Biomathematics

Dr. RanjithPadinhateeri

Department of Biotechnology

Indian Institute of Technology, Bombay

Lecture No. # 14

Vectors

Hello. Welcome to this lecture of Biomathematics. In this lecture, we will be discussing a new concept, very important concept, which, it is a very basic concept and important concept to understand any physical system. So, since we, our aim is to understand Biology and Biological systems using Mathematics, it is very important to understand this particular concept. So, the idea, the concept that we study today, is called vectors. So, today's topic is Vectors. We will discuss what is vector, what are vectors and what is the importance of this and what exactly, why exactly, we need to know this vectors. So, before discussing that, first…So, the first question comes to anybody's mind when we come to any topic is that, why should we learn this topic at all? Why should we learn about vectors at all?

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So, that is the next question that we want to ask. Why should we learn about vectors? So, let us take…To answer this question that, why should we learn about vectors, let us take something, some very common things in Biology and then, let us see, if, let us try to understand those things and then, we will learn, we will see that, to understand them, we need to understand a new idea and that idea called, it is called vector. So, the most important part of Biology, in some sense, are proteins and proteins and DNA. Many molecules, bio-molecules are actually charged molecules. So, they all have charge and we have been discussing various things about charges and all that. And, when there are charged molecules, they attract, repel and all that. So, let us think about, a bit about charged molecules and ask a few questions and let us see, to understand, to answer those questions, whether we need something new or not; and then, we would need something new and then, we will see, we will go ahead and learn this. So, let us think about, a bit about charges today, to begin with, and let us see, how, where we go from there. So, the first question about charges actually is that, have a look at it here.

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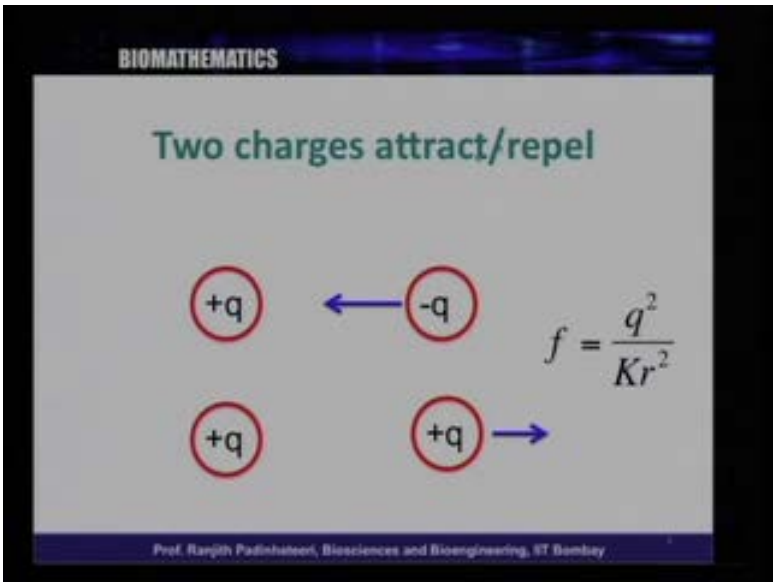

So, we all know that, two charges, whether they are, sometime, if they are positive and negative charges, opposite charges, they attract and when they are opposite charges, is the same charges, they repel. So, charges either attract or repel. So, if you have a look at here, if you have a plus q and a minus q, then, they attract; that means, this minus charge will go, will move towards the plus charge. There will be a force on this charge and the force on this, both this charge will be, the magnitude of the force will be q square by

some constant K times r square, where r is the distance between this two charges. This, we know.

This is the famous Coulomb's law. So, we also know that, this is the magnitude of the force and this, the force on this negative charge is such a way that, they are attracting; that means, the negative charge will be pulled towards the plus charge. The force on this charge, due to this charge, will be such that, this charge will be repelled away. So, this charge will be repelled in this direction, this direction of arrow. So, this two arrows here, represent, this arrow represents…So, you can see that, this arrow, from this arrow, you will understand that, this charge is being pulled in this particular direction. From this arrow, you understand that, this charge being pushed away in this direction.

So, you have two charges. When they are opposite, this charge will push this charge away in this direction. When they are opposite, this charge will pull this charge towards this. So, this are, this are some things that we know. So, charges either can repel in one direction or it get attracted in the opposite direction. So, there is a direction coming here. We had to put some arrows. So, look at, in the, to tell you, to convey to somebody that, this is being attracted in this direction, this is being repelled in this direction, we had to put some arrows. So, have a look at this slide here. You have to put arrows to convey to somebody that, it is being attracted in this direction; it is being repelled in this direction. So, we started saying that, Mathematics is a language.

If mathematics is a language to tell you that, it is being repelled in a particular direction, or it is being attracted in a particular direction…In a picture, we can put arrows…In, if you speak in English, we can say that, it is going towards this, towards this and all that. But in Mathematics, how do we do that? So, the answer to that, is vectors. So, vectors will help you precisely, telling in which direction things move, or in which direction the force is being applied. So, in which direction the attraction is happening, or in which direction the repulsion is happening. So, all this, if you want to precisely convey to somebody, you have to use this idea of vectors. So, that is the need of vectors. So, we will go ahead and then, see more examples and precisely understand, how do we use this vector idea, to convey this direction, that this…

In this particular, this thing about direction, in which direction things are moving, or in which direction the force is being applied…But the need of the vector is basically, to

convey the direction of any quantity, that would be either moving, or…So, what are the quantities that are having direction, so, that is the question. So, that is something, we will come. So, the vectors are some quantities that are having direction. So, force is a vector. You saw that, force has either they are, the (( )) we can apply a force in this particular way or you apply a force in this way. So, force has direction. So, force is a vector. So, in Mathematics, vector is defined as a quantity that has direction. So, let us look at this force little more. How do we precisely put this direction thing using Mathematics? So, that is the question we have. How do we put this direction in a, using a mathematical symbol?

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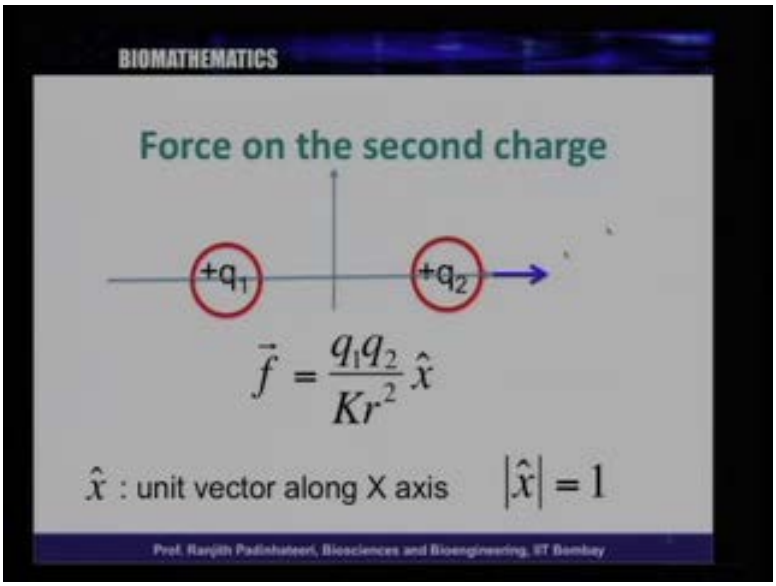

So, let us look at the next slide, and the answer is here. So, let us look at the force again. So, you have two charges q 1 and q 2 and we want the force on the second charge at q 2 due to the first charge. So, that force is represented by f with a sign here, which is an arrow here; this is, this is saying that, f is a vector. So, any vector is typically represented by an arrow on the top. So, this is the mathematical representation of a vector. If any quantity is a vector, you typically put an arrow on top of this. And, the value of the force is basically, as we know, q 1 q 2 by K r square. That is the value of this and this is being repelled in the direction of the X axis. So, you can see here, there is the X axis and Y axis in the slide. Have a look at this slide. There is a X axis and Y axis and you have two charges and the charge is being pushed in this direction of the X axis. So, look at this slide. In the direction…So, this is your X axis and this is your Y axis. So, in the direction

of the X axis is where, in the direction of the plus X axis, as this is the X axis, increase in direction of X axis.

