Biomathematics

Dr. Ranjith Padinhateeri
Department of Biotechnology
Indian Institute of Technology, Bombay

Lecture No. # 20
Diffusion

Hello, welcome to this lecture of Biomathematics. In the last few lectures, we have been discussing about Diffusion like last couple of lectures, we discussed the diffusion equation and then, we discussed how do we calculate the rms distance from this equation without really solving the equation, even though diffusion equation is a second order differential equation. At this moment, we will not discuss how to solve it, it is a little more you need to learn some more mathematics too.

Figure out, how to learn this or to understand, how you solve this diffusion equation, but without really solving it something very useful, some relation that is very useful we got, which is the rms distance and time the relation between time and the x square average. So, this we will continue to discuss something about the relation diffusion, initially the first part we will discuss a bit about another quantity, which is average x average.

And then, we will go on to discuss with the one another very important relation as far as the diffusion is concerned. So far, we took diffusion coefficient as a constant, today we will discuss, what is diffusion coefficient related to, that is how is that how diffusion coefficient related to temperature, viscosity, of the etcetera of the medium. So, this relation is a famous Einstein relation, so we will discuss the Einstein relation in today's lecture, towards the end of this lecture.

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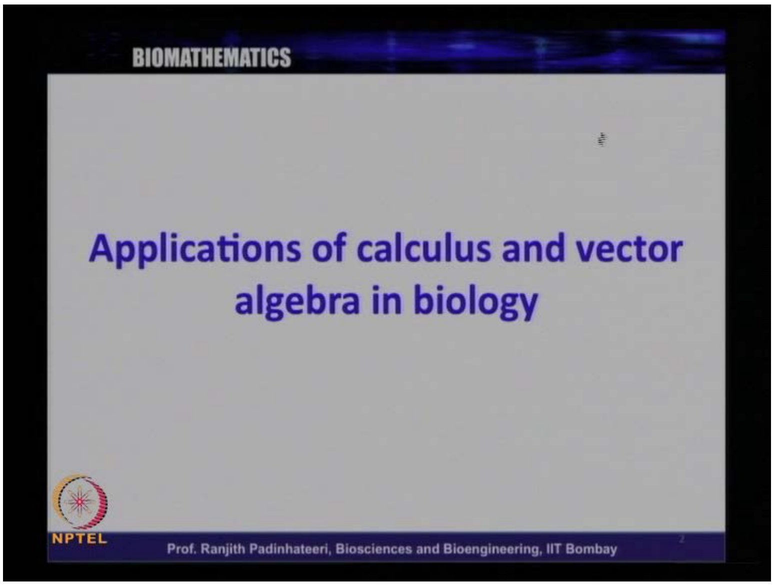

So, the topic of we are continuing to discuss on section on Applications of calculus and vector algebra in biology and under this, we are discussing this diffusion.

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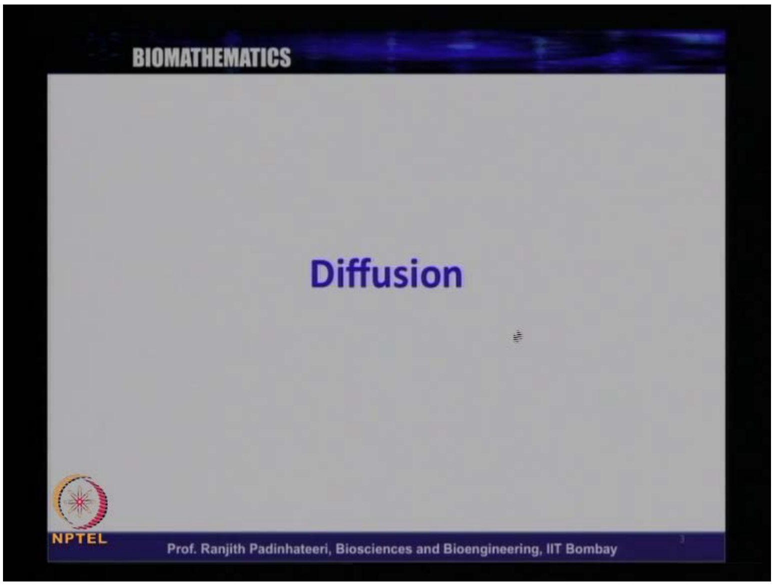

So, we said that, if you start with, if you take a tube as you see here.

