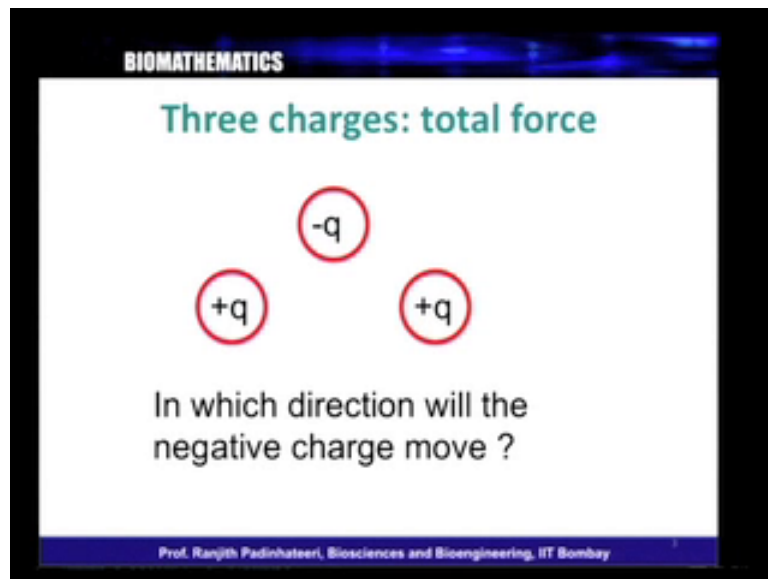


Biomathematics
Dr. Ranjith Padinhateeri
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Lecture No. # 15
Vectors - 2

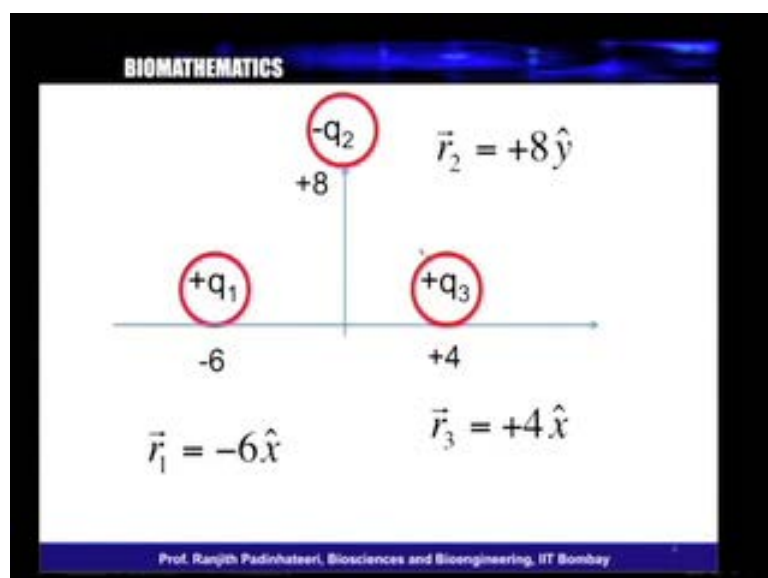
Hello. Welcome to this lecture of Biomathematics. In the last lecture, we have been discussing about vectors. We introduced the idea of vectors and we discussed why would we need to use vectors at all and we found that, for example, if you want to calculate the force between charges in a, on a protein or any system, any charge system, you would need vectors. To also represent, like, the motion, flow, all those things are very important thing in Biology. For all these to mathematically represent, to convey this idea mathematically, one need to use the idea of vectors. So, in this context, we also asked this question, if you have three charges, what will be the force on one of the charges, due to other charges. And, as an exercise, you wanted to calculate this clearly, explicitly. So that, as an example, you all will know, how to calculate force on a charge, due to other charges. And, through this, we will learn many ideas of vectors. By calculating this force, we will learn many things about vectors. So, let us go and see, what the question we are asking, just to remind us, ourselves, what exactly the question we were asking.

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So, this lecture is Vectors part 2 and the question you are asking, is basically, three charges and calculating the total force. And, our, the aim of, the thing what we want to understand is that, if you have this negative charge and two plus charges, in which direction the negative charge will move? Will it move this way? Will it move this way? Will it move this way? Will it move this way? So, in which direction will it move, that is the question and that is what exactly we are trying to answer.

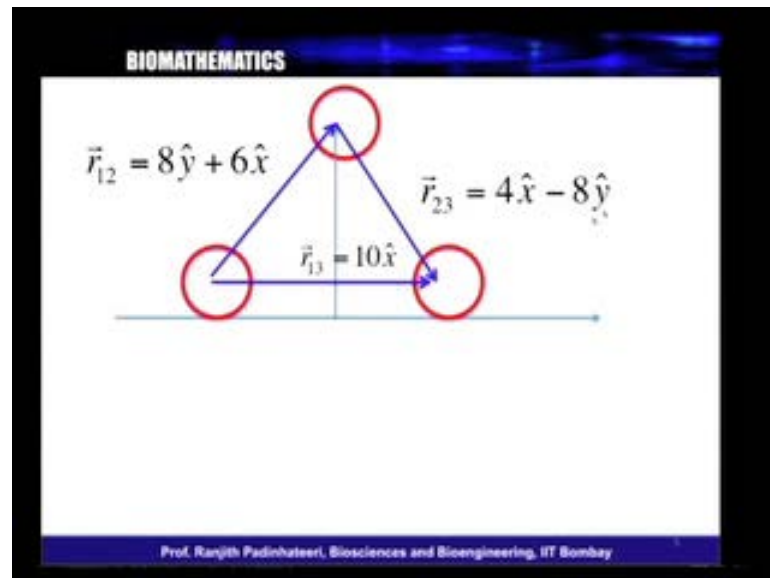
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So, while answering this, we did, to begin with, to do this, we did a few things. We calculated the position of each of this charges. So, if you say that, this is an x y plane and this is sitting at the plus 4 on the X axis, this q 1 is sitting on the minus 6 position on the

X axis, if q_2 is sitting on the plus 8 position of the Y axis, we said that, r_1 , which is the position of the first charge, this minus 6 x; r_2 is plus 8 y and r_3 is plus 4 x. So, we said this and we also calculated the distance between first charge and second charge, the distance between the first charge and the third charge and the distance between the second charge and the third charge.

