Biomathematics

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Lecture No. # 12
Differential equations

Hello and welcome to today's lecture on Biomathematics. We have been discussing calculus and we discussed differentiation and integration. Now, we will go to a new topic, which is very closely related to what we discussed so far, very closely related to differentiation and integration and it is called differential equations. So, today, we will discuss differential equations. What are differential equations? That we will come to know as we go along. What exactly one means by differential equation etcetera will be clear, as we go along. So, let us start with taking some example. So, what could be…So, let us take one of the examples that is familiar to you, when we studied, when we discussed integration or when we discussed calculus. So, let us take this simplest example, one of the simplest examples of something polymerizing.

So, one example is actin in Biology. So, actin polymerizes or micro-tubules polymerize or many, there are umpteen number of polymers in Biology and they all polymerize and when they polymerize, as you know, their length increases. So, polymerization is nothing, but adding monomers and the length increases. So, the question we have to ask is, how exactly we will describe this mathematically; and the answer you will get is that, this can be described by a differential equation. So, we use something called a differential equation to describe this phenomena of polymerization of a filament. So, that is what we will discuss to begin as a first example and once we discuss you will get an idea that, what exactly differential equation means; and once you get that idea, we will discuss couple of examples today, two or one or two examples at least and once you get this idea, we can go to the proper definition and so on and so forth. So, first I want to convey you the idea. So, let us take this example of actin polymerization.

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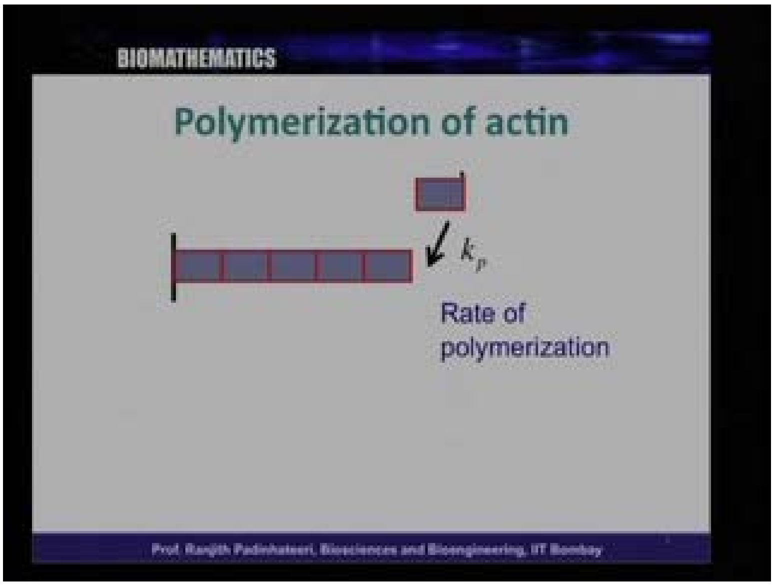

So, have a look at this slide we want to study the polymerization of actin. So, actin is a filament as is shown here, made it of N monomers. So, there are many monomers 1, 2, 3, 4, 5. And then, you can add monomer with some rate k p. So, k p is the rate of polymerization. So, with this rate, this monomers or the sub-units are being added to this filament and the length of the filament increases. So, this is the phenomena. So, we are, we want to describe a simple phenomena and the question is, how do we describe this using mathematics. So, as we see here, we have a polymer. And, as the monomer, as we, the monomers keeps adding to this, the length of the polymer increases.

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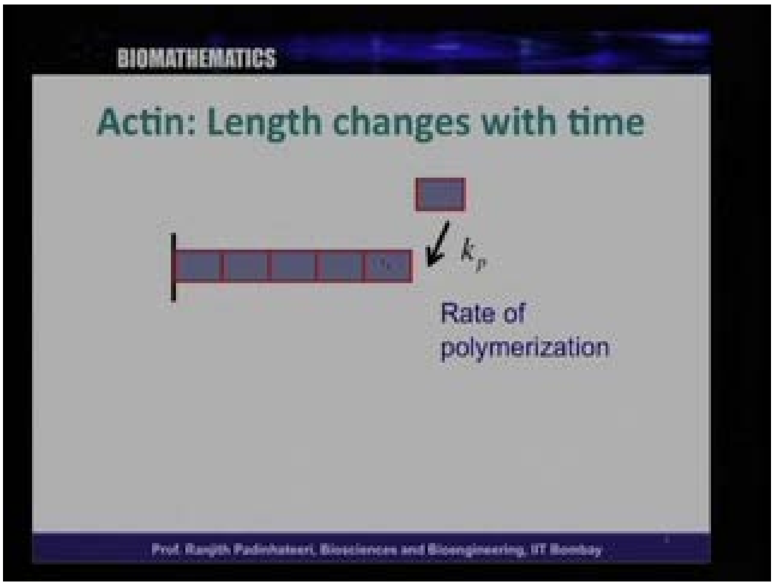

So, let us look at the next slide. So, what, what happens is that, the length changes with time. So, the, on the quantity that changes with time is that length. So, we want to describe this mathematically.

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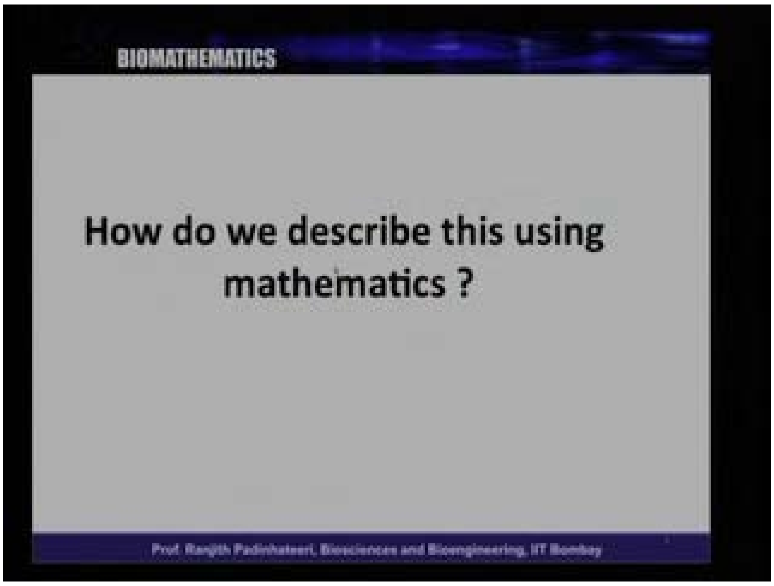

So, let the question is, how do we describe this using mathematics? So, this change in length that we see, how do we describe this mathematically. So, how do we describe the length, change in length mathematically is a question and the answer to that is the following.

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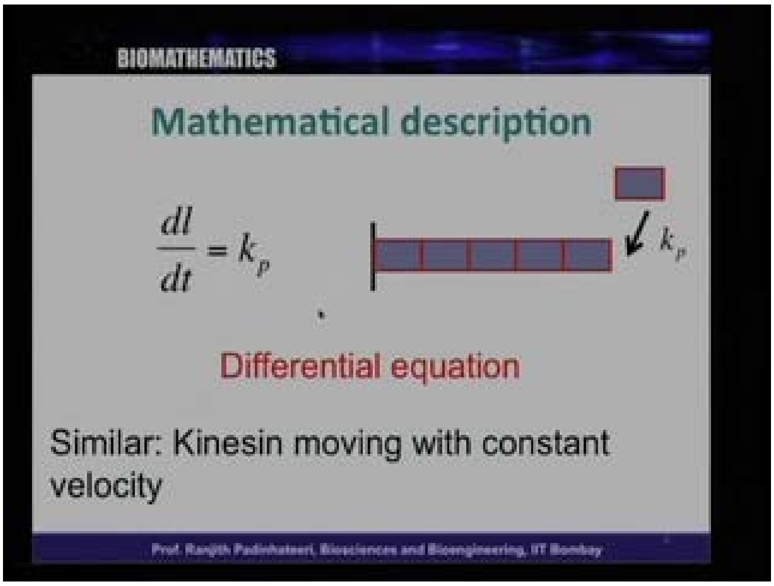

So, have a look at this slide. So, we can say the mathematical description be, we will have something like this, where d l by d t is equal to k p. So, this is what written here. Now, what does this mean? d l by d t, as we saw, it means, the change in length d l, when the time changes by an amount dell, d t. So, the time, when, how much is the length change, when the time changes by an amount d t. So, when the time changes by d t, the length will change by an amount d l and this happens with a rate k p. So, that is what it means. So, the mathematical description of this phenomena of polymerization with the rate k p can be the, mathematically, it can be described by using this equation, which is d l by d t equal to k p; that means, the change in length with time is k p. So, now, this, such equations are called differential equations. As we see, the more the k p, the more the change in length will be; you can see that. So, if the, because the k p and d l are directly proportional; if you increase the k p, the d l the length and change in length will increase, if you fix the d t constant. In other words, let us take d t as 1 second. And, k p, let us assume, k p is 1. So, d t is 1, k p is 1.

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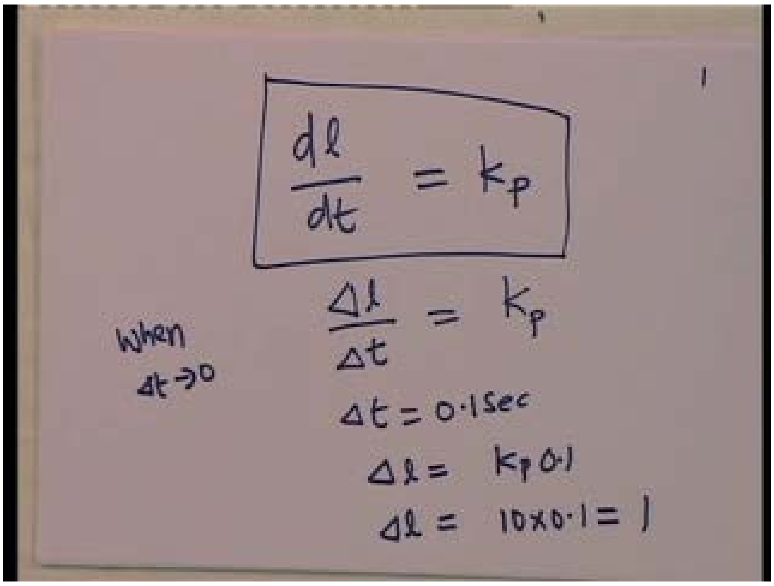

So, let us write it down here. So, d l by d t is equal to k p. This is the equation we said. So, let us make some sense out of this equation to begin with. So, let us take…So, as we saw in the calculus, when we studied differentiation, this is actually, means, the change in length by change in time, when time is very small. So, we can write, when time is very small, we can write this equation, in this particular way, d l by d t. So, when, when time is very small or delta t is very small, we can write this particular way. So, now, let us say that, delta t is very small, let us say, a fraction of a second. So, let us say, delta t is 0.1 second. So, then, what does it mean, delta l is k p into 0.1.

