Biomathematics

Dr.Ranjith Padinhateeri
Department of Biotechnology.
Indian Institute of Technology, Bombay

Lecture No. # 11
Integration – Part 2

Hello. Welcome to this lecture on Biomathematics. We have been discussing integration. We talked about, a bit about definite, indefinite and definite integrals. So, today, we will continue discussing that and we will discuss a few more things related to calculus. So, what we said previously...So, today's lecture is basically, Integration part 2; mostly we will discuss integration and towards the end, we will bit, we talk a bit about something related to calculus, something related to differentiation. And, we will see, when we reach that. So, what we basically discussed in the previous lecture is two things. We discussed indefinite integrals and definite integrals.

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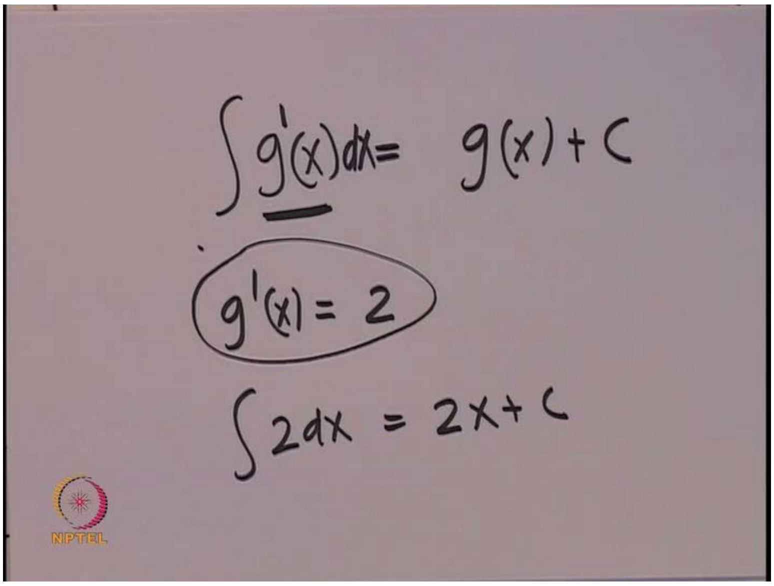

So, we said in the class that, if you have a function, which has an integral, let us say, integral g of x, the answer could be like, let us say, this is g prime of x, g prime and the answer could be like, g of x plus some constant c. What does this say is, g of x d x, what does this say is that, if, this tells you to find a curve, which has the slope g prime x. For example, if g prime of x is 2, integral 2 d x is basically, 2 x plus c, which is basically, a line with slope 2. So, this basically, this kind of integral basically, telling you to find a curve, that has a slope g prime of x. We also said that, you can have, you can ask, you can calculate definite integrals. So, such, such integrals are called indefinite integrals; we discussed this.

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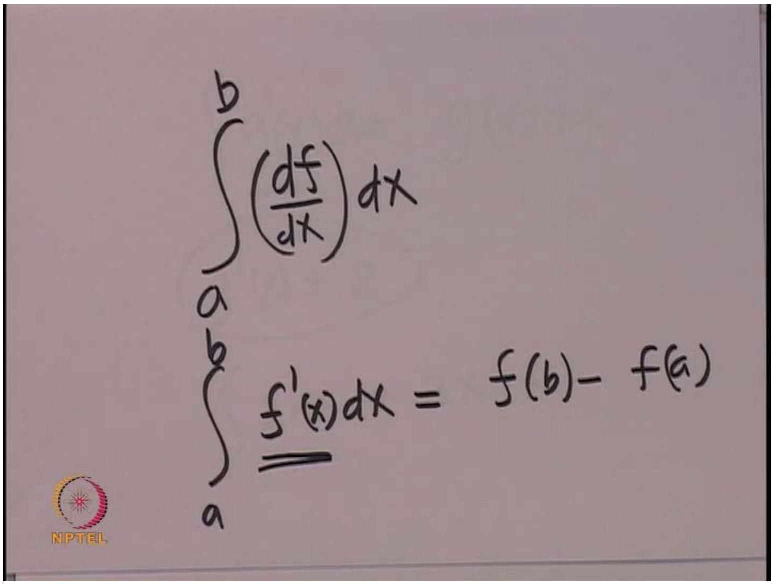

Definite integrals, we could have two limits, like, let us say, within this limits, let us say, a to b, some function, which is d f by d x into d x. So, the function is f prime. So, or in other words, a to b, f prime of x d x and the answer to this, we said is that, d f by d x, f prime is d f by d x. So, d f by d x is integral f, so the answer is f of b minus f of a. So, we said this; and the answer, this is nothing, but a number and this represents area under this particular curve. So, this is, this is kind of…Definite integral, basically, tells you, if you have a curve, given by this particular function; so, let us say, let us call this f prime of x, or some, some curve and then, what will be the area under that curve between a and b, this is what this tells us, because you can see, this is a number.

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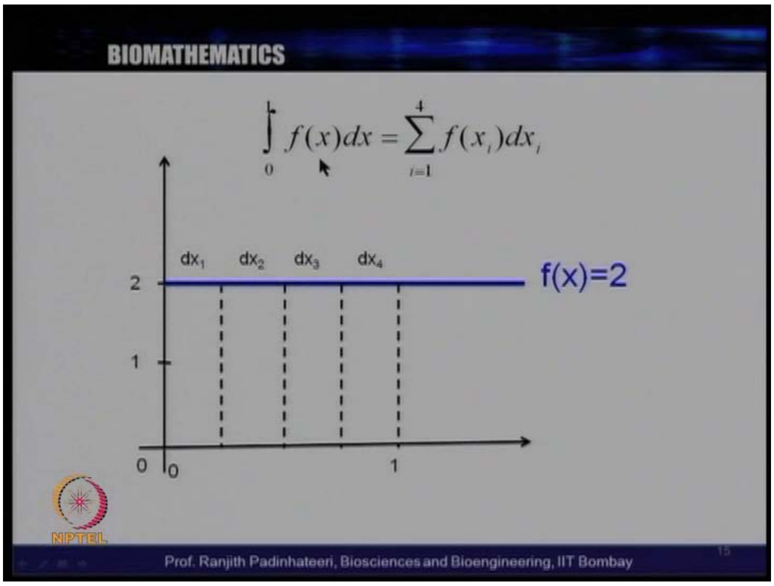

So, this is what we saw for example. We said that, 0 to 1 f of x d x can be written as, can be divided into many parts, like this d, which has width d x 1, d x 2, d x 3, d x 4 and if you do this, f of x i d x i, sum over i, you will get the area under this; because, area is nothing, but this width into this height. So, the width is d x 1, here and the height is, the function f of x here. So, f of x in to d x. So, this is f of x into, f of x 1 into d x 1, plus f of x 2 into d x 2, plus f of x 3 into d x 3, plus f of x 4 into d x 4 will gives this area and this is nothing, but this integral. So, this integral is area under this curve. But, then, this is, we understood this for a simple curve, which is like a constant value; because area here is, length into breadth, which is area of a rectangle, which is easy. But, you can think of more complex curve, which is, which has some curvature. At that point, this kind of thing will be little more tricky to do. So, let us think of some, some kind of a curve, some kind of a way, wiggly curve like this.

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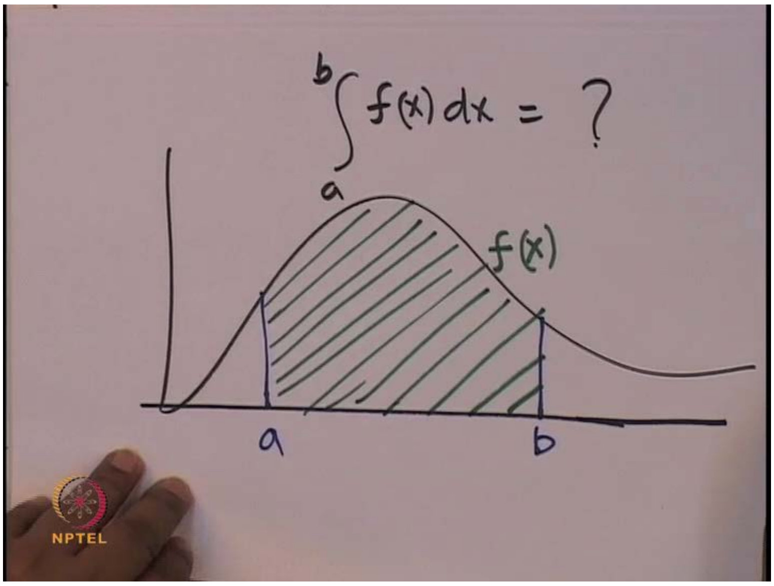