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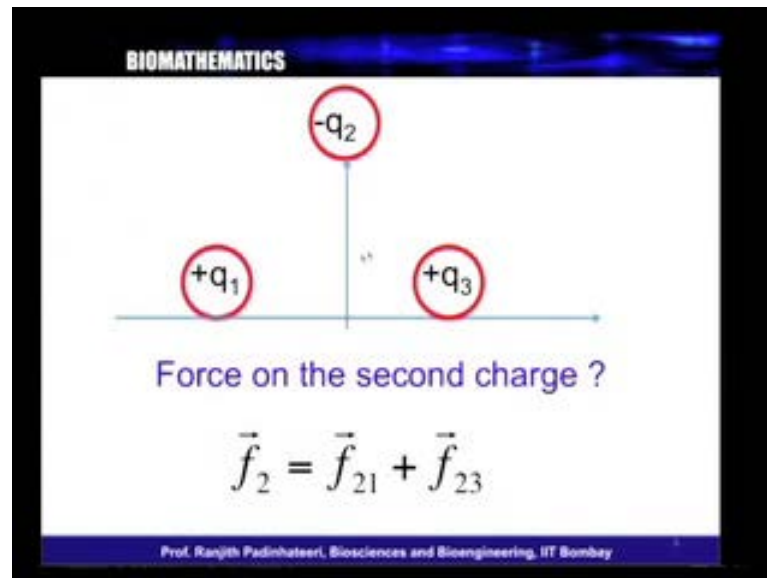
So, this distance is basically, shown here. So, what is shown here is basically, r_{12} . So, this is where we stopped in the last lecture. We calculated r_{12} , r_{12} is the distance between this charge and this charge; that is, the first charge and the second charge; r_{23} is the distance between second charge and the third charge, and r_{13} is the displacement vector of this charge, $10x$, between first charge and third charge. So, all these are vectors and there is a direction for all this. Now, it, there is a meaning for all this. Like, think about, what exactly this $8y + 6x$ means. So, in our, or $6x + 8y$, right. You can either call it $8y + 6x$ or $6x + 8y$, they are the same. Now, what does this mean? If you think about it, there is a $6x$; that means...So, this says that, to go from first charge to second charge, you have to go 6 along the x, x direction and 8 along the y direction. That is what it means; 6 along the x direction and 8 along the y direction. If you go from 6 along the, if you go, if you travel, or if you move 6 along the x direction, that is, from here to here and 8 along the y direction, that is, from here to here, you will reach the second charge; that is what precisely this means.

Which way you have to move, such that, you will get, from the first charge to the second charge, and how much you have to move; that is what it is. And, r_{23} is from this second charge to third charge, which way you have to move; this is the way you have to move and how much you have to move. That is, that is what is coded into this vector r_{23} . So, when somebody says r_{23} is $4x - 8y$, so, first, you have to do minus $8y$; that is, you have to come down minus 8 , the decreasing direction of y , you have to come 8 . So, 8 along the minus y , that is what minus $8y$ means; 8 along the minus y , you have to go. Then, 4 along the plus x . So, minus $8y$ plus $4x$. So, that is what this is; minus $8y$ plus $4x$, that is what this means; that is, to go from 2 to 3 , you have to travel 8 unit along the negative direction of the y axis and 4 unit along the plus direction of the $4x$ axis.

What is r_{13} equal to $10x$ means? To go from 1 to 3 , you have to just travel 10 unit along the X axis in the positive x direction; that is what it is. You do not have to go along the Y axis. So, there is no y component here. So, from this, when you say a displacement vector r_{12} , r_{23} , etcetera, it has some meaning. Take this slide, hold it and think about it. Just spend some time with this slide and convince yourself that, r_{12} is nothing, but the distance to go from 1 to, the way to go from 1 to 2 . What kind of, where you have to, which direction you have to travel and how much you have to travel.

That is, that is what precisely encoded in this r_{12} . When you say that, r_{12} is $6x + 8y$, that means, to go from 1 to 2 , you have to go 6 units along the X axis and 8 units along the Y axis. So, it is the recipe to move. If you want, if you wish, this is some kind of a recipe given to you, to know, which direction, how exactly you have to move; precise information. So, if somebody says $6x + 8y$, this is the precise information, how much you have to move, in which direction and all that. So, there is some meaning. If you wish, take a graph paper and plot, mark all those in a graph paper and then, do it yourself and convince yourself and the only way to learn I think, is to do this yourself and convince yourself. So, we understand the distance or the distance between this. Now, we have to calculate the force. So, again, let us go back to the question.

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So, here, if you look at this slide here, the question we wanted to ask is, the force on the second charge, what is it? The force on the second charge f_2 is f_{21} plus f_{23} . What is the f_{21} ? f_{21} is force on the second charge, due to the first charge; that is, f_{21} ; force on the second charge, due to the first charge. So, the first charge will try to attract this one with some force. So, that is this f_{21} . f_{23} is force on the second charge, due to the third charge. The third charge will exert some force on the second charge. As you know, these are plus and minus charges. So, this plus charge will try to pull this, attract this. So, there will be some force along this direction, due to the third charge; that is, this one and there will be some force along this direction, due to this charge, that is the f_{21} . Now, if there are two forces, in which way will it move? Precisely you have to know, where exactly, precisely in which direction will it, it will move. So, this is what we will calculate. So, now, of course, we will start calculating with f_{21} . We will precisely calculate this number f_{21} . If we have this f_{21} , then, we, we can calculate f_{23} and sum this and you get the answer f_2 . So, let us go ahead and see how do we calculate f_{21} .

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BIOMATHEMATICS

Force on charge q_2 due to q_1

$\vec{r}_{12} = 6\hat{x} + 8\hat{y}$

$|\vec{f}_{21}| = \frac{q_1 q_2}{K |\vec{r}_{12}|^2}$

$|\vec{r}_{12}| = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$

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So, here, have a look at this slide. In this slide, you have two charges again, q_1 , q_2 , as we had and we knew that this r_{12} is $6x$ plus $8y$. And, the magnitude of the force f_{21} , we know that, $q_1 q_2$ by $K r^2$ is the magnitude of the force. This is the Coulomb's law. The $q_1 q_2$ K , k is a constant which is $4\pi\epsilon_0\epsilon_r$. And, r^2 square, this is the distance square between these two charges. Now, how do we calculate this distance between these two charges r . So, you can see here two lines. So, this lines represents, this lines is called magnitude. So, this two lines, if you put a vector between these two lines, that means magnitude of a vector.