So, if k p, the more the k p, the more the delta l is. So, the k p is 10. What does it mean, if k p is 10, what does it mean? delta l is 10 into 0.1 is equal to 1. So, the change in length will be 1 monomer per second; that could be, that is what it means. So, this differential equation has the meaning and the phenomenon that we wanted to describe, can be described by this particular equations; and such equations are called differential equation. As the name suggests, there is a differential here; there is a difference that…You write a differential equation because there is a difference; this equation is written as difference between two length and two time. So, this is what one typically means by differential equation. We will come, examples, a little more carefully, we will define all this; but you, I hope you understand the concept, the idea. Now, given such equation what does it immediately tell you? So, that is the next question one wants to ask. So, you have this equation, which is d l by d t is equal to k p, but what does that mean? What information does that give you?

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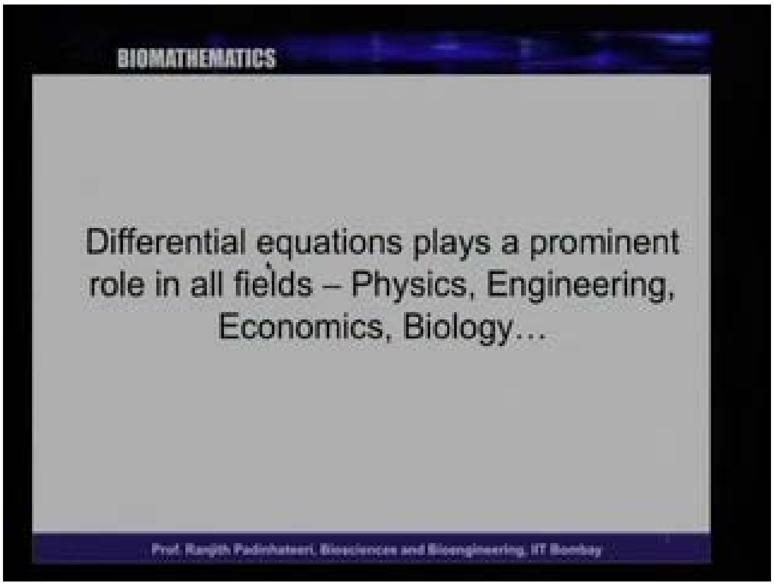

So, let us have a look at this. So, before that, we want to understand an important point that I want to describe here. The differential equation plays a very important role, very prominent role in all fields; let it be Physics, let it be Engineering, Economics, Biology and the weather prediction or whatever be the field you think of, differential equations plays a prominent role. So, understanding differential equation is very important, whatever be the field you are interested in, including Biology. So, we will, we will spend some time carefully understanding differential equation, because it is one of the crucial, very important, prominent techniques used to understand or used to describe various phenomena and understanding differential equation will help you in understanding how to describe various day to day phenomena that we see, using mathematics.

And, at the beginning of this lecture, we said, mathematics is like a language. So, this language is used to describe things in a precise manner, in a very quantitative, precise manner. So, as you know…Once you understand this differential equation properly, you will get some idea, you will have a good idea how to describe various phenomena that we see around us in a very quantitative, precise manner, in a, using a scientific language which is mathematics. So, this, you should keep this in mind, when we study this differential equation. So, keeping this in mind, let us go and study and understand this particular differential equation that we have in hand.

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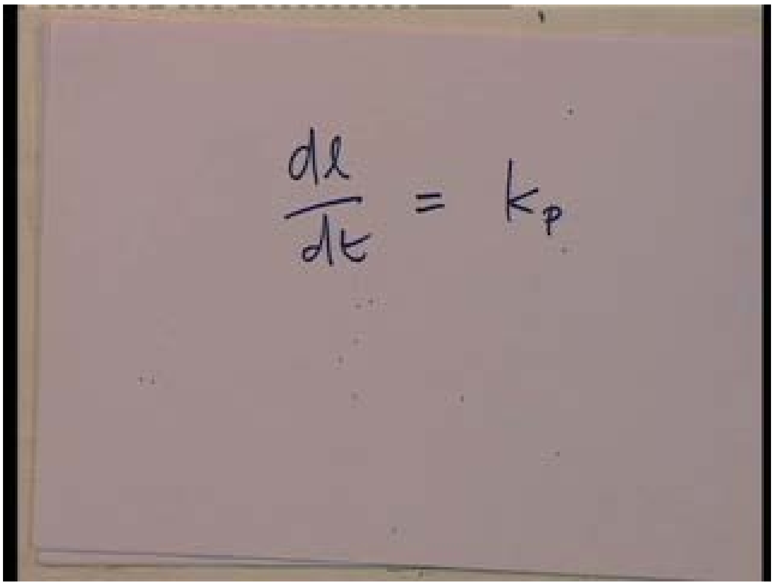

So, the differential equation that we have in hand is, as we said is, d l by d t is equal to k p. So, when we see such an equation, what does it immediately comes to your mind? Something we studied in the calculus, in the earlier differentiation, in the previous lecture or many of the previous lectures, we discussed something. So, that is something related to that of a straight line. So, let us just go back and see what we, quickly see, what we studied in the last lecture; something we saw in the last lecture, which is the following.

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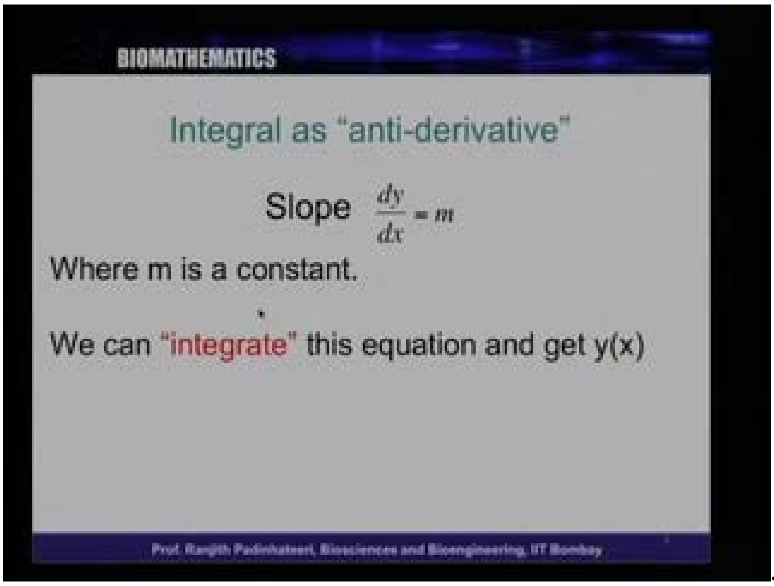

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So, when we studied, discussed integral, we said it is like an anti-derivative. So, integral is an anti derivative is something we discussed. So, for example, if we have a slope m, we can say, slope is nothing, but d y by d x. So, this is the something that we discussed before. Slope was nothing, but d y by d x. Let, when we discuss differentiation, we said the slope is d y by d x.

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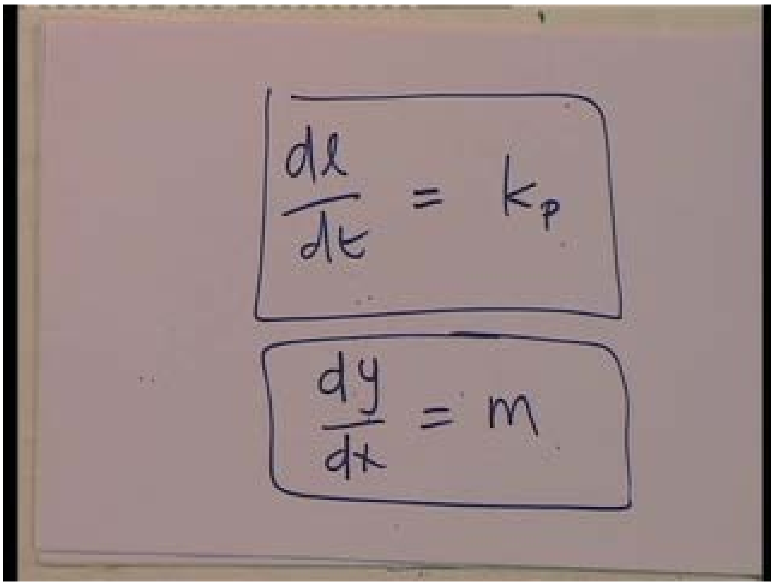

So, now, this looks very much similar to what we described here; this d l by d t is equal to k p, this is same as d y by d x is equal to m. So, there is, like, you can see, these are like similar equations; this is slope and d y by d x is slope. So, this looks very similar. So, this itself, give you some hint, what can we understand something about it. This has to do, this will have something to do with straight line. So, let us try and understand, what is, what is straight line here, but, let us go back. So, in this slide, when we discussed d y by d x is equal to m, m is the slope, which is the constant; here also, k p is a constant and we can integrate this equation. We said that, we can integrate this equation and get y of x. So, we can integrate this d y by d x and get y of x.