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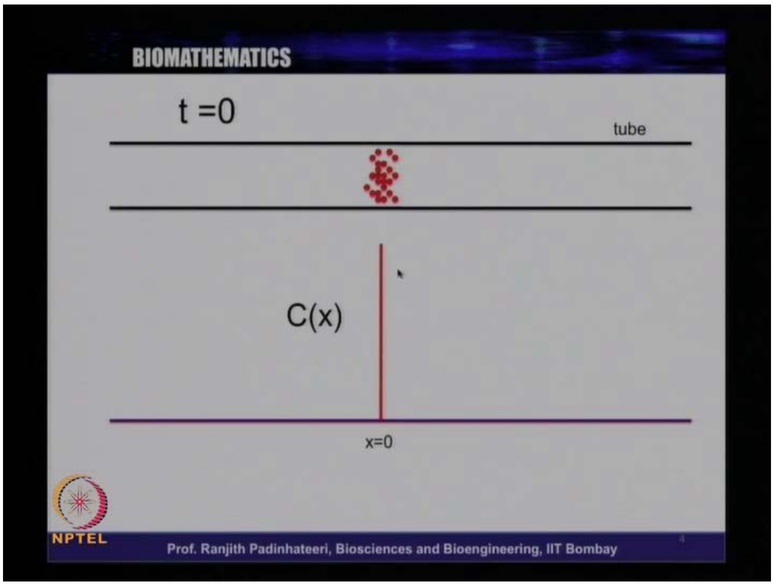

If you take a tube and if you put a few number of particles in this or a protein molecules on the middle of this tube, the concentration is only t is equal to 0, the concentration is only at the middle of the tube and as we go along it will spread, the molecules will diffuse.

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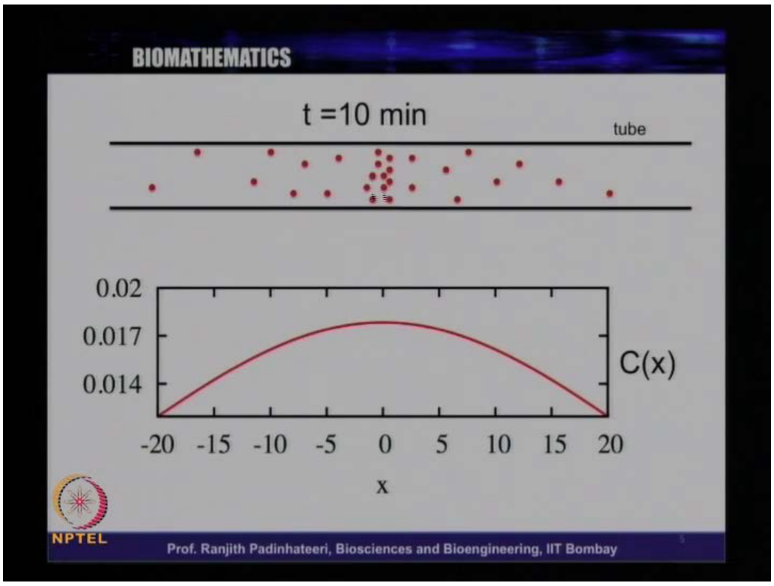

Even at this time, maximum number of molecules larger bigger number of molecules, the concentration is still more here at the center, but there are still there are some still there are some molecules as we go along the tube to the left or to the right; so, this is the concentration of profile and the diffusion equation is the equation that deals with this spread of this concentration is the diffusion equation.

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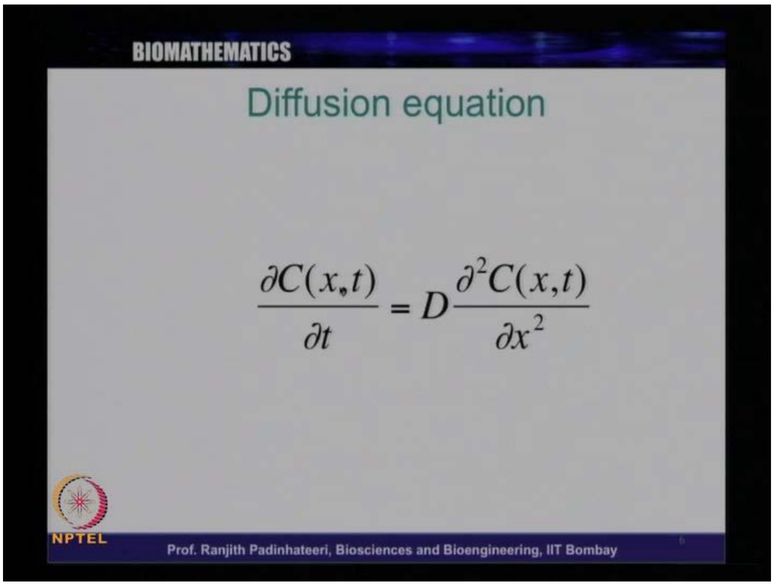

