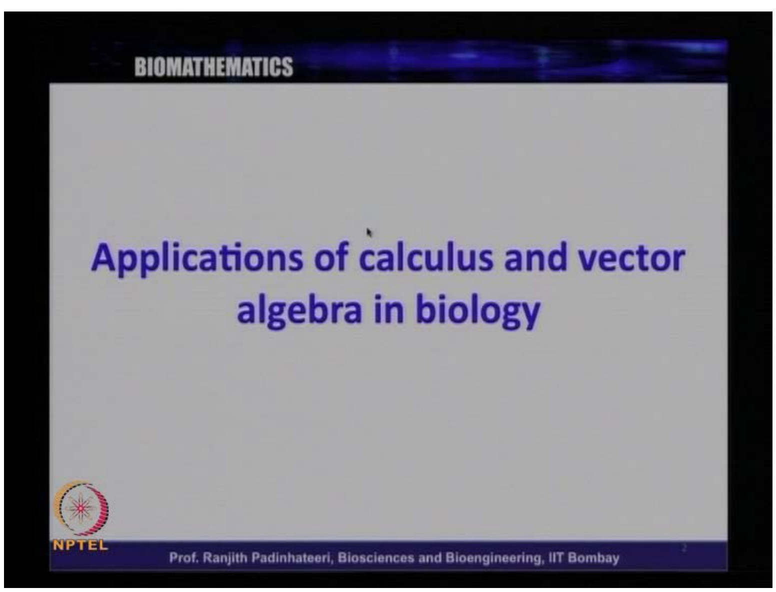
Biomathematics
Dr. Ranjith Padinhateeri
Department of Biotechnology
Indian Institute of Technology, Bombay

Lecture No. # 17
Applications of calculus and vector algebra in Biology.

Hello! Welcome to this lecture of Biomathematics. We have been discussing vectors for the last few lectures. And, also we discussed calculus; how do we do differentiation, integration and all that. Today, we will discuss an application of calculus and vector. The idea of that we learnt in calculus and the ideas we learnt in, on related to vectors. We can combine together and learn a very important equation in Biology or a very important concept in Biology that is related to, something related to membrane potentials.

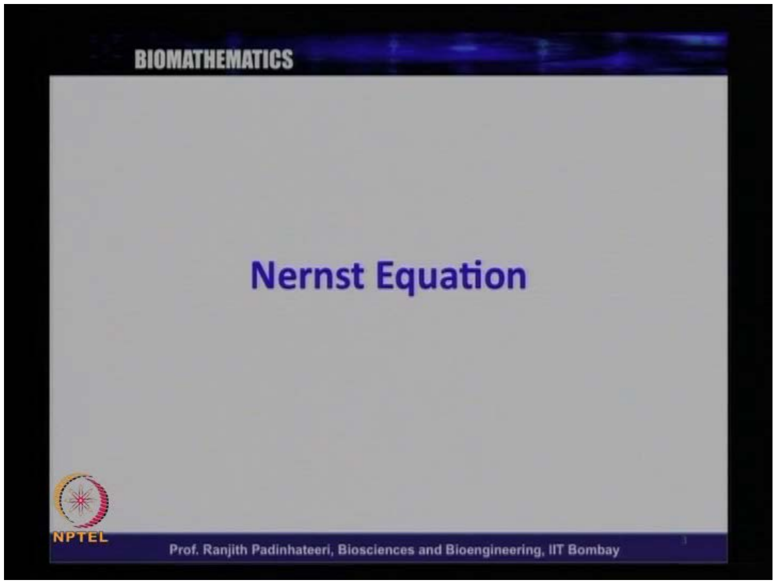
So, this is our aim today. Our aim is to basically understand the new interesting idea related to membrane potentials. And, to understand that we will be using the ideas that we learnt from Calculus and the ideas that we learnt from Vectors.

(Refer Slide Time: 01:28)



So, today we are going to discuss Applications of Calculus and Vector Algebra in Biology.

(Refer Slide Time: 01:33)



And, the important concept, I think we are discussing is called Nernst Equation. So, this is an equation that relatesthe potential across a membrane to the concentration of ions.So, if you have some ions, it is a very, if you know that, there is NaCl, KCl; salt solutions in the cell. So, they are in water, typically K appear as K plus; they in solution in cells, they will be K plus and Cl minus. So, there will be ions of K plus and Cl minus. And, you know that there are, they can go; K plus for example, they can go across the membrane, cell membrane. And, this can lead to some kind of electrostatic potential.

So, as you learned in the school, when you have charges, there is a potential can develop. And, this can lead to some electrostatic potential; this movement of ions, the flow of ions across the membrane channel.

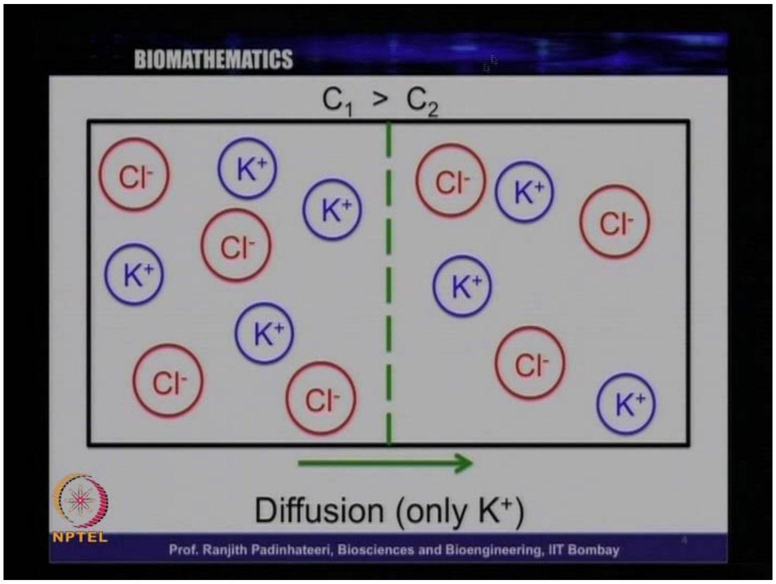
And, to really understandit; the Nernst Equation is an equation… many of you might of heard of equation called Nernst Equation. This governs, this tells you if we have some amount of ions on one side of the membrane and some amount of membrane and you have some amount of ions here and some amount of ions here, what is the potential across this membrane?

What is the equilibrium potential? You can have, you can put some amount of ion here, some amount of ion here, and then they will reach equilibrium; because you can flow this way and that way. And, this flow will lead to equilibrium. And, what is the potential at equilibrium across this membrane? That is what Nernst Equation will tell you. So, this is what we will understand.

This is an interesting and important idea in Biology, which is used in many places. So, let us start thinking about what we just said. That is, ions across a membrane, like both sides you have a membrane separating the space into two parts. You can imagine one part is inside the cell and the other part is outside the cell, if you wish. Essentially, what you have is a membrane that is separating, physically separating two regions in space. That is, outside the cell or inside the cell; inside the cell or outside the cell or either way you like.

So, there is a physical separation. That is important concept. That is membrane. And then, membrane is a semi permeable membrane. That the what is; that means.... So, let us have a look at this picture, here this slide.

(Refer Slide Time: 04:20)



So, you can see this is KCl. K plus and Clminus ions. And, here also K plus and Cl minus ion. Now, if you look at here, you can see that there are more K plus ions in this side. So, if you count here.This is a just a pictorial representation in this. So, you can count here, you can see four of them here, but does not mean four. So, you can see three here. So, you can, what is it only mean, what I want to represent in by this picture is that, in this part of this partition, we have more K plus than this part. So, the concentration of the K plus or Cl. So, equal number of K plus and Cl. This is a four K plus and four Cl minus in this side.

So, on a total, this part of the box is neutral;equal number of K plus and equal number of Cl minus. Here also equal number of K plus and equal number of Cl minus. This is also neutral. So, the concentration of KCl here is C 1 and the concentration of KCl here is C 2. And, as you see in this picture, C 1 is greater than C 2.