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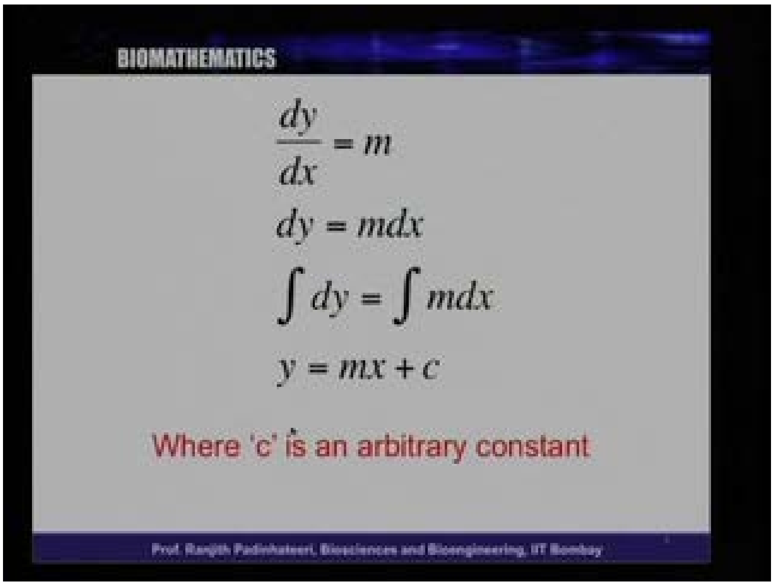

So, how did we do that? We did it in the following way, in the last lecture. We said that, in the previous lecture, we said that, the d y by d x is equal to m. So, and we took this d x this side and we said d y is equal to m d x and we said, we can integrate both sides now. So, we write integral d y is equal to integral m d x. So, which has nothing, but integral d y is y and integral m d x is m x, and integration has a constant always and that constant is c. So, this can be anything. So, this decides, what is the value of y. So, this is an arbitrary constant as far as the integration is concerned. So, this is what we said, the solution of d y by d x equal to m. So keeping this idea in mind, keeping what we did in the previous classes, previous lectures in mind, we can try and go and understand this d y, our polymerization problem.

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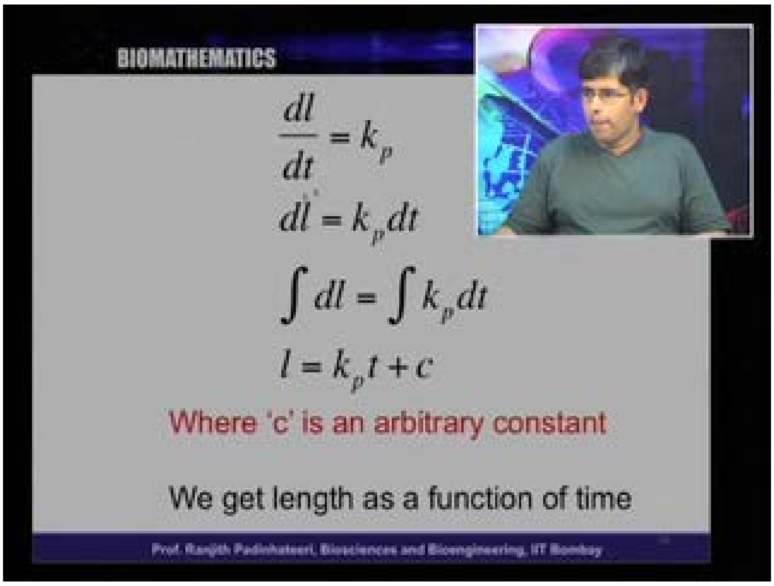

So, we, let us go this polymerization problem, which is we have d l by d t is equal to k p. So, we can write, as we did in the previous case, d l is equal to k p d t. And, you can integrate both sides. So, integral d l is equal to integral k p d t. This immediately tells you that, l is k p t plus some constant; just like we said, y is equal to m x plus c, where c is the constant and we get the length as a function of time. So, what does it mean, for any given value of t, there is a value of l; if you change the t a bit, you will change the l. So, what does it mean? So, this is the solution of this differential equation that we discussed; and, if the t is 1 second, it will give you what is length. If the t is 2 seconds, we will give you what is length. So, given a value of t, any number, you will get a value of l. So, if we can write down, from this equation to, if we, the way of going from d l by d t to l is equal to k p t plus c, we call the solution of differential equation. So, in this slide, we have solved the differential equation and we wrote down the solution. So, the solution will give you, how does the length change; the length changes as a function of time. So, that is the solution of the differential equation. So, we have solved differential equation and got the length for any value of t. So, let us look little more carefully and let us try and understand this.

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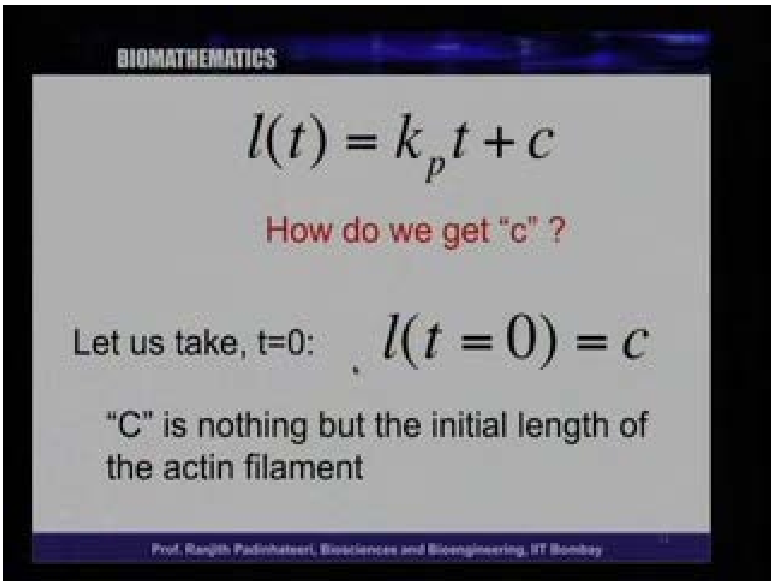

So, let us look at this slide. So, what we have got? We have got length as the function of time. So, this bracket t means, length depends on the value of time. So, length is the function of time, that is what it means. We discussed what a function is, in the early classes. So, length is the function of time is given by k p t plus c. Now, how do we get this c? So, let us take, in this case, t equal to 0 in the above equation and what do we get. If we take t equal to 0, we get l at t equal to 0. When t equal to 0, the length is nothing, but c. So, c, it turns out that, c is nothing, but the length at t equal to 0; or, you can say initial length. When we start the experiment, or, when we start the discussion, what was the length. So, if you imagine, if you are doing an experiment, you have actin with a length, polymer with a length to begin with. You immediately add things and let us start polymerizing. So, the length at, when the polymerization starts, whatever be the length, that is, you can call it as a initial length. So, if you, the c turns out that, c is nothing, but the initial length, as you can see here. C is nothing, but length at t equal to 0. So, the length, initial length of the actin filament. So, knowing this, we can write down the following.

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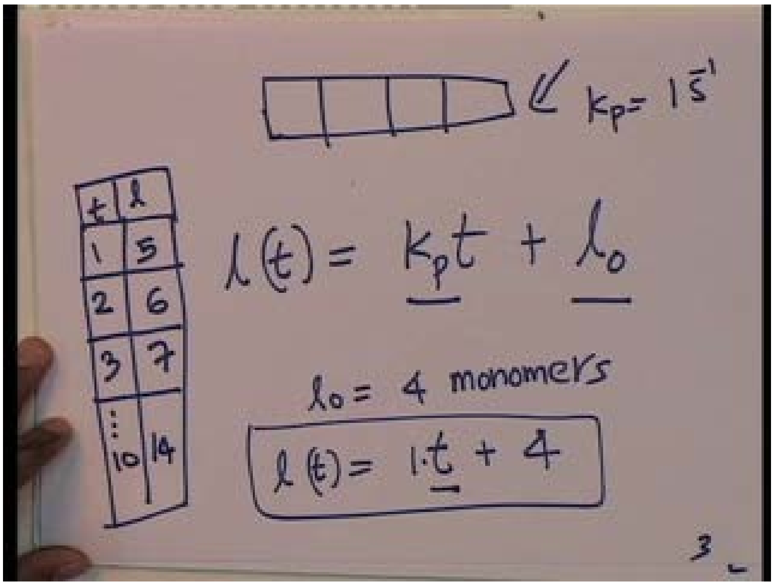

So, what did we get now? We got l of t is k p t plus length at t is equal to 0. Let me call it l 0. So, this is length at t equal to 0. If you know the length at t equal to 0, and if you know the k p, you can get the length for every time. So, if you…So, let us say, l 0 is 4 monomer. So, l 0. So, let us say, to begin with, we have 4 monomers. So, to begin with, we have a polymer, which as 4 monomers. Now, you add monomers with a rate k p.

And, let us call this k p as 1 per second. So, in every second, 1 monomer will be added; that is what k p equal to 1 per second means; the rate of addition in every second, every monomer; every second, 1 monomer will be added and it will polymerize. So, then, the length at any time t is given by, k p is 1; so, 1 into t plus, l 0 is 4. So, now, this is, this, this is the solution; that is, for a, if it is, t is 10, that is after 10 seconds, you will have 10 into 1 plus 4, 14. So, let us say, let us think of a column here, right. So, we can think of a table. So, we can have time here and the length here. So, when t is equal to 1 second, 1 into 1 is 1, 1 plus 4 is 5; the length is 5. When t is 2, that is, t is equal to 2 seconds, t into, 2 into 1, 2, plus 4 is 6; when t is 3 seconds, 3 into 1 plus 4 is 7. So, l becomes 7; 3 into 1 plus 4, 7. So, this is similarly, if it goes and let it be 10; then, t is 10, 10 into 1 plus 4, 14. So, for any value of t, there will be a value of l. What does it mean? That means that, we can plot this as a function of time.

