now we add another twist to the whole thing and ask what is the mixture ratio corresponding to evaporation?

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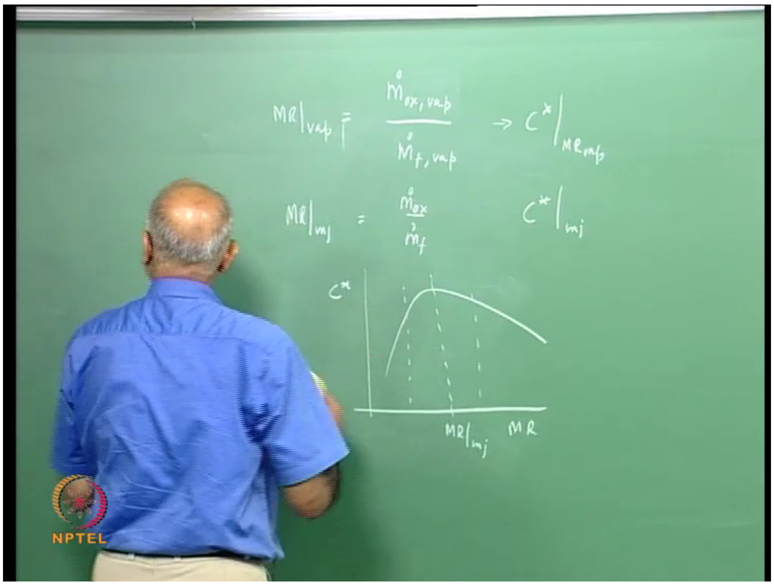

The value of mixture ratio corresponding to evaporation is equal to m°oxidizer which has vaporized divided by m°fuel which has vaporized. Now, what is the value of mixture ratio at injection? Is equal to m°oxidizer which is injected divided by m°fuel which is injected. And this is going to be different from the mixture ratio corresponding to vaporized mixture. What would it lead to?

Let us re-plot the figure of C* versus mixture ratio. The C* increases initially and then drops. The maximum value of C* is in the fuel rich region. It has a shape as shown. This value C^*_{inj} corresponds to the injection M R. The point is whether oxidizer evaporates faster, fuel evaporates faster and depending on which evaporates faster and whether the vaporization is complete, the MR changes. We could have a C* corresponding to mixture ratio corresponding to vaporization and another C* corresponding to injection which is the nominal value. And now, what is the efficiency due to incomplete evaporation and how do we put it together? We have the fraction here which evaporates and which effectively causes a reduction and we have the fraction corresponding to the vaporized and injected mixture ratios.

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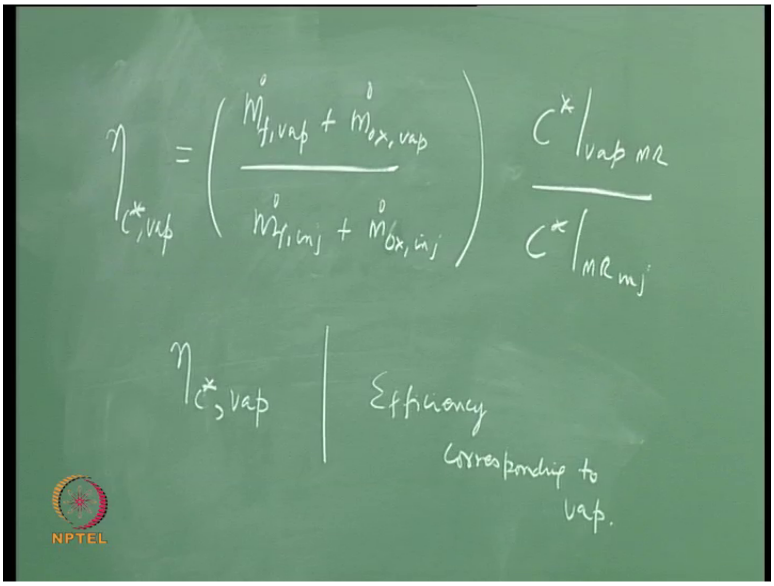

$$\eta_{C^*,vap} = \left(\frac{\dot{m}_{f,vap} + \dot{m}_{ox,vap}}{\dot{m}_{f,inj} + \dot{m}_{ox,inj}}\right) \frac{C^*|_{vap\,MR}}{C^*|_{MR\,inj}}$$

$\eta_{C^*,vap}$ | Efficiency corresponding to vap.

The C* due to evaporation being limited can be written as fraction of the total propellant that vaporizes and is equal to m°fuel which is vaporized + m°oxidizer which is vaporized ÷ m°fuel corresponding to injection + m°oxidizer corresponding to injection. This fraction is to be multiplied by the value of C* corresponding to the vaporized mixture ratio and the C* corresponding to mixture ratio at injection. This gives the efficiency due to incomplete vaporization.