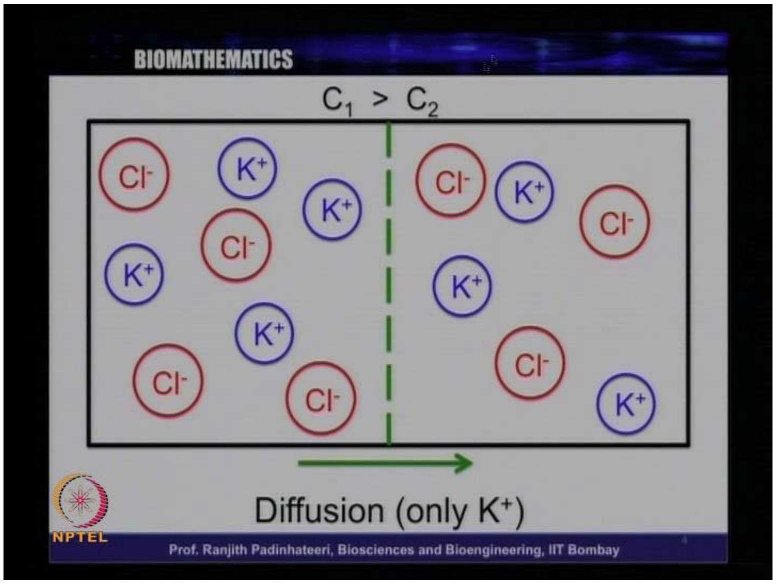
That is, the concentration here or the number of ions in a given volume is more here and less here. So, the volume is equal in both sides. So, the number is more here; so, more concentration here and less concentration on this side. And, this green line, this is the line with dashed lines or there are holes here. So, this through this hole, this smaller ion K plus can pass through. So, what is this mean? The K plus can go either way.

So, this is a semi permeable membrane. It does not allow both K plus and Cl minus to go through. It only allows K plus to go through. Either way, K plus can go from here to here and here to here. Whichever is permitted by the laws of Physics, it will go that way. If it is feeling some force, if it is pulled in this way it will go this way. If it is pulled this way it will go this way.

So, now we have to see the Physics and find out whether it is…which way it is going. But, remember C 1 is greater than C 2.That is, more concentration here and less concentration here. Whenever, there is something having more concentration in one side and less concentration in other side, you know that something like, called diffusion will happen. Diffusion is the concentration will change. Things will flow from higher concentration to a lower concentration. Something like there is diffusive flow.

So, this will happen now because you know that one side it is more concentration; the other side is less concentration. So, as you can see from your intuition, things will go from higher concentration to lower concentration. Why this? All this we will understand later, why is this happening. We will learn about diffusion little more carefully in the coming lectures. But, at the moment, you just need to understand that it will go from higher concentration; it will flow from higher concentration to a lower concentration,which is intuitively, probably, you know.

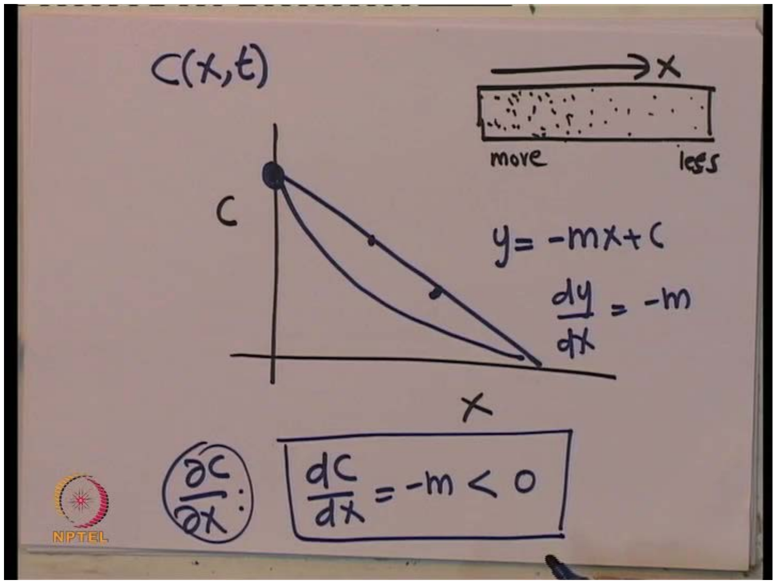
(Refer Slide Time: 07:55)



So, that is what shown here in this arrow. So, the only K plus and the K plus can… will go from here to here. There will be a flow of K plus ions. So, flow; as we said is a vector. So, to say something about the flow; as you can guess, we will have to use some ideas about vectors because flow of K plus ion in this direction is a vector.

So, what does this means? So, there is a flow. And, how do we say this mathematically? How do we say that there is a flow from here to here? Before that, let us just understand a big more about the concentration. So, you can see that concentration decreases as we go from here to here. So, let us plot for a minute here. So, have a look at this, here.

(Refer Slide Time: 08:46)



So, let us plot a minute here; the concentration and the x axis. So, you have this here. So, more concentration here, less concentration here; so, what does it mean? It is like lot of particles here and less particles here.

So, this is many ions here and as it goes, it will like less and less. So, there is a higher concentration. C here is greater than C. So, if you take this as the x axis, so, if this is if you take this as x axis, as you go along the x axis, the concentration is decreasing. So, the concentration... So, let me plot concentration versus the distance from the left end. So, at the left end there is big large concentration.

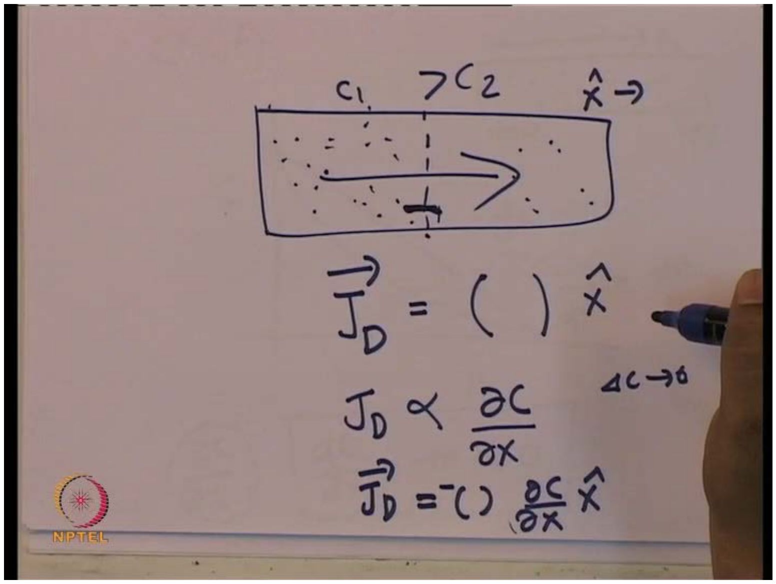
So, let me mark here. At the left end, you have large concentration and as we go little here, the concentration decreases and further you go concentration decreases and further your concentration decreases. So, the concentration decreases. It need not decrease in this particular way. It can decrease in this way also. So, we do not know which way it is decreasing, but it is clear that the concentration is decreasing as we go along. That much is clear.

Now, if, let us imagine the first case. The first case is, you know, you, we learned in calculus that if I have a straight line like this, this is an equation of y is equal to minus mx plus C. Where this m, this slope is negative; what is it mean? As x increases the C decreases. So, that is why this is negative. dy by dx; the change in x and change in y. This is minus m,it is a negative number. So, dy by d x is negative. So, that means, here dC by d x is minus m .This is a negative number. So, this is less than zero. It is negative.

So, this much, as we said we also we can also represent this by partial derivative del C by del x. This also means, if c, which is a concentration can be a function of the distance, the space and it can be also be the function of time. So, if you are only finding the derivative with respective to x, it is a partial derivative. So, we will, we can represent del C by del x. So, but in this lecture, most of the times we are not considering time.We are, we want to discuss the equilibrium properties of the system .Where, the time is independent of time. So, even though we will write dC by d..;del C by del x, the partial derivative.

Essentially, it is same as the, not full derivative dC by dx here; because we are not considering about time. So, we will interchangeably,we will use del C by delx and dC by d x. We will interchange and use del C by del x and dC by d x. But, any way the point is that dC by d x is negative. If you take this or even if you take this, the dC by d x, the C decreases. So this, as we go along the x, the change in C is less. This is negative. That is, the del dC is negative; the C change d, the C decreases. So, this is the point to remember. The point to remember is this; dC by dx is negative. So, keep this point in mind. Keeping this in mind, let us look at; now, what do you want to look at? We want to look at the flow.

(Refer Slide Time: 13:05)



So, let us look at we want to look at the flow. So, we have this membrane and we have ions here and ions here and C 1 is greater than C 2. And, we want to know about this flow from here to here .This is a larger concentration to a… So, this side is like one end of diffusion. So, you have a diffusive flow if you wish. So, let me call this J D.It is the diffusive current and it is a vector. It has a direction. So, we know that it has; the direction of this has to be along the x direction. We know from this. So, there is some magnitude and some direction. And, we do not know what the magnitude is.