So, this is the direction of the force and so, now, we have to put that. So, that is what we have put here, x with the hat like sign. So, this, we can call it x hat or x cap. So, this symbolizes that, the direction of the force is this, is in the direction of the X axis. The direction of the force is in the direction of the X axis, this is what this means. So, x is a unit vector along the X axis. So, this is the vector, which is along the X axis. And, the magnitude, if I say, if I write this symbol, that means, the magnitude. The magnitude is just 1. So, its direction, which is along with the X axis and the magnitude, is 1. So, that is what it means. So, any vector that we are going to represent, will have a magnitude and a direction.

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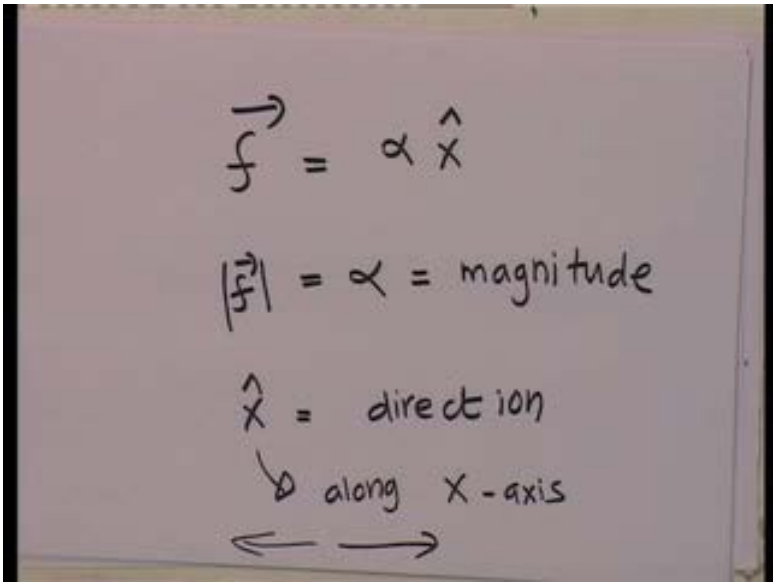

So, have a look at this, here. So, the vector f will have some magnitude. Let me call this alpha, the magnitude and some direction. So, the mod of the, mod of f will give you the magnitude alpha. The X…So, this is the magnitude. This is the direction. What does this imply? This implies is that, this implies is that, the force is along the X axis. So, this is what it implies. Now, if you want to say that, it is along the minus x axis, but…So, this typically means that, along the increasing direction of the X axis, because here, this, there is plus x, this is no, there is no particular, but…So, now, if you want…So, this

particularly means, in this direction, as we said. Now, let us say you have a force in this direction, which is the decreasing direction of the X axis, then…

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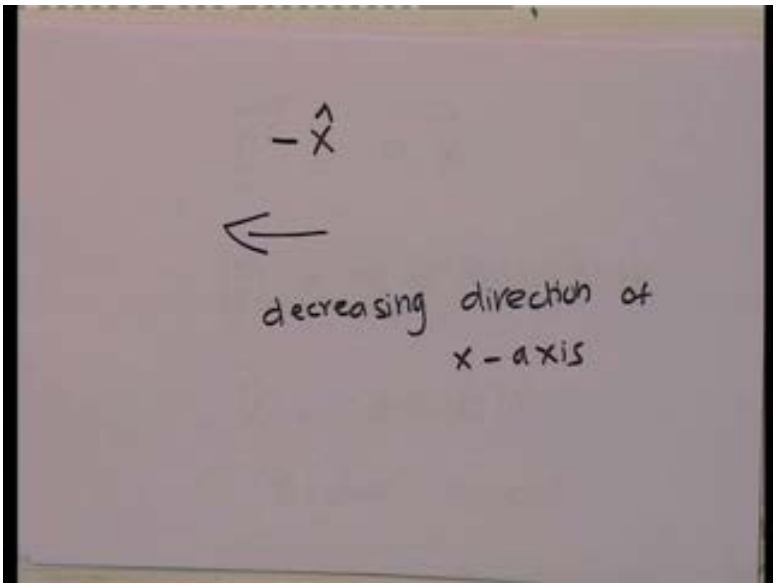

So, if there is a force in the decreasing direction of the X axis, you can say that, minus x cap; means, in this particular direction, which is, we can call it decreasing direction of the X axis. So, now, what is…So, this is basically, the another notation to show that, something is decreasing direction of the X axis. Now, let us have a look at this next slide.

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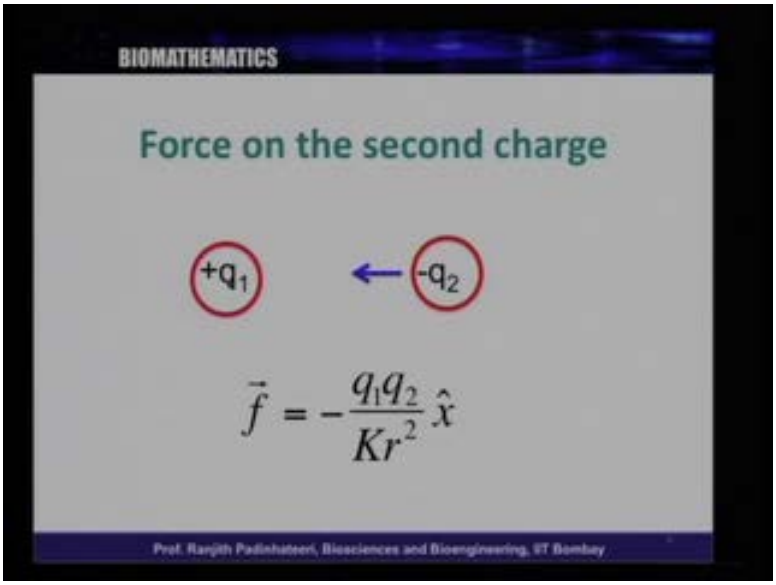

Here, what you have is q 1 and q 2, which are plus, one is a plus charge and the next one is a minus charge. And, the force on the second charge due to the first charge is again, q 1 q 2 by r square, but it is in this direction of the minus X axis; it is in this direction; it is the direction along the X axis, but they are attracting. So, the direction is this. So, see the direction of arrow. So, this direction is towards the minus, decreasing X axis, just, as we just discussed. So, you have to put a minus x cap. So, that is why minus x cap, here. So, this minus x, the magnitude is q 1 q 2 by K r square, but it is in the direction of the minus x cap. So, that is what this represent now. So, if, we learnt a new way of telling the direction way by putting x cap, which is a unit vector along the X axis. We can also, similarly, we can put a unit vector along the Y axis.

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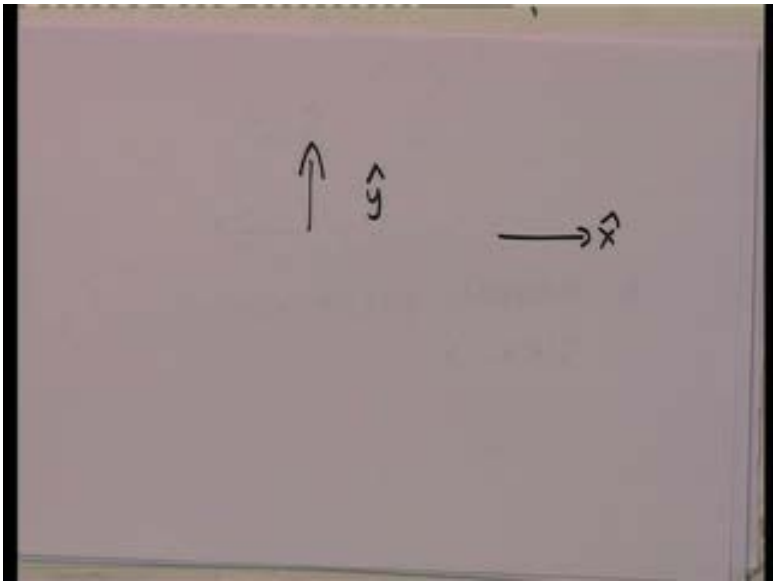

So, if there is something along the Y axis, have a look at this paper, here. If something is along Y axis, you can put y cap. So, something is along X axis, you put x cap. So, now, let us look at this next question, what we are interested. We learnt about two charges and either they can attract, which is along the minus X axis, or they can repel which is along the increasing direction of X, you will say, which is the x cap. Now, let us go to a little more complicated situation. Let us look at three charges. Let us see what happens, when there are three charges.