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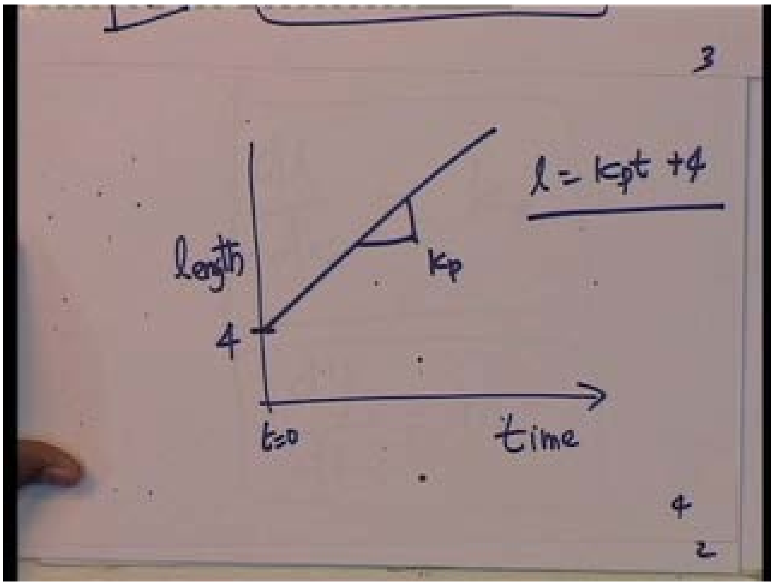

So, let us plot here. So, let us plot length as a function of time. So, this is the length of the polymer as a function of time and t equal to 0 we had 4 monomers. So, this is t equal to 0. We had 4 monomers. So, this is 4. And, as the time increased, this increased like a straight line. So, we had l is k p into t plus 4. This was our equation. This is the equation that we have plotted, where k p is the slope of this; k p is the slope of this straight line. So, just like the equation looked similar to that of y is equal to m, d y by d x is equal to m, which immediately gives you y is equal to m x plus c, which is equation of a straight line. This is just like that. This differential equations will give, the equation will give you d y by d x is equal to k p will give you, l is, d l by d t is equal to k p will give you, l is equal to k p t plus c, which is nothing, but a straight line. If you plot the length of the function of time, it is a linear; it is, it is a line having the slope k p.

So, the differential equation d l by d t is equal to k p will give you a straight line and physically, biologically, it means that, if you have a constant polymer, k p is a constant, then, the length will increase in a linear manner with the constant; like, it will increase, it will increase as a straight line, as we see; as we see in this plot, it will increase like this; so, this is like a linear function, as we see here. So, this is, this is what you see here is the linear function, where the length increases with time as a straight line. So, we understand this now. We understand polymerization and we understand, how do we describe polymerization using a mathematical description. But now, let us ask, if we want to describe the, let us say, de-polymerization; how does this change? Like, so, we had this equation, differential equation; will it change, if you had to describe de-polymerization what, what change will it have?

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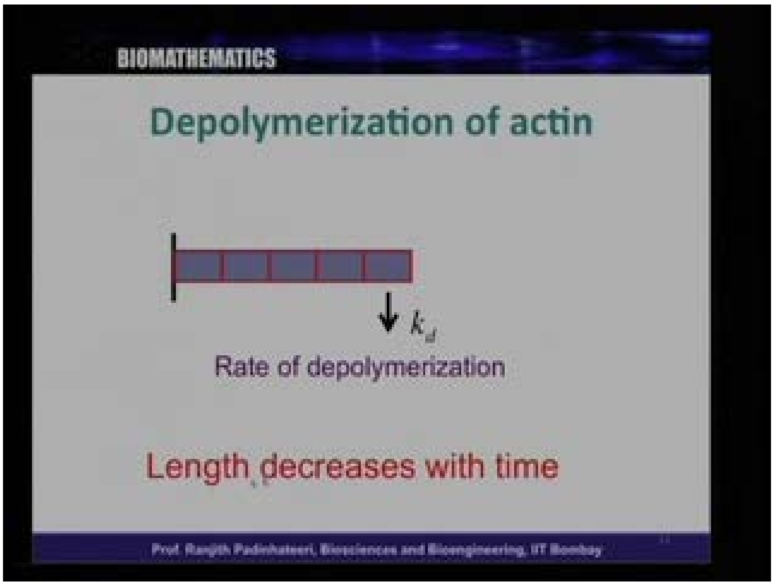

So, let us think and understand, in the case of de-polymerization, have a look at this slide. What you have is, you have a polymer, as we said; instead of polymerizing, this polymer is de-polymerizing. So, k d is the rate of de-polymerization. So, when the de-polymerizes with the rate d, the length of the polymer decreases. So, this is something which is, which we all know that, when it is de-polymerizing, what will happen, the length decreases with time. So, this is what happens. If you have a polymer, which is de-polymerizing, so, the length comes, decreases one by one. So, it just shortens the polymer, shortens and shortens and shortens and shortens. So, this is what physically happens. So, now, in the previous case, it was length increase; in this case, it is length decrease. So, that is the difference. If you polymerize, the length increases; if you de-polymerize, the length decreases. But in the both the cases, the length changes.

So, there is change in length in both the case. There is a number rate, with which it increases or decreases; but in this case, it is decrease. So, we have to somehow put in this difference, increase and decrease. So, they are opposite things. So, how do we put in something mathematically, increase and decrease? So, we will, as you know, like, increase, we always use like, positive is increase and negative is decrease. You add, 2 plus 3, addition, is increase in the value and subtraction 2 minus 3 is decreasing the value. So, then, minus or the negative implies, will give you this idea of decrease. So, we know this from the numbers and we will use exactly the same idea here. So, whatever we had previous, plus a negative.

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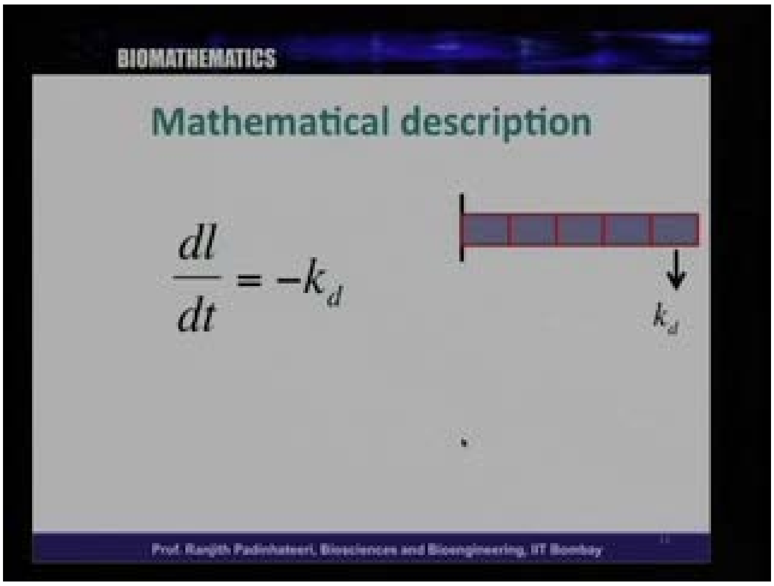

So, let us see what do we get. So, have look at this slide. So, we will write d l by d t, which says, which also, this left side only says that, the length changes with time; d l by d t means, d l, the length, change in length with d t, when the time changes by an amount d t, the length changes by an amount d t; but this change is decrease here. So, we have to say specifically, the change is decrease. So, when it is decreasing, it is the minus sign. So, look at this slide here, it is the minus sign here. So, d l by d t is minus k d, where k d is the rate of de-polymerization. So, this is the crucial difference, this minus sign; then, the meaning changes completely. Now, we are describing a phenomena, where things, the change in length, it is decreasing. Previously, it was a plus here. We had d l by d t is equal to plus k p. So, then, it was an increase; here, it is a decrease. So, this is the crucial difference. So, when we want to describe something is increasing, we will use plus; something is decreasing, we will use minus. This is the standard thing in mathematics. We will make use of that and then, let us see, what does that give, how does that change our answer.

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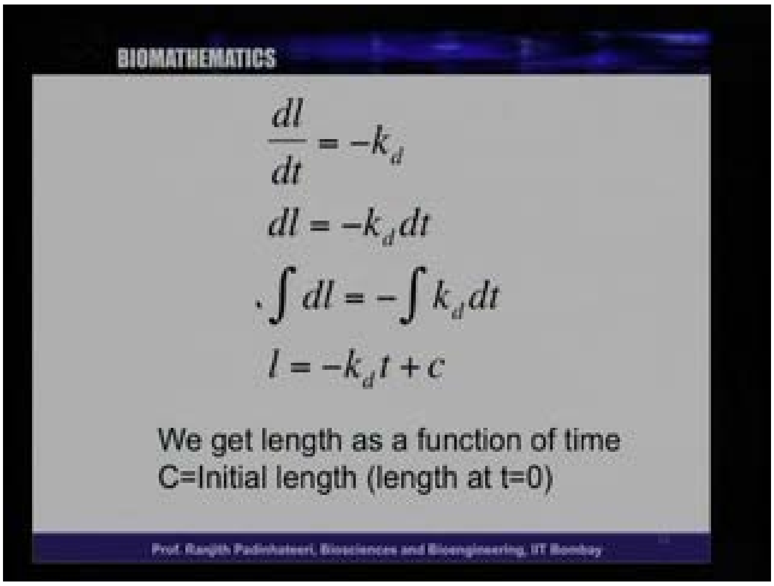

So, let us look at the slide, next slide. So, we had this equation, we had this equation d l by d t is equal to minus k d. So, look at this solution here, what is written in this slide. So, as you see here, what do you see, d l can be written as minus k d t. Now, we can integrate on both sides. So, you can integrate here and integrate here; then, you get integral d l is equal to minus integral k d t. So, if, since you have this minus sign here, the integral of d l is l, but integral of k d t is, since k d is a constant, it is like just k d t plus a c. So, since k d is a constant; so, you can take it outside and then, you can write it as k d t plus a constant. So, we get length as the function of time here, again, but there is a minus sign here. So, what does that mean? We will see what does that mean. So, what does that mean is that…So, let us have a look at what does that mean here.