And, we know the direction it has to be along this; because here it is more concentration; here it is less concentration. So, it has to be along this. We can also guess that more the concentration difference, more the flow is. That is, a large concentration here and small concentration here, the more it will flow. So, this is proportional to del C by del x. This much is also clear. The more the concentration, the more the flow. This is clear because the more the concentration difference, if the del c by del x is the large, like that means, the huge difference in concentration. There will be the, flow will be more.

If the del C remains, if del C approaches zero; that means, if del c goes to zero, the flow will go to zero.Right that is clear. So, there are two things. The direction is this. We know it has to be proportional to this. But, in this case, we just discussed the del c by del x is negative. So, if you say that J D is some constant times del C by del x, this is negative. So, this will be delC by del x is negative. So, what is, what would this mean? This would mean, so this is some proportionality constant we have to find. So, this would mean that, this is a negative quantity. So, negative x.This would mean that this goes along the this direction because del C by del x is already negative. We found that.

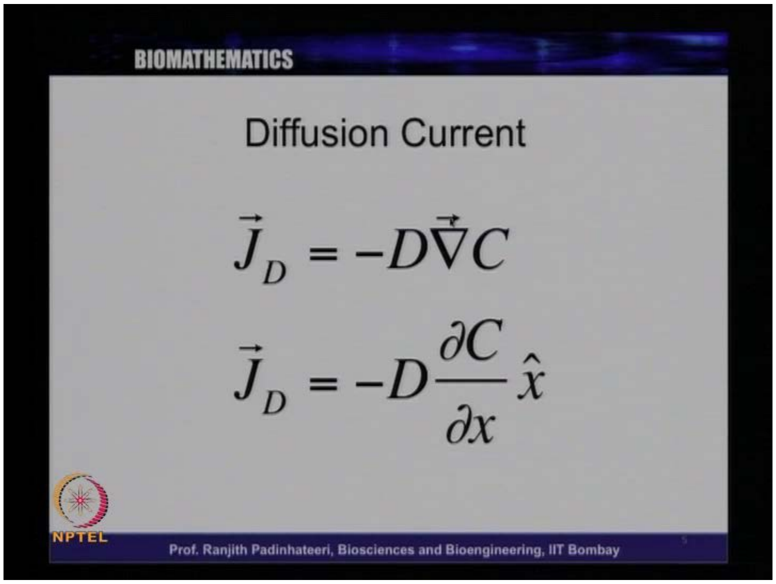
So, if this negative, this would say that, it goes in the minus x direction because negative times x cap would means that it goes in the negative x direction. But, that is not true. It is going in this direction. So, there has to be a negative sign here, extra. So that, del c by del x is negative. There is a negative sign. So, totally it is positive.

So, J D is along the x cap. So, just from mere common sense, we deduce this much. What did we, how did we deduce? First we said that C 1 is greater than C 2. So, it will flow from here to here. So, if since this is the x axis direction, so we call this direction as x cap direction. This direction of the flow, J D is the current or the flow. So, this is the current of ions in this direction. So, this current, due to this concentration difference, has to be in the direction of x. That much is clear. It is also clear that it has to be proportional to del C by del x.

Del C by del x is the change in the difference in concentration. The change in concentration as we go along x axis. So, if you look at this point, at this interface and look, this is the concentration difference between here and here. The more the concentration difference; the more the flow is. We are interested to calculate the flow at this interface. At this particular interface, how much is the flow? That is our interest. Whether it is flowing this way or this way, and how much is the flow. That is the thing that we are interested in. So, if you look at this point, the more the concentration difference; the more the flow.

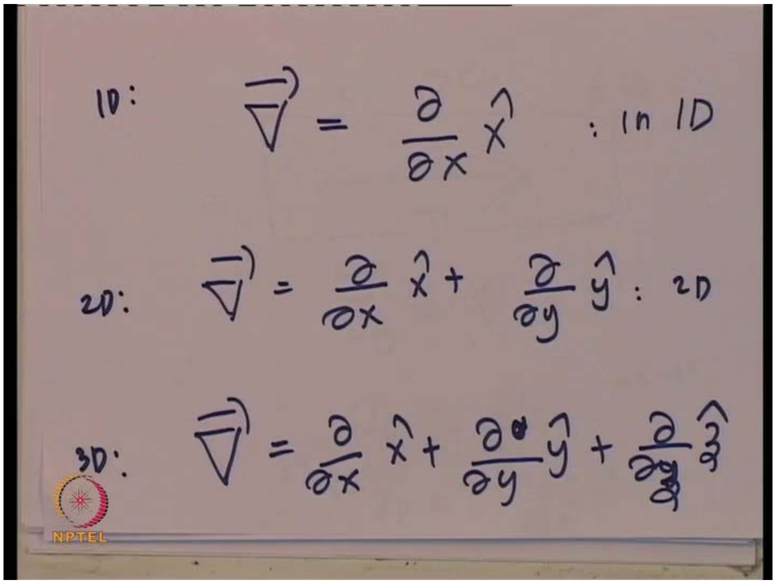
Now, you can write this proportionality constant as some constant into del c by del x. Now,del c by del x is negative which we just said. Because since the concentration decreases as we go along the x, del c by del x is negative. So, if this is positive, it will mean some negative x cap; that means, it would mean the direction of a J D is along this, which is not true. So, you have to add a negative sign here.

(Refer Slide Time: 18:13)



So, this leads to our next slide, where we said the current J D is minus D times del C. Where del is something, which we learned in the last previous lecture. So, one of the lecture is, a del is, for one dimension it is del C by del x into x cap. Del is the vector; is a gradient vector.

(Refer Slide Time: 18:41)



So, we said thatdel is a gradient vector. This means, del by del x into x cap in 1 D. In 2 D the del would mean, del by del x into x cap plus del by del y into Y cap. In 3D the del would mean, del by del x into x cap plus del by del y into y cap plus del by del y, del z by del z into z cap.

So, there is, this is in 1 D, 2 D and 3 D. This is 2 D and this is 3 D. So, 2 D and 1 D. One dimension, two dimension and three dimension. So, if you are talking about a flow in a plane, if things are flowing in all the direction in a plane, we have to apply this derivative because there is things, concentration can change along x and y. And, if it is flowing in three dimensions, all over the three dimensions, then we have to use the third derivative that we discussed.

But, here for simplicity we will only consider flow along a line because that is easy to understand. And, whatever we learn is very easy to extend to three dimensions, if you wish. It is easy. So, here we will use one dimension to make everything simple. So, that is easy for you to understand. And now, once you understand the simple idea, we can always go ahead and do the more the little more complicated generalized version of three dimension.

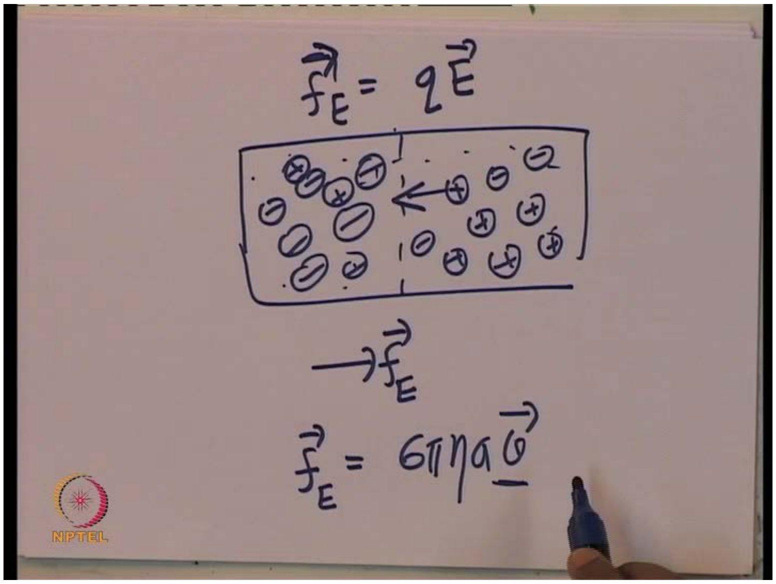
So, let us look at here. So, what we said so far is that the current, the flow due to the diffusion or the concentration gradient. The concentration gradient flow is essentially, minus d del c by del x x cap. So, this is the current. Now, what will this lead to? So, what did we say? So, we had this situation where K plus will go from here to here because it is more concentration here and less concentration here. And, what would that lead to? That would lead to this. So, more K plus here and more K plus here and less… So, the it will lead to the…K plus going here.