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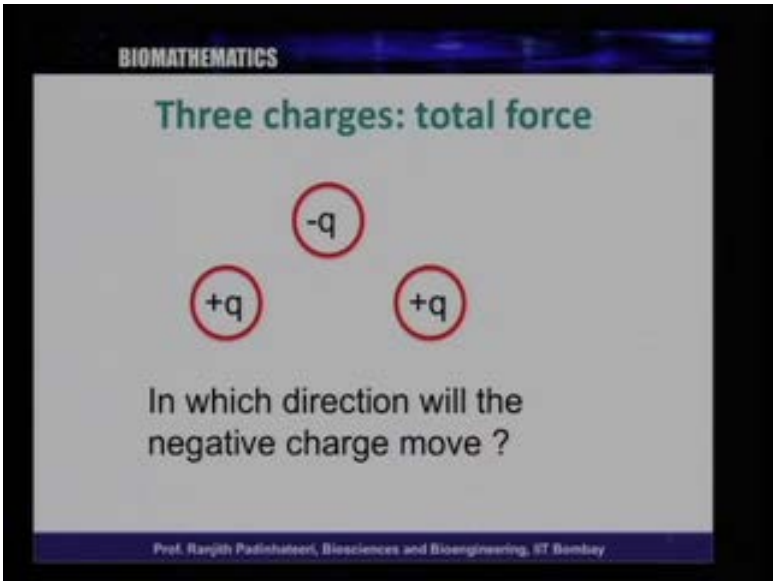

So, have a look at this. So, you have three charges there. There is a plus q, there is another plus q and there is a minus q. Now, let us say, let us ask this question. Let us say this charge and this charge are fixed there. And, let us ask the question, in which direction will the negative charge move. So, the negative charge either has to move, this charge will attract this negative charge; this also will try to attract this negative charge. So, the negative charge will move this way or this way or which way will the negative charge move. So, that is the question that we want to ask. Which way will this particular negative charge move?

To find the answer to this particular question, we need to understand little more about vectors. So, you have a force along one way and two, this two particular, different directions, we have to find out, what is the sum of this forces. Here, you need to some, learn something about adding vectors etcetera. You have to learn something about subtracting vectors. So, let us go and understand, how do we add vectors, how do we subtract vectors etcetera. So, this is some interesting concept, that we, which we have to learn; but before that, before just doing this, let us, let us see, what we, what we understand, what we discussed so far. So far, we said…So, there are, we said something about vectors and we said that, vectors are something having direction.

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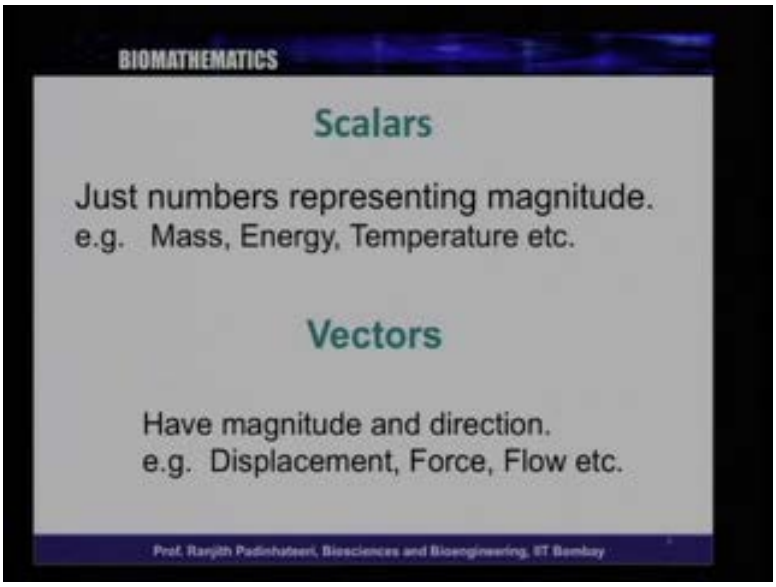

So, what are the other quantities, other than vectors? So, there are two types of quantities. There are scalars and vectors. Have a look at this slide. So, there are something called scalars and there is something called vectors. So, scalars are just numbers representing magnitude; for example, mass, energy, temperature, these things have no direction; mass has no particular direction. Energy has no particular direction. Temperature has no particular direction. So, these things are scalar; called scalars. They are just numbers. Mass can be like 3 Kg, 4 Kg, 3 gram. There is nothing called 3 gram upwards, 3 gram downwards; 3 gram is 3 gram. There is no direction for it. Similarly, we can say, energy is 23 joule. There is no 23 joule up or 23 joule down; there is no direction for energy.

Similarly, similarly you can have, for example, temperature. You can say it is, temperature is 23 degree Celsius or 40 degree Celsius. So, again temperature is a scalar. So, it has just a number representing it. So, that is the magnitude of temperature. Now, we have vectors. Have a look at this. Vectors have magnitude and direction. Examples are displacements, that is, how far you moved. So, when you, displacement is, you… So, let us say, I move, I go 2 meter northwards. So, there is the direction, towards North, towards South, towards West. They are direction, which we do, we use in everyday life. So, these are vectors. The quantities, this quantities are vectors, displacement. I go northwest direction. So, when you ask somebody, if I want to go to this particular place, how do you go? Then, you will say, you go there, turn left, then, go straight and then, turn right. So, the left, right, straight, back, these are like, directions. So, displacement, or displacement is basically, a vector, which has direction. You can either displace straight

or you can get displaced into the left; you can get displaced to the right; you can get displaced in the north-south direction; you can get displaced in the North.

Sorry, you can get displaced in the northeast direction, or northwest direction; sorry, you cannot get displaced in north-south direction. You can either get displaced in the northeast direction, or northwest direction. Similarly…So, to say all this mathematically, we need to have, use, use the idea of vectors. So, as we saw just now, force is also a vector. Have a look at this. Force is another quantity, that is vector; another quantity that is vector is Flow. Something is flowing; you can either flow in this direction, flow in this direction. Whenever there is a flow, there is always, to tell this flow, we have to have vectors. So, in Biology, flow is very common. Like, let us say, protein concentration; when the things flow from, proteins move from one place to other place…So, there is always direction associated with this. There is complicated 3D structure of proteins. So, you have to have, this atom is in this direction, and the another atom is in this direction; some other atom is some other direction. So, you have to tell this directions. So, to say this, the position of atoms properly, again, you have to use vectors.

So, you have to use positions; you have to have a force, again; in Biology all this atoms or proteins get pulled in a particular direction or they…So, they move, this vector, they have a force; they get, they feel a force, which is also a vector. And for example, proteins like actin monomers, actin monomers they flow, they flow from one part…So, there are many things that flow in Biology. So, you know, you heard of diffusion, where they flow from higher concentration to lower concentration. So, that is also a vector; there is a vector involved there. So, to learn all this, we have to have understanding, good understanding of vectors. So, as we know, the simplest vector is position. So, let us understand a bit about this vector called position vector.