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\vec{A}

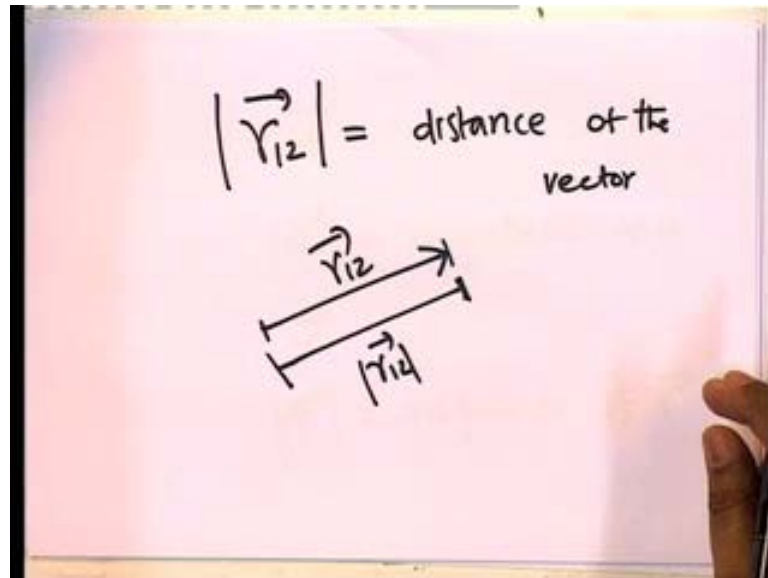
$|\vec{A}| = \text{magnitude of } A$

\vec{r}

$|\vec{r}| = \text{magnitude of } \vec{r}$

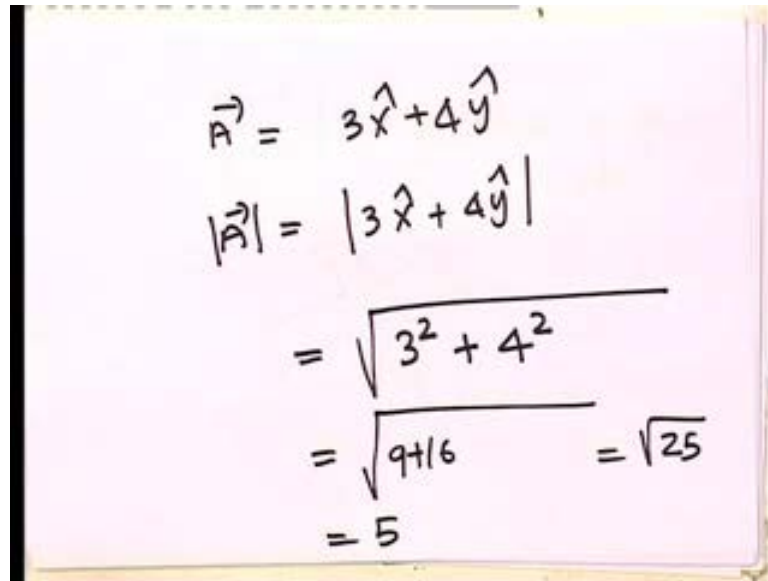
So, have a look at here. When you say A is a vector and if you say, this means, what is the magnitude, how...Or, if you want, like, r is a vector and if you say $\text{mod } r$, this is the magnitude of r . Or, we can also, if this is a vector, this is a distance vector like we had...

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So, let us say, we had a distance vector here, r_{12} and we had this magnitude of r_{12} . So, **while we**, since we are going to learn how to calculate the force when we do this, we will go ahead and learn a few things through this. So, while we learning the calculation of the force, we will learn some concepts in vectors also and this, one of this concepts is magnitude of the vector. So, this is the displacement vector and the magnitude of the vector actually means...So, this also would mean the magnitude. So, this is the, another concept in the vector algebra. It is a magnitude of a vector. How do we calculate this? So, while we calculate the force, we will learn how to calculate this and which will be used to calculate the force. So, that is the strategy we are going to use. So, let us learn how to calculate this r_{12} . So, r_{12} as I said, is magnitude. It is also the distance of the vector; that is, if you, if you have a vector like this, from here to here, this distance, if this is r_{12} vector and this distance is called $\text{mod } r_{12}$. $\text{Mod } r_{12}$ will give you the distance from here to here. So, this is what will be represented in $\text{mod } r_{12}$. Now, how do we calculate this mod ? That is what we will discuss.

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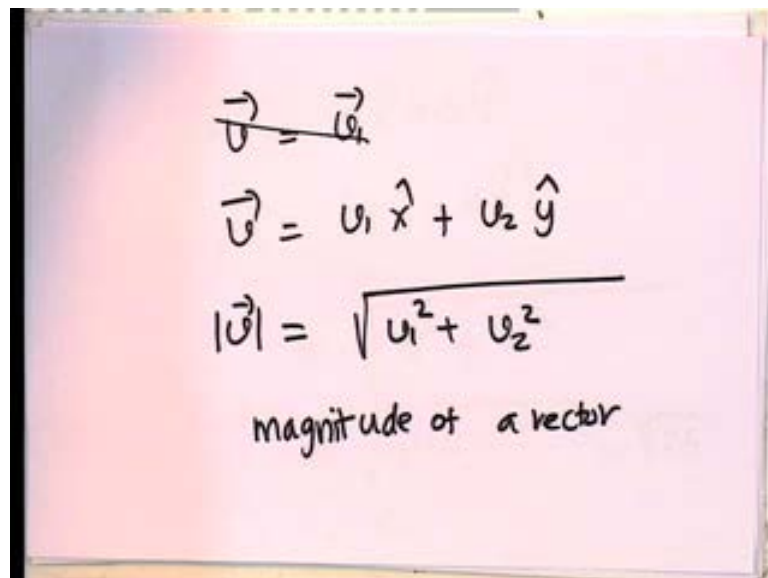


A handwritten note on a pink background showing the calculation of the magnitude of a vector \vec{A} . The vector is defined as $\vec{A} = 3\hat{x} + 4\hat{y}$. The magnitude is calculated as $|\vec{A}| = |3\hat{x} + 4\hat{y}|$, which is equal to $\sqrt{3^2 + 4^2}$. This is further simplified to $\sqrt{9+16} = \sqrt{25}$, resulting in a magnitude of 5.

$$\begin{aligned}\vec{A} &= 3\hat{x} + 4\hat{y} \\ |\vec{A}| &= |3\hat{x} + 4\hat{y}| \\ &= \sqrt{3^2 + 4^2} \\ &= \sqrt{9+16} = \sqrt{25} \\ &= 5\end{aligned}$$

So, if you have any vector, let us say A and if the vector is 3 x plus 4 y, mod A is mod 3 x plus 4 y. So, this is calculated as root of, take the coefficient of x, square it, plus, take the coefficient of y, square it. So, this is the mod a. So, this is, 3 square is 9. This is 16. So, this is root 25. So, this is 5; 5 is the magnitude of this vector A.

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A handwritten note on a pink background showing the general formula for the magnitude of a vector \vec{v} . The vector is defined as $\vec{v} = v_1\hat{x} + v_2\hat{y}$. The magnitude is calculated as $|\vec{v}| = \sqrt{v_1^2 + v_2^2}$. The text below the equation reads "magnitude of a vector".

$$\begin{aligned}\vec{v} &= v_1\hat{x} + v_2\hat{y} \\ |\vec{v}| &= \sqrt{v_1^2 + v_2^2}\end{aligned}$$

magnitude of a vector

That is what it means. If we have any vector v, which is v 1 along the X axis and v 2 along the Y axis, mod v is equal to square root of v 1 square plus v 2 square; that is the

formula for magnitude of a vector. So, this is magnitude of a vector. So, in the case where we are discussing, what we want to calculate is actually, the magnitude of force.

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A photograph of a pink sheet of paper with a handwritten formula for the magnitude of force between two charges. The formula is:
$$\vec{f} = \frac{q_1 q_2}{K r_{12} |r_{12}|^2}$$

So, we had force is equal to $q_1 q_2$ by $K r_{12}$. This is r_{12} . This is r_{12} magnitude square.