But, only K plus can go. So, here this will be more and more positively charged. This side of the semi permeable membrane will be more and more positively charged. And here, there are more and more Cl minus; because Cl minus cannot go. So, what will what will we end up with? We will end with more positive charges here and more negative charges here. What will happen if we have more positive charges here and more negative charges here? This K plus will be attracted back by this negative charges; this negative charges will attract this positive charges. So, we have more Cl, more negative charge here.

So, that will attract this positive charges. So, there will be an electrostatic attraction on K plus. Just because many K plus flew, were flowing in this direction, so the concentration or the number of K plus increased here. So, the number of positive charges increased here. So, number of.., therefore number of negative charges increased here, as you can see from the picture. Do look at this picture carefully.

So now, this positive K, for example, this K plus has four K and Cl minus here and three Cl minus here. So, this K plus will feel more attraction towards this side on an average. So, this K plus would want to go this way because there is an electrostatic attraction. So, what will happen? So, the K plus will start now flowing in this way. So, now how much is this electrostatic attraction? How much will be this flow of k plus in this direction due to the electrostatic attraction? So, let us see how much that will be.

(Refer Slide Time: 22:59)



So, what we have now is more positive ions here and a few negative ions only. And, more negative ions here and only a few positive ions. So, this positive ion will have to flow; will be feeling an electrostatic force and there will be a flow. They will flow with a… So, if something is, if there is a force, so if you push something in water. So, this is water here of course, there is water everywhere.

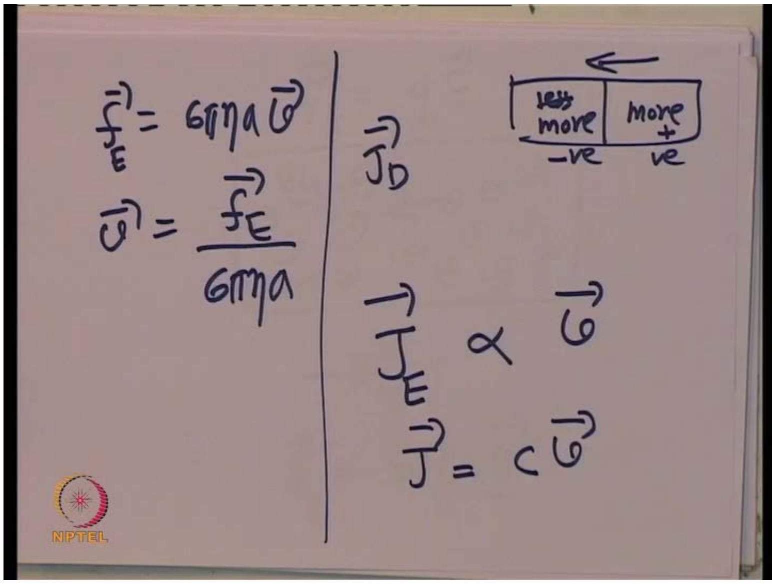
So, if you something… here pushing this electrostatic forces or pulling this electrostatic forces by a force and this force is an electrostatic force. So, you know that if you push anything in water, by a force f, especially in… by very small ions in highly viscous. So, compare to... So, this is something called a lower Reynolds number regime. What it is mean? Is that friction is very high. So, high friction regime; because there is a compare to the size of the ions, the friction is very large.So, that means this small ions will be feeling lot of friction.

So, the force, whatever the force you feel the electrostatic force. So, you will, each at this plus ions will be feeling some electrostatic force. This electrostatic force will be proportional, will be know, you know, anything moving in this spherical particles, the force the drag force is“6 pi eta a v”; where v is the velocity of this. So, this will move with a velocity v given by this formula. If we have this much force, electrostatic force, the velocity will be given by this formula;“f is equal to 6 pi eta a v”.

So now, f E is the electrostatic force and this is the drag force. So, when they are equal there will be like a… there will be a uniform flow. It will be a steady state flow or it is a steady flow. And, this will the… velocity of this flow will be given by this v, which is related to this electrostatic force by this formula. So, we know that there is a electrostatic force. Now, what is this electrostatic force? How much this electrostatic force?there will there will be an electric field. So, if you have an electric field e, the force is charge times electric field.

So, force f is q times E. q E charge times electric filed is the force. So, this is the force. So, the force is charge times electric field. And, this force will be equal to this flow, this 6 pi eta a v because there will be friction. So, the velocity of the flow will be given by this. So, if you know the velocity of the flow… so, what do we know? We know the velocity of the f. We know that f is the electrostatic force will be equal to 6 pi eta a v.

(Refer Slide Time: 26:40)

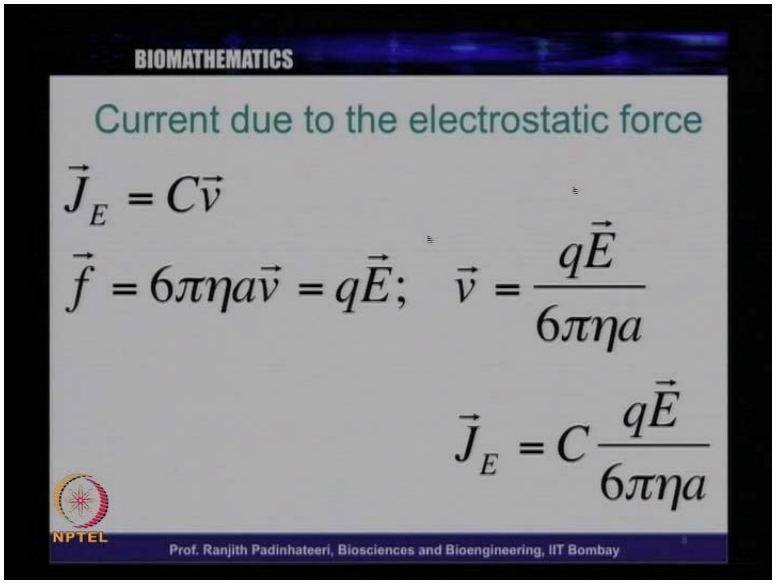


That means v is “f E divided by 6 pi eta a v”. So, we know that velocity is a vector, force is a vector, this way we have vector signs. Now, we also know the direction of the velocity. It will go from more positive charge to less positive charge or here more negative I should say; more negative, more positive. So, in this way it will flow. There will be, the velocity direction will be in this direction. And, the force direction also will be in this direction.

Now, the current due to the electric electrostatic interaction what will be the current? We found out in the previous case, the current;we found out the current due to the concentration difference we called it J D. Now, we will find out the current due to the electrostatic interaction and we will call it J E. So, this J E has to be proportional to this velocity because more the velocity or more the force; the more the current .You know that velocity is proportional to the force; so, more the force; more the current.

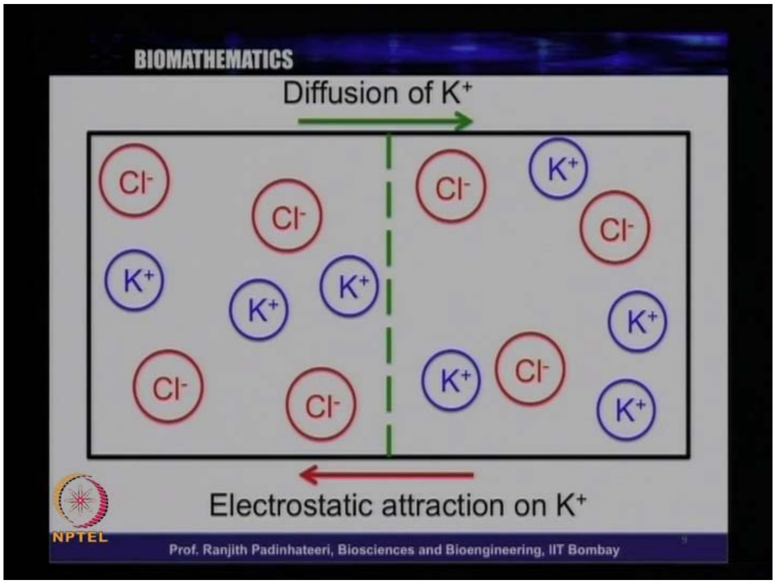
So, by just looking at the dimension by dimensional analysis we will know that this proportionality constant has to be concentration. So, the proportionality constant here, so J has to be written as Cv; where C is a concentration at this particular point. This is Any point the flow will be the concentration at point times the velocity at that point. If you do not understand how this c is coming,just look at the dimension of velocity and current, the J. And, you will see that they have, they are related to this Cv. Now, known this, let us look at this slide.