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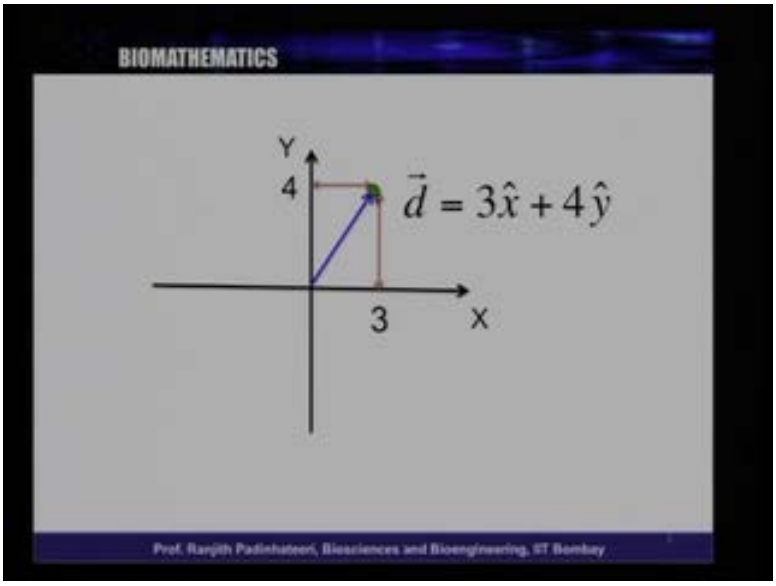

So, have a look at this slide, next slide. So, here, the green point in this slide, the green point is represented by a vector d. So, we can ask the question, how do we tell the position of the green point? How do we tell the position of this particular point? So, to reach this particular point, starting from this 0, 0, which is the origin, you have to go 3 units along the X axis and 4 units along the Y axis. So, if you go 3 units along the X axis and 4 units along Y axis, you will get this particular vector d; this position of this green dot. So, that is represented as d is equal to 3 along the X axis that is 3 x. So, x is the direction you have to go along the…So, you go to 3 meters. Let us say, this are meters. So, if you go to 3 meters along the X axis and 4 meters along the Y axis, you will reach this. So, you can say this position Vector d is nothing, but 3 x plus 4 y. So, this is 3 x plus 4 y. Then, you can ask the question…So, this two numbers, 3 and 4 are the crucial, important points. They represent this particular green point. Now, let us look at some other point.

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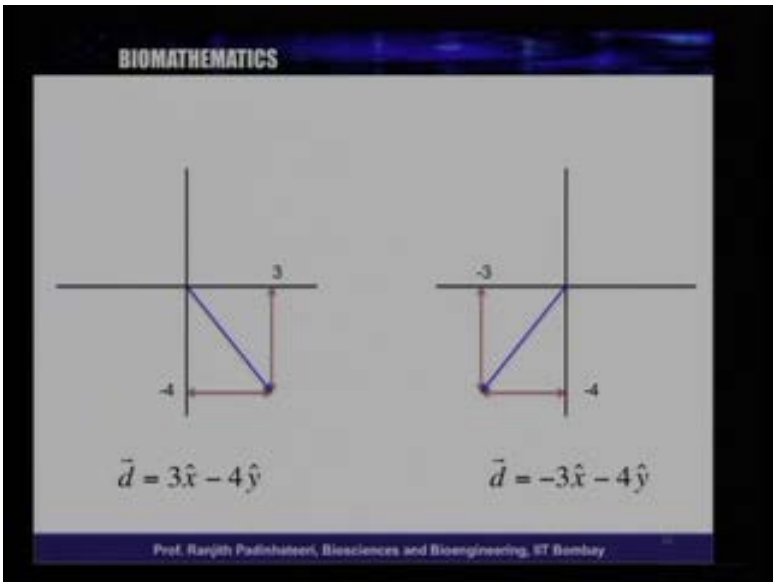

So, let us say, a point here. To reach to this point, you have to go 3 along the X axis and 4 along the minus Y axis, in this minus direction; in the decreasing direction of y, you have to go 4. So, how do you write 3 along the X axis? So, the 3 x, in the increasing direction; 4 along the decreasing direction of the y, minus y. So, that is, 4 along, minus 4 y. So, this point is represented by 3 x minus 4 y. This minus represent, along the y, you have to, 4 unit you have to go in the decreasing direction of the y, in a minus y direction. Similarly, what about this point here? This point here is…To reach this point, you have to come 4, 3 units along the decreasing direction of the X axis and 4 units along the decreasing direction of the Y axis. So, the total is 3 x minus 4 y. So, this minus 3 and minus 4 are two quantities that basically, describe this particular point. So, as you see here, there is, there are two quantities, two numbers you need to describe a vector in two dimension. So, these are x component and y component. So, that is the next point, that I want to make.

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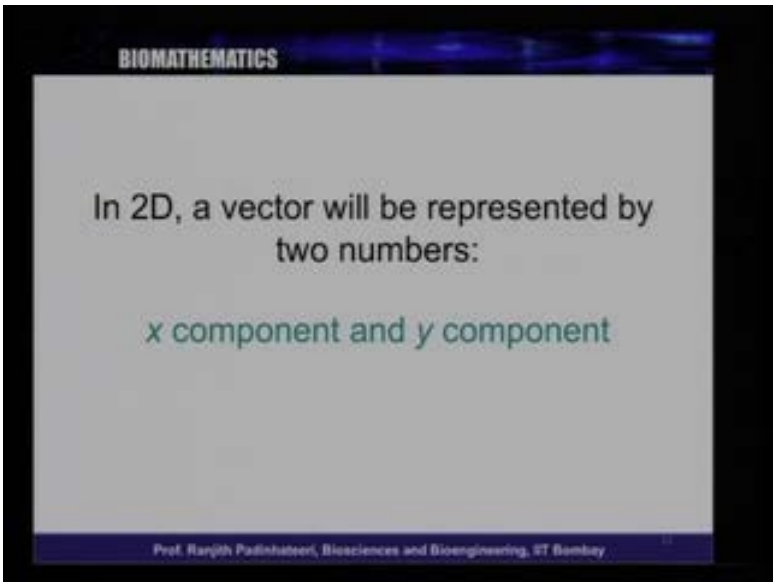

In 2D, a Vector will be represented by two numbers, its x component and y component. So, let us look at a few more examples. So, let us look at this papers here, paper here.

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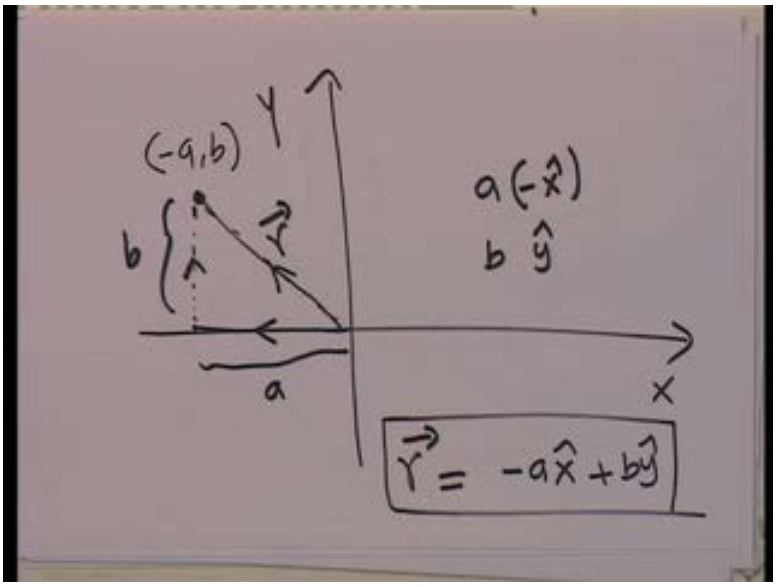

So, let us, let me draw here. So, this is our X axis and this is our Y axis. Any number here, any number on the, any point on this, let us say, some point here. To reach this point, we have to go negative X axis something and then, along in this particular direction. So, you have to go this way and this way. So, this point. So, the distance we have to travel here is, first, you travel this way. So, you have to go to some distance. So, let us call this distance a. So, a distance you have to go along the minus X axis and you have to go b distance along the plus Y axis. So, b distance along the plus Y axis. So, the

total distance for this Vector r, r is this particular Vector, which is going from here to here. So, this r Vector will be minus a into x, a into minus x is minus a x, plus b y cap. So, this is, this r; a x, minus a x plus b y. So, I can represent this point by two numbers, minus a comma b; that is, minus a along the X axis and b along the Y axis. So, any two numbers, you need two numbers to represent a Vector, the x component and y component.