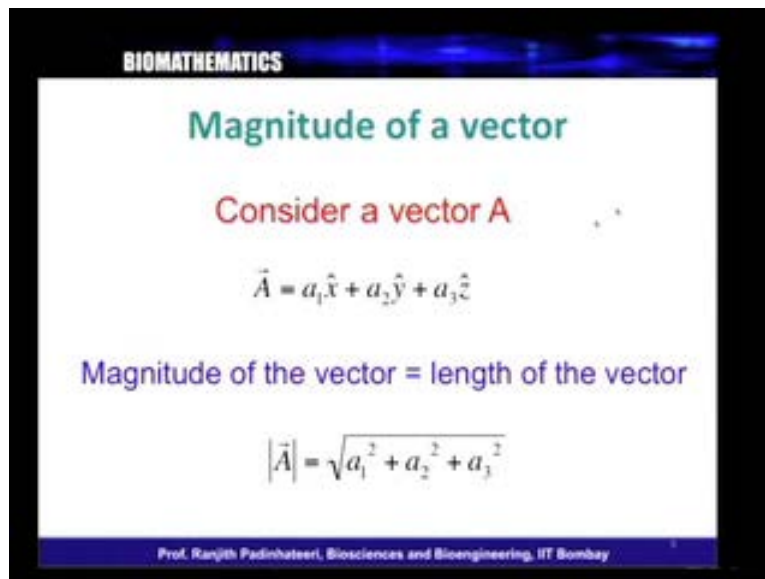
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The slide is titled "BIOMATHEMATICS" and "Force on charge q_2 due to q_1 ". It features a diagram with a positive charge $+q_1$ at the bottom left and a negative charge $-q_2$ at the top right. A vector \vec{r}_{12} points from $+q_1$ to $-q_2$. The components of this vector are given as $\vec{r}_{12} = 6\hat{x} + 8\hat{y}$. The magnitude of the force is calculated as $|\vec{f}_{21}| = \frac{q_1 q_2}{K |\vec{r}_{12}|^2}$. The magnitude of the vector \vec{r}_{12} is calculated as $|\vec{r}_{12}| = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$. At the bottom, it credits "Prof. Ranjith Padinhateeri, Biosciences and Bioengineering, IIT Bombay".

So, have a look at this slide again. So, we had r_{12} square here. So, when you have this r_{12} square, the mod r_{12} , as we just discussed...So, r_{12} is nothing, but $6x$ plus $8y$. $6x$ plus $8y$ is r_{12} . The way to calculate r_{12} is basically, mod r_{12} can be calculated, as

we said, as we just discussed, as square root of the coefficient of x, which is 6 square, plus coefficient of y, which is 8 square. mod r 1 2 is equal to root of 6 square plus 8 square, which is root of, 6 square is, 6 square is 36; 8 square is 64. So, 36 plus 64 is 100. So, square root of 100 is 10. So, mod r 1 2 is 10. So, mod f 2 1 is q 1 q 2 by K into mod r 1 2 square is 10 square, which is 100. So, essentially, q 1 q 2 by 100 K is the magnitude of this vector; that is, the force, the magnitude, how much force this q 2 will feel, is q 1 q 2 by 100 K. If you know the q 1, if you know the q 2, this is the force.

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So, let us just discuss what we discussed about magnitude once more. If you have any vector A in 3D, let A be a 1 x plus a 2 y plus a 3 z; that is, a 1 is the component along the X axis; a 2 is the component along the Y axis; a 3 is the component along the Z axis. Magnitude of the vector or the length of the vector is mod a, which is equal to root of a 1 square plus a 2 square plus a 3 square; coefficient of x square plus coefficient of y square plus coefficient of z square. So, this is the way to calculate the magnitude.

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BIOMATHEMATICS

What is the direction of the force ?

$\vec{r}_{12} = 6\hat{x} + 8\hat{y}$

$|\vec{f}_{21}| = \frac{q_1 q_2}{100K}$

q_2 will be attracted towards q_1
Direction is opposite to the direction of \vec{r}_{12}

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So, now, let us go back to the force. We know the magnitude of the force now, which is $q_1 q_2$ by $100 K$. Now, the question we want to ask is, what is the direction of the force. We know the magnitude of the force. We know the, how much force, the q_2 will be felt; but which direction will the q_2 be pulled? So, as you know, q_2 is negative; q_1 is positive. The q_2 will be pulled towards q_1 . There will be an attraction; q_1 will attract q_2 . So, if you fix q_1 here, q_2 will be pulled towards q_1 . So, q_2 , the force will be in the direction opposite to the direction of r_{12} . Have a look at here, r_{12} . r_{12} is from 1 to 2; r_{12} is from, in this direction, to the, point, arrow is pointed in this direction; arrow is pointed from 1 to 2. The r_{12} is pointing from 1 to 2. But we know that, the force will be pointing from 2 to 1, because, the second charge will be pulled from 2 to, from here to here. So, the direction of this force will be opposite to the direction of the r_{12} . So, q_2 will be attracted towards q_1 , which is nothing, but opposite to the direction of the r_{12} . So, we know the magnitude. Now, we know the direction. But, how do we... We know the direction, now, in sentence, we know it will be opposite to the direction of r_{12} . But, how do we say that in mathematically? This is how we do it. So, have a look. The way we do is the following.