(Refer Slide Time: 29:07)



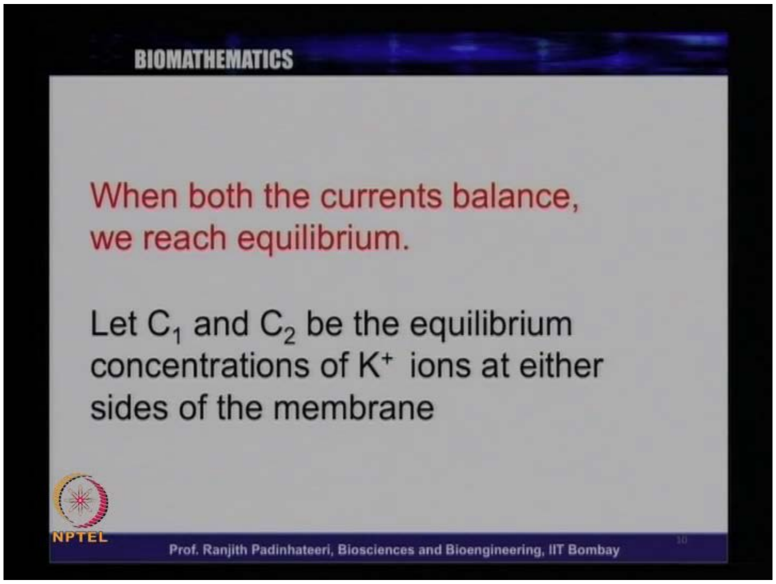
So, this is essentially message which I am giving here that the current J is Cv and the force is 6 pi eta a v. Which is, the force is also q times E; where E is the electric field. And, from this we know that qv is qE by 6 pi eta a.I just take qE this way. So, you get qE is equal to 6 pi eta a. So, essentially what you have is, J E is C times qE by 6 pi eta a. So, this, just I substitute this v in this equation. So, v is qE by 6 eta a. So, I substitute for this v from here. For this velocity, I just simply substitute this. So, I get J E is C times qE by 6 eta. So now, we found out two currents J E and J D. This is simple algebra. I just rearranged all these and then I can get this.

(Refer Slide Time: 30:16)



So, I have now two currents. The diffusion or the concentration gradient will lead to a current in this direction and electrostatic attraction will lead to a current in this direction. So, we have two currents. One current in this way, which is the concentration gradient which we said. And the other current, as this flows in this way, as the concentration gradient leads some flow in this way, the more positive charge will build up here and more negative charge here, which will generate in an opposite flow. So, there will be two flows this way and this way.

(Refer Slide Time: 31:08)



So, when this, both this flow balance; that means, the flow this way equal to the flow this way or the total flow here, if the total flow is 0. So, when the flow is equal, then the total flow is 0. Because it is not, it is flowing this way and flowing in this way so that vectors,some of the flow at this particular point. When that becomes 0, when the vector is the flow, some of the total flows is 0 in this; along this membrane,you have equilibrium.

We call it in, you can call it an equilibrium. That means the flow in this way, balances the flow in this way. So, when this, if five of them go this way and five of them go this way, five of them go this way and five ions go. Let us say, in 1 minute five ions go this way and five ions go this way. Then, on an average, the number of ions and both size will remains the same because same number goes this way and same number goes this way. So, we can call it as equilibrium because a number on an average, will remain the same. And, the flow will be equal and opposite.

So, when the flows are equal and opposite, what we get is equilibrium. So, we want to know, at equilibrium what happens. What is the force and what is the concentration difference, etcetera at equilibrium. We want to know that.

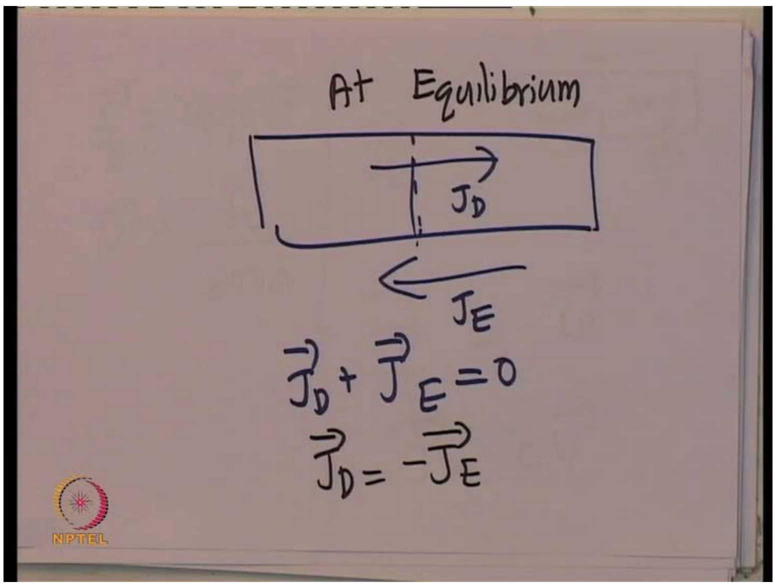
So, let us go ahead. So, we understand, what is the…? At equilibrium, the flows should balance. The flow in this direction should be equal to the flow in this direction. The flow should be equal and opposite. So, let us go here. So, what…. when equilibrium,… when both the currents are balanced, we reach equilibrium

So, and let us assume that C 1 and C 2 are the equilibrium concentration of K plus ions at either sides of the membrane. At one side at equilibrium, that is, when the flows are balancing on an average there is C 1 be the concentration on one side of the partition. The other side of the partition we have C 2, if the concentration at equilibrium. So, these are equilibrium concentration. This is not the initial concentration. Initially, we had a some concentration. So now, at let me call this initial concentration C 1 0or C 2 0.

So now, this is the C 1, C 2; the concentrations at equilibrium. So, if C 1, C 2 is the concentrations at equilibrium, the question we want to ask is that will there be any potential across a membrane or not. So, C 1 and C 2 may not be equal. We do not know.

If they are not equal, what will be the potential? That is what we are going to calculate. So, let us let us go and calculate this. But, before that, let us understand the idea of equilibrium a bit more. So, what did we say just now?

(Refer Slide Time: 34:31)

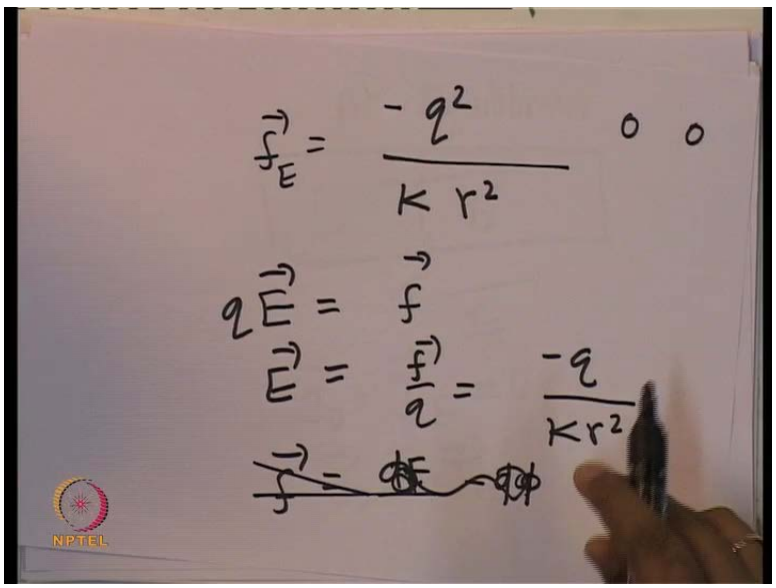


We said that the flow, there is two flow. There is J D and in this way there is J E. So, they have been equal and opposite. Or in other words, at this point the total flow, the J D plus J E has to be 0. The total flow has to be 0. Then only we will get this equilibrium.

So, at equilibrium what we expect is that, this flow will balance this flow. The total flow at this point will be 0.In other words, J D will be equal to minus J E. The flows will be equal and opposite. So, this is minus sign; gives you some direction. So, there will be equal and opposite.

So, that is what we want at equilibrium. So, now let us have a look at a few more details. So, what did we say a minute ago? We said that, when there is charges there will be an electric field. So, there is one more idea that you want to understand about electric field and charges etcetera.