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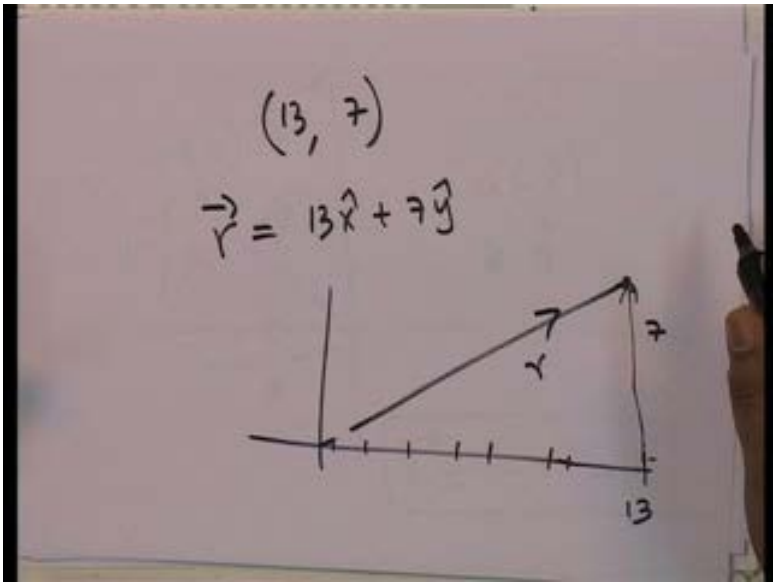

So, similarly, if I tell you, if I tell you that, you have a Vector, which distance is 13, 7. So, two numbers, if I say, this immediately, Vector means that, the Vector r is 13 x plus 7 y. If I tell you what does it mean; that means that, you have to go 13 units along the X axis. So, you have to reach here, 13 units and 7 units along the Y axis. So, you will get this is 7. So, you get this Vector r, which is 13 x plus 7 y. Similarly, if I, I tell you another one.

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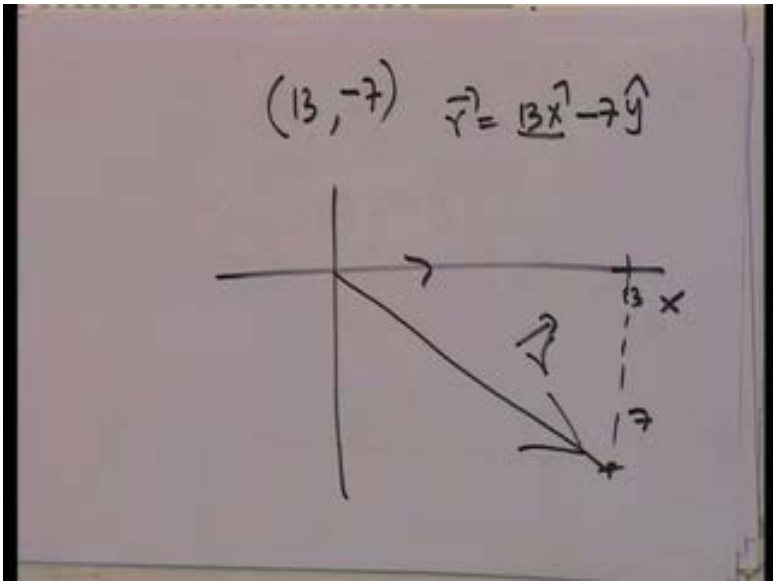

13, minus 7. What does it mean is that, you have to go 13 along the X axis. So, you go along the X axis 13. So, this Vector r will be 13 along the X axis plus 7 along the minus Y axis. So, minus 7 y. So, 13 along this X axis; minus 7; that is in this direction, 7 along the Y axis. So, then, you have this particular Vector r. So, two numbers represent a Vector and this two numbers are x component and y component. So, any point, you can see on the, on a 2D plane can be represented by two numbers. They are called x component and y component. Now, similarly, if you go to 3D, in a 3, in a 3D, a Vector will be represented by 3 components - x component, y component, z component. We will come and discuss 3D Vectors in the coming lectures soon. But today, we will stick to 2D and try and understand the idea of vector in 2D properly, so that, it is very easy to go and do it in the 3D.

Now, let us say, we want to tell you something. Let us say, we want to say something in the, using the language of Mathematics, we want to say that, I want to lift this. I have this box with me and let us say, I want to lift this box up or I want to slide this box along in this direction. So, let us say, I want to lift this box in this direction. I might want to slide this box in this direction. So, I have to say this things mathematically. So, how do we say this? Lifting in this direction, sliding in this direction. I have to apply some Force to lift it; I can apply some Force to slide. So, now, I have to tell somebody, you apply some Force in this direction. How do I say this? So, just like we saw this for Force, for the

charges, we can say exactly similar way. So, let us have a look at this. So, the question what we want to ask is, how do I say? Have a look at this slide here.

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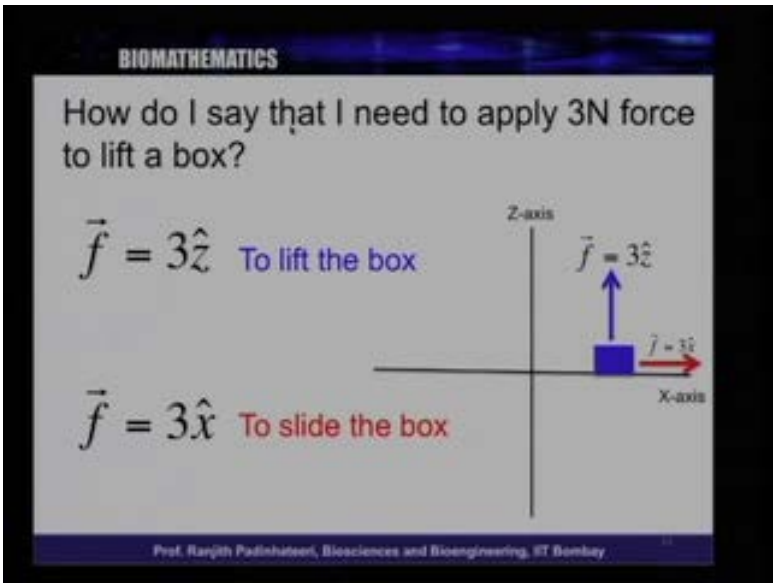

How do I say that, I need to apply 3 Newton Force to lift a box? So, to lift a box, you have to lift along the X axis, Z. So, I define this axis as Z axis, for it is just a name. I can call it Z or Y or whatever I want, I want. I am calling this Z axis and I want to lift it; lifting means, moving along the Z axis. So, the way I say is that, if I say f is 3 z, that means, you have to apply 3 Newton Force along the Z axis to lift the box. If I say f is equal to 3 x, I have to apply 3 Newton Force along the x direction; this is my X axis. So, if I tell somebody, f is equal to 3 z in Mathematics, they will understand that, you have to apply a Force of f, along the Z axis. If I tell you, tell somebody that, f is equal to 3 x cap, they will understand that, you have to apply a Force along the x direction. So, when you do an experiment, if you want to clearly say that, please apply a Force along this direction, you can precisely write in Mathematics. So, we understand, telling the simple things, how do we apply a Force, etcetera. Then, we have to come and understand, how do we know about three charges. That is something which we, we had this question that, we have three charges and we want to know about this force. So, that is one question that we asked in the beginning. So, let us go back to that question.