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BIOMATHEMATICS

Opposite to the direction of \vec{r}_{12}

$\vec{r}_{12} = 6\hat{x} + 8\hat{y}$

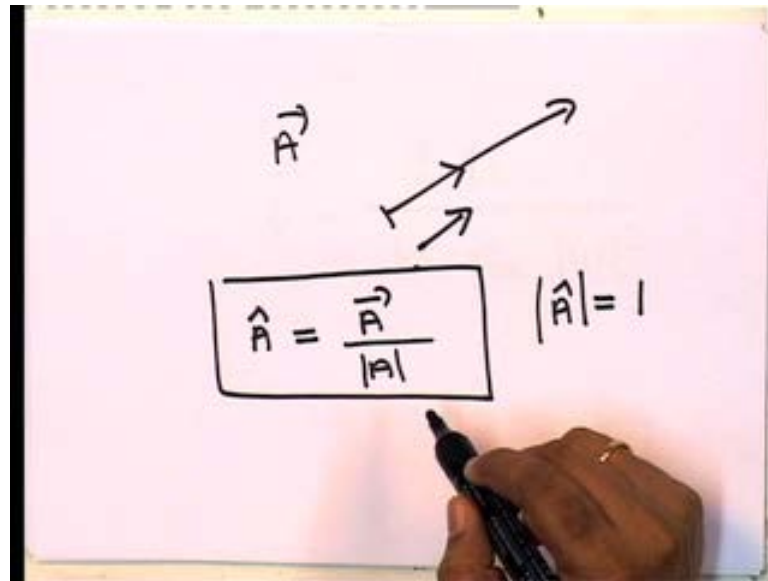
$\hat{f}_{21} = -\hat{r}_{12}$

A unit vector represents direction

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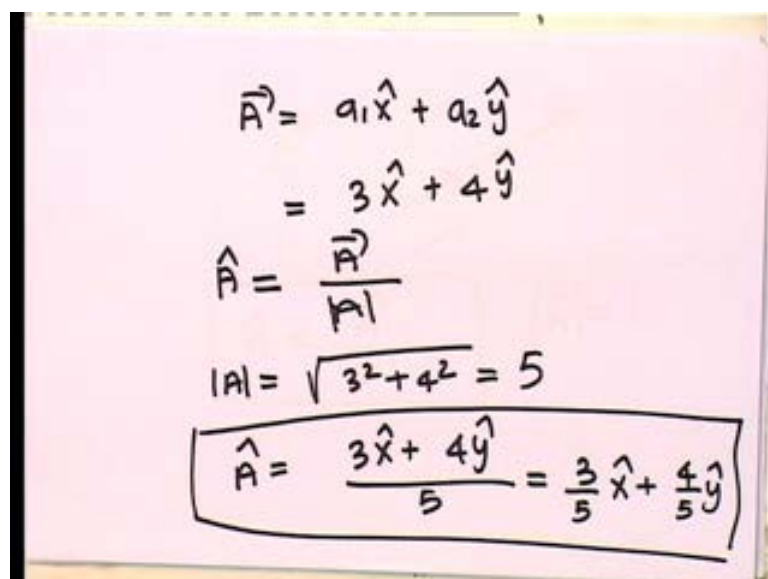
We say that, the direction \hat{f}_{21} or a hat. So, when you say hat as the z, so, it is like a unit vector and unit vector represents direction. So, the direction of \hat{f}_{21} is minus \hat{r}_{12} . This is opposite to the direction of \vec{r}_{12} . We just saw that, \vec{r}_{12} is from here to here and \hat{f}_{21} will be from here to here. It will be opposite; minus shows the opposite sign; represents the direction, minus is in this sign. So, in opposite direction of \vec{r}_{12} . So, the point here to remember is, hat or cap means that, it is representing the, representing the direction. If you have some magnitude and a cap, it represents the direction. So, if you, if you had \hat{x} means, the direction along the X axis; \hat{y} means direction along the Y axis. So, \hat{z} would mean direction along the Z axis. So, \hat{r}_{12} would mean direction along the \vec{r}_{12} , but with a minus sign, $-\hat{r}_{12}$ with the minus sign would mean, opposite to the direction of \vec{r}_{12} . But now, how do we calculate this \hat{r}_{12} ? What exactly this \hat{r}_{12} is? We know what is \hat{x} and \hat{y} . How do we calculate this \hat{r}_{12} ? So, the way to calculate \hat{r}_{12} is again, definition of a unit vector. So, we will again learn, we will learn now, how do we define a unit vector. So, the definition of a unit vector...

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So, let us say you have a vector A , which is from here to here. Now, we want to know that something is going in the direction of A . If you want to know this direction, so that can be represented by unit vector A , which means, it is a, it only shows the direction; unit vector only shows the direction; this magnitude is 1. So, we know that, mod A cap, that is, modulus of any, magnitude of any unit vector is 1. So, now, what, how do we define? This is defined as A vector by mod A vector. This is the definition of unit vector. Any unit vector is that vector divided by its modulus. So, let us think about this a bit carefully.

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So, let us say you have vector A, which is nothing, but, a 1 along the X axis plus a 2 along the Y axis. So, let us say, in our case a 1. So, this is, let us say, as we said, 3 x plus 4 y. Now, mod. What is A cap? We said A cap is A vector divided by modulus of A vector. Now, we said that, modulus of A vector is nothing, but root of 3 square plus 4 square; which is this, coefficient of this square plus coefficient of this square. So, this is 9 plus 4, 16, is root of 25; this is 5. So, A cap is 3 x plus 4 y divided by 5. What is this? 3 by 5 x plus 4 by 5 y. So, this is how we calculate A. So, this is 3 by 5 x plus 4 by 5 y. So, the unit vector along A is nothing, but...Some...You have to go...If you say...If I am pointing in this particular, if I, if I show you, in that direction, that means, a little bit north or there will be, like, if you come little bit right, and then, straight, or, north south. So, this is like, when you say, something x plus something y, you have to go something along the X axis and something along the Y axis. So, similarly, if I say here, if you look at this, 3 by 5, which is a number, this much you have to go along the X axis and 4 by 5, which is another number, you have to go this much, 4 by 5 unit along the Y axis. If I, if you go this, you will get the direction of the vector A. Now, as we said, mod of A cap has to be 1; that is the definition of A cap. Now, will this mod of this will be 1? We can have a check. We can, we can check.

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$$\hat{A} = \frac{3}{5} \hat{x} + \frac{4}{5} \hat{y}$$

$$|\hat{A}| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$$

$$= \sqrt{\frac{9}{25} + \frac{16}{25}}$$

$$= \sqrt{\frac{25}{25}} = 1$$

So, we had A cap as 3 by 5 x plus 4 by 5 y. Mod A is mod of this, which is square root of 3 by 5 square plus 4 by 5 square; that is the mod of finding any vector; mod of anything. So, 3 by 5 square plus 4 by 5 square. What is it? So, 3 by 5 square is 9 by 25

plus 16 by 25. What is it? This is root of 25 by 25, which is 1. So, we defined mod A, A cap in a particular way. And, we defined in that particular way, because at the end of the day, we want to get mod A equal to 1. And here, as we can see here, if you define, if you had, if you define A as 3 by 5 x plus 4 by 5 y, mod A is essentially...If you, 3 by 5 square plus 4 by 5 square, square root, it is essentially, 1. So, we get that, mod A is 1. We calculate the direction, the unit vector representing the direction. So, now, we go ahead and calculate r 1 2 cap.

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BIOMATHEMATICS

How do we calculate \hat{r}_{12} ?

$\vec{r}_{12} = 6\hat{x} + 8\hat{y}$

$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|}$

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{6\hat{x} + 8\hat{y}}{\sqrt{6^2 + 8^2}} = 0.6\hat{x} + 0.8\hat{y}$$

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So, have a look at here. So, the r 1 2 cap. So, we know that, r 1 2 is 6 x plus 8 y. r 1 2 cap is r 1 2 divided by mod of r 1 2. r 1 2 is 6 x plus 8 y; mod of r 1 2 is root of 6 square plus 8 square. So, this is what it is. So, what is 6 square plus 8 square? 6 square is 36; 8 square is 64. So, 64 plus 36 is 100. And, root of 100 is 10. So, what you have in the denominator is 10. So, 6 by 10 is 0.6; 8 by 10 is 0.8. So, essentially, mod r, cap r 1 2 or r 1 2 cap or r 1 2 hat is nothing, but 0.6 x 0.8 y. Let us have, quickly look, whether mod r 1 2 is unit vector or not.

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$$\begin{aligned}\hat{r}_{12} &= 0.6\hat{x} + 0.8\hat{y} \\ |\hat{r}_{12}| &= \sqrt{0.6^2 + 0.8^2} \\ &= \sqrt{0.36 + 0.64} \\ &= \sqrt{1} = 1\end{aligned}$$

So, what did we get? We got mod r_{12} is $0.6x$ plus $0.8y$. Mod r_{12} cap, this is nothing, but square root of $0.6x$ square plus 0.8 square. So, this is equal to, square root of 0.36 , 0.6 square is 0.36 plus 0.8 square is 0.64 . What is this? 0.36 plus 0.64 is 1 . So, this is essentially, 1 . So, we got mod r_{12} is 1 . So, again r_{12} is a unit vector, which represents the direction. What does it mean to say is... You can, here, you can, you have it here $0.6x$ plus $0.8y$, that is mod r_{12} . So, putting all these together, what is the force? So, we know the magnitude. We know the direction. Now, we can put together, magnitude and direction, to get the force.