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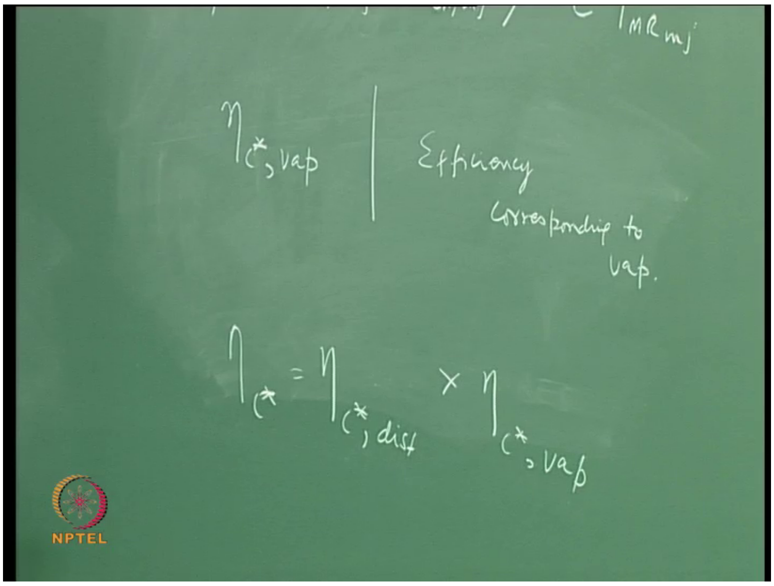

$|_{MR\,inj}$

$\eta_{C^*,vap}$ | Efficiency corresponding to vap.

$$\eta_{C^*} = \eta_{C^*,dist} \times \eta_{C^*,vap}$$

In other words, this was the value of C* modulated by the total mass of propellants which evaporates and also alters the mixture ratio and thus contributes to combustion and thrust. This divided by the value of C* at injection gives the C* efficiency. You have a value of C* corresponding to vaporization divided by the C* corresponding to injection. Depending on the choice of the injection mixture ratio, it could be either less than 1 or greater than 1. In general the C* vaporization will be less than C* at injection since we normally choose the optimum mixture ratio at injection which gives the maximum value of C*. The fraction vaporized will always be less than or equal to one. In this way we get the efficiency corresponding to incomplete vaporization.

Therefore, what will be the total efficiency due to the mal-distribution of mixture ratio and incomplete vaporization? The total efficiency would be equal to $\eta_{C^*,\, dist}$ due to distribution and that due to incomplete vaporization $\eta_{C^*,vap}$.

$\eta_{C^*,dist}$ was the efficiency due to distribution of mixture ratio and we get the next one due to incomplete vaporization $\eta_{C^*,vap}$, and the net value of C star efficiency in the chamber would depend on both. We did not consider diffusion or mixing of the fuel and oxidizer vapors and we did not consider chemical reactions, because chemical reactions and diffusion are somewhat much faster than evaporation. The evaporation tends to be the controlling event, and that is why we say the total efficiency of C star is equal to that due to the mal-distribution and that due to incomplete vaporization. We should keep this in mind.

We just made a case saying that we need vaporization to be completed in the chamber. Normally we choose L* of a chamber, and what did we say was L*? L* = volume of the chamber ÷ the throat area. This has units of length: $\text{meter}^3/\text{meter}^2$ = m, and this length scale is something which tells us whether vaporization could be completed in the chamber.

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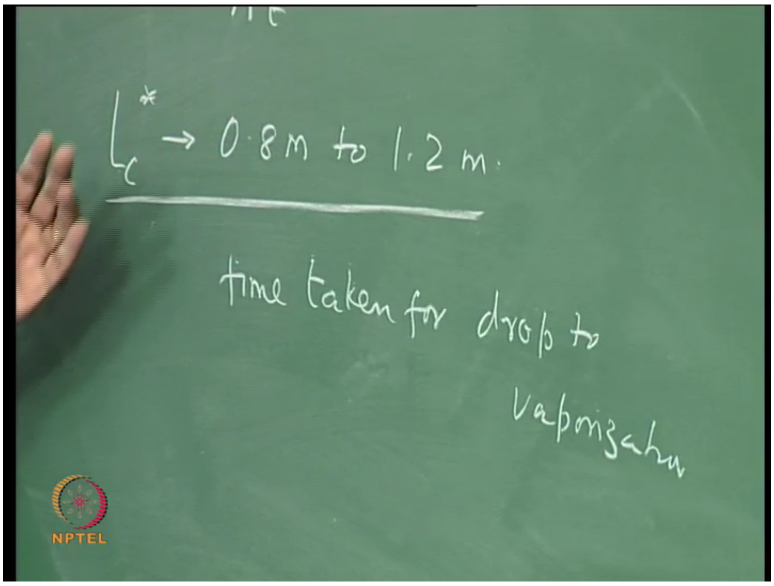

$L_c^* \rightarrow 0.8\,m$ to $1.2\,m$.

time taken for drop to vaporization

L* varies between 0.8 meters to 1.2 meters for the different propellant combinations. L* is about 0.8 meters for a hydrogen oxygen propellant system and about 1.2 m for hypergolic propellants. But we should really find out the time taken for a drop to vaporize. I think that is a more reasonable way of ensuring complete vaporization. However, these thumb rules on values of L* are based on experience and there is no way it could be wrong. What is the physical significance of L*? L* tells us the distance taken for vaporization and combustion to be completed and therefore is an indication of residence time during which the combustion events could be completed.