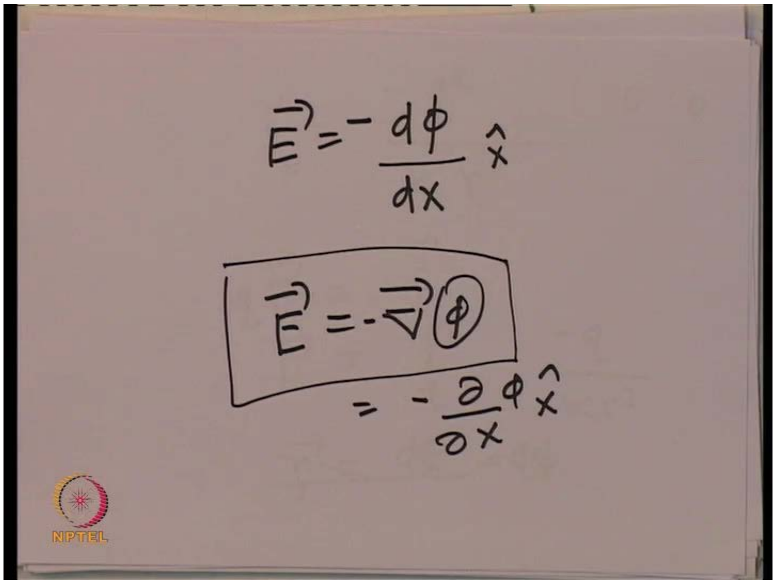
(Refer Slide Time: 36:15)



So, from the Coulombs law, we know that the force the coulomb force is, if you have charges q and q square; if you have two charges, you have q square by K r square. This is the.., if you have two charges q, then this will be the electrostatic force. If it is plus and minus, it is it will be minus q into minus. So, this will be the minus sign, but this will be the force.

Now, we also, just a minute ago we said that q E is equal to f. So, what does this mean? E is equal to f by q. So, which is q by K r square. This is our E. And, we said in previous lectures that the force is minus dele by del rdel energy by del r, d energy by del r. So, let me call phi as the energy here. So, let me… we said that f is, generally the force is derivative of energy. So,we said that ,derivative of energy.

(Refer Slide Time: 37:48)

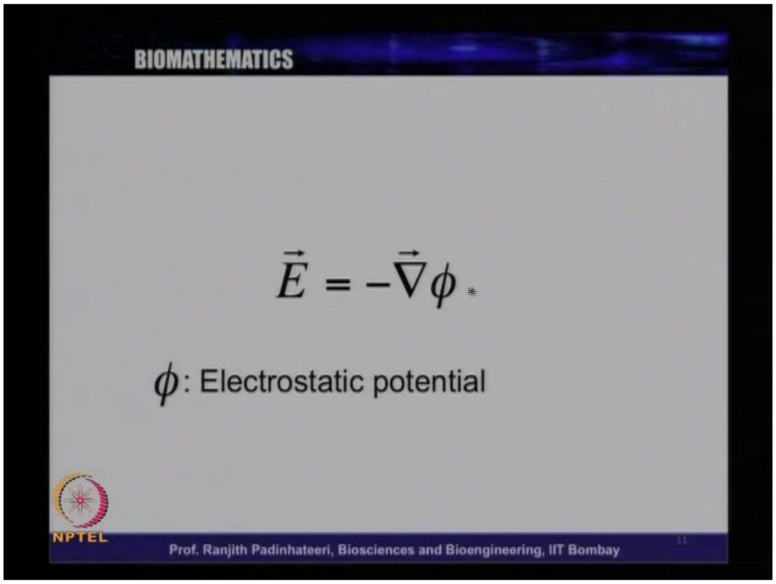


So, if you use that idea that the force is derivative of energy in this, what you will end up is that electric field is the derivative of electrostatic potential with the minus sign. But, there has to be some direction associated with this. So, there has to be some unit vector. So let me have some unit vector here, in some particular direction.

So, in other words, E can be written as minus del phi; where del is… as we said this means minus del by del x of phi along the x. So, this is the electric field. So, phi; where phi is the electrostatic potential. So, the potential energy. So, if you know the potential energy, the electric field and potential energy are related in this particular way.

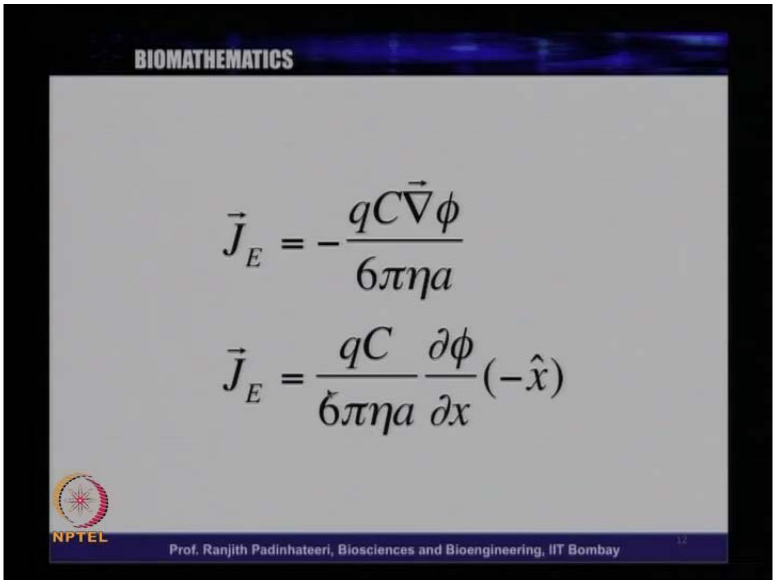
So, knowing this… this is some simple idea that we remind ourselves. From that, we have to remember from electrostatics. Now, if we know that electrostatic field is derivative of potential, we can go back and do the algebra of all the equations that we learned.

(Refer Slide Time: 39:18)



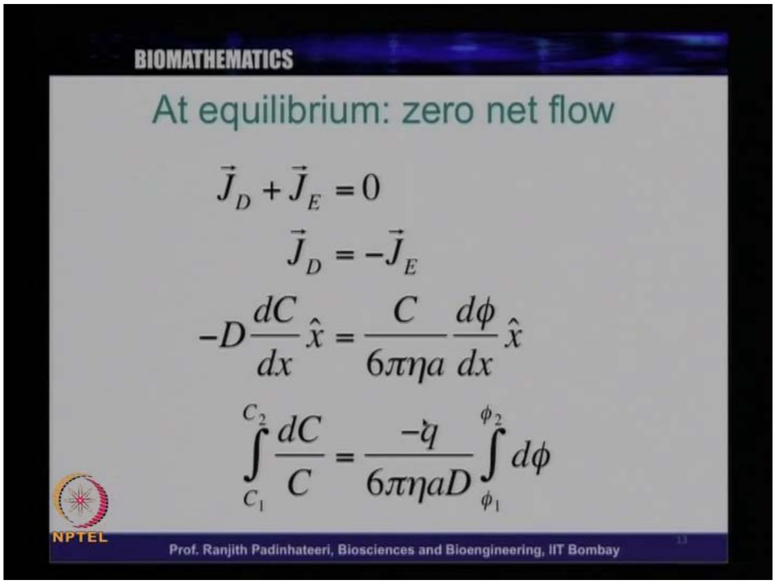
So, we have to just remember that electric field is minus del phi; where phi is Electrostatic potential. This is another vector relation. So, the E is a vector and del is a vector and phi is a scalar. Potential, it is like energy is a scalar. So, this is just like force is derivative of energy, something equivalent of that here. So, now we can substitute this E in the J E we had.

(Refer Slide Time: 39:48)



So, J E, we have said that J E is q C E by 6 pi eta a.And, E is substituted with del phi.So, for electric field, wherever we see electric field,we substituted as minus del phi. And,del phi is del d phi by del x d phi by del phi by del x with x cap and there is a minus sign here, which I put here. Which is only means that, this current is along the minus x direction. It is along this direction. This is what we have been telling all the while that the current is along this particular direction. So now, what does this mean?

(Refer Slide Time: 40:47)

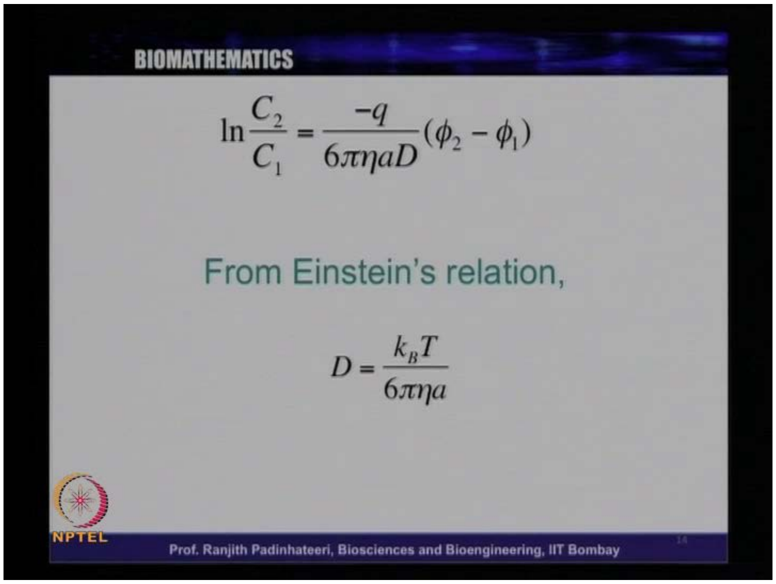


So, we have a current, we have an expression for the current, we have an expression for the J E and we just said that at equilibrium zero net flow. There is no net flow at equilibrium. This means as we just said "J D plus J E equal to 0". That is,the net flow at the partition is 0. That is, J D equal to minus J E. Now, we can substitute for J D and J E.