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BIOMATHEMATICS

Force on charge q_2 due to q_1

$\vec{f}_{21} = -\frac{q_1q_2}{K|\vec{r}_{12}|^2}\hat{r}_{12}$

$\hat{r}_{12} = 0.6\hat{x} + 0.8\hat{y}$

$\vec{f}_{21} = -0.6\frac{q_1q_2}{100K}\hat{x} - 0.8\frac{q_1q_2}{100K}\hat{y}$

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So, the force f_{21} will be in this particular direction. And, the magnitude is $q_1 q_2$ by $K r_{12}^2$ and the direction will be \hat{r}_{12} with the minus sign. So, this is the total vector, which is $q_1 q_2$ by $K r_{12}^2$ whole square, $K r_{12}^2$ square, \hat{r}_{12} cap with a minus sign. And, \hat{r}_{12} cap, we saw that, it is $0.6x + 0.8y$. So, if I substitute for \hat{r}_{12} cap here, what do I get? I get f_{21} is equal to 0.6 into $q_1 q_2$ by $K r_{12}^2$ square, which is... So, we saw that r_{12}^2 square is 100 . So, $q_1 q_2$ by $100 K x$, minus 0.8 into $q_1 q_2$ by $100 K$ into y . So, this is, what this tells us that, f_{21} is this. Now, what does this mean? So, f_{21} is, what does it mean? f_{21} has some x component and f_{21} has some y component. This means that, the q_2 will feel some force of this amount. So, you substitute for q_1 ; substitute for q_2 . Let us, let us say, this is number 11 . So, this is 1 and let us say, K is some number. So, essentially, you will get a number here, which is a negative number after all. So, that much, minus x direction you have to come; you have to, force will be, some force will be felt along this direction and some force will be felt along this direction, which is along the minus y direction.

So, this much force will be felt along the minus y direction and some force will be felt along the minus x direction. So, minus x direction some force; minus y direction some force. So, that is the total force that will be felt on this charge 2 , due to charge 1 . Charge 1 , some direction... So, it will pull a bit down, it will also pull a bit sidewise. It will get, get pulled a bit downwards; it will get, get pulled a bit sidewise. So, it will go, come here a bit, go here a bit. Where exactly this will go, that, we will calculate later. But how much force is being felt, is what we have calculated. The amount of force is this much and the direction of the force is this much. So, we know now, how much is the force being felt, on the second charge due to the first charge. Similarly, we can go ahead and calculate other forces also; that is, force on the second charge due to the third charge can be calculated. So, that is what our aim is. We want to calculate now, force on the third charge due to the, force on the second charge due to the third charge. So, let us go ahead and calculate this force on the third charge due to the second charge.

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BIOMATHEMATICS

Force on charge q_2 due to q_3

$\vec{r}_{23} = 4\hat{x} - 8\hat{y}$

$$|\vec{f}_{23}| = \frac{q_1 q_2}{K |\vec{r}_{23}|^2}$$

q_2 will be attracted towards q_3
Direction is the same as direction of \vec{r}_{23}

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So, let us have a look at this. So, here we know that, r_{23} ... So, you have charge 2, charge 3 and we know that, r_{23} is $4\hat{x} - 8\hat{y}$; that is, to go from 2 to 3, you have to come along the X axis 4 and minus y, 8 unit along minus y. 4 unit along x; 8 unit along minus y, you to reach here. Now, the force, what is the force? We know that, the magnitude of force is $q_1 q_2$ by $K r_{23}$ whole square. This is the magnitude of the force. What is the direction of the force? You know that, since, if you fix this charge here, this charge will be attracted towards this charge; q_2 will be attracted towards q_3 , so that, the direction will be, this direction. And, which is the same direction as that of r_{23} . So, r_{23} is along this direction and f_{23} also will be in the, along the same direction. f_{23} will be in the direction of r_{23} . So, we said that, we showed, we, we, a minute ago, we discussed that, to show the direction of anything, we have to use unit vector. So, we use unit vector r_{23} to show the direction along r_{23} . So... So, what, how do we define unit vector r_{23} ?

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BIOMATHEMATICS

Force on charge q_2 due to q_3

$$\vec{f}_{23} = \frac{q_1 q_2}{K |\vec{r}_{23}|^2} \hat{r}_{23}$$

$$\vec{r}_{23} = 4\hat{x} - 8\hat{y}$$

$$\hat{r}_{23} = \frac{\vec{r}_{23}}{|\vec{r}_{23}|} = \frac{4\hat{x} - 8\hat{y}}{\sqrt{4^2 + 8^2}}$$

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So, let us look at this next slide. So, r_{23} is this and unit vector \hat{r}_{23} , as we have been defining, r_{23} vector divided by mod r_{23} . What is r_{23} vector? r_{23} vector is $4x$ minus $8y$. So, which is... And, mod r_{23} is root of 4 square plus 8 square; 4 square and plus 8 square. So, this is the mod of r_{23} , because minus, even if it is, call it a minus 8 , minus 8 square is like plus 8 square only. So, see, this is basically, 4 square is 16 ; 8 square is 64 ; 16 plus 64 , 80 . So, the denominator is square root of 80 . So, \hat{r}_{23} , if you look here...

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$$\vec{r}_{23} = 4\hat{x} - 8\hat{y}$$

$$|\vec{r}_{23}| = \sqrt{4^2 + 8^2} = \sqrt{16 + 64}$$

$$= \sqrt{80}$$

$$\hat{r}_{23} = \frac{\vec{r}_{23}}{|\vec{r}_{23}|} = \frac{4\hat{x} - 8\hat{y}}{\sqrt{80}}$$

So, we had r_{23} as $4x$ minus $8y$ and mod r_{23} is root of 4 square plus 8 square, which is root of 16 plus 64 , which is square root of 80 . And, \hat{r}_{23} , r_{23} cap, that is, the unit

vector along r_{23} is r_{23} divided by mod r_{23} . So, this is $4x$ minus $8y$ divided by square root of 80. So, this will be, this r_{23} cap.

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BIOMATHEMATICS

Force on charge q_2 due to q_3

$$\hat{r}_{23} = \frac{\vec{r}_{23}}{|\vec{r}_{23}|} = \frac{4\hat{x} - 8\hat{y}}{\sqrt{4^2 + 8^2}}$$

$$\vec{r}_{23} = 4\hat{x} - 8\hat{y}$$

$$\vec{f}_{23} = \frac{q_1q_2}{80K} \left(\frac{4}{\sqrt{80}} \right) \hat{x} - \frac{q_1q_2}{80K} \left(\frac{8}{\sqrt{80}} \right) \hat{y}$$

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So, have a look at here. So, essentially, f_{23} is some magnitude into 4 by root 80 x . This is the unit vector along the, r_{23} unit vector, if you substitute and this is just like we did in the case of f_{123} , we substitute here and we get q_1q_2 by $80K$, it is 8 by root 80 y . So, this is an x , some component along the x direction and some component along the minus y direction. So, there will be some force along the x direction, in this direction; there will be some force downwards along y . So, that is what this means. Essentially, this means that, there will be some...All these are numbers; some, all this is a big number. So, there will be some number of, some amount of force along the plus x direction. So, this can be pulled; some amount of, some amount towards right, to rightwards in this direction, towards the plus x direction. Some amount of it, it will get also pulled towards minus y direction.