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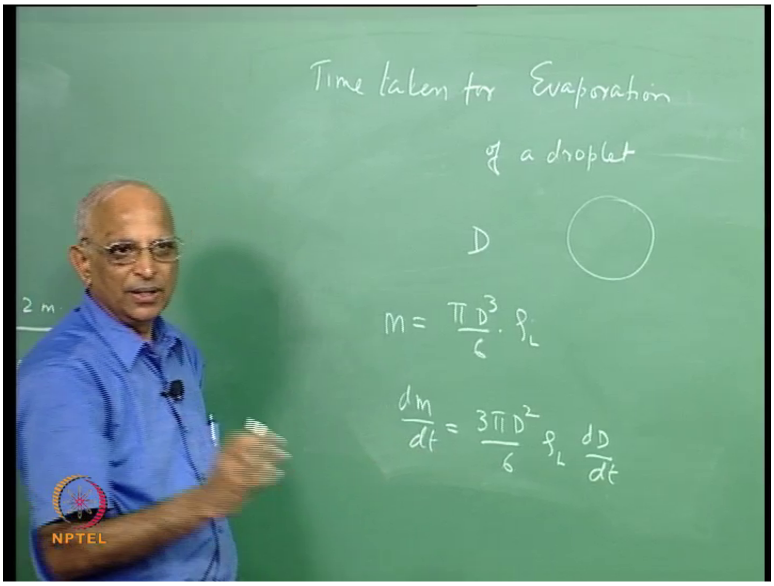

Time taken for Evaporation of a droplet

D

$$m = \frac{\pi D^3}{6} \cdot \rho_L$$

$$\frac{dm}{dt} = \frac{3\pi D^2}{6} \rho_L \frac{dD}{dt}$$

Therefore, let find out how to estimate the time taken for burning to occur or initially the time taken for evaporation to occur? Now, we introduce the word droplet. We have a series of droplets, which are produced by the injector. The mean droplet size is let us say D. We want to find out what is the time taken to for this droplet to vaporize? So, we find the mass of the droplet is equal to, assuming the droplet is spherical as $\pi D^3/6 \times \rho$. $\pi D^3/6$ is the volume, where D the droplet size is in meters and ρ is the density of the liquid that is liquid fuel or liquid oxidizer.

Now, we want to find out the rate of evaporation. We look at dm/dt and since density of the liquid fuel and oxidizer is reasonably constant, this is equal to $(3\pi D^2/6)\times\rho\times dD/dt$. Thus the diameter of the droplet changes. How the droplet diameter changes with time is a relevant parameter. But we must also keep in mind that the droplet vaporizes at the surface; therefore, you should have something to do with the surface area. What is surface area of a sphere of diameter D? It is πD^2. Therefore we should rather look at πD^2 i.e., d/dt (D^2). Since the process of evaporation is happening at the surface, we are more interested in the variation of D square rather than D with time. Therefore, let us convert it to this parameter D into D square.

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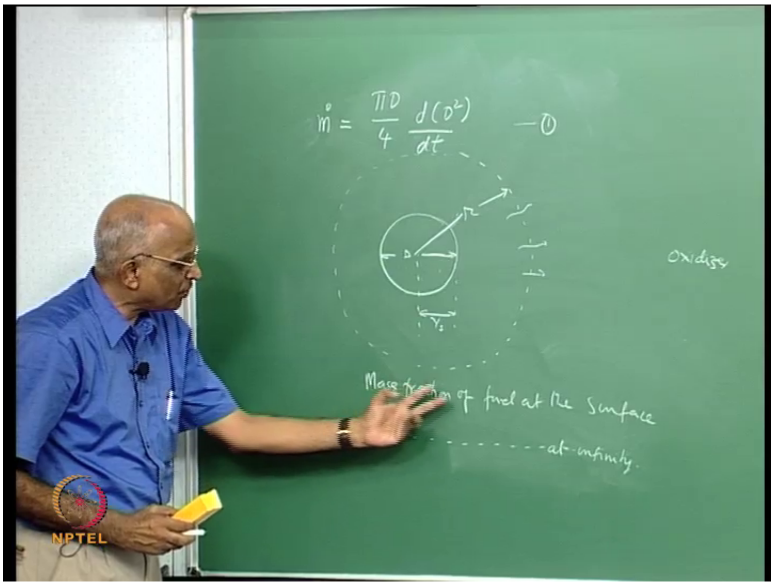

We can write therefore, $dm/dt = m° = \pi D^2/2\times\rho\times dD/dt = (\pi D/4)dD^2/dt$.

This is the first equation $dm/dt = (\pi D/4)dD^2/dt$. We need to find out what this value would be and therefore, let us picture in our mind that we have a droplet let us say of diameter D: a fuel droplet of diameter D in an oxidizing medium. Let us say the radius of the droplet is the r_s. How does this fuel droplet evaporate why should it evaporate in the first place? At the surface of the droplet maybe we have a high value of concentration of the fuel, while far from the droplet we have only oxidizer vapor. This is because we are considering the evaporation of a fuel droplet in an oxidizing medium. Let us say I have a diesel droplet in air; therefore, why should diesel droplet evaporate? I have a higher concentration of diesel at the surface, I have a very low concentration further away, and because of the concentration gradient I have the fuel migrating out of the surface.

Therefore, in other words we are interested in something like a mass fraction of fuel at the surface; we are also interested in mass fraction of fuel at infinity. Let us consider the following: If we consider the diffusion talking place at a radius r as shown and consider the area over which the diffusion takes place, the mass of vapor coming out of the droplet surface crosses this surface. We have to write an expression for the mass fraction at r_s and also at the surface.