So, let us think of this curve, some curve like this; some curve, which is like this. Now, we wanted to have some integral of this function. So, this function, let me call this function f of x and we want to integrate this function, between this point and this point. So, this, let us call this a and this, let us call this b. So, what we want to calculate is, integral f of x from a to b; we want to calculate this integral from a to b of this function f of x d x. What is this? So, from what we learnt last time, it is nothing, but the area under this curve, between this two points. If you can calculate the area between this two points, we will get the answer. So, to… So, the area of this shape. So, let me shade this region. So, area of the shaded region, essentially, will give you this answer. So, if you can calculate the area of this region, that will give you this answer. Now, how do we calculate the area? We can divide this into many parts like we did. So, let us go ahead and draw this, little more carefully. So, again, if we divide this into many parts…

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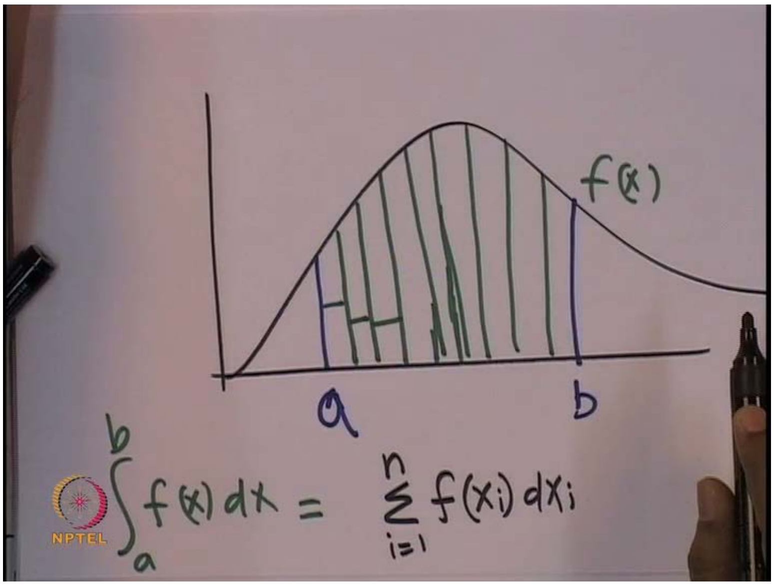

So...So, we had some function, which look like this and we had a and b here, a and b, a and b, and now, we can divide this into many parts. Now, how many parts will we divide, that is the question. Can we divide just to 2, 3 parts or 4 parts? Is that enough? So, let me divide this to, like a few, n parts, like, I am not going to count this; but let us say, there are n, I divide this to n parts. Let us assume that, all this width are equal. What I have drawn here is not clearly equal, but we could draw this equal. So, let us say, you divide this into n equal parts.

Then, this integral f of x d x…So, this is our f of x, from a to b, can be written as, sum over i, f of x i d x i, i is 1 to, let us say, I divide it into n parts; so, this is 1 to n. Now, what should be the value of n? Should I divide this into, like 5 intervals, 10 intervals, 20 intervals? So, we will answer this question, but assume that, you divide it to n intervals, this is the answer. Now, you will get the correct answer, if the n is very large. So, in the limit n going to infinity, that means, the larger, the better. So, you will get more accurate answer, when, when you, let us say, you do for a practical curve; practically you have a curve, you really want to divide this; the more you divide, you will get the more accurate answer. The real, the correct answer will be in the limit n going to infinity. Like, if you take a million intervals, if you divide this into million intervals, you will get pretty accurate answer for such a curve; but, if it is much more complex curve, with lot of wavy natures, you will have to divide this into more than a million maybe, sometime. So, the correct statement to make is that, as the number n goes to infinity, the sum will be equal to this integral. So, integral and the sum.

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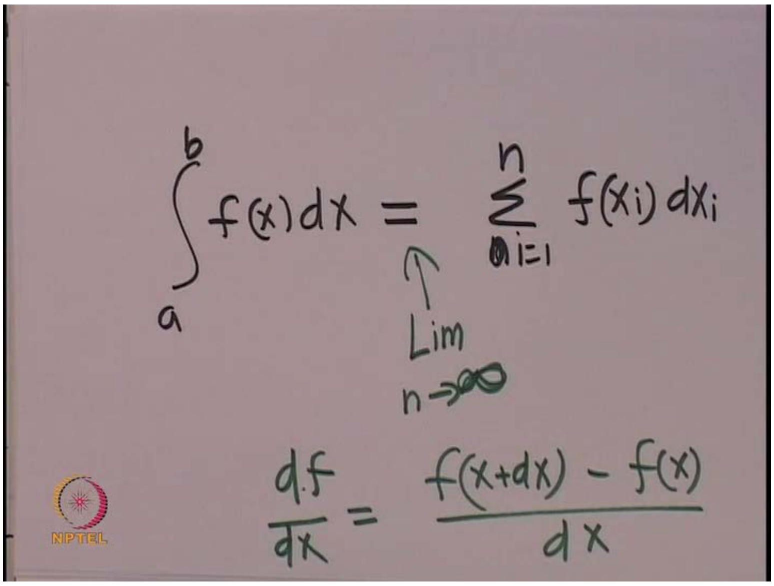

So, what we wrote, we wrote that, integral a to b f of x d x will be equal to sum over n, sum over i is equal to 1 to n, f of x i d x i. So, if you take all equal intervals, you will have again here, n of such intervals; so that, anyway, let me write this d x i. So, if you do this, in the limit, in the limit n going to infinity, this will be equal; this will be equal when n goes to infinity; when n is very large, this two will, this two, this sum will be equal to this integral. So, now, we will see some examples. So, the correct thing to do, even, it is just like in the calculus, we had like f of x minus…We said that, d f by d x, we said that, f of x plus d x minus f of x divided by d x. So, this is equal in the limit d x tending to, delta x tending to 0. So, d x tending to 0. When the d x is very small…So, when this quantity is very small, then, this two will be equal. So, similarly here, the limit n tends to infinity, this will be equal. So, the correct thing is to divide this into large number of this and then, calculate the area. So, right now, we will do some examples for this curve. So, let us think of a simple function that you know and see, whether we can calculate, divide this into many parts and do the sum; and when the n goes to very large, will we get the answer we know.

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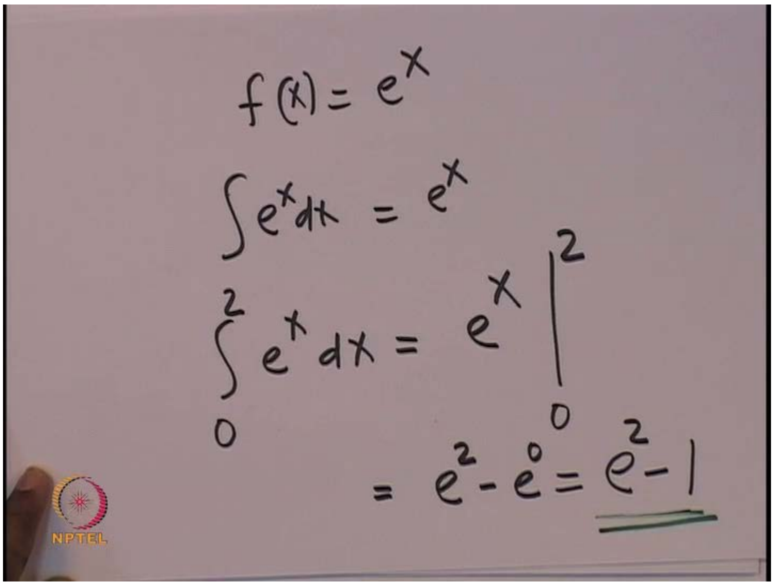

So, let us think of this function f of x is equal to e power x. So, we learnt that integral of e power x d x is e power x itself. So, now, let us divide, let us say, this is…Now, we want to calculate integral of 0 to 2, let us say, e power x d x. Now, what is this? So, this we said that, this integral in the limits, this is e power x in the limit 0 to 2. So, this is the answer. Now, how do we apply the limit? This is, what does this mean? This means that, this is equal to, you substitute x is equal to 2 first; that is, e power 2 minus e power 0. So, if you substitute x equal to 2, that is, e power minus, evaluate the function at… First, evaluate the function at the upper limit, that is, x is equal to 2; then, evaluate the function at x equal to 0 and this difference is the answer here. So, this is nothing, but e 2, e power 2 minus e power 0 is 1. So, this is the answer. So, we know this answer, we know the final answer. Given that, we know the final answer, let us see, we can think of this, like, as dividing this into many parts and can, can, can we do this, can we divide, write it as a sum of many, can we divide this into many intervals and can we find the sum and get the same answer; that is what we want to check. So, we want to check, if you can divide this e power x into many parts and we find the sum of this function, will we get the same answer. So, let us draw e power x for a minute here. So, let us draw e power x. So, let us say, we have e power x here. So, let us say, e power x.