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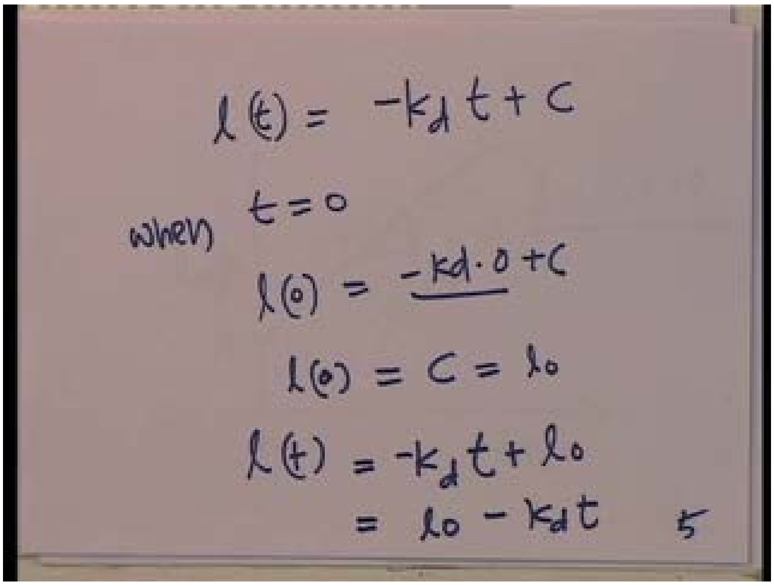

So, let us look at this here. So, we have l as a function of time, which is minus k d t plus c. So, just like we had in the previous case, when t is equal to 0, what do we have? When t is equal to 0, we have l equal to, l of 0 is equal to minus k d in to 0 plus c. So, this is 0. So, l at 0 is equal to c. So, we get that, c is again the initial length. So, we can write, l at c, we can substitute this back to the c. So, we can write c as l 0. So, let me call l of 0 as l 0; I just write l of 0 for convenience. So, l of t is nothing, but minus k d into t plus l 0. In other words, it is l 0 minus k d t. So, previously, it was l 0, here it is plus; now, it became minus. So, whatever be the initial length, from that, it decreases, as the time goes. So, this is again a straight line with a negative slope. So, this is like, y is equal to c minus m x. So, m has a minus sign here. So, this will look like a decreasing function. So, let us see how does this plot; when we plot it, what do we get. So, let us look at, how do we plot this. So, have a look at here.

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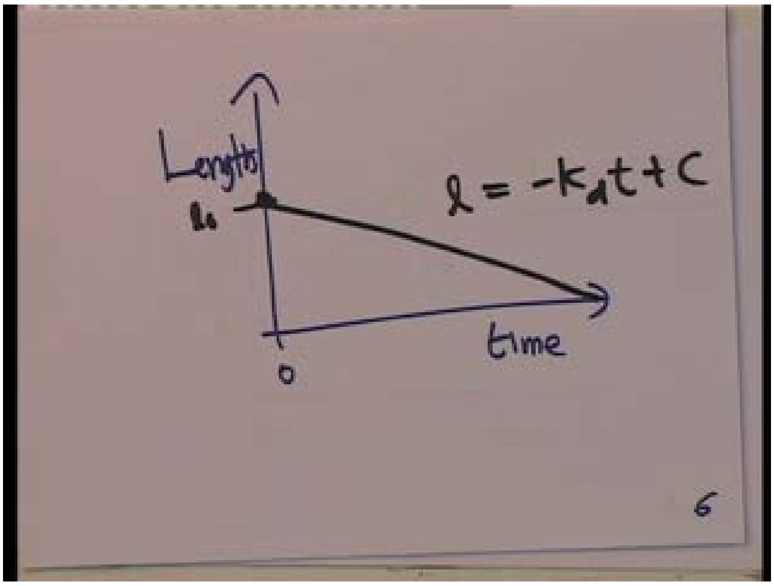

So, we can plot length versus time again, here, length of the polymer versus time. So, at t is equal to 0, we have some initial length, which is l 0. So, let us, let me mark this initial length as l 0. So, this is our l 0. And, as the time goes, it decreases. So, it decreases like a straight line. So, l is equal to minus k d t plus a constant. So, in the previous case, we had an increasing, a function, that is increasing; here, we have a something, that is decreasing. So, this is the way differential equations work. So, we saw two differential equations, but essentially, the part of the same idea; one was polymerization; the other one was de-polymerization. Now, given this two, you understand, let us go to a slightly different case; little more complex case and let us see, for that phenomena, how do we write down the differential equation. So, what is that? Again, we will use another example, that is we are very familiar with; that is the example of, like, cell division or the bacteria…We will take, for a example here, the case of bacterial cell division. So, the number of bacteria, as they divide and divide, and divide and the number increases. So, how do we describe this phenomena using a mathematical description? Again, we will come and we will have to use differential equations and that is how we will try and understand about…

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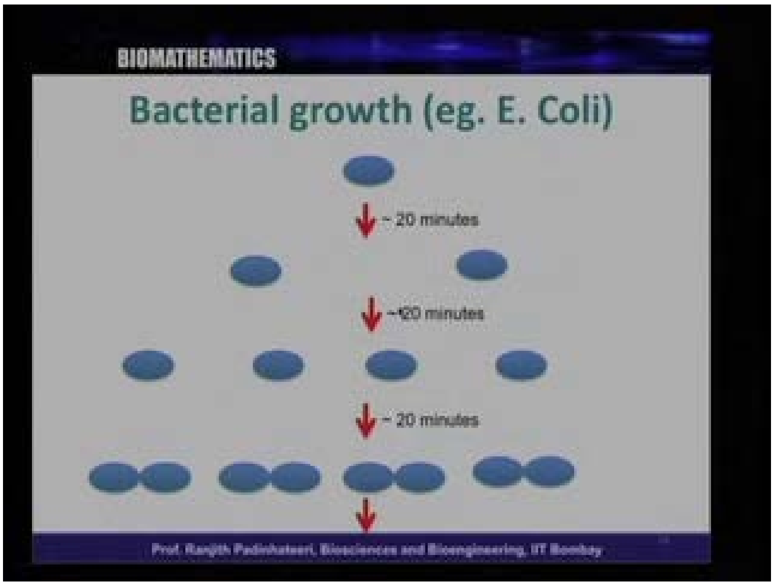

So, let us look at the next slide. So, we want to say bacterial growth. So, the example is, we could say, example is e coli. So, when you have e coli, you start with some, let us say, 1 cell; it divides into 2 in about 20 minutes. So, that is the typical time with which bacteria divides and it again divides into 2, in 2, each of them divides into 2, so, it is again 4; they again divide into, each of them again to 2, so, 8 and so on and so forth. So, they divide 2, 4, 6, 8, 16, 32. So, then, it just keeps increasing. Each of them can divide. So, if there are 3 of them, all 3 can divide and become 6; and if there are 6 of them, all 6 can divide and become 12; and if you have 12 of them, all the 12 can divide and become 24. So, in this case, there are two points to note.

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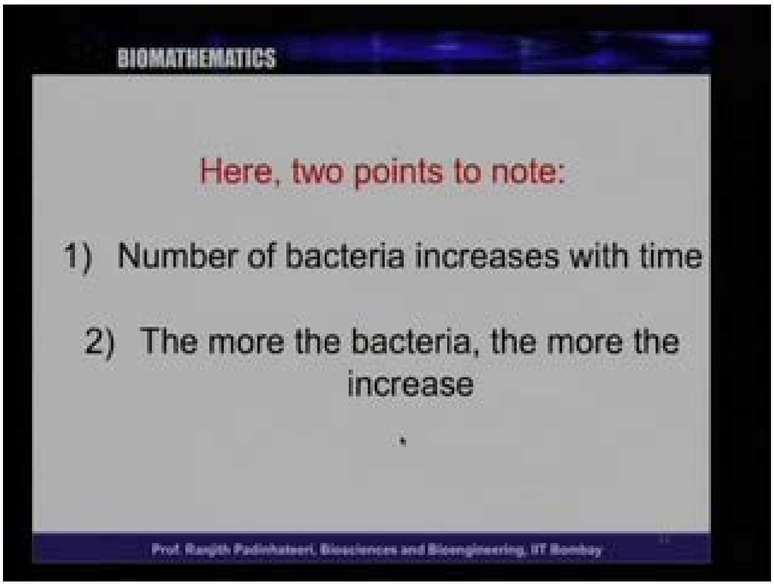

So, let us have a look at what are the two points to note. So, one is the Number of bacteria increases with time. So, number increases. There is no question about it. As the time goes, the number increases. And, the more the bacteria, the more the increase. If there are, that is what just now we were discussing; if there are like, 10 bacteria, all the 10 can divide. So, 10 will become 20; if there are only 2 of them, 2 can become 4. So, they increase only by 2. So, let us try and understand this a bit more; what we mean by the more the number, the more the increase is.