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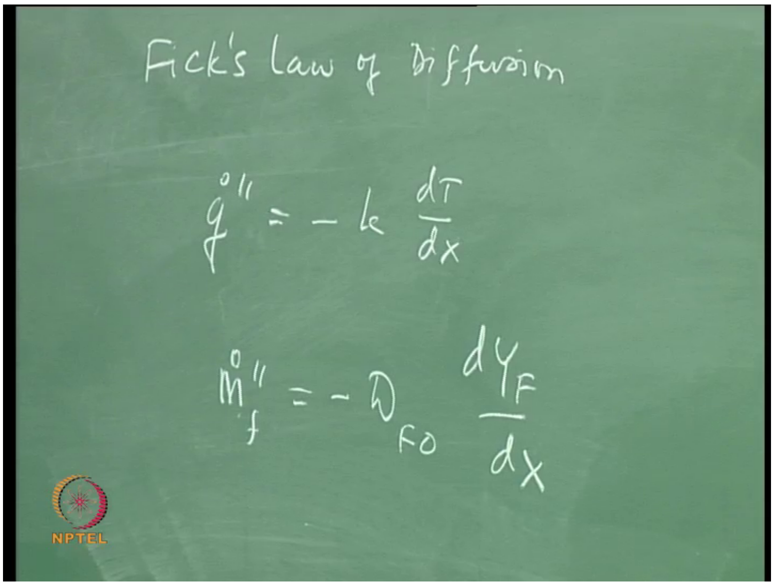

Fick's Law of Diffusion

$$\dot{q}'' = -k\frac{dT}{dx}$$

$$\dot{m}''_f = -D_{FO}\frac{dY_F}{dx}$$

We denote the mole fraction and mass fraction by *x* and Y respectively. And what is the mass fraction Y. It is equal to mass of the fuel that we interested in, mass of the fuel divided by the total mass of the fuel and oxidizer.

At infinity the value could be mass fraction of the fuel would be 0 or a small number. At the surface the value could be a large value. Let us assume that the value at the surface that we have to calculate is $Y_{F,s}$ while at infinity it is equal to $Y_{F,\infty} = 0$. This is the variation in the mass fraction. What is the law, which tells us how the mass flux diffuses? It is the Fick's law of diffusion. In case some of us are little bit rusted on this diffusion equation, let us take a look at heat transfer instead of the mass transfer. What does heat conduction equation tell you? That the heat flux is given by Fourier law as the thermal conductivity k into the temperature gradient dT by dx. Similarly if we were to write an equation for mass flux it is equal to the diffusion coefficient of the particular medium, which is getting diffused into the concentration gradient.

The diffusion of fuel equals the diffusion coefficient of fuel in oxidizer or fuel in oxygen into let us say the value of the relevant gradient. But you will also recall that the equation for diffusion is given in terms of concentration gradient. We are writing as a function of mass fraction and therefore let us clarify what the unit should be? The diffusion coefficient diffusivity has units of m^2/s while the mass flux on the left is equal to kg/m^2s.

The mass concentration has no units, because it is a fraction. The mass fraction and therefore the unit of concentration gradient is 1/m, and therefore to get the units of mass flux on the left side we must multiply the diffusivity it by the density, which has units of kg/m^3. Therefore, in terms of concentration gradient, Fick's law of diffusion states that mass flux at any distance is equal to the diffusion coefficient of the particular substance in the medium multiplied by the density of the medium at that particular point into the concentration gradient. This is Fick's law of diffusion viz., $m^\circ_f = - D_{FO} \rho \, dY_F / dx$. We have to use this law to determine the rate at which the fuel vapor is leaving the droplet.

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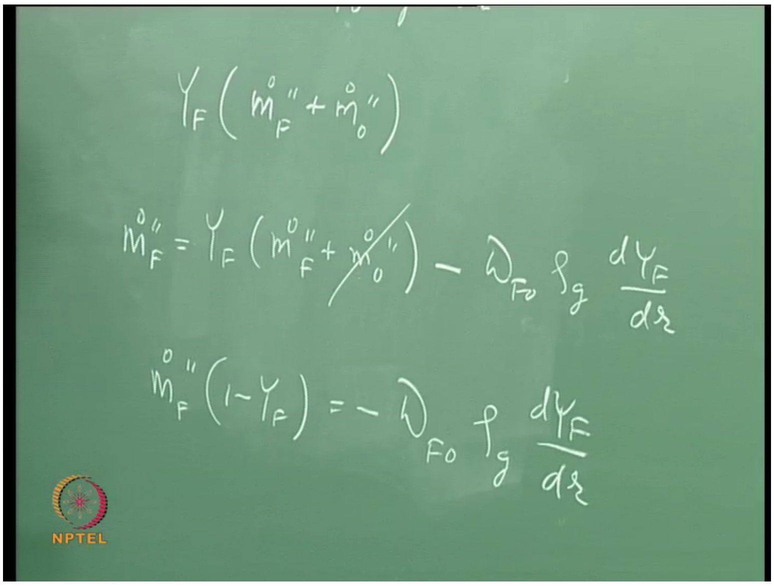

$$Y_F\left(\overset{\circ}{m}{}_F'' + \overset{\circ}{m}{}_O''\right)$$

$$\overset{\circ}{m}{}_F'' = Y_F\left(\overset{\circ}{m}{}_F'' + \cancel{\overset{\circ}{m}{}_O''}\right) - D_{FO}\,\rho_g \frac{dY_F}{dr}$$

$$\overset{\circ}{m}{}_F''\left(1 - Y_F\right) = -D_{FO}\,\rho_g \frac{dY_F}{dr}$$

Therefore let us now write out the relevant equation. We say because of diffusion due to the gradient we have minus diffusion coefficient of fuel in oxygen multiplied by the density which is the gas density. Let us be very clear. For the mass of the droplet, we should have multiplied this by density of liquid fuel ρ_L because we had mass of droplet = $\pi D^3/6\times \rho_L$, whereas here it is the density of the gas which we am considering. Let us put it as density of the medium gas $\rho_g \times dY_F/dr$, because our coordinate is along the radius r instead of x. We are considering a droplet and we am considering a symmetric spherical coordinate system and therefore the direction or cordinate is r.