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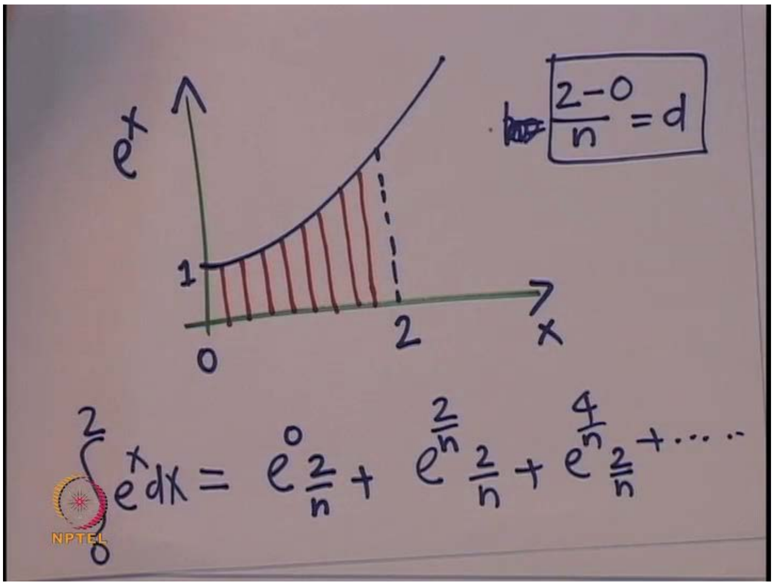

So, the e power x at x equal to 0 will be 1. So, this will be 1 here and it will just go like this. So, at 2, it will have some value here. So, this is 2 and this is 0. So, this is your x and this is your e power x. Now, basically, what we want is the area between this two. So, we can divide this into many intervals. So, let us divide this into like, n intervals, equal. So, let, let us call this 1, 2, 3, 4, 5, 6, etcetera. So, what does this mean? What does this mean? What did we say? We, according to what we said, integral 0 to 2, e power x d x will be equal to…So, you divide into many parts. First, let us take at 0. So, this is e power 0 and let us divide this to equal intervals. So, we divide this to n intervals, between 0 and 2. So, there are n intervals between 2 and 0. So, the each interval will be 2 by n, because, this is, let me call h, which is the length of this interval.

No, let me call this d, d is the length of the, width of this interval. So, so, this d will be 2 by n, because between 0 and 2, I divide it into n intervals. So, each interval will have width, all of them are equal intervals. So, each interval will have a width 2 by n. So, this will be e power 0 into 2 by n plus, now, next part, this will be at 2 by n. So, this will be e power 2 by n into 2 by n plus, this will be 4 by n, e power 4 by n into, d x is 2 by n; the width is 2 by n; plus dot dot dot. If you do this sum, for large n, n tends to, upto infinity, you will get this. So, this part, we know. So, let us try doing this. So, we can take this 2 part, 2 by n common, outside. So, look at this carefully, for a minute. So, I can take this 2 by n out in common. So, let me do this. Let me take this out here.

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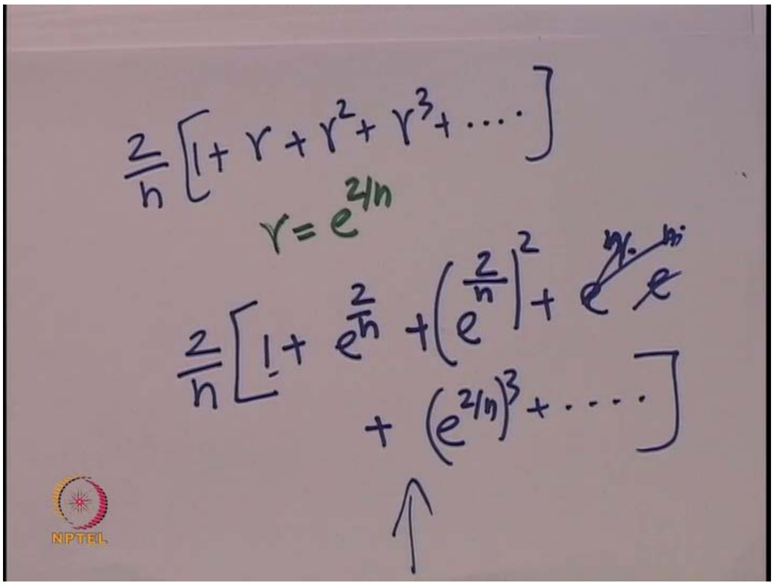

So, so, this, if I do this here, I take 2 by n out in common and what I have is, e power 0 is 1, and 1 plus e power 2 by n plus, this is e power 4 by n, which is basically, e power 2 by n whole square. So, let me write it as e power 2 by n whole square. The next term is going to be e power 6 by n. So, which is going to be e power 2 by n, sorry; this is going to be e power 2 by n. So, let me write the term here, e power 2 by n whole cube, plus dot dot dot.

This is going to be the sum. So, think about it. So, you can, you try this yourself; this sum, like this, can be rewritten, as, in this particular way. So, if you look at this, look a minute. So, what do you have, 1 plus e power 2 by n, plus e power 2 by n whole square, plus e power 2 by n whole cube plus, I can write e power 2 by n whole power 4, plus e power 2 by n whole power 5. So, this is something that you learned in school; this is something called geometric series. So, this is a series; this is like a geometric series; this is like, 2 by n into 1 plus, let me call this r, plus r square, plus r cube, plus dot dot dot. So, this is a geometric series, where r is nothing, but e power…The r here, is e power 2 by n. So, if you see...So, this is a geometric series, with r is equal to 2 power n. So, if you do this sum of this geometric series, we should get e power x. So, we know how to do sum of geometric series; we learnt in school.

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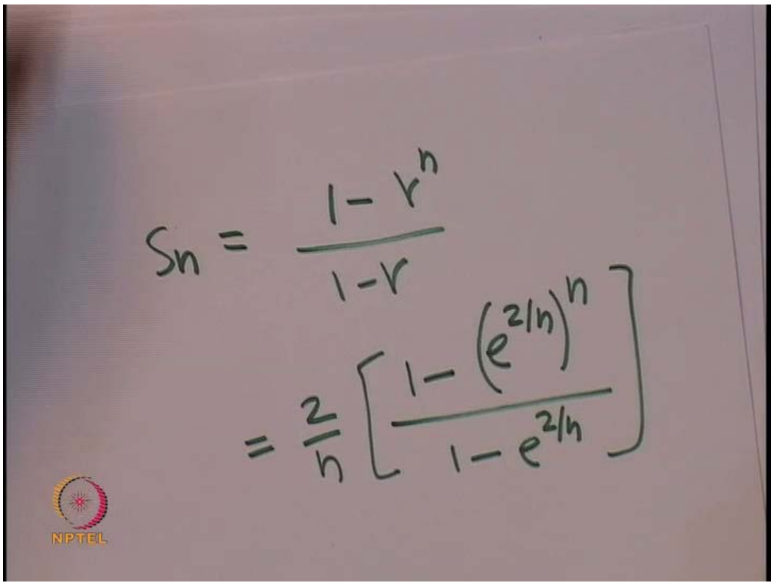

So, sum of geometric series, the answer of sum of geometric series is like, what we call s n, upto n term is basically, 1 minus r power n divided by 1 minus r. So, we, we have learnt this kind of a formula. So, or, I can, I can rewrite this. So, what…Here, in our case here, r is e power 2 n. So, we have a 2 by n here. So, let us, let us rewrite this 2 by n into 1 minus r power n, 1 minus e power 2 by n whole power n divided by 1 minus e power 2 by n. So, this is the sum and this, in the limit n going to infinity, we should get the, our integral. So, this is the theorem, we said, this is what we said that, the integral is nothing, but the sum, in the limit n going to infinity. So, we know this. So, you look, look at here, e power 2 by n whole power n is nothing, but e power 2. So, I can take a minus sign from both numerator and denominator outside and rewrite this as…So, let me rewrite this term slightly and you can see the answer emerging from this. So, let me rewrite this term a bit. So, let me take this minus sign out from here and minus sign out from here. So, I can write this e power 2 n by n whole power n minus 1.