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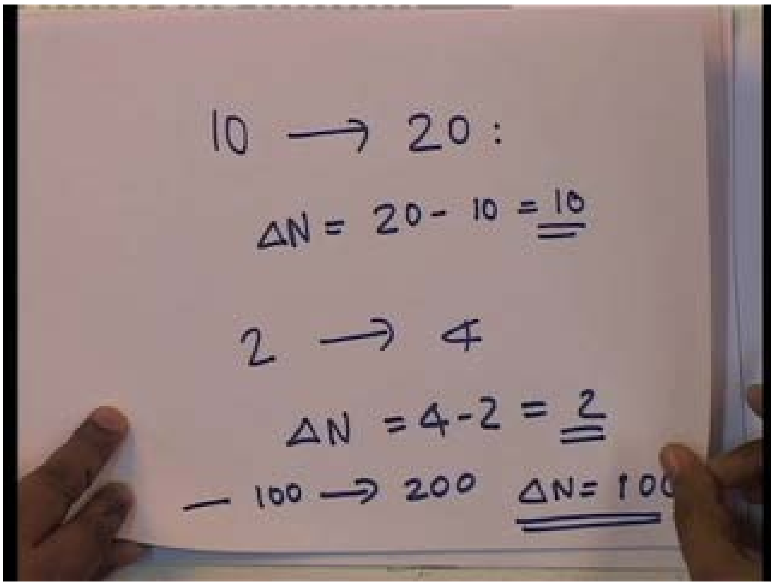

So, let us look at here. If there are 10 bacteria, they divide and go to 20. So, the difference in numbers…So, then…So, I begin with the 10 and then became 20. So, the difference in number is 20 minus 10 is 10. So, the increase or the change in number is 10. If there were only, like 2 bacteria, they will go to 4. So, the change is just 4 minus 2, which is only 2. If there are 100 of them, they will go to 200 so, they increase by 100. So, the delta N will be 100. So, the more the number, the more the increase is. So, what does that, how do we say this mathematically? The most…So, here, we had 100, the delta N was 100; here, we had 2, the delta N was 2; here it was delta N is 10 and delta N was 10. So, 2 has delta N 2, 10 has delta N 10, 100 has delta N 100. So, the delta N, what we have here is, proportional to the N.

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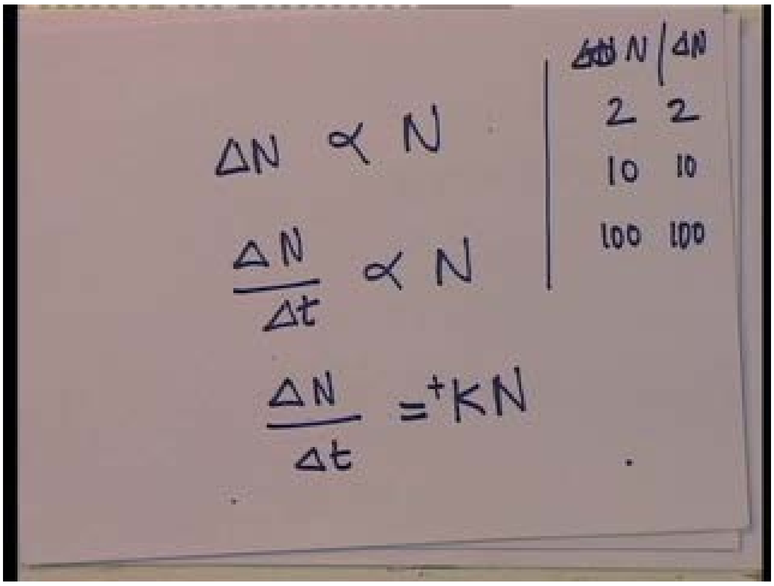

So...So, what does this…How do we say this mathematically? We can say that, delta N is proportional to the N itself; because we said that, when, when we had N and delta N, we had previously, 2 had delta N 2, 10 had delta N 10, 100 had delta N 100. So, the delta N is proportional to N. In other words, the change in number, when the time changes, increases with time; in 20 minutes, in a given time delta t, they increases proportional to N; if there are more of them they will increase more. So, this proportionality constant can be, anything that is proportional in mathematics, can be changed to equal to, by adding a constant k. So, you can write d delta N by delta t is k N. Now, you have to say this is a plus sign here, because, this is increase. As we said that, there are two points to note. So, look at this slide. The two points to note is that, first it increases with time; the second point is that, the more the bacteria, the more the increase is. So, combining these two points, we get the following equations.

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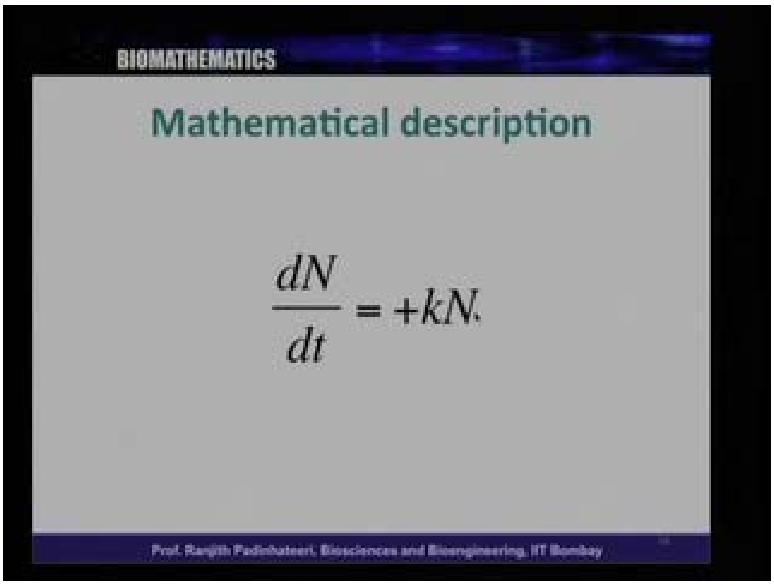

So, let us look at the next slide. The equation, the mathematical description is the following. The d N by d t is plus k N; the plus represents the increase; not the decrease. It is increasing and the fact that it is, k N is proportional to N, implies this phenomena, that we see that, the more the N, the more division will happen. So, the more, there are more numbers to divide, so, the increase will be more. So, we combine these two ideas into one line and we got this equation d N by d t, the change in number when the time changes, that is d N and by d t, is equal to plus k N. So, we have a new mathematical equation, new…So, this is another kind of differential equation. Previously, we had only some equation, there was no N; something like d N by d t equal to k. So, we had d l by d t is equal to k. Then, we knew how to solve it. Now, we have d N by d t is equal to k times N. So, how do we solve it? So, let us look at the next slide. We will, we will try and do this in a similar way that we did.

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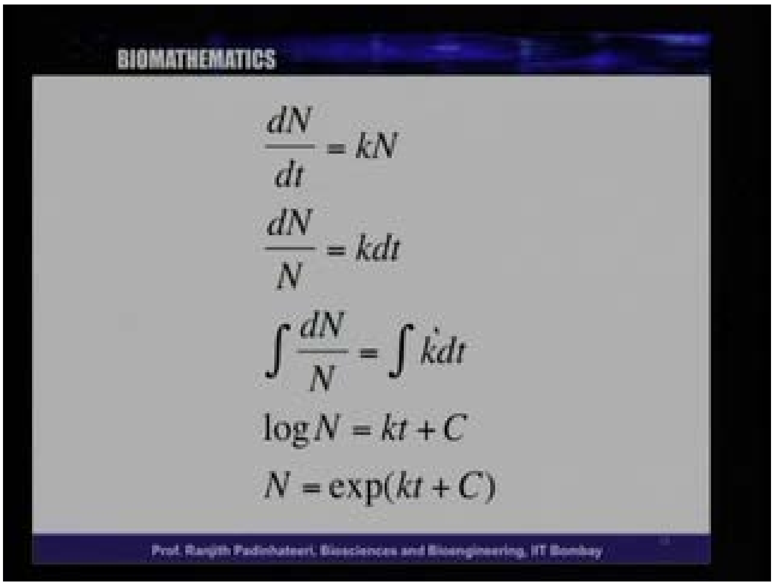

So, let us look at the next slide. So, we have d N by d t is equal to k N. So, that is the equation that we have. We can rewrite this as d N by N. So, we can take this N below and d t the other way. So, d N by N is equal to k d t. So, that is what this says, d N by d t is k d t. Now, you can integrate both ways. So, d N by N is integral k d t. So, that is what this says. So, d N by N is integral k d t. Now, what is integral d N by N? So, we, as we studied before, integral d N by N is log N or we also studied the derivative of log N is 1 over N. So, integral of d N by N is log N and integral of k d t, since k is a constant, integral of k d t is k t, plus we have a constant of integration, k t plus c. Now, this log is a natural log, or this is log to the base e or ln. So, we can, anything log y is equal to x, you can write y is equal to e power x, exponential.

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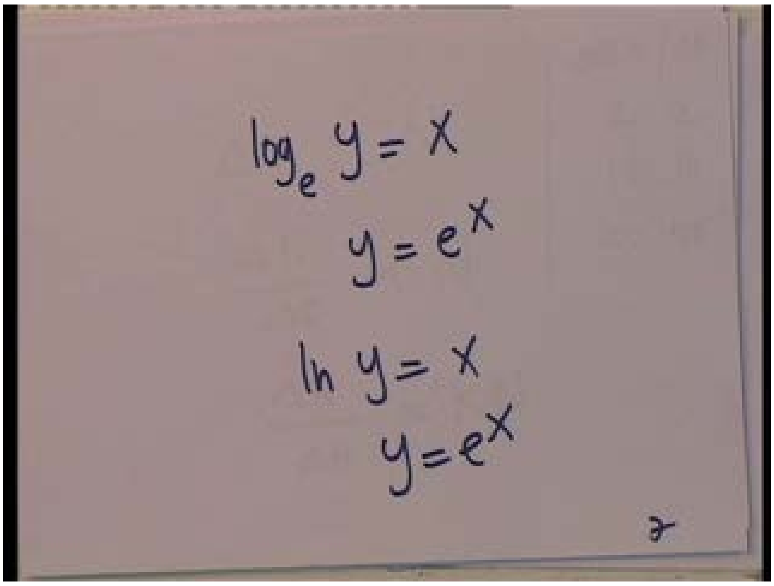

Like, we studied this, you know, let me say. We studied that, log to the base e y is x; y is e power x. In other words, ln y is x; y is e power x. So, this is something that we studied before. So, keeping this is in mind, we end up this answer, that is, log N, if you have log N is equal to k t plus c, as we have it here, we can say N is equal to e power k t plus c. Now, what is c?