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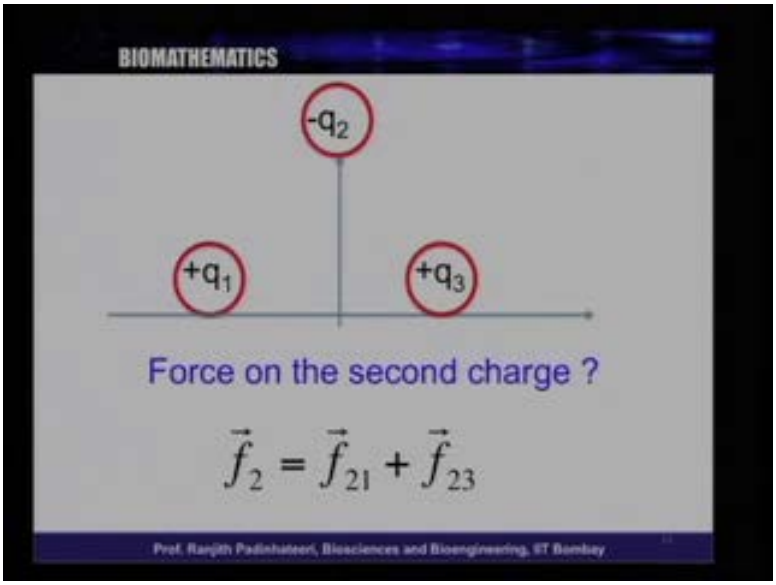

So, let us go to the slide. So, we have three charges q 1, q 2 and q 3, and we want to know the Force on the second charge; due to this charges, what is the force? So, it turns out that, the Force on the second charge f 2, due to other charges is f 2 1 plus f 2 3. What does this mean? This means is that, Force on the second charge is the sum of Force on the second charge due to 1, plus Force on the second charge due to 3. f 2 1 means Force on the second charge due to the first charge. So, Force on this charge due to this charge, plus the Force on this charge due to this charge. So, there are two Vectors and the sum of these two Vectors is the total Force, is the resultant Force. Now, to learn this, you, of course, you have to understand, how to sum the Vectors; how to sum two Vectors. So, how do we sum two Vectors? This is something we will understand. So, the way of summing two Vectors is basically, summing their components. Just simply sum the component. So, we, we said that, any Vector can be represented by two components.

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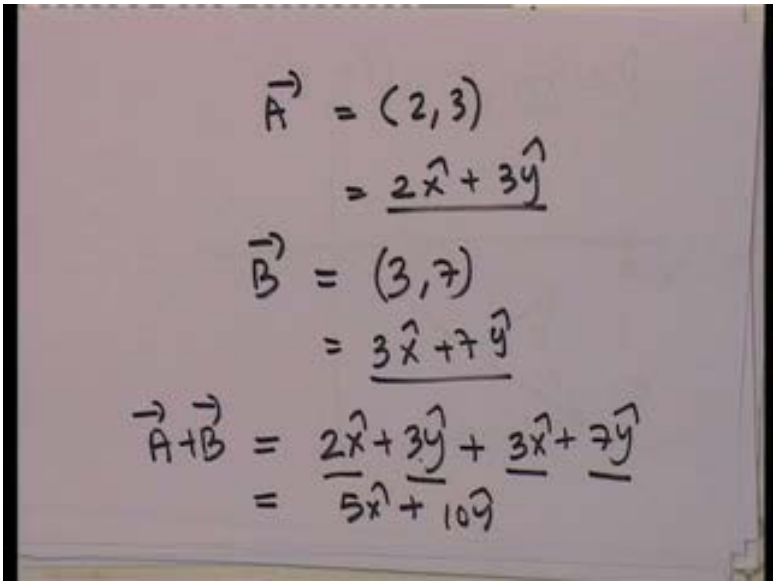

So, Vector A, I can write is 2, 3. What would it mean? It means 2 along the X axis and 3 along the Y axis. Now, I have another vector B, which is 3, 7. These are two components. This is means, 3 along the X axis and 7 along the Y axis. A plus B is basically, 2 x plus 3 y, this is this Vector, plus this Vector, which is 3 x plus 7 y. Now, when you have such a situation, the one, the thing you should do is that, add, this component and this component are x, this and this are y. So, add the components. So, the answer is 2 x plus 3 x is 5 x; 3 y plus 7 y is 10 y. So, answer is 2 plus 3, 5 x, plus 3 plus 7, 10 y. So, this is the answer. So, adding Vectors is basically, adding the components. So, let us, we will discuss about adding the Vectors little more detail. So…But adding Vectors, is essentially, adding the components. Now, in the case of charges, let us see, what are the Vectors. So, we have to calculate essentially, the Force on the second charge. To now calculate the Force, we know, we know something about the position of this Vectors, because Force is q 1 q 2 by r square, where r is the position, the distance. So, how do we know about distance? To know about the distance, let us try and understand something more about this distance.

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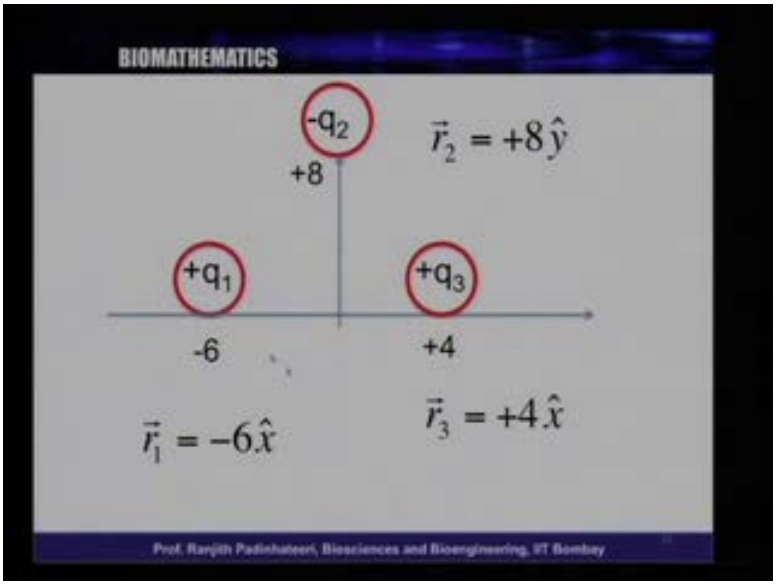

So, let us have a look at this particular slide. Have a look at this slide. So, you have q 1, q 2 and q 3. Now, you have, I place q 1 in the X axis, at a position minus 6. I position q 2 in the Y axis, at a position plus 8. I position q 3 along the X axis, at a position plus 4. So, I have positioned three charges in the X Y plane. Now, what is r 1? Essentially, it is a position of the X axis, the Vector, where essentially, this Vector starting from here to here; starting from, that is basically, minus 6 x; that means, to get to the position of the first one, you have to go to, go minus 6 along the X axis. So, that is, you can say r 1, which is position of the first charge, is minus 6 x. You have to go 6 units. Let us say, this 6 meters along the X axis. Now, this is q 2. We have to go 8 units along the Y axis. So, let us say...So, then, how, this is written as, this is mathematically written as, r 2 is equal to plus 8 y. You have to go plus 8 in the plus y direction, plus 8 y. Now, what about this third charge? Third charge is at 4; so, that means, r 3 is plus 4 x. You have to go plus 4 along the X axis; that is what this means. So, you know r 1; we know r 2; we know r 3.

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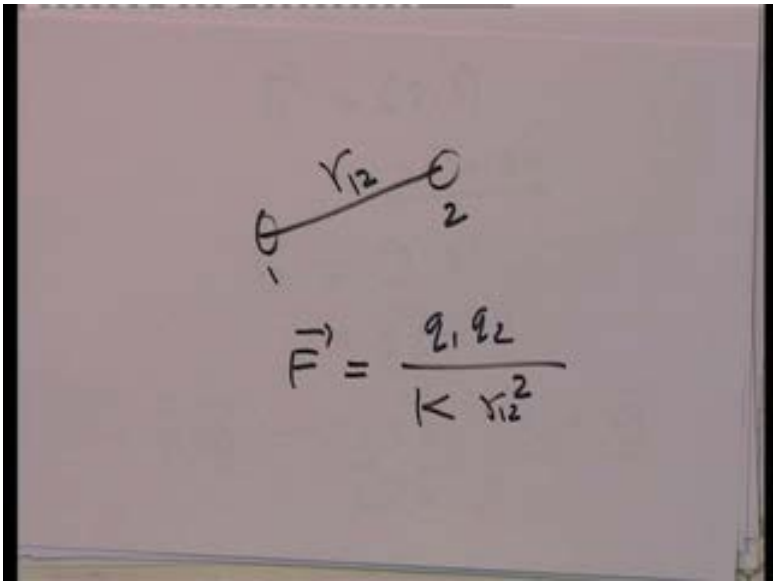

Now, we know that, we have two charges. Have a look at here. You have two charges. And, the Force, if this is charge 1 and this is charge 2, and let r 1 2 be the distance between the charge 1 and 2. And, the Force then is, q 1 q 2 by K r 1 2 square. So, you have to know, if you know, you have to know, what is the distance between the charges. We know now, in this slide, in this, we know the position of charge 1; we know the position of charge 2; we know the position of charge 3. But we do not know, what is the distance between 1 and 2, what is the distance between 1 and 3, what is the distance between 2 and 3. We have to know this. So, how do we calculate this? That is the next question. So, now, we will go ahead and calculate the distance between the two charges.