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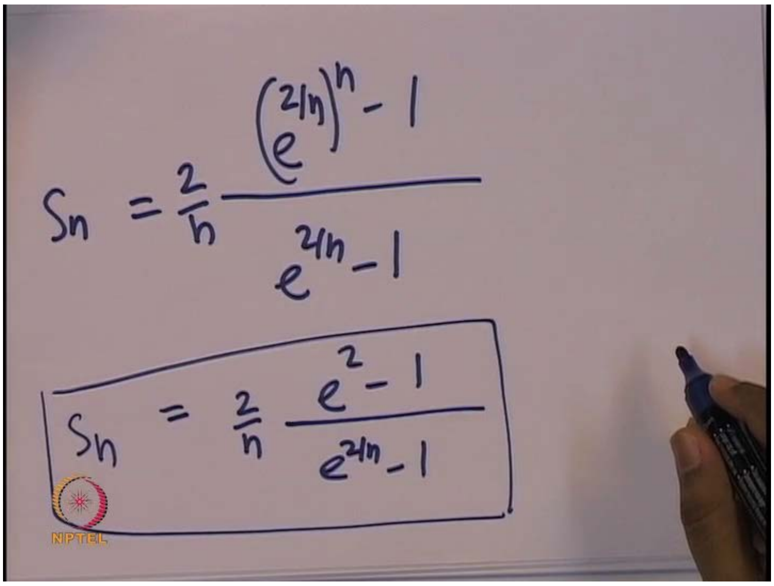

So, let me write this. e power 2 by n whole power n minus 1 divided by e power 2 by n minus 1. So, I took a minus sign out from both and I have a 2 by n here. This is my s n, the sum of n terms. Now, what is this e power 2 by n whole power n? This is nothing, but… So, this we have 2 by n into e square minus 1 divided by e power 2 by n minus 1. So, see, like, you know, we know, the answer is e square by n and this is emerging from this. And, what we have to show now is that, the rest of it goes to 1, when n goes to infinity. So, this is our s of n. Now, we have to see, when n goes to infinity, what is this? So, let us see this. So, what we have here is this. So, let me rewrite this in a slightly different way. So, let me rewrite the whole thing in a slightly different way.

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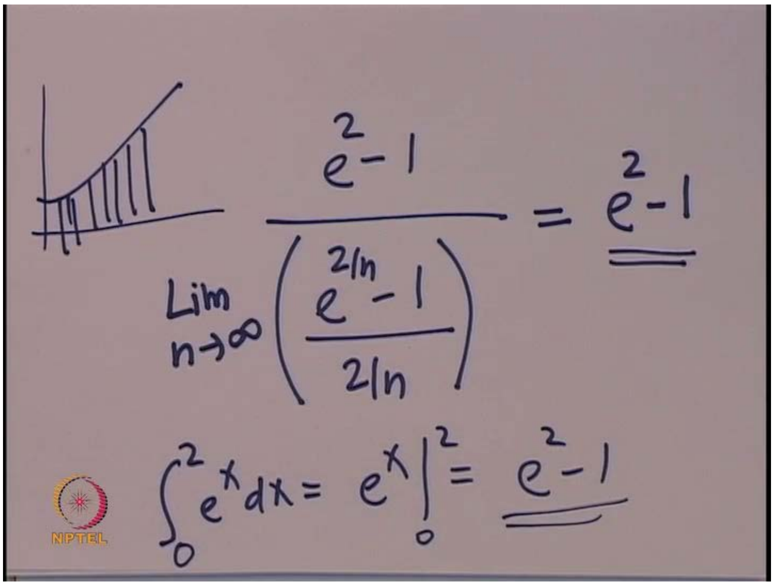

So, I can rewrite the whole thing as e square minus 1 divided by e power 2 by n minus 1, e power 2 by n minus 1 divided by 2 by n. So, I wrote 2 by n here, is divided the whole thing. Now, this is my s of n and I want to do this, limit n going to infinity. So, it turns out that, when limit n going to infinity, this term will go to 1. So, then, you will have the answer e square minus 1. So, we saw previously that, integral 0 to 2 e power x d x is basically nothing, but e power x, 0 to 2, which is e square minus 1. So, we got the answer, both the answers. By doing the sum, we got the answer e square minus 1, which is basically, the area under this curve; area under the curve that we saw; like, we drew this is a curve; area under this, divided into many parts and the area, we found as a sum and we found that, the answer is e square minus 1.

So, what did we find now? We found that, if you sum up to infinity, you will get exactly the same answer. So, this is one way of proving the, or, saying you could do it yourself of some other function that you know, and see that, if you take large number of, divide into large number of intervals and calculate this, the sum, you will get this answer. So, we know that, definite integral is nothing, but area under this particular curve. So, we will do now, one or two examples of, let us say, one simple example of some integral and we will see some examples of… So, let us, let us, let us do a function, which is…So, we did many, many examples. So, let us do one simple example.

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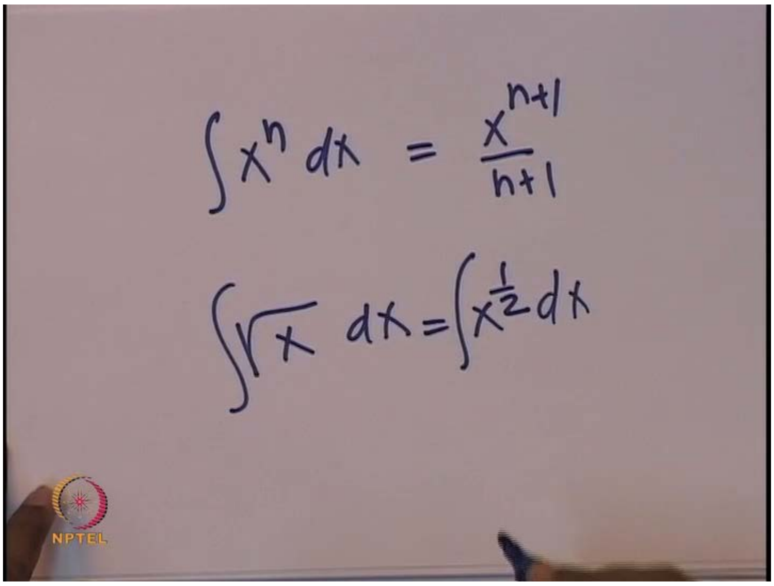

So, we, we already know, by examples like e power n. So, we said that, x power n d x is x power n plus 1 by n plus 1. Now, it can be like root x. So, the root can be written as x power half. So, then, similarly, we can use the, you substitute n is equal to half here. So, this is, you can substitute n equal to half here and you will get the answer here, for this particular function. So, you can even think of some other examples like, like, let us say, we will do, we will do more complex examples as the coming lecture, but as a one example of area under the curve, let us think of a circle, something that we already know.

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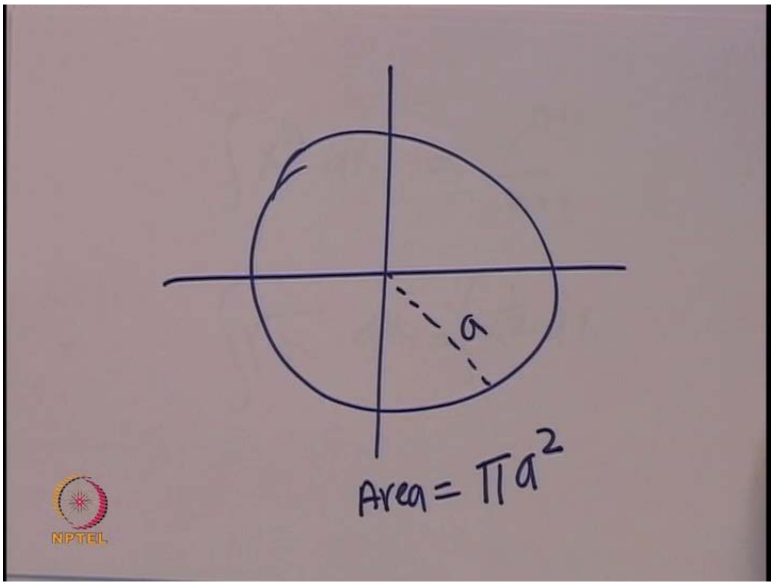

So, we know this, that this circle and we want, we know the area under this. So, let us say, we have a circle of radius a. So, let us say, this circle has a radius a. Then, we already know that, the area of this circle is pi r square, which is pi a square. So, we know that, area of this circle is pi a square. Now, let us check that, by doing integral, do we get the same thing? Do we get the same thing or do we get something different. So, let us, let us think of…So, what is the equation of a circle? The equation of a circle is, x square plus y square is equal to the radius square is the equation of circle.

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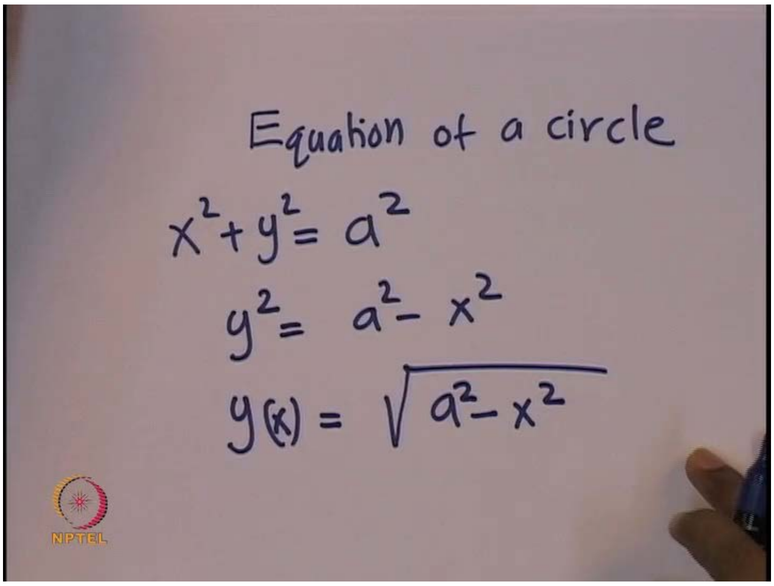

So, we know that, equation of a circle is x square plus y square is equal to, let us say, you have a radius a, which is a square. In other words, I can write y square is a square minus x square. In other words, y of x is root of a square minus x square. So, this is my y of x.