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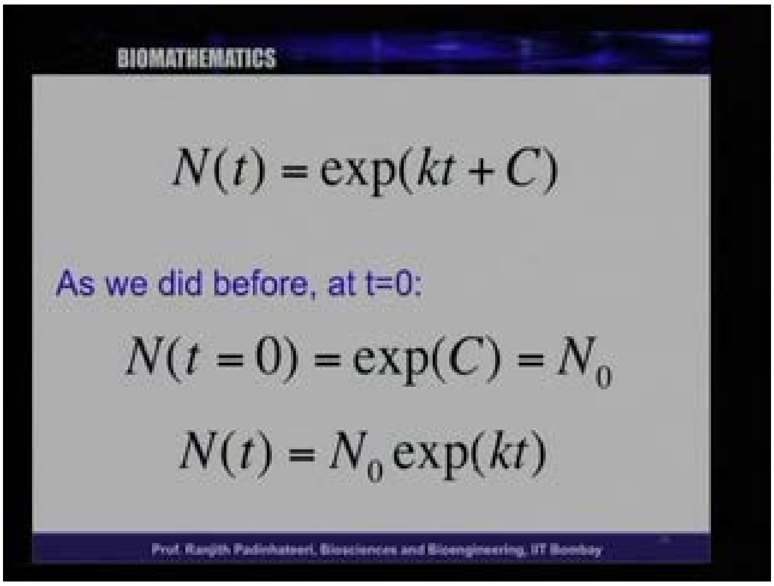

So, let us have a look at, as before. So, what do we have? We have N, the number of bacteria as a function of time. So, for a given value of t, given k and c, you have N. Now, as before, when t is equal to 0, we get N of t equal to 0; that is, N 0 is equal to exponential c; whether this k t becomes 0, k into t becomes t 0. So, k t is 0, this term was 0. So, e power c is equal to N 0. So, we can write this equation, the substituting this e power c back. So, you know, trick in mathematics that…

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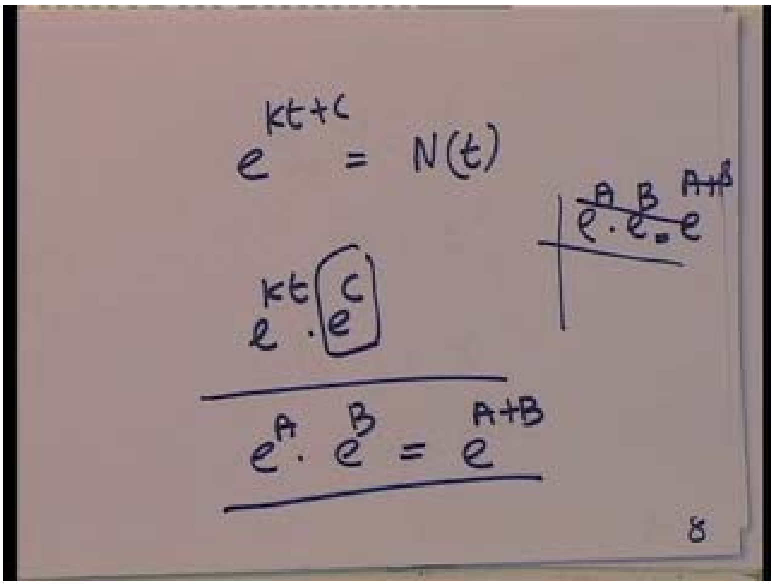

So, what do we have here? e power k t plus c is equal to N of t; that is what we had and e power k t plus c can be written as e power k t into e power c. In mathematics, e power a into e power b is e power a plus b. So, knowing this identity, you can write e power k t plus c as e power k t into e power c. So, that is what we did, and this e power c is nothing, but N 0. So, that is what we did here. So, we know that, e power c is N 0. So, N of t is N 0 e power k t. So, we can write it in this way. So, what does this mean now? So, what does this N of t is equal to N 0 e power k t means? How does this graph look like? If you plot this, how does this look like?

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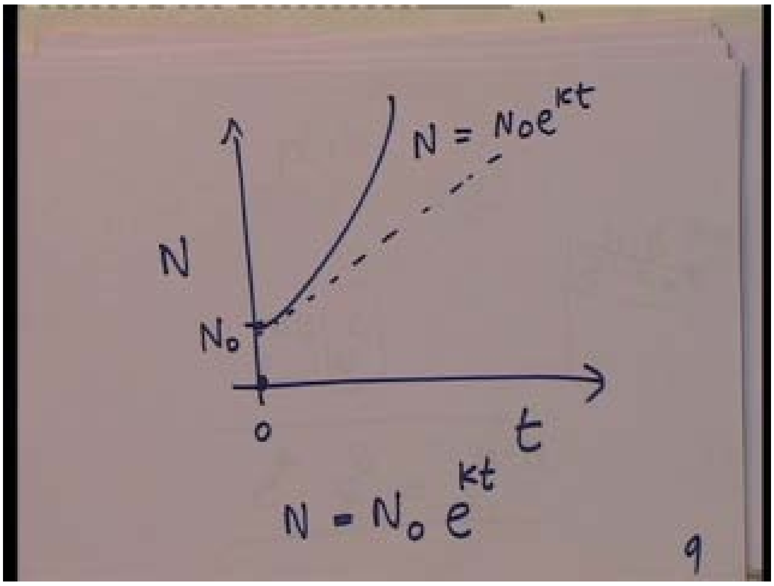

So, let us plot this. N versus t. For every t value, there is an N and our equation is N is N 0 e power k t. So, when t equal to 0, N equal to N 0. So, when t equal to 0, this is t equal to 0, N has some particular value; let me call this N 0. So, this is 0 and the t increases. Now, this is an e power, this is an exponential function and we have seen how do we plot exponential functions. So, this is, they increase like, very fast; the straight line will be something like this; but straight, if a straight line, it will looks something like this. But this increases very fast, much faster than a straight line. So, this is an exponential increase. So, this is N is equal to N 0 e power k t; exponentially increases. How fast they increase, that will depend on the value of k. We will discuss how does the value of k depends on many other things etcetera; that is something we can discuss. But at this moment, for you to understand the differential equation, it is sufficient to say that given a value of k, you can write down such a solution and that will give you this exponential growths, that we study in any growth curve, right; when we have any growth curve, we have this exponential growth. When…

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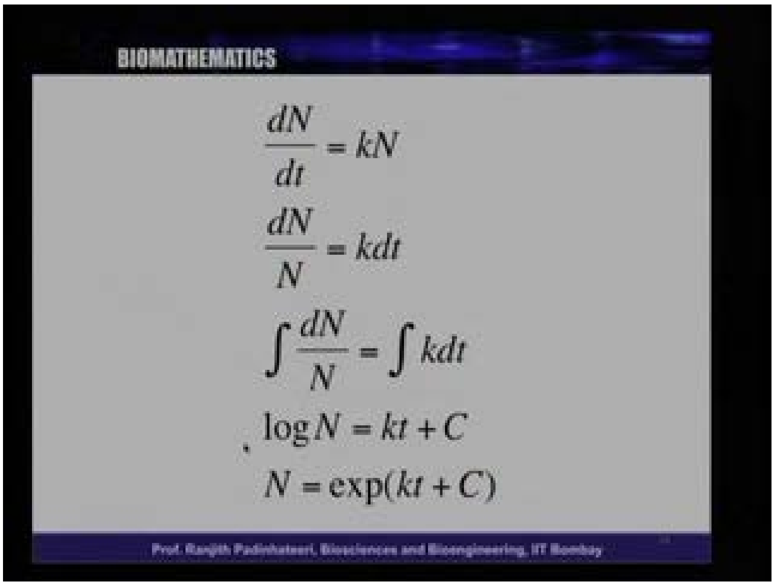

So, let us, let us for the moment, go back to this slide, the previous slide. We had this log N is equal to k t plus c. So, let us see, what we get, if we plot this equation. This is also, we had, we started from d N by d t is equal to k N and we got log N is equal to k t plus c.

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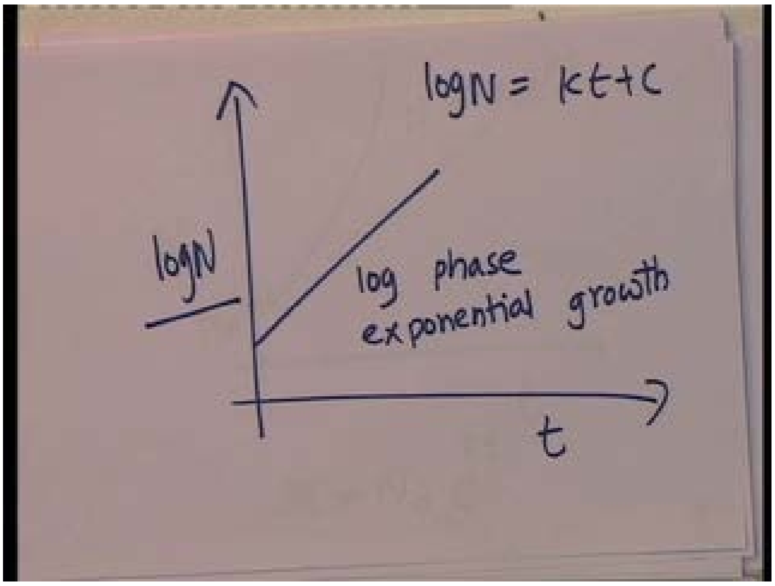

So, when we have this log N is equal to k t plus c, what do we get. So, let us plot log N here and t here. And, if you plot log N is equal to k t plus c, which is actually the solution of this equation which we wrote for the bacterial growth, this part, we will get, this is like, y is equal to m into x plus c. So, you know that, this will look like a straight line. So, this is what typically, you call the log phase; in Biology, we call log phase, right. So, the growth, there is a log phase of growth. So, we have such an equation; there is a growth, which we draw; most of the time we draw straight line; the way we saw…The reason that we are drawing straight line is precisely this, that in the x, in the y axis, we are plotting the log N not N. So, we plot log N in the y axis and t, this part will look like a straight line. So, this is, we can call the log phase of bacterial growth. This is what we discussing, the second or log phase or exponential phase.