Now, we have to solve this equation. But we also realize that have a concentration Y_F of let us say due to m°fuel + m°oxidizer which is available at any particular point. And the fraction of that is also available. Therefore, we must include the above and the mass of the mass flux of the fuel which is leaving = Y_F from the mass which is available of the oxidizer and fuel + the value due to diffusion viz., the diffusion coefficient × $\rho_g \times dY_F/dr$. Why did the two terms come? We of diffusion of due to the gradient but at any point you have m°ox + $m°_F$, and mass fraction of this is also available for the migration.

Therefore, the total mass flux which is available is the sum of these two, and again we note that the fuel is what is migrating; m°ox is not migrating therefore, this can be dropped, and therefore, we can write m° of the flux of fuel which is migrating × $(1 - Y_F)$ = $-D_{FO} \times\rho_g \times dY_F/dr$.

We would like to convert the mass flux into the mass flow rate. What is the value relation between mass flux and mass flow rate? We immediately find that mass flux of fuel is equal to the rate of amount of fuel which is migrating / $4\pi r^2$, this is the flux; the area is $4\pi r^2$; this fuel rate is kg/s; flux is kg/ s m^2. Therefore, we substitute the mass flux by mass flow rate m°_F/ area and the equation simplifies as follows.

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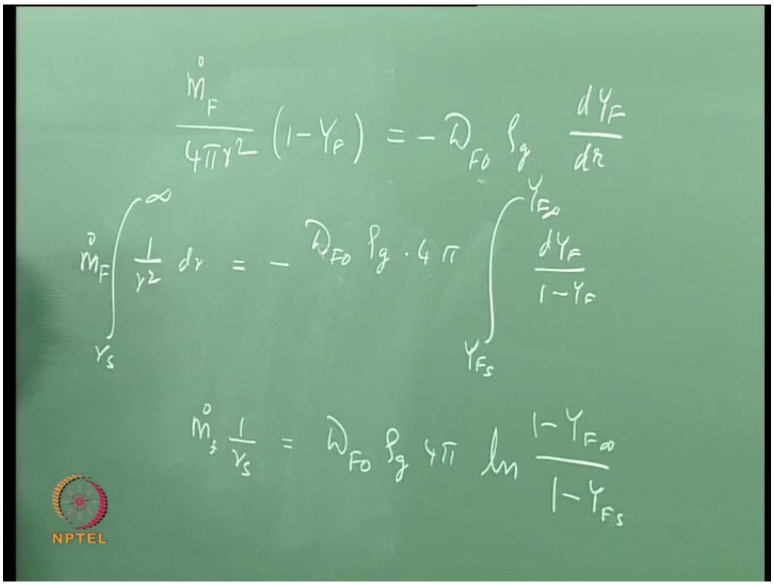

$$\frac{\overset{\circ}{m}_F}{4\pi r^2}(1-Y_F) = -D_{FO}\,\rho_g\,\frac{dY_F}{dr}$$

$$\overset{\circ}{m}_F\int_{r_s}^{\infty}\frac{1}{r^2}\,dr = -D_{FO}\,\rho_g\cdot 4\pi\int_{Y_{Fs}}^{Y_{F\infty}}\frac{dY_F}{1-Y_F}$$

$$\overset{\circ}{m}_f\,\frac{1}{r_s} = D_{FO}\,\rho_g\,4\pi\,\ln\frac{1-Y_{F\infty}}{1-Y_{Fs}}$$

We the value of m°_F fuel which is leaving / $4\pi r^2 \times (1 - Y_F) = -$ of diffusion coefficient D_{FO} of the fuel in the medium × density of the gas $\rho_g \times dY_F/dr$. We simplify it and get $dY_F/ (1-Y_F)$ on the right hand side with a minus sign. Please be careful with the signs.

The diffusion coefficient $D_{FO} \times \rho_g \times 4\pi$ also comes on the right side. The mass flow rate m°_F at the different radii is the same and this divided $r^2 \times dr$ is left on the left side. This is what we get.

What does the equation suggest? It shows the mass of fuel that migrates as a function of the mass concentration. The concentration changes from the value $Y_{F,s}$ at the droplet surface r_s to $Y_F = 0$ at infinity. Therefore, we integrate this equation between the surface of the dropius changes let at r_s to infinity, and therefore, what is it I get? I have Y_F at the surface $r = r_s$ and the corresponding $Y_{F,s}$ and $Y_F = 0$ at infinity. This is what should give me the value of m°_F. I simplify it and get the value of $m^\circ_F = -$ 1/r being infinity is 0, − of

($-1/r_s$). We integrate the right hand side to give D_{FO} ×−× dY_F by 1 minus Y_F; we have minus sign minus log of 1 minus Y_F, minus and minus sign cancels into ρ_g into 4π × natural logarithm of ($1 - Y_F$) at infinity divided by ($1 - Y_F$) at the surface.