So, we go back…. and if you go back and look the expression for J D, we had written minus D dC by dx x cap. And, J E was 6 into sorry C into C by 6 pi eta a into d phi by dx x cap. So, we might have changed the partial derivative to full derivative because the phi and C at this moment, only depends on x. It does not depend on time. It does not depend on time or any other quantity at this moment. They all at equilibrium, when we sayequilibrium is independent of time, there is no idea of time. It is like forever. This is true. That is, for irrespective of time, this is true. So, dC by dx is d phi by dx. So, now we have such an equation; this is a differential equation. So, there is derivative here and the derivative here.

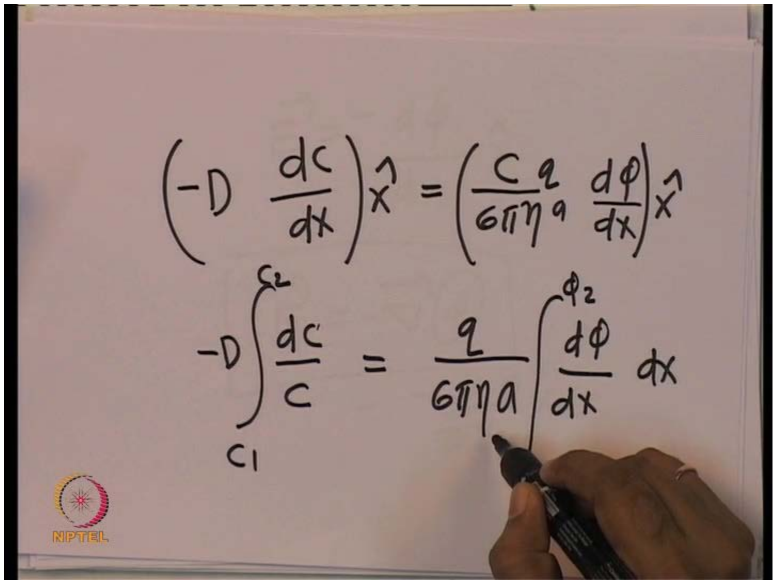
Now, what we can do? We can take this dx this side and this c this side. So, if I take this C to the left hand side, I will get dC by C. So, there is why here dC by C.And, I take this dx, this space also dx into dx, so and essentially, what you have is dC by C is equal to minus q by 6 pi eta a D into integral phi 1 to phi 2 d phi. And, here C 1 to C 2 dC by C. So, you can integrate like this and you essentially end up with this integral.

(Refer Slide Time: 43:18)



So, what would this lead to?If you do this algebra, what you get is that lan C 2 by C 1 is equal to minus q by 6 pi eta a D into phi 2 minus phi 1.If you do this integral, so there is a minor type… here. So, the right way to write these equations. Let me write. So, have a look at this equation. So, let us look at this equation. So, this equation let me write here.

(Refer Slide Time: 43:52)

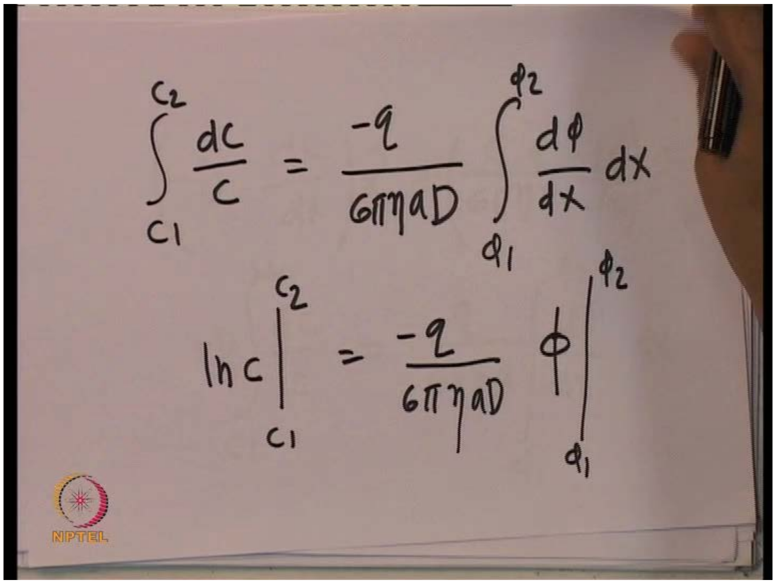


So, if you write here, you have minus D dC by dx x cap is C by 6 pi eta a d phi by d x. So, this is also x cap. So, I… this quantity in the brackets, they will be equal. So, this would… now I can take this C here and this dx there. So, I take this C here. So, you have minus D dC by C is equal to…I said one, there is a q here. So, q by 6 pi eta a. So,.. type… the q here. So, this is a correct one. q by 6 pi eta a into d phi by d x times d x.So, this is the equation.

Now, we have to integrate. Now, I can… if you wish I can take d also this side. So, I can write this d here if you wish. But, I can integrate both sides. This C 1 to C 2 and this phi 1 to phi 2. So, if you do this integral properly, you know that integral of 1 by C is log C. integral of 1 by C is log C. So, let us let us do this carefully this integral. So, I take this d this side and rewrite this equation.

.

(Refer Slide Time: 45:49)

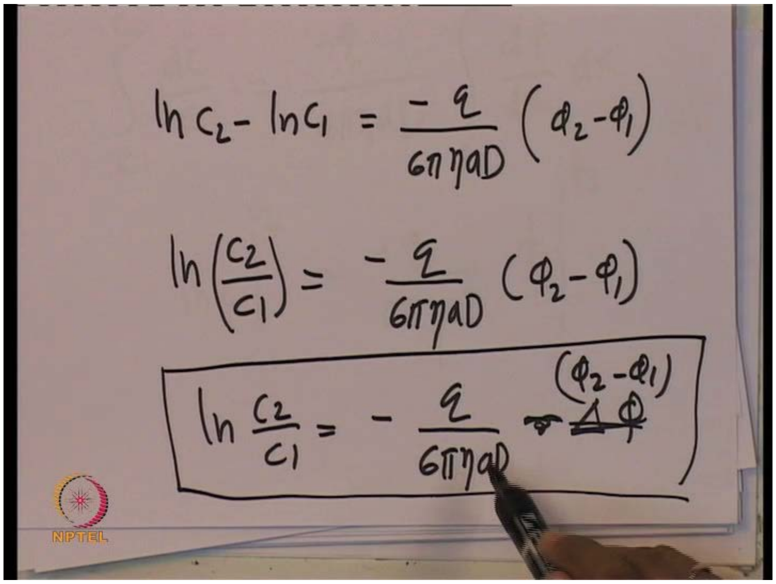


So, if we rewrite this equation, what you get? We have integral C 1 to C 2 dC by C. So, that is the... And, I take minus sign this side. So, if I take if I take the minus sign this side, what you get is minus q times 6 pi eta a D and there is this integral;integral phi 1 to phi 2 d phi by d x into d x.

So, you know that this d C by d s is log C. So, log C in the limits C 1 to C 2. So, integral of this d phi by d x, d x is equal to minus q 6 pi eta a D into phi, in the limits phi 1 to phi 2.

So, this is just d phi by dx.dx is integral. It is essentially phi because if you integrate d phi by d x, you will get phi. So, your phi r, in the limits phi 1 to phi 2 and log, in the limits C 1 to C 2.