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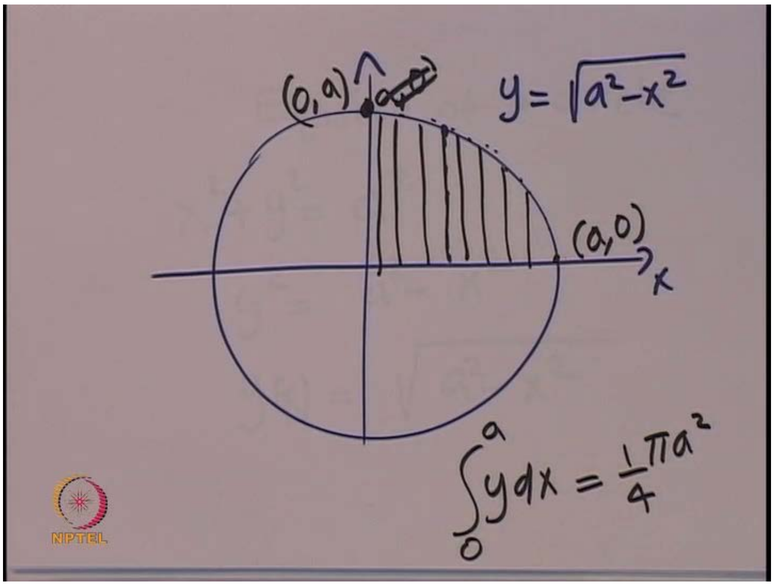

So, now, let us take a small sector, let us think of a circle, we just we did. So, let us think of this circle, like, I did not draw this very nicely, but, now, this has an...So, this is our x axis and this is our y axis. So, any point here, will have a y value given by root of x square minus, a, root of a square minus x square. So, we just said that, y is root of a square minus x square. So, depending on…So, at this value of x, you will have this particular y value; at some other value, we will have some other value. Now, we want to calculate the area of this circle and show that, this is nothing, but the integral of this function. Now, how do you calculate area? So, you can divide, let us say, this into many small parts like this.

So, let us first do, area of this quadrant and we know that 4 times of this…So, if you do area of this part and multiply, same as the area of this, same as area of this. So, the area here is same here, for a circle. So, assume that, this is a circle, like, I have not drawn it very nicely; actually, it is a circle, let us assume. So, area here, will be equal to area here, will be equal to area here and the same, it will be the same area, here. So, if you calculate the area here and multiply it by 4, we will get the total area of a circle. So, we know this y and if you can do this integral y d x, from here to here…So, y, you can start from here and y will go along this point, upto here. So, this is x equal to 0 and this is a equal to a. So, sorry, yes, this is x is equal to 0 and this is, x equal to a.

So, sorry, this is y equal to 0, from y equal to a. So, let us integrate this from 0 to a. So, if you do this, y going from, height y equal to 0 to y equal to a, which is this. So, y, here the y, at this point, y is nothing, but the radius. Like, a here, the height is, y is nothing, but the height, here. Here, x is equal to a, but y equal to 0. So, here x is equal to a, sorry, x equal to a and y is equal to 0; here y is equal to a; here y, x is equal to 0, but y is equal to a. These are the points; this point is 0, a and this is a, 0. So, integral 0 to a, y d x. If you can calculate this, it has to be area. According to what we said, it has to be area under this and it has to be pi a square 1 by 4; this has to be pi a square one fourth, because 4 of this has, 4 times is this has to be pi a square. So, now, let us think about, what is integral y d x for a minute and let us do this.

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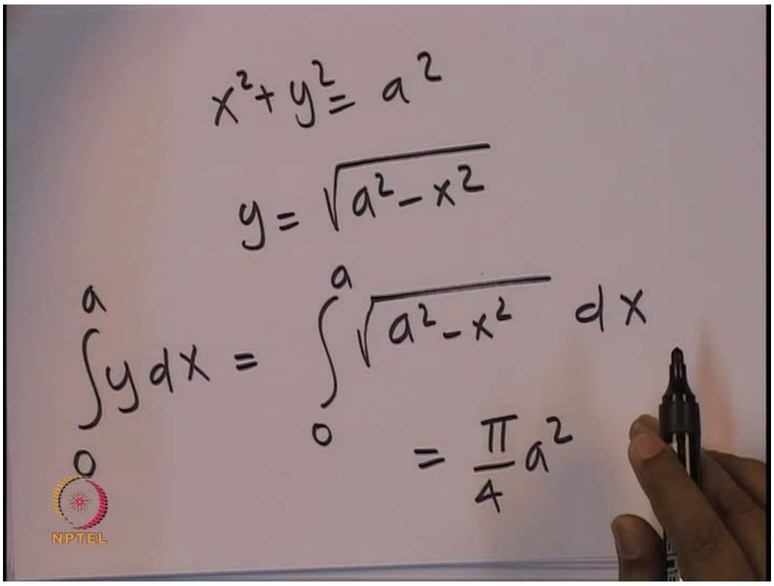

So, we said that, our equation is x square plus y square is equal to a square and y is root of a square minus x square. So, integral y d x 0 to a, will be, integral 0 to a, root of a square minus x square d x. So, this is what you have to calculate, root of a square minus x square d x, 0 to a. So, how do we calculate? We can, you can do this many ways; whatever way you do, you will, at the end of it, you will find that, the answer of this, is nothing, but pi by 4 a square. So, now, let us, let us do one way of doing it. Let us, so, let us do this, x square. Let us substitute. So, one, one simple way of doing it is, substitute x is equal to, let us say, sin theta. So, let us, let us substitute x is equal to sin theta. So, this is 1.

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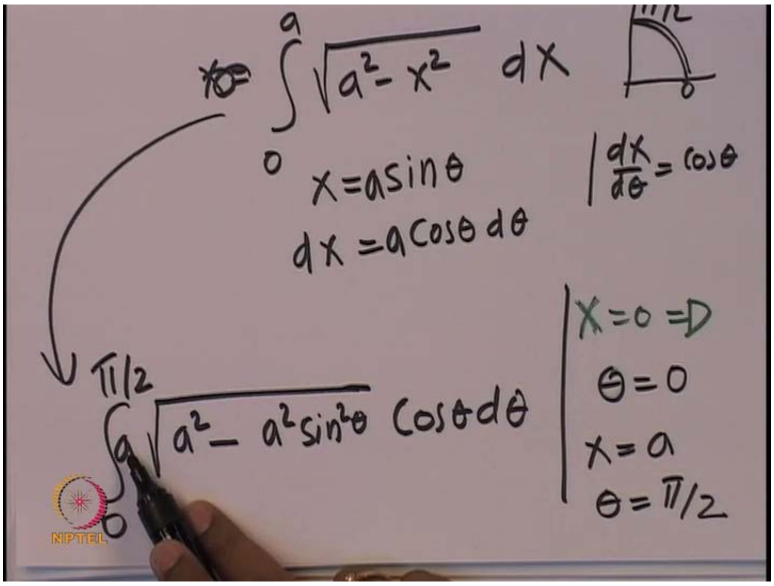

So, let us take x is equal to…So, what we want to calculate? We want to calculate, 0 to a, root of a square minus x square d x. So, and let us substitute x is equal to sin theta. What does this mean? This means, d x is, derivative of sin theta is cos theta d theta; because d x by d theta will be cos theta. So, d x is cos theta d theta, right. It is clear for you that, if x is equal to sin theta, d x by d theta has to be cos theta, the derivative of this with respect to theta, as sin theta has the derivative cos theta. So, that means, d x is equal to cos theta d theta. Now, so, we can rewrite this integral. So, we can write this integral root of a square minus, substitute, let us substitute x as…So, small, sensible to substitute x is equal to a sin theta. So, this will have d x is a cos theta d theta. So, you will have…So, let us say, here, we have a square.