So, it will get pulled this way and this way. So, how much the lower it will move, that we will see later. But the amount of force f_{23} , is this much, this. f_{23} is now calculated. So, we calculated f_{12} , f_{23} . So, if we know that, f_{21} and f_{23} , what we need is the total force and as we have been saying, the total force f_2 is f_{21} plus f_{23} . So, what is f_{21} ? f_{21} is minus $6q_1q_2$ by $100x$ plus $0.8q_1q_2$ by $100Ky$, that is f_{21} . f_{23} is q_1q_2 by $80K$ 4 root $80x$ minus q_1q_2 8 root $80y$. So, this is the f_{23} . Now, the total

force is the sum of these two forces. So, how do we get the sum of these two forces? So, let us just do this, calculating the sum of this two forces a little more carefully.

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$$f_{21} = -0.6 \frac{q_1 q_2}{100 K} \hat{x} - 0.8 \frac{q_1 q_2}{100 K} \hat{y}$$

$$f_{23} = \frac{q_1 q_2}{80 K} \sqrt{\frac{4}{80}} \hat{x} - \frac{q_1 q_2}{80 K} \frac{8}{\sqrt{80}} \hat{y}$$

$$f_{21} + f_{23} = \left(\frac{q_1 q_2}{K} \left(\frac{1}{100} + \sqrt{\frac{4}{80}} \frac{1}{80} \right) \right) \hat{x}$$

So, what did we find is that, f_{21} is equal to, we got that minus 0.6 $q_1 q_2$ divided by $100 K$ \hat{x} cap minus 0.8, minus 0.8 $q_1 q_2$ by $100 K$ \hat{y} cap. Similarly, f_{23} is $q_1 q_2$ by $80 K$ into square root of 4 root 80 along X axis, minus $q_1 q_2$ by $80 K$ 8 by root 80 along minus Y axis. So, this is this force and f_{21} plus f_{23} . So, when you say f_{21} plus f_{23} , this is the x component and this is the x component. So, if you sum this x component, that will be the x component of the total force. So, the, what is the x component of the total force? There is $q_1 q_2$, there is $q_1 q_2$ by K. So, let us take it common, $q_1 q_2$ by K and there is 1 by 100 here, 1 80 here. So, 1 by 100 plus this, of root of 4 by 80 here, there is a 1 by 80 here; this much along the X axis. This is the total force along the X axis. If you substitute all these number and calculate, you will get some number. So, I urge you to calculate this number by putting q_1 equal to 1, q_2 equal to 2 and the value of K, which you can estimate. So, if you take the epsilon 0, epsilon r; epsilon r of water you can take, because proteins are in water. So, I urge you to calculate this number, what is the force along the X axis, as an exercise, but we can calculate this, by substituting all these number. Now, similarly, there is some force along the Y axis and that is this much. So, this is the force along the y, this plus this, if you sum these two, you will get the force along the Y axis. So, how much is the force along the Y axis?

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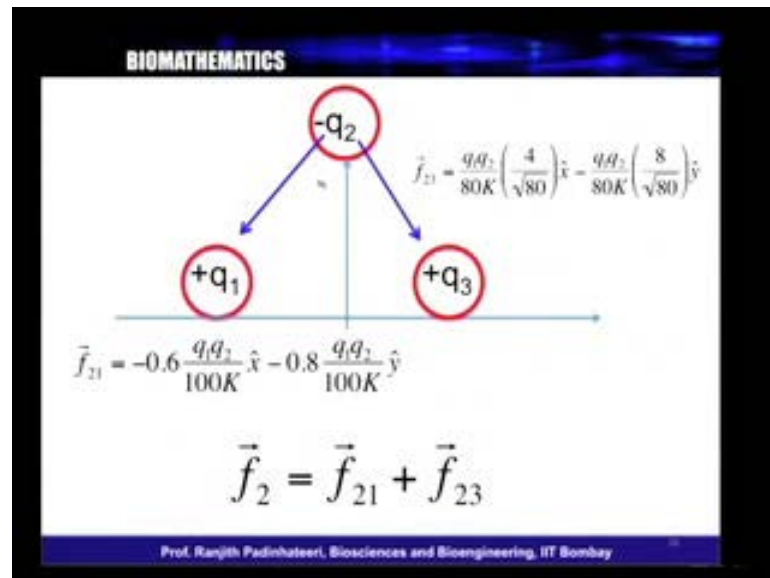
$$\vec{f}_2 = \frac{q_1 q_2}{K} \left[\frac{0.6}{100} + \sqrt{\frac{84}{80} \frac{1}{80}} \right] \hat{x} - \frac{q_1 q_2}{K} \left[\frac{0.8}{100} + \sqrt{\frac{8}{80} \frac{1}{80}} \right] \hat{y}$$

$$\vec{f}_2 = (\alpha) \hat{x} - (\beta) \hat{y}$$

So, let us write, the force along the X axis is $q_1 q_2$ by K into 1 by 100 plus square root of 4 by 80 into 1 by 80 ; this is along X axis; minus, along Y axis... So, we had $q_1 q_2$ by K here also. So, and there is a 0.8 by 100 . So, there is a, there is a 0.8 here actually. There is a, there is a 0.6 here, actually, 0.6 , because there is a $q_1 q_2$ and then, there is a 0.6 here. So, $q_1 q_2$ by K into 0.8 by 100 ; there is a minus sign and this is also with the minus sign. So, I can take the minus sign also common out. So, and, there is minus, plus root of 8 by 80 into 1 over 80 along y cap. So, this is the force on the second charge. This much along the plus X axis, and this much along the minus Y axis.

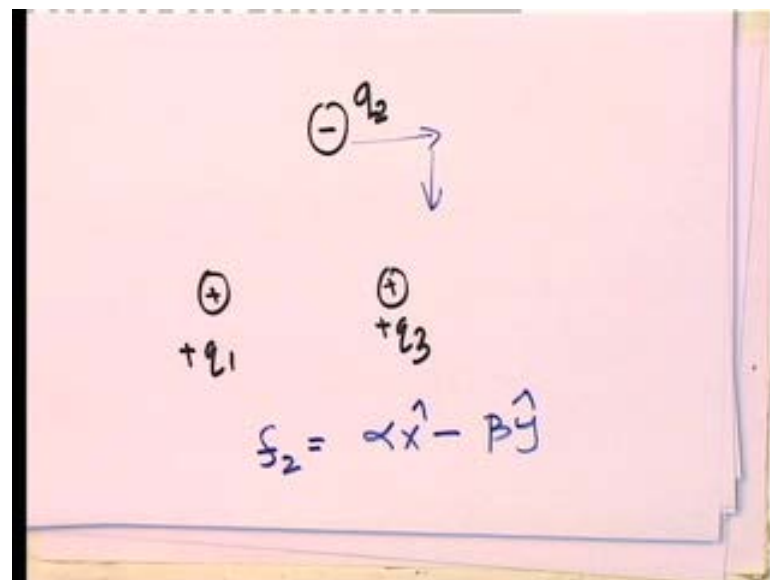
So, there is, as you can see, f_2 is some number along the plus x , plus, minus some number along the y . So, this is some positive number, this is some positive number, here. So, let me call this alpha; let me call this beta. So, some number alpha, along X axis; some number beta, along the Y axis, with the minus Y axis, where alpha is given by this particular number and beta is given by this. Where alpha... So, this is our alpha; this is alpha. So, this is alpha for you and this is beta. So, we know alpha and we know beta and we know that, $\alpha \hat{x} - \beta \hat{y}$. So, what does this mean? This is a, this itself gives you some idea. It means that, some amount of force, it, there is some force along the plus X axis and there is some force along the minus Y axis. The total force is along the X axis and something along the minus Y axis.