So, this is exponential growth. So, exponential growth, some, most, sometime, we can see it as the straight line people draw; that is because, you plot log N versus t. If you plot, on the other hand, N versus t, it will look like an exponential graph. So, it depends on what you draw. People sometimes call it exponential phase; sometime people call it log phase. So, essentially, this is the same thing. So, this equation that we said, d N by d t is equal to k N, this differential equation that we described, describes the log phase or the exponential phase of growth. So, again we used a mathematical description and equation, to tell you something about this phenomena. And, it precisely tell you, after 10 minutes, if you tell me at t equal to 0, how many cells you start with and the rate of cell division, we know that, we can say that, this will tell you, this equation will tell you, how many of them are there, as a function of time, ok.

So, we discussed polymerization; we discussed cell division or bacterial growth, any growth; this can be applied to any growth. As we can see, it is very general. It can be any number; it can be bacteria, it can be something else. So, that is why you use k, k is some general rate, rate of growth. It could be even like rate of, like, number of, population, right; the population can grow exponentially. If we had, nobody was dying, you had only growth and things would exponentially grow. We will not take any death in to account; we get only the growth. So, we exponentially grew. Now, let us think of how do we take the death into account or it is, in the case of cell, it is like apoptosis, that is cell death. So, how do we take this cell death in to account? How do we describe that mathematically? We had polymerization, de-polymerization; the difference was, in the polymerization things increased; the length increased.

In de-polymerization, length decreased. In the case, again, the number increases when it is cell growth; in apoptosis, this, it is cell, the number decreases or cell death, the number of cells, live cells, decreases. So, how do we describe that? Just like we did, a minus sign will do. Like we had this minus sign idea, for this…

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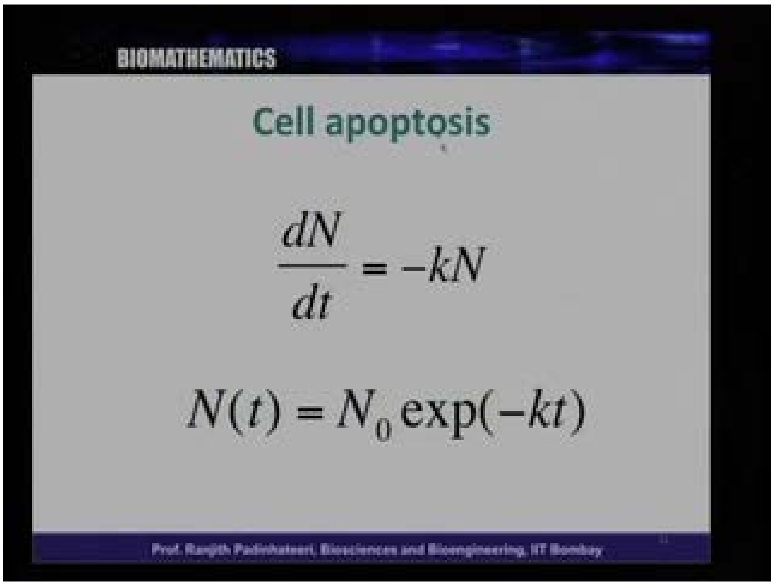

So, let us look at this here. When we have apoptosis, we can describe that, by writing d N by d t is equal to minus k N. And, as we just saw, we can solve this again, the way we did and we will get a solution, N of t is N 0 e power minus k t. So, the solution has a minus sign here; e power minus k t. We had previously, N 0 e power plus k t. So, now, it is a minus k t, here. So, now, how do we draw this, if we want to draw. So, let us say…

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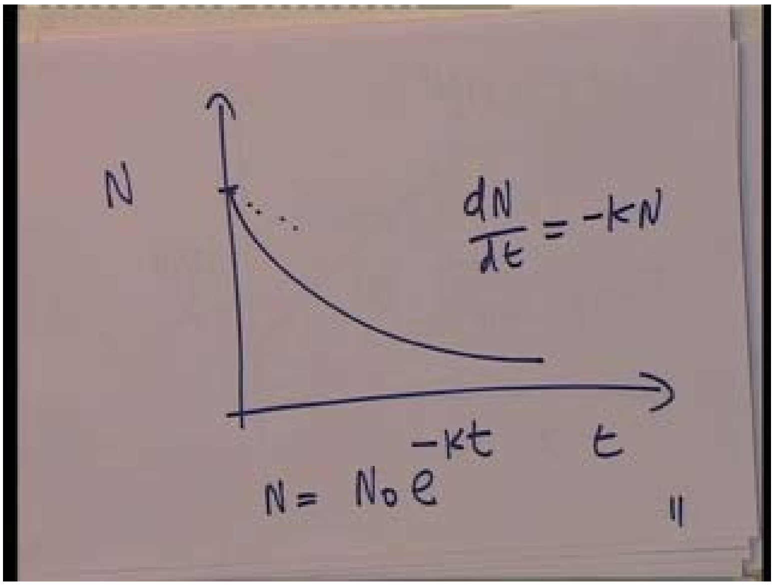

Let us draw this N 0 e power minus k t. So, let us look at here. So, you have N and t and at t equal to 0 we have N 0; that is this. And, what we want to say is that, N 0, N is equal to N 0 e power minus k t, exponential minus k t and that will look something like this. As the time goes, the number increases exponentially; if it is a straight line, we could have got, we would have got something like, straight line like this. But here, the exponentially decreases. So, this is, this will describe this equation N is equal to N 0 and…So, this is the solution of d N by d t is equal to minus k times N. So, this is the case that we discussed.

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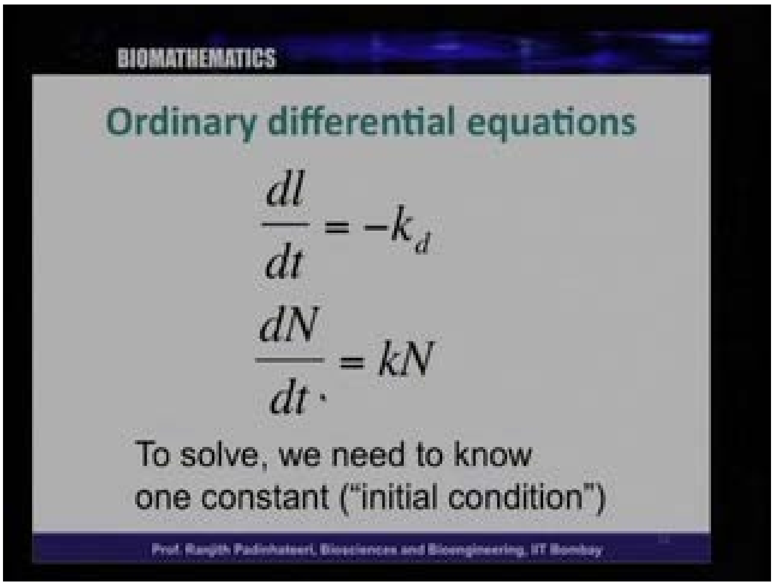

So, let us see, what we have. So, we have, what we studied is like two kinds of equations and such equations are called ordinary differential equations. Now, why this ordinary? We will discuss this, because when we discuss some other type of differential equation, we will understand the difference, and we will see what is ordinary differential equation and what is some other equation. So, such differential equations, two examples, d l by d t is equal to minus k d or d l by d t is equal to plus k d, this describing the rate of change of length, the length increase or length decrease, depending on the minus or plus sign, plus or minus sign, is describe this first equation; the number increase is described by this equation. So, the first equation precisely describes the decrease in length of anything, a polymer.

If l is the length, d l by d t equal to minus k d is the rate of de-polymerization k d, then, this will describe the de-polymerization of any polymer, for example, actin. This d N by d t is equal to k N, will describe the number increase of anything that is growing. So, the bacterial growth or a, any yeast cell growth; the log phase or the growth phase or the exponential phase of it will be described by this equation. So, basically, what we said, we found that, to solve this equation, as we see here, in the slide again. To solve this, we need to have one constant, this c thing and that we call initial condition. Since it is involving time, this is always at time equal to 0; if we know, we know this solution, so that, we call it as a initial condition. At the beginning of the experiment, how many numbers are there, what is the length; by knowing that, we can solve this.

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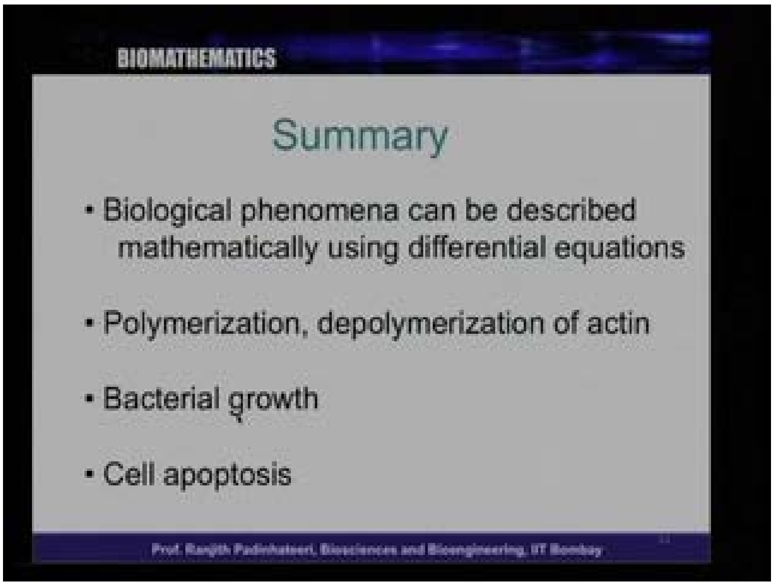

So, let us summarize what we saw. So, to summarize, the biological phenomena can be described mathematically using differential equations. And, for examples we described polymerization and de-polymerization of actin, bacterial growth and cell apoptosis. And, knowing one constant, which is the initial condition, in this case, we will describe, we will, you can solve this equation and get length or number, as a function time. So, this is what we learnt in this class. In the next class, we will describe more differential equations and find, try and learn, how do we solve them and thereby, how do we describe various mathematical phenomena. Thank you very much .