So, x is equal to a sin theta; a square sin square theta and d x is a cos theta d theta. So, you will a here and you will have cos theta d theta. Now, we have to think of the limit. So, when…So, let us think of limit a minute. So, what we have is x is equal to sin theta when x is equal to 0, this limit, lower limit means, x is equal to 0. So, when x is equal to 0, this means, sin theta is equal to 0; that means, theta is, theta has to be, theta has to be 0; because, when x is equal to 0, theta 0 sin 0 is 0. So, this is value. This is correct. When x is equal to a, when x is equal to a, sin theta has to be 1. Then only, like x…So, when x, x is equal to a means, sin theta has to be 1. So, sin theta is 1, when theta is equal to pi by 2. Otherwise also, we know that, we have to integrate like half of this circle, which is 0 to pi by 2. So, this limit has to be 0 to pi by 2, here. So, now, think about this. What,

what do we have here? You have an a square, a square common. So, I can take out a square outside. So, root of a square is a. So, you will have…

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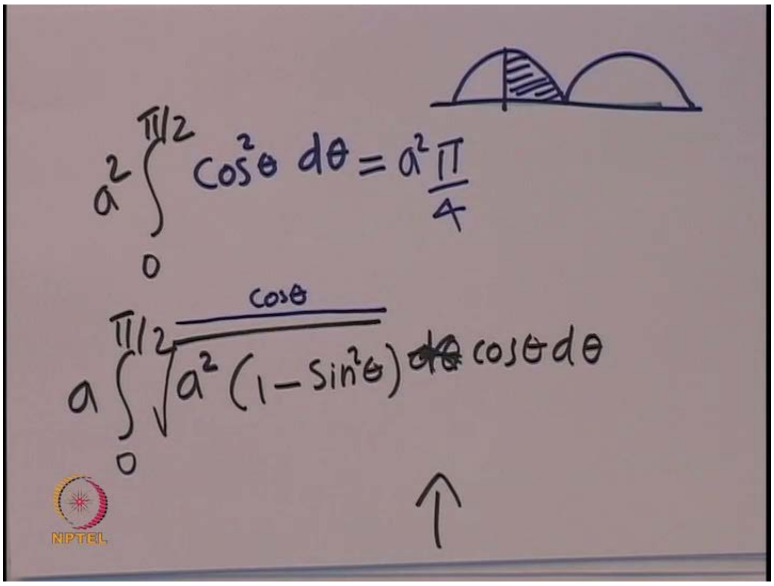

So, if I rewrite this…What do I get, if I rewrite? So, let us rewrite here. You have an a here. So, let us say, 0 to pi by 2. I have this a, which I am taking out and I have a square here, a square here; I take it out common. So, a square and there is a root into 1 minus sin square theta d theta, cos theta. So, there is cos theta. So, let me write it cos theta d theta. So, I have this here. So, root of a square, I can take out is a and a times this a square. So, I have a square, 0 to pi by 2, 1 minus sin square theta is cos theta, cos square theta; root of cos square theta is cos theta. So, this whole part here, this root part is cos theta. So, you have 1 cos theta here and you have another cos theta here. So, cos theta into cos theta is cos square theta. So, what you have is cos square theta d theta. So, 0 to pi by 2 cos square theta d theta. So, you can do this. So, if you think of, how do you plot cos square theta.

So, again, you can plot cos square theta. So…It will, plots will be like...So, 1, 0 to pi by 2. So, this is 0 to pi; 0 to pi by 2 only will be a part of this. So, at 0, cos theta is 1. So, 0 to pi by 2 will be one part of this. So, this will be like, one quarter of a circle again. So, this is another way of thinking about it. But, you can do this. You will see again, this answer is a square into pi by 4. So, we can do in different ways. You will end up seeing that, the answer is pi a square by 4. So, always you will see that, the answer of one quarter of a circle will have a area pi a square by 4 and when you multiply with 4, you will get the whole circle. So, you can make this, in other words, 0 to 2 pi cos square theta and you will get the answer as pi and therefore, the answer will be pi a square.

So, what we saw right now is that, area under this circle is pi a square or one quarter of a circle is one fourth pi a square. So, this is same; this is consistent with what we have been saying that, area under the curve integrally, integral is nothing, but area under that curve we have. So, by doing integration, we found that, area under a circle is pi a square. So, with this, we will go to a different…So, we could do…We can now do simple integrals and what you have to do is to practice this, by going to some books and looking at many examples and doing them as much as you can, because, the more you do, the better you will be and you will have, you will know more things about integrals. One, another way of doing integral is something very useful and important. So, that is called integration by parts. So, we will learn how to do integration by parts because this is going to be very useful.

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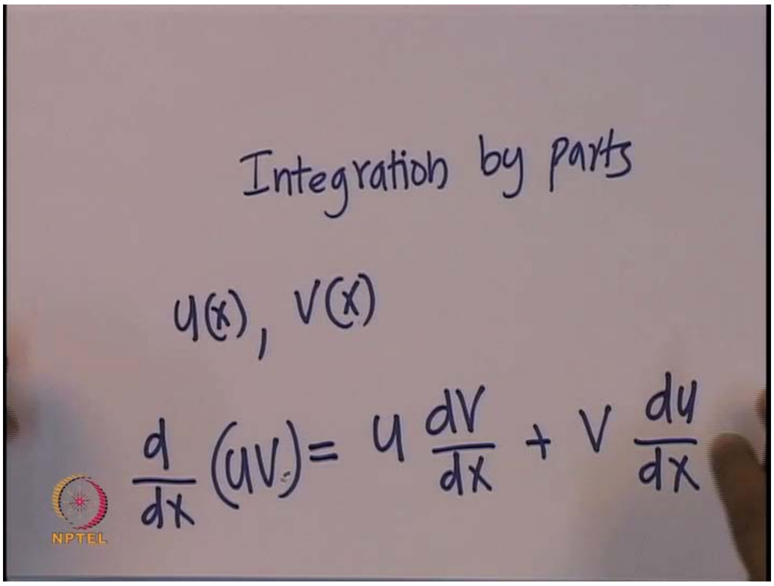

So, this is something that we are going to learn next, is called integration by parts. So, let us do this. So, while to do this…So, let us recall something we learnt in, during different, when we learnt differential calculus. We learnt that, if we have a function, let us say, u of x and another function v of x, we can write the derivative of this function, product of this u v as u d v by d x plus v d u by d x. So, it is good to recall this, because we will use this idea now, to think of integration by parts. So, we learnt that, if you had product of two functions, this could be f and g, if, or u and v, two functions, the product of this, derivative of this product is first function times the derivative of the second function plus the second function times the derivative of the first function. So, from this, we can get some interesting formula for integral of, for doing some, certain kind of functions; like, whenever you have a product of two functions, in functions, in particular, we can, we can get a interesting formula. So, let us have a look at this for a minute. We will make use of this formula, that, that we learnt, that, for the Integration by parts. So, let us look at, what we can, extend this formula here. So, we had learnt this formula earlier.

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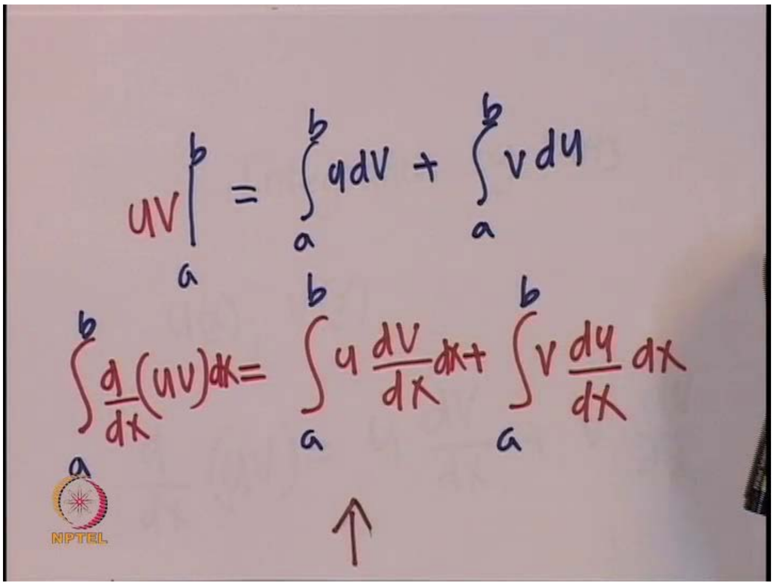

So, now, let us see, go ahead from here, let us go ahead from here and write this as an integral. So, let us, let us, let us write this as an integral. So, let us write, integral d by d x of u v. So, I, I wrote an integral here; integral of this, which has to be integral of this. So, integral of u d v by d x. So, I integrate on all sides here, plus integral v d u by d x. So, what did I do? I just put integral here, here, here. So, I integrate on both sides of this equation. This equation is something that we had learnt. Then, what do we get? We get something like this. So, when I integrate, I have to put a d x everywhere. So, I have a d x here and d x here. So, what is d by d x u v d x? So, the integration, this has to be u v itself. So, this has to be u v itself. So, let us say, if we are doing this between a and b, a and, with two limits a and b, then, you will have to apply a limit a and b; here, in the limit a to b is equal to integral u d v. So, this can be written as integral u d v by d x d x or you can write this as integral u d v, plus integral v d u, integral a to b, v d u. So, we have an interesting formula here. What do we have in formula? Let me write this in a slightly different way. So, if I write in a slightly different way, what do I get?