This is the value of the mass, which is leaving the surface. But the mass which is leaving we already saw was equal to πD /4 × d D^2/dt, and therefore, we can substitute it in the relation for $m°_F$ and we get the following:

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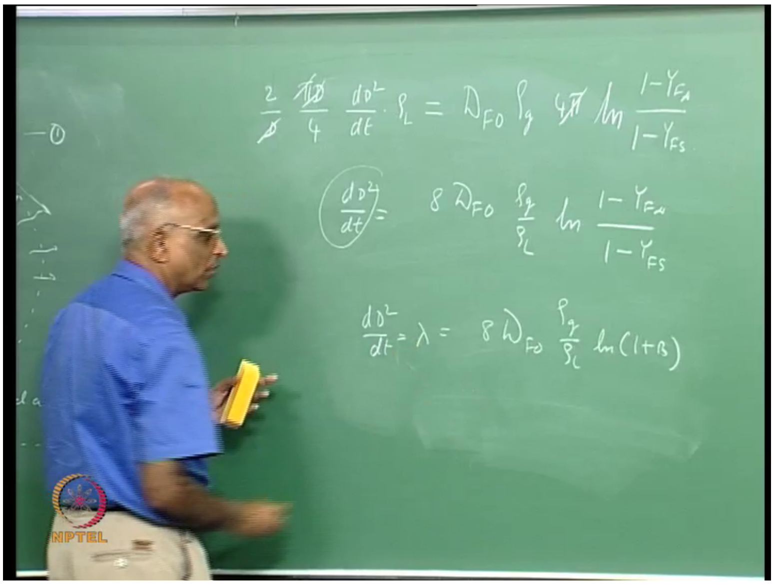

$$\frac{2}{\cancel{D}}\,\frac{\cancel{\pi D}}{4}\,\frac{dD^2}{dt}\cdot\rho_L = D_{FO}\,\rho_g\,4\pi\,\ln\frac{1-Y_{F\infty}}{1-Y_{Fs}}$$

$$\boxed{\frac{dD^2}{dt}} = 8\,D_{FO}\,\frac{\rho_g}{\rho_L}\,\ln\frac{1-Y_{F\infty}}{1-Y_{Fs}}$$

$$\frac{dD^2}{dt} = \lambda = 8\,D_{FO}\,\frac{\rho_g}{\rho_L}\,\ln(1+B)$$

The value of $m°_F = \pi D^2/4 \times dD^2/dt \times \rho_L \div r_s$ and $r_s = D/2$ while the right side is the diffusion coefficient due to the fuel in the oxidizer × the density of the gas × 4π into natural logarithm of $1 - Y_F$ far away from the droplet surface ÷ by $1 - Y_F$ at the surface.

What is it we are interested in? We are interested in solving for the rate of evaporation which is dD^2/dt. It is equal to ; D and D cancels, π and π cancels and we get 4 × 4 = 16, and divided by 2, it becomes 8 ×diffusion coefficient of fuel in oxygen × ρ_g/ρ_L × natural logarithm of ($1 - Y_F$ at infinity) / (1 − the mass fraction at the surface).

Now, when we look at this expression we find that the diffusion coefficient depends on the type of fuel and oxidizer and therefore for a given propellant combination at a given mixture ratio this is a constant. The density of the medium is specified and cannot change while density of the liquid is constant. We are left only with only two parameters. Let us put this together. We have a term (1 − the mass fraction of fuel at far away)/(1 −

the mass fraction at the surface) . Supposing we were to define a term B, which we call as equal to mass fraction at the surface minus mass fraction at infinity divided by 1 minus the mass fraction at the surface and this represents the transport of the vapor and we call this as a transport number B.

Then immediately we find that $1 - Y_F$ at infinity divided by $1 - Y_{F,s}$ in terms of B becomes equal to $1 + B$. Therefore, we can write the expression for the rate of evaporation as this particular expression = 8 ×diffusion coefficient of fuel in the medium of oxidizer × ρ_g/ρ_L × ln (1 + the transport number B). We find it really is a function of fuel and oxidizer which we find is equal to dD^2/dt which is a constant for a given combination of fuel in a given medium of the oxidizer. It is termed as evaporation rate constant. Therefore, we find evaporation rate is a constant for a given fuel in a given oxidizer medium and is denoted by λ.

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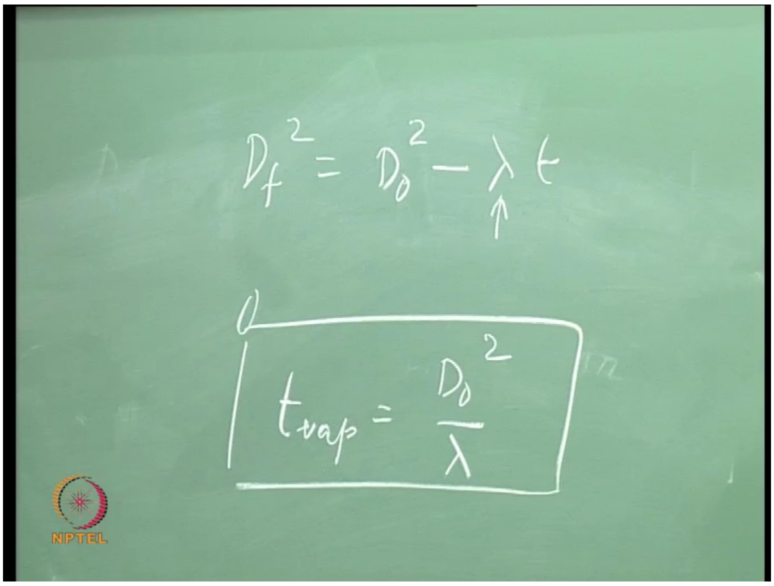

If the initial droplet diameter is D_0, the time taken for it to evaporate will go as D_0^2; that means, we looking at the final value $D^2 = 0$ giving $D_0^2 - \lambda t = 0$. The value of lambda is known from the expression derived for it and therefore, the time taken for evaporation is equal to D_0^2/λ Therefore, we know the time taken for evaporation. If the droplet is not completely evaporated, we can find out the reduction in diameter over a given time. This all about evaporation and time taken for vaporization of fuel droplets and likewise for oxidizer droplets. We should learn to play with these equations viz., the transport equations and find out the vaporization times.