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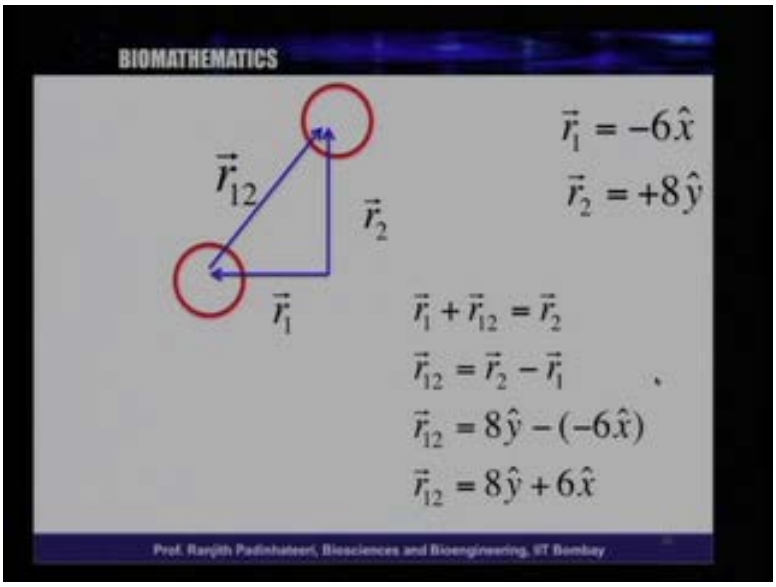

So, have a look at this slide. So, r 1 is the position of this charge; r 2 is the position of this charge and r 1 2 is the position from the first charge to the second charge. So, look at this arrows carefully; r 1 is the distance from here to here; this is the arrow direction. r 2 is a distance from here to here. r 1 2 is the distance from here to here; that is, from the first charge to the second charge. So, the distance from, the Vector from first charge to the second charge is r 1, r 1 2. And, we know that, r 1 is minus 6 x. You have to go 6 in the minus x direction. r 2 is plus 8 y. You have to go plus 8 along the Y axis. Now, what is r 1 2? That is the question we are asking. What is r 1 2? Now, if I look at this diagram, I can immediately say, if I go here and then, from here to here, if I go along this arrow, I will, would have, I would have reached this position r 2. So, that means, if I go r 1 plus r 1 2 is nothing, but r 2. If I walk along this way and this way, I would have reached r 2.

So, r 2 is nothing, but r 1 plus r 1 2. So, that is what is written here in the first equation. So, have a, have this first equation and this diagram a few times, slowly and carefully, and then, you will understand that, what this equation is correct. That is, what is written here is that, r 2, this distance, this is a Vector, this distance, is nothing, but this plus this. If I go this way and if I go this way, I get r 2. r 2 is r 1 plus r 1 2. This is something we just write down; just common sense, actually. But by looking at this picture, you can write it down. So, this is essentially, essentially, the common sense. So, once we know this r 1 2…So, from this equation, we can calculate r 1 2, actually. So, I can take this side, I can say r 1 2 is r 2 minus r 1. And, we know that, r 2 is 8 y and r 1 is 6, minus 6 x and r 2 minus r 1 is 8 y minus, minus 6 x; because r 1 is minus 6 x. So, minus, minus is plus. So, you get this r 1 2 is 8 y plus 6 x. So, the Vector r 1 2 is essentially, 8 y plus 6 x.

So, you have a new Vector r 1 2 as 8 y plus 6 x. Similarly, we want to calculate other Vectors like r 2 3 and all that. So, let us go ahead and see, what next.

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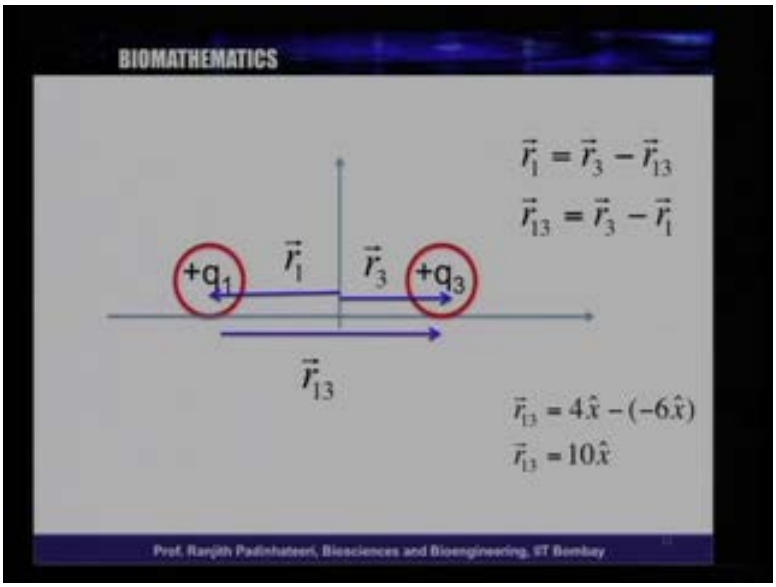

How do we calculate r 2 3? Have a look at this slide. So, here q 1 is the first charge and r 1 is the distance from here to here. Going from here to here, in this direction is r 1 and going from here to here, in this direction, is r 3 and r 1 3 is going from here to here; that is, going from first charge to the third charge, this distance, this position Vector is r 1 3. Be careful, just look at this carefully. r 1 3 is the distance from here to here. So, now, how do we understand this r 1 3? r 1 3 can be understood, r 1 3 is equal to…So, we have to get the r 1 3. But…Let us see, let us do a walk along this and let us see what we get. If we walk r 3 along this direction, I will reach q 3. Then, I walk back, that is, in the opposite direction of r 1 3, I walk this distance. If I walk back, that is a negative direction, if I walk, r 1 3, if I distance, walk this much, r 1 3 distance along this, I will reach q 1. So, if I walk r 3 here, I will reach q 3. Then, if I walk r 1 3 along in this direction, opposite direction, I will reach q 1. So, if I, r 3 minus r 1 3, r 3 minus r 1 3 in the opposite direction, I will reach r 1.

So, what, what does this equation says? The equation says that, I go from this point, I go here, then, go back certain distance. So, r 3 minus r 1 3, r 3 minus r 1 3. Then, I will reach here. I want to reach here, first, I go certain distance, then, I go back a lot more distance. So, that is the, what is written here. Then, I will get from this, I can get r 1 3 is r

3 minus r 1, where I can take r 1 3 this side and r 1 this side; rearrange this. So, I would get r 1 3 is r 3 minus r 1. So, if I know r 3 and if I know r 1, I can calculate r 1 3, which is r 3 minus r 1. r 3 is 4 x, r 1 is minus 6 x. So, 4 x minus, minus 6 x is 10 x. So, r 3 is 10 x and we know that, if this is 4 and this is x, the distance has to be 10 and this is 10 along the X axis. So, this is 10 x. r 1 3 is 10 x. So, we got r 1 3 also. Now, what we want is, the next r 2 3.