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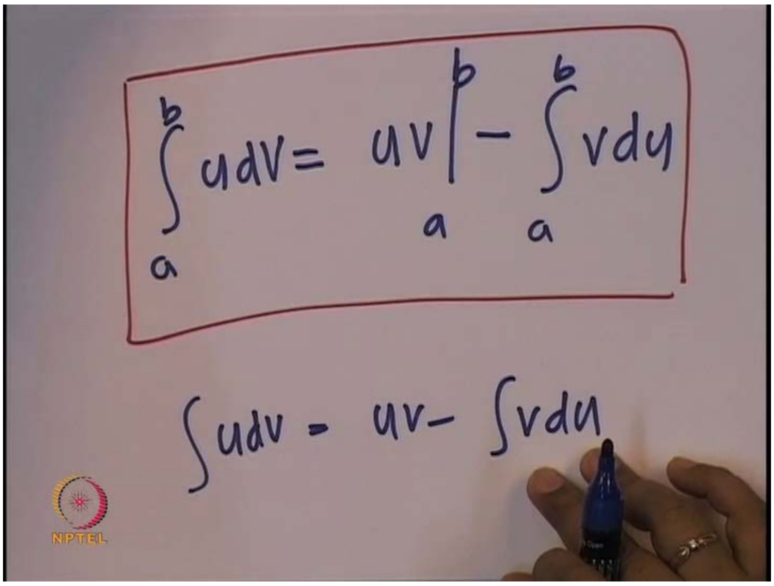

So, let me write this first. Let me write integral a to b, u d v is equal to…This is nothing, but u v in the limits, u v in the limits a to b minus integral a to b, v d u. So, this is an interesting formula, which will be very useful and we will be using it later. So, integral u d v is u v in the limits minus integral v d u. So, what did I do? I did not do much; I just used the fact that, the way, we learnt derivative of a product of a function. Here also, you have two functions and let us say, you want to find the integral of this. We can use this kind of a formula. So, let us, let us do this in an example. So, if you do not want to take a limit, you can also write this as integral u d v is u v minus integral v d u. If you, with this as a, this is definite integral. So, this is an indefinite integral. Now…So, let us do an example, to be clear, here.

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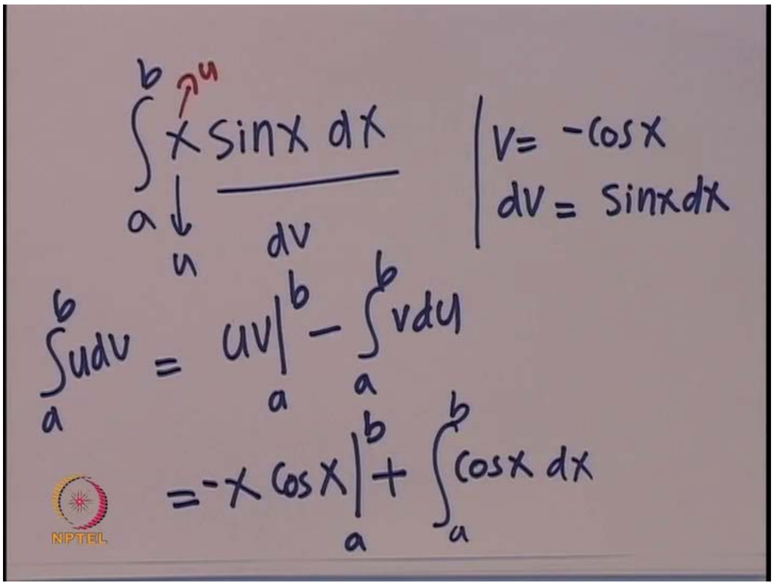

So, let us do an example, integral x sin x. So, let us say, in the limit a to b, d x. This is what you want to do. So, what is sin x d x? So, this sin x d x, you can think of as...So, you can think of this as u and this as d v. So, this is like integral u d v. So, if this is d v, the integral of sin x, that is, v is nothing, but cos x, right. So, we have integral of this, which is cos x. So, actually, it is minus cos x; so, we will put a minus sign. So, we can use the formula now, here; the formula that we learnt that, integral u d v is integral...So, the formula that we learnt here that, integral u d v is equal to u v minus integral v d u. So, here, what we have now here is that, u is x. So, this is our u and sin x d x is d v. Let us take it as d v. So, then, what you have is, integral u d v. So, you have to find out v. So, d v is sin x d x. So, v has to be minus cos x, because, what does this mean? This means that, v…So, this would imply that, v is minus cos x, so that, d v can be sin x d x. If v is cos x, minus, the derivative of this is sin x d x. So, the formula says that, integral u d v, a to b, is u v in the limit a to b minus integral a to b v d u. So, let us do this.

So, this is u into v. So, u is x and v is cos x with a minus sign, in the limit a to b, minus integral v, v here is minus cos x. So, minus and minus will become plus; cos x and the derivative d u. So, d u you can take it as here, derivative of x which is 1. So, this is nothing, but integral of cos x, which we already know, the answer. So, this is a to b. So, this will be integral of cos x d x. So, this can be done and this answer we know. So, this integral can be easily done. So, this is the trick of doing this integral. This is the, you can use this particular idea, which we will use in the coming lectures. So, with this, like, we have some good, quick look at the integrals. Now, we will quickly say something about, like…We said that, the integral can be written as sum of many, sum of many parts, like sum of area under the curves, can be written as sum of things with the equal areas, say f of x i d x i, sum over i. Now, if we go back to this derivative and think a bit about derivatives, and let us say, you are, you are given a function; instead of function ,what you have given is a table, let us say, x and f of x.

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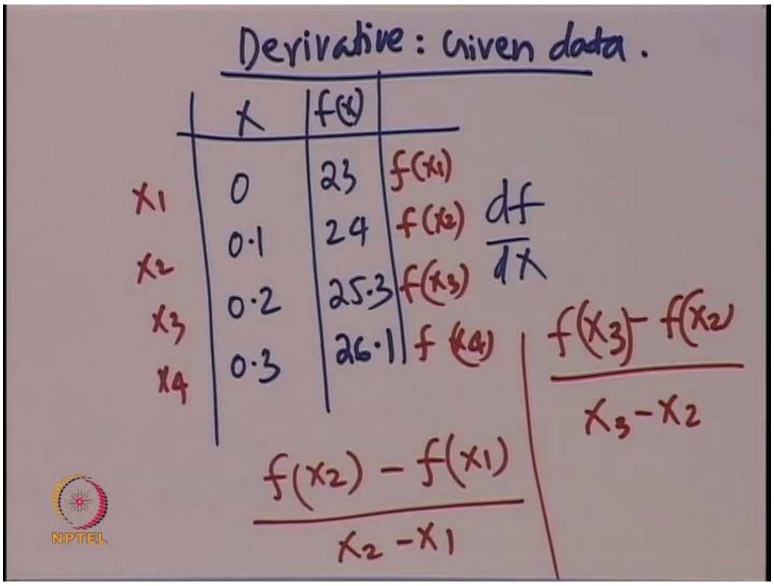

So, let us say, you are given some function, let us say, 0, 23, 0.1, like, 24, 0.2, 25.3, 0.3, 26.1 and so on and so forth. So, this is the function and this, x and f of x. So, how do we calculate the derivative of such data? For this, if you could know the discrete form of…So, we know, if you have, know the functional form, if you know that f of x sin x, we know how to calculate d f by d x. So, how do we calculate the derivative of such function? So, what we are going discuss quickly is that, derivative, given data. So, if you are given data like this, how do we calculate the derivative. So, it turns out, this is easy; we can extend the results, that we already learnt.

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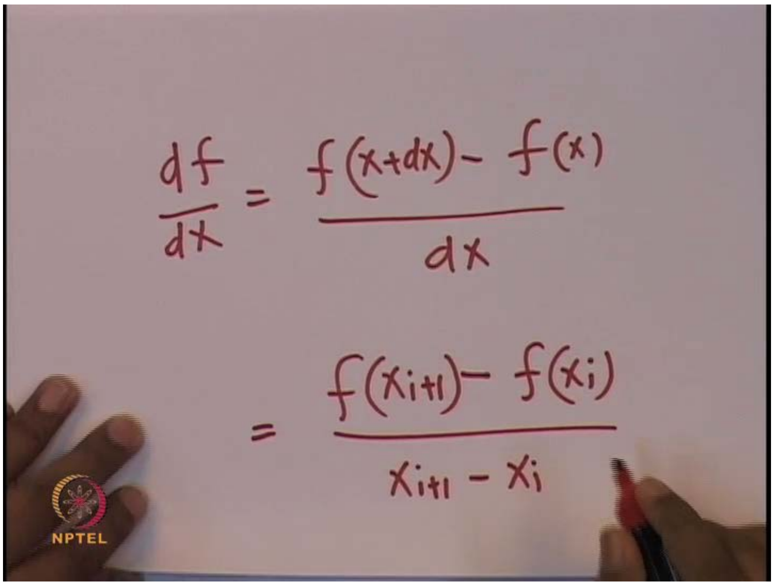

So, we said that, the derivative d f by d x is f of x plus d x minus f of x by d x. So, the same thing can be used here. So, here d x. So, here, instead of d x, this can be written as f of x i plus 1 minus f of x i divided by x i plus 1 minus x i.