Let me quickly summarize what we have been doing in this class.

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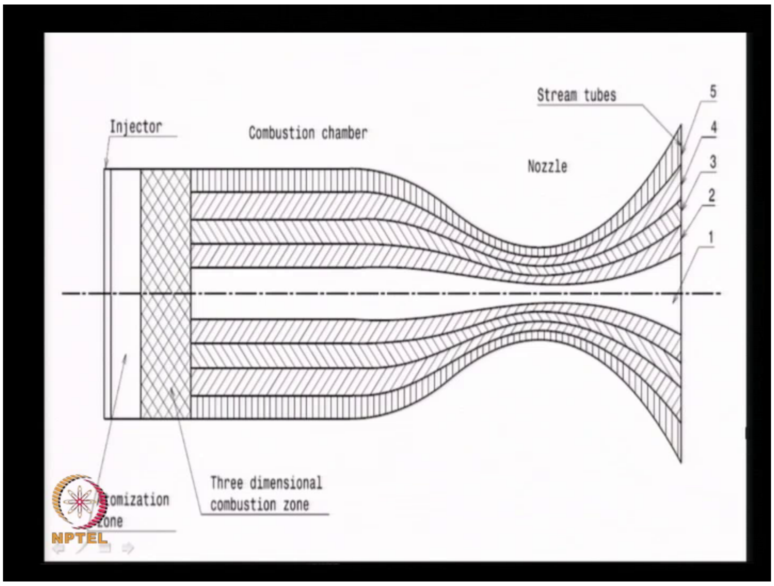

We started with a stream tube model, we said we have something like several stream tubes at different mixture ratios.

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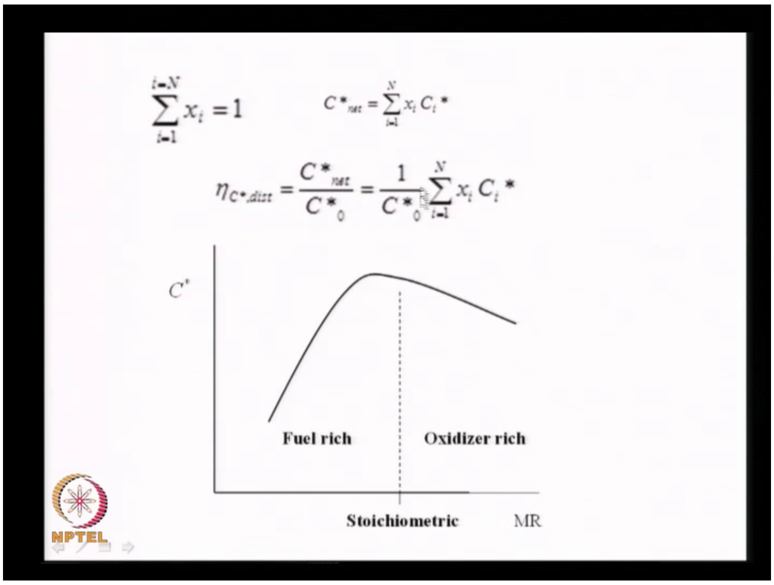

$$\sum_{i=1}^{i=N} x_i = 1 \qquad C^*_{net} = \sum_{i=1}^{N} x_i C_i^*$$

$$\eta_{C^*,dist} = \frac{C^*_{net}}{C^*_0} = \frac{1}{C^*_0}\sum_{i=1}^{N} x_i C_i^*$$

And give a value of η_{C^*} due to distribution is equal to the net value of the mass weighted C* values divided by the C* value corresponding to injection.

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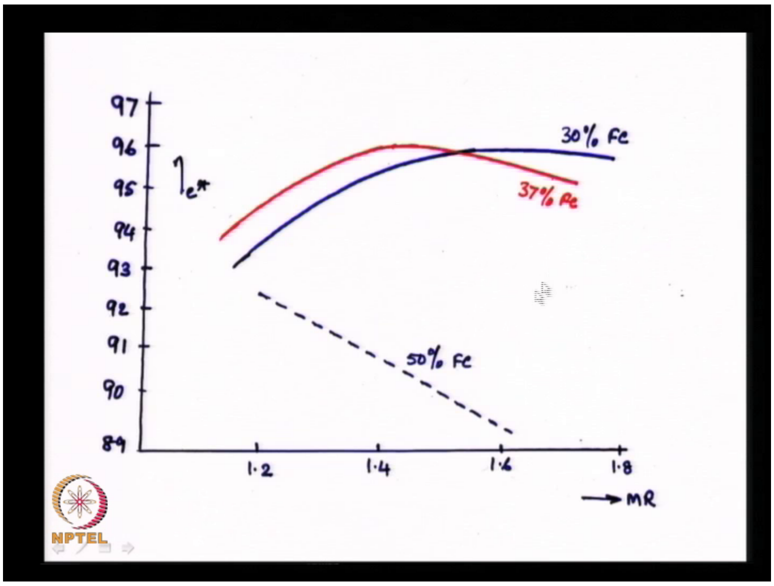

I showed you this figure and said we will revisit it later. When we have different values of mixture values at injection and different distributions due to changes in film cooling, you find that the C* falls rapidly due to mal-distribution. It changes at different values of mixture ratios. It is so happened in this experimental engine the peak value was about the same, but normally we would expect a lower value of C* as the percentage of film cooling increases.