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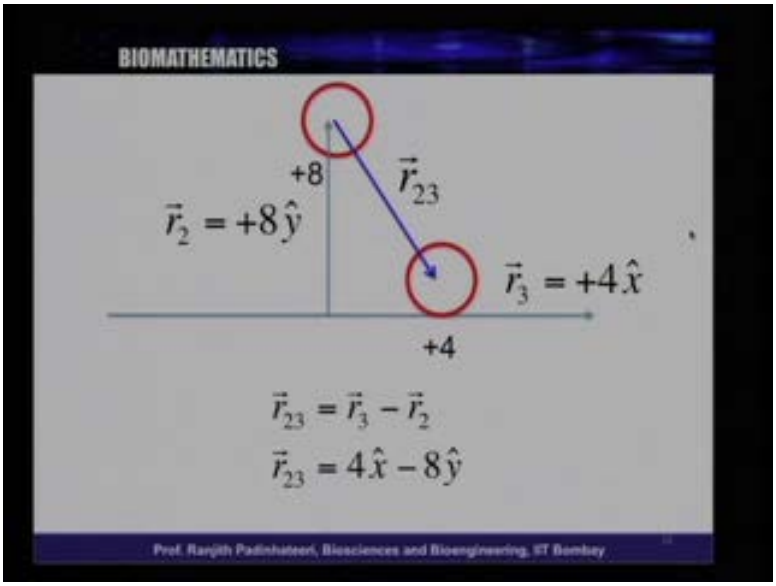

So, have a look at the next one. If I look at this slide again, r 2 3, in a similar manner, we can calculate. r 2 3 can be calculated as r 3 minus r 2. So, r 3 is 4 x, r 2 is 8 y. So, r 3 minus r 2 is 4 x minus 8 y. So, what do we have? We have r 1 2, r 1 3 and r 2 3. So, these are three Vectors that we got. So, let us write down this here, what we got. So, we got a few things. So, let us, we got basically, r 1 2.

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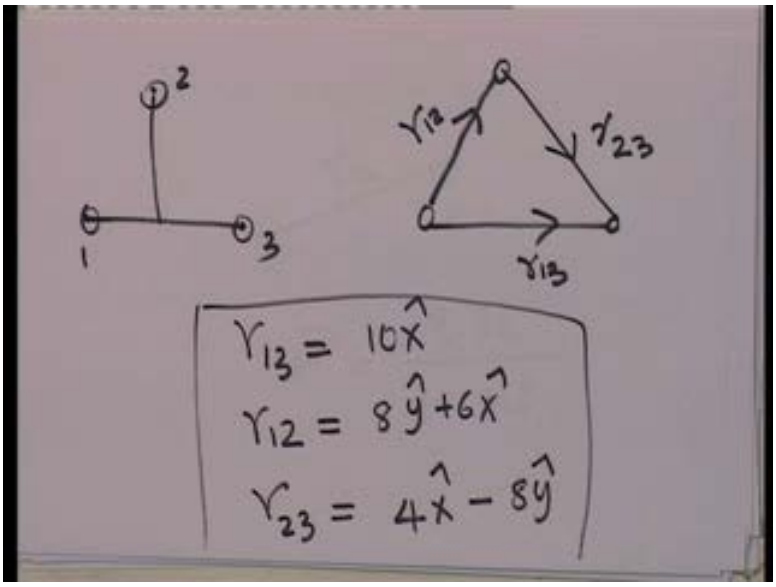

So, we got, so, we have this charges, right. So, we have this three charges here and r 1 2 is basically...So, we have, I call this 1 and this 2 and this is 3, three charges, and r 1 2 is this distance; r 1 3 is this distance and r 2 3 is this distance, from 2 to 3, r 2 3. So, we have this three Vectors we got. And, what did we get, r 1 3 is equal to...So, let us write down all this Vectors. r 1 3, we had got r 1 3 as 10 x; r 1 2, we had got as 8 y plus 6 x; this is something we had got; and, r 2 3, we had got, r 2 3, we had got r 2 3 as basically, 4 x minus 8 y. So, we had got three Vectors and these are the three Vectors. Now, our aim is to calculate the Force, which is a little more elaborate process. So, we will discuss the calculation of the Force in the next class. But we have this three Vectors. We know all the distances between the charges. If we know the distance, we know how to calculate the force. But to get the direction of the Force, we have to go, things, a little more in detail. So, just remember this three Vectors that we got. We will, knowing this three Vectors, how to calculate the Force, that is, that we will discuss in the next class. So, we have r 1 2, r 2 3 and r 1 3.

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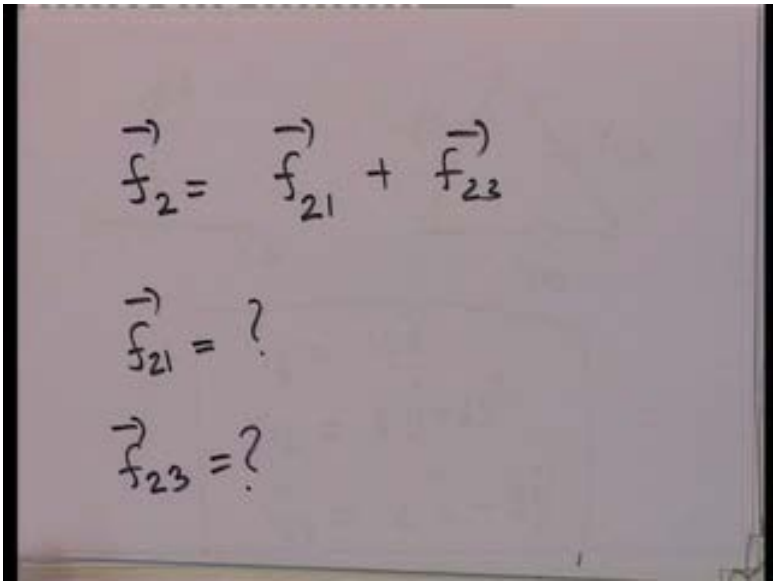

And, what we essentially, will do is that, we know, as we said, f on the second one, Force, we wrote is, f on second, 2 to 1 plus f 2 to 3 and we know that, we have to know what is f 2 1, what is f 2 3; these two things we have to know. We will calculate that and we will calculate the resulting Force. That is in the next class. To Summarize what we learnt today, let us Summarize this.

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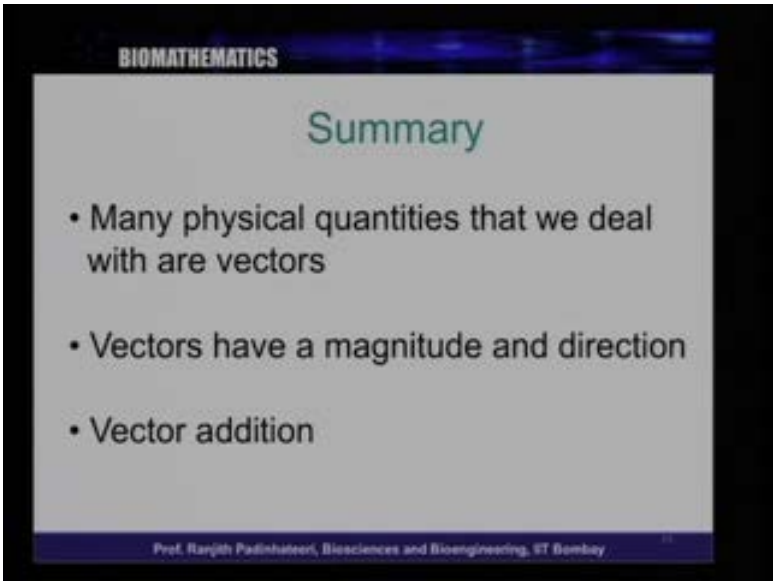

So, this is our Summary. Many physical quantities that we deal with are Vectors. For example, we said Force, etcetera. So, there are many physical quantities that we deal, are

Vectors. So, and Vectors have magnitude and direction. So, have a look at this. And Vector, we know how to add Vectors. So, this Vector addition, we learnt a bit about. So, there are three things that we learnt. Many quantities like Force, Flow, position are Vectors and Vectors have a magnitude and direction. And, Vector, how do we add two Vectors, we briefly saw. We will see them in greater detail in the coming class. So, this is the Summary of what we learnt today. Now, we will go ahead and calculate the Force between the charges. But we learnt today, how to calculate the distance between the charges, charges if they are placed in different places in a 2D plane. So, with this, we will stop today's lecture and see you for the next lecture. Bye.