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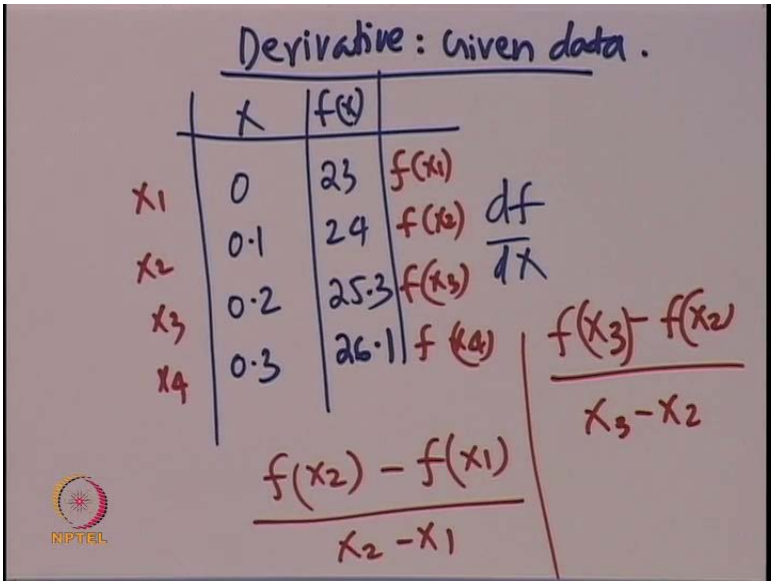

So, what we have here, this is x 1, x 2, x 3, x 4 and what we have is f of x 1, f of x 2, f of x 3 and f of x 4. So, if you have this, what we can do is that, to find the derivative with, between, the slope between this two, what I can do is that, I can do f of x 2 minus f of x 1 divided by x 2 minus x 1. Similarly, the slope, between this two will be, f of x 3 minus f of x 2 divided by x 3 minus x 2. So, what essentially, what you have is basically, this formula f of x i plus 1 minus f of x i divided by x i plus, x i plus 1 minus x i. So, this can be used; you can extend this to higher derivative. So, you can just say that, higher derivative…

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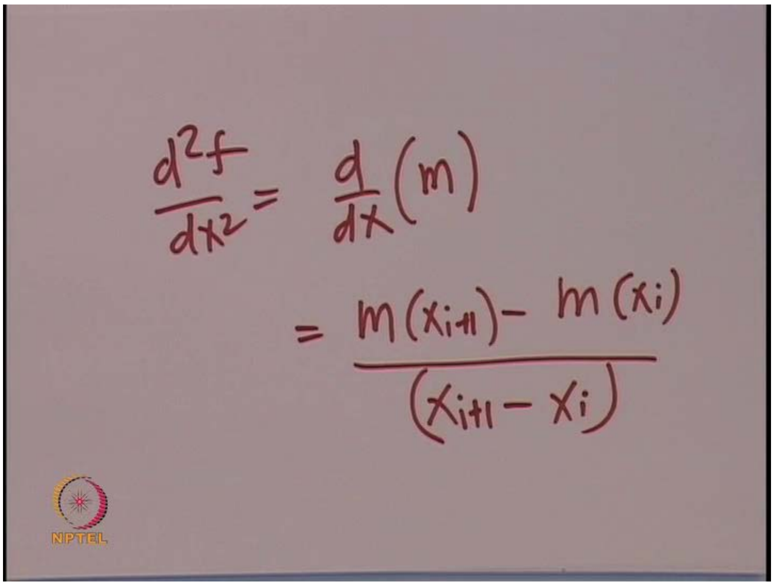

So, let us say, d square f by d x square is nothing, but d by d x of this, d by d x of this slope. So, you can write this as m at x i plus 1 minus m of x i divided by x i plus 1 minus x i, so, where m is the slope. So, I can substitute for m of x i plus 1 and m of x i, because m, this itself is derivative and you can go ahead and do this.

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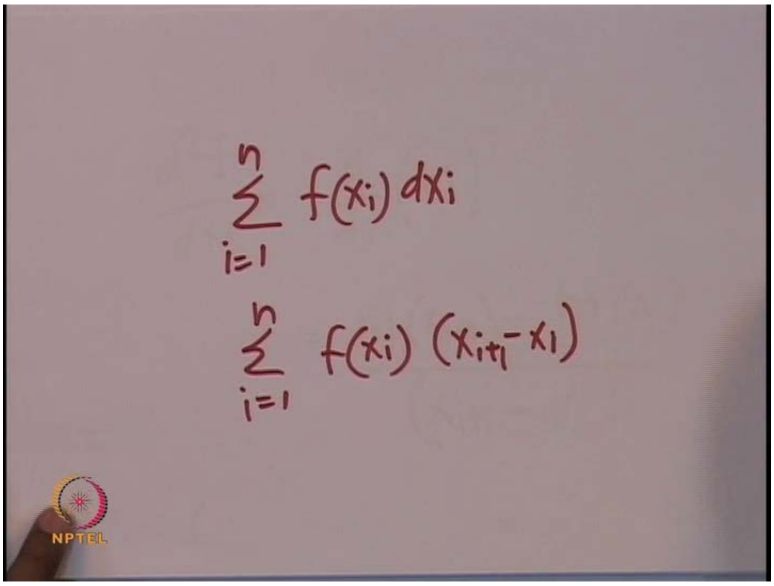

So, similar things can be done for given data. You can also integrate by the same rules that we said, integration of a data can be written as, like as we, as we did, i is equal to 1 to n, f of x i d x i. So, in other words, sum over, i is equal to 1 to n, f of x i into x i plus 1 minus x i. So, if we have data like given here, x 1, x 2, x 3, we can use this formula to do the integral of this numerically also. So, this ideas can be extended to numerical part, which is actually a different course in itself; that numerical methods, like, how do you use numerical methods to do the integration. There are like, very, very efficient; we can make this methods very efficient by doing the various, various techniques, which is the course in itself. So, I would not go into that.

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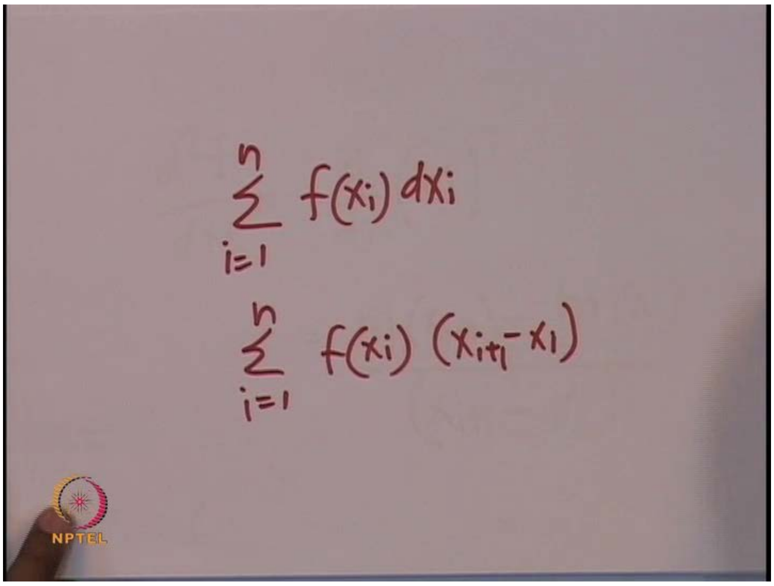

So, what we learnt today is two, three things; like, we talked about integration. We discussed integration. We talked about area under the curve, area under the curve f of x, for definite integrals, right. So, this is, this will give you definite integrals. And, we saw a couple of examples like, we saw e power x and we saw the area under the circle and we also said that, quickly said something about derivative and integral of data. So, if you are given some data, you can use the formula to do the derivative and integral of this data. So, with this I will stop this integral and in the coming classes, we will see something called differential equations and use this integrals. So, we will discuss the integrals in details there, as an examples of solving differential equation. So, integrating is nothing, but solving some kind of different equations. So, which is very useful in Biology, like, which is, you can see very often in Biology. So, we will discuss in the coming classes the differential equations and how do we use integration to integrate the equation, differential equation and get something that is very useful in Biology. With this, I will stop today's lecture. Bye.