(Refer Slide Time: 47:26)



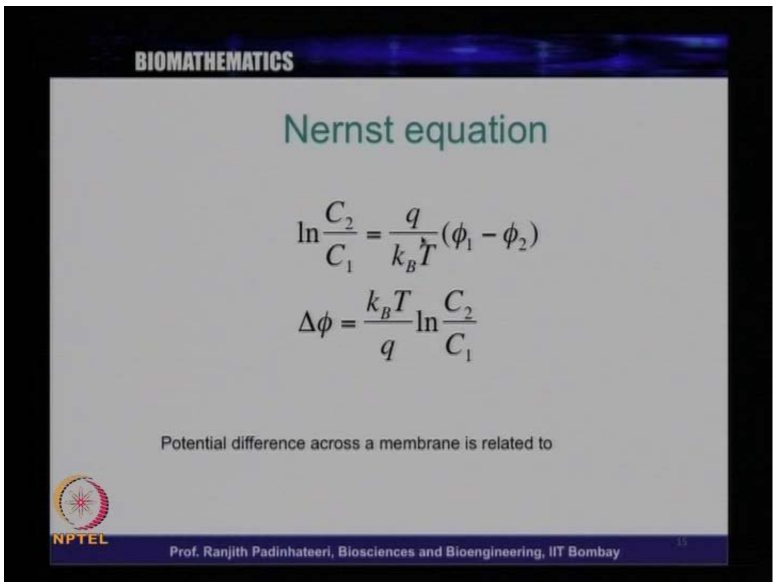
So, if you have this integral you can say that… apply this limits. So, you have, log lan C 2 minus lan C 1. That is, if you apply limits on the left hand side, you get this minus q 6 pi eta a D phi 2 minus phi 1.If I apply limit here, phi 2 minus phi 1. So, log a minus log b is log of a by b. So, I can write this thing. Log of a minus b as log of C 2 by C 1 is minus q by 6 pi eta a D into phi 2 minus phi 1.

So, as some of you are familiar with this equation, you can already see the Nernst equation is appearing. So, what is this ?log C 2 by C 1 is equal to minus q by 6 pi eta a D into.. I can write delta phi. Delta phi is nothing but phi 2 minus phi 1. If you want, I can I can write this. Just delta phi, if you wish.

So, this is for change. This is, let me write it phi 2 minus phi 1 itself. So, the change in potential is related to the change in concentration. So, this potential difference, the difference in potential between the two sides is related to the change in concentration in this particular way. So, this is essentially the Nernst equation. Now, if you want I can beautify this constant a bit by substituting something for D.

So, let us do that. This is essentially the Nernst equation. We have already obtained the Nernst equation. Now, I am just re or rewriting in a different, more familiar fashion. So, let us have a look at here. So, what we have is this. And now, there is something called famous Einstein's relation. Einstein, in his 1905, very famous paper, derived the relation between diffusion coefficient D and temperature and viscosity. So, k BT is k B is Boltzmann constant, T is temperature, eta is the viscosity,a is the radius of the particle. So, if you have this many variable; so, this is a relation between the diffusion coefficient D and temperature and eta which is the viscosity.

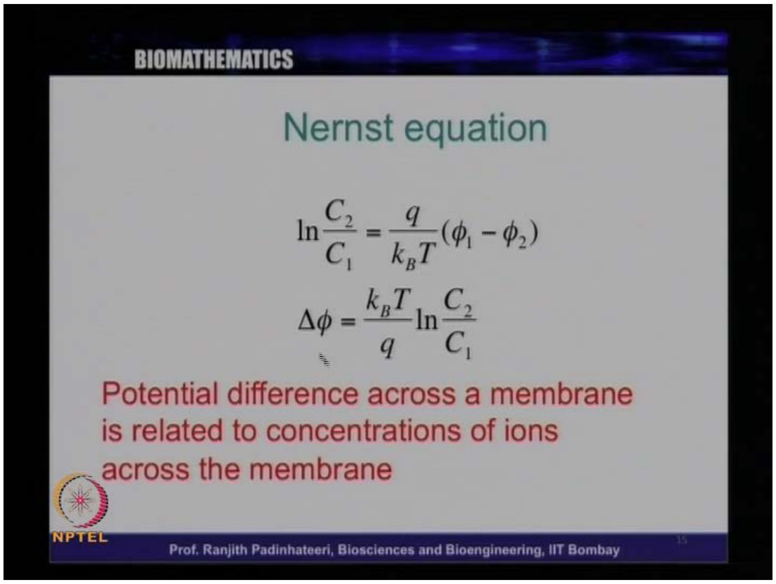
(Refer Slide Time: 50:31)



So, if you just borrow this relation and substitute here for this particular D, What you would get is that, you will get that lan C 2 by C 1 is q 1 by k B T because as you can see here, D is k B T by 6 by eta away. So, if I substitute here, 6 pi eta a and 6 pi eta a, they will cancel each other and you will end up with this equation. q by k B T into phi 1 minus phi 2.

So, if I call phi 1, phi 2 minus phi 1 as del phi. I can write this particular relation, they can be phi 1 minus phi 2 as del phi. I can write this particular relation there can be phi1 minus phi 2 as del phi I can write this particular relation. So, if k B T by q into lan C 2 by C 1. So, this is the Nernst equation. Basically, here what is happening is that the Potential difference is related to the constant relation across the membrane. I repeat this slide.

(Refer Slide Time: 51:40)

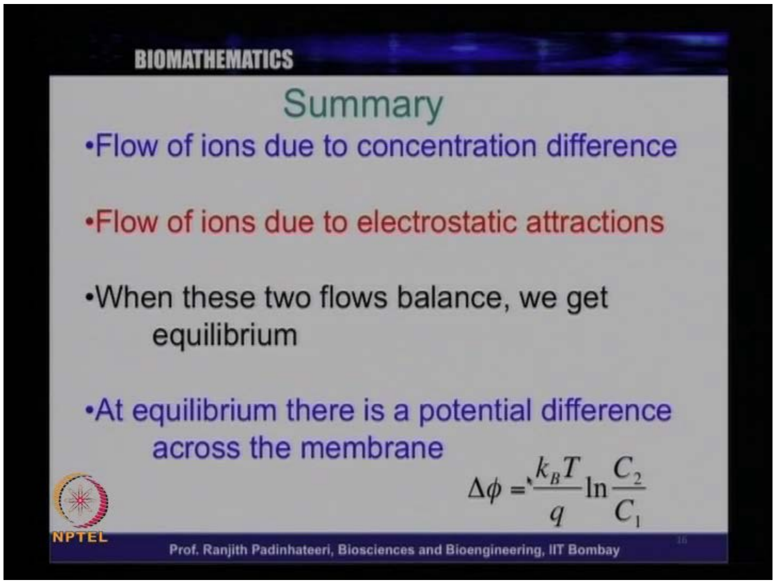


So, what we say, what we essentially will end up,getting is this equation as you see here, log of C2 by C1, which is the concentration the on both sides. C 2 and C 1 are the equilibrium concentration at either side.…is a is a is related to, is equal to q by k B T into phi 1 minus phi 2.

Phi and phi 2 are the potential. This is the potential difference across the membrane. So, phi 1 minus phi 2 is the potential difference. So, if I call this deltaphi, so the delta phi is equal to k B T q by k B T by q into lan C 2 by C 1.

So, the potential difference across the membrane is to related to the concentration difference. In other words, so the… this potential difference across a membrane is related to the concentrations of ions;equilibrium concentrations of ions across the membrane. So, this is, at equilibrium if there is a concentration difference C 1and C 2, they will lead to some equilibrium. If the equilibrium concentration C 1 and C 2 that will have a potential difference, that will develop a potential differencedelta phi given by this particular equation. So, this equation is called the Nernst equation. So, we learnt how to derive the Nernst equation.

(Refer Slide Time: 52:59)



So, to summarize what we said so far. In summary,in summary we first… that the flow of ions due to concentration difference we discussed .We said that they go in a particular direction. And, in the opposite direction there will be a flow due to the… So, this flow of ions due to concentration difference will lead to some kind of charge differences. It will lead to positive charge ending up in one side and the negative charge ending up in other side.

And, that will lead to a counter flow of ions due to electrostatic attractions because this will lead to charge separation. And, charge separation will lead to electrostatic attractions. And, there will be a flow in the opposite direction.

So, when these two flows balanced, we get equilibrium. And, at equilibrium there is a potential difference across the membrane. And, the potential difference is given by this particular equation;delta phi is equal to k B T by q into lan C 2 by C 1.

So, this is the potential difference. And, this is the famous Nernst equation. So, by just knowing, by using the ideas from vector as well as calculus, so, vector algebra like direction vector and ideas from calculus, we could derive this equation. This Nernst equation; which is the very important equation in Biology because potential across membrane; the membrane potential is the thing that drives lot of movements of ions across the membranes and this can, this has lot of role that we go along. We might come and discuss some part of this; a role of this in Biology.

So, at…with this lecture we are stopping here. We are concluding with this Nernst equation. So, we understand the Nernst equation and we will go ahead and learn new, more new things in the coming classes. So, Bye.