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So, have a look at here. What does it mean? There is some force along the plus X axis; some force along the minus Y axis. So, let us look at this little more carefully.

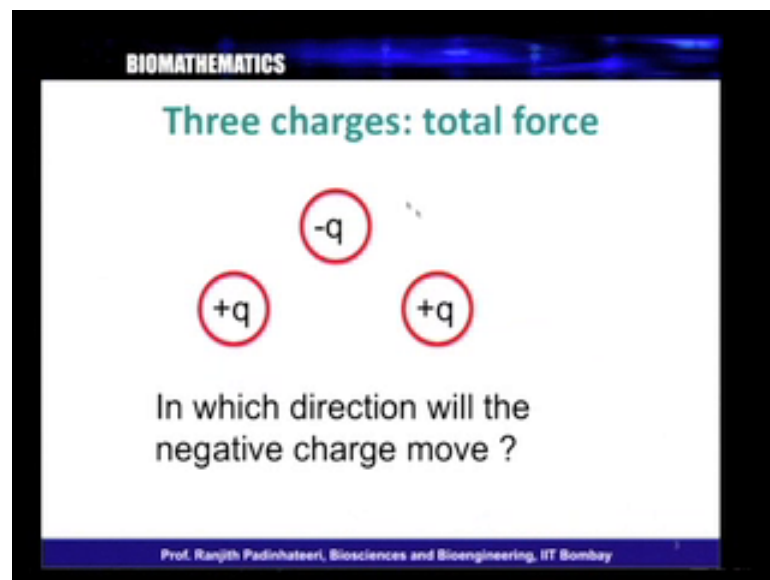
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So, when you look at this carefully, see, we have three charges and you had this plus q 1, so, this is q 2, q 3; this is q 1 and this is q 2. And, we asked the question, what is the force on this charge, due to this charges. This is plus, plus and what is the force. And, we found that, the force f 2, we found, we calculated this f 2 and we got f 2 is some alpha x cap minus beta y cap, where alpha and beta are some numbers. We got a number for

alpha; we got a number for beta. What does this mean? This means that, there will be some force along plus x direction; there will be some force along this direction and there will be some force along the minus y direction. So, essentially, it will slightly move, this direction and this direction. So, we got the already, the precise answer; we got, we know now that, it will not move this way; it will not move this way; it will not move this way; it will only move a little bit here and a little bit here. So, that is the force. Some force, and a bit of force along X axis; a bit of force along Y axis. So, we got the idea about the force and that precisely tells you, in which direction this charge will move. That is the question where we started. So, if we go back and see where the question we started.

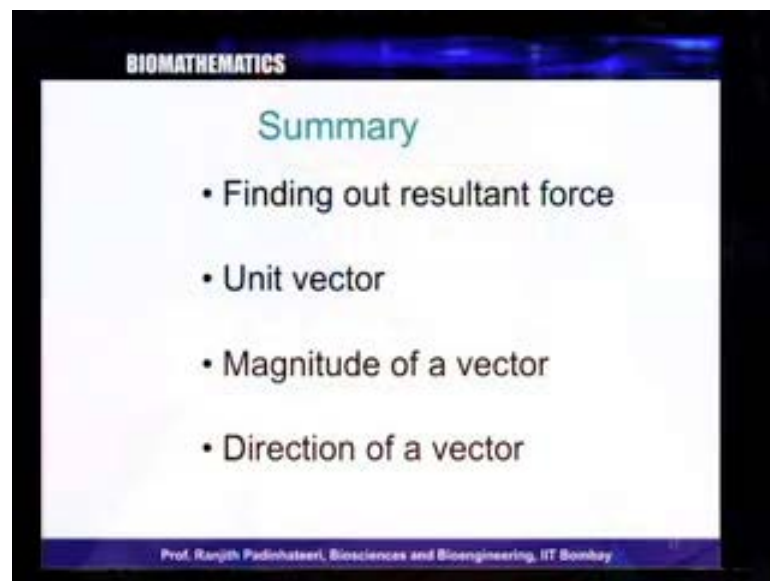
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So, we started with this question. In which direction, the negative charge will move? Now, we know the answer. It will move a little along the y, X axis and a bit towards, downwards Y axis. So, it will move a slightly towards this plus charge essentially. This plus q here, this charge, this is what essentially, it will move a bit. How much it will move, that we will calculate later. But the aim was, of this exercise was, to calculate the force on this, due to other charges. What is the resultant force? What is the total force? Whether it is the total force is in this direction, or, in this direction, or, in this direction. So, we saw that, it is none of this; it is along this direction; the total force is along this direction.

So, that is what it means. So, essentially, we have now discussed, what is... We have now understood, how to calculate the force. So, let us summarize whatever we learnt today. While we calculating, we learnt many things. So, let, **let** us have a quick summary of what we learnt. Let us summarize what we learnt today. In the last many minutes, we basically learnt, how to calculate the force and while learning that, we learnt to calculate the magnitude of a vector. We learnt to calculate the unit vector. We learnt that, unit vector means direction. So, three ideas from vector algebra, we learnt. And, we learnt how to calculate the force.

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So, essentially, the summary is this. We learnt finding out the resultant force. We learnt what is a unit vector. We learnt the magnitude of a vector. We learnt the unit vector represents the direction of a vector. So, knowing all this, we can now, we are equipped now, to calculate... If you have, for example, many charges or many forces acting, we know, what is the resultant force. We know how to calculate the... If you know, if you have a particular vector in a particular direction, what is the unit vector... What is the unit vector, how to calculate that, we know.

So, we will make use of all these, when we go along. The many, **many** phenomena as we said, like diffusion, etcetera, to understand all these in a proper manner, we have to get some sense of direction. And, we have to know precisely, how to calculate this direction of a vector. So, this is the first step and what one should do is, one should sit for a long

time and do all these calculation carefully; spend long time with pen and paper, doing all these calculations carefully, many times; so that, it will be very, very clear in your mind. So, learning this much, we will stop today's lecture here and we will continue a few more ideas of, we will learn a few more ideas of vectors, in the coming class. So, and we will, we will go ahead; using this ideas of vector, we will learn more interesting Biological concepts.

So, with this, we end today's class. Thank you.