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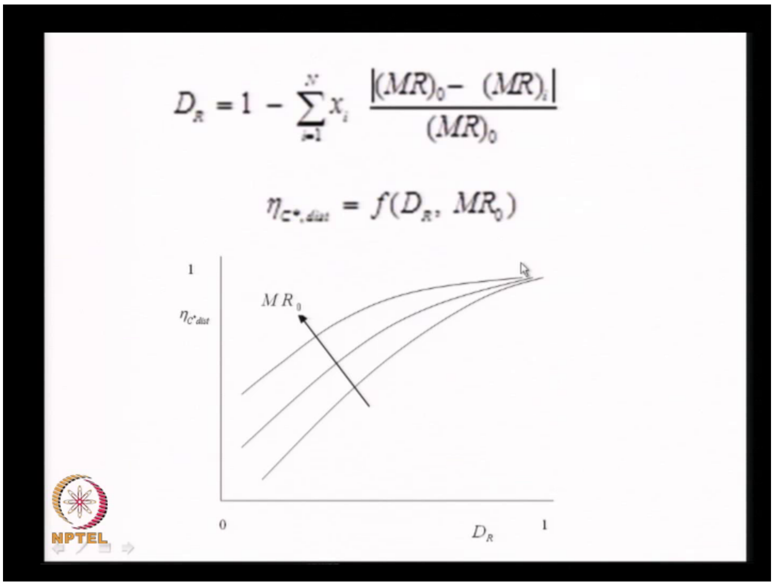

$$D_R = 1 - \sum_{i=1}^{N} x_i \frac{|(MR)_0 - (MR)_i|}{(MR)_0}$$

$$\eta_{C^*,dist} = f(D_R, MR_0)$$

Well this was the distribution parameter D_R which influenced the value of C*. We also found the effect of incomplete vaporization on C*.

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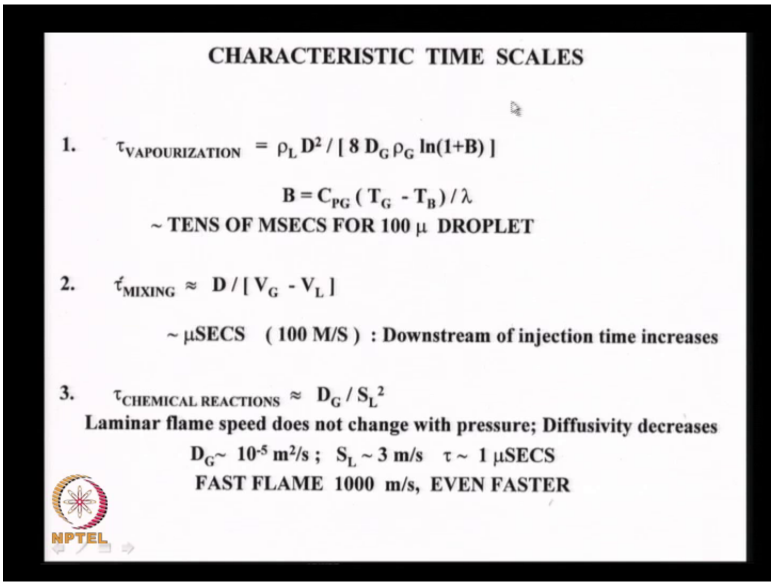

CHARACTERISTIC TIME SCALES

1. $\tau_{VAPOURIZATION} = \rho_L D^2 / [\, 8 D_G \rho_G \ln(1+B) \,]$

   $B = C_{PG} (T_G - T_B) / \lambda$

   ~ TENS OF MSECS FOR 100 μ DROPLET

2. $\tau_{MIXING} \approx D / [\, V_G - V_L \,]$

   ~ μSECS (100 M/S) : Downstream of injection time increases

3. $\tau_{CHEMICAL\ REACTIONS} \approx D_G / S_L^2$

   Laminar flame speed does not change with pressure; Diffusivity decreases

   $D_G \sim 10^{-5}$ m²/s ; $S_L \sim 3$ m/s τ ~ 1 μSECS

   FAST FLAME 1000 m/s, EVEN FASTER

And now we consider the above slide on characteristic time scales. We found that the time for evaporation in terms of the transport number B; we recall B was defined in terms of mass fraction, it is also possible to define it in terms of enthalpy divided by the latent heat of vaporization. The enthalpy we define as the difference in enthalpy between the hot gas temperature and the boiling temperature of the liquid, and therefore, B becomes this enthalpy difference divided by the latent heat of vaporization of the liquid fuel.

In this expression lambda is the latent heat of vaporization and not the evaporation rate constant. The time taken for vaporization typically for a 100 micron droplet is around tens of milliseconds. We can talk in terms of mixing time as equal to the diffusion coefficient divided by velocity, meter^2/second by meter/second this is much lower than vaporization time, and chemical reaction time is something like the characteristic time like let us say the diffusion term divided by the laminar flame velocity square, which is again much smaller than this and therefore, vaporization essentially controls the energy release from the propellants in the combustion chamber. We know how to take care of vaporization to get high C star value.

We will complete liquid propellant rockets in the next class.