So, the diffusion equation is d C x by d C by d t is equal to D del square C by del x square, where C is a concentration as a function of position and the time and by solving this equation we expect to get concentration as a function of position for any time, that is what you will get.

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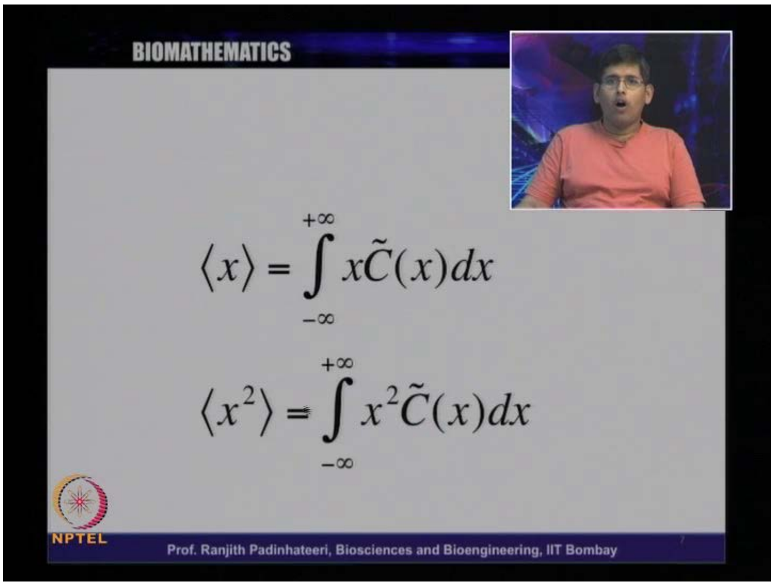

And we defined two quantities, x average and x square average as x is C tilde the average integral d x C tilde d x minus infinity to infinity, and x square C tilde minus infinity to infinity as x square average. And we also saw that x square average is 2Dt, this is a very interesting relation and important relation, the square of the position goes as time or in other words.

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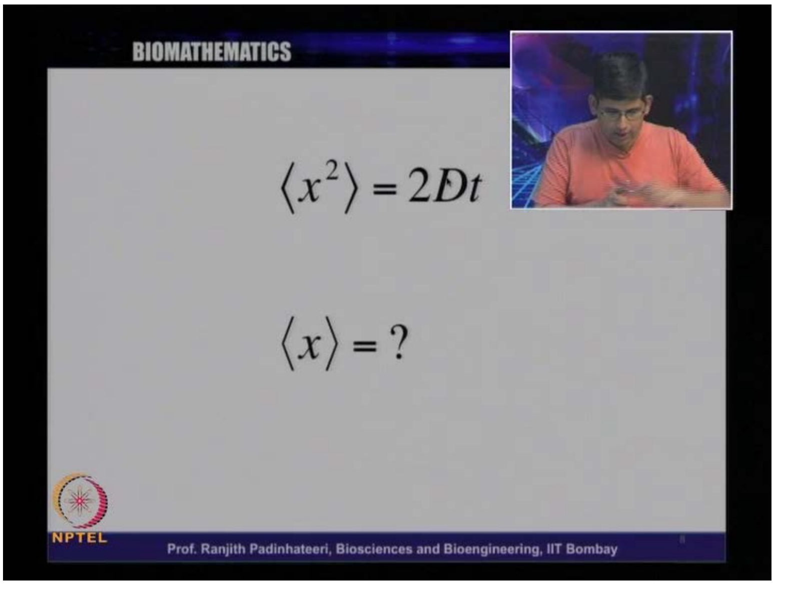

If you take square root both sides, the square root the rms distance goes as the root of time, so that is what the important relation that we saw, that is X rms which is, this is proportional to the square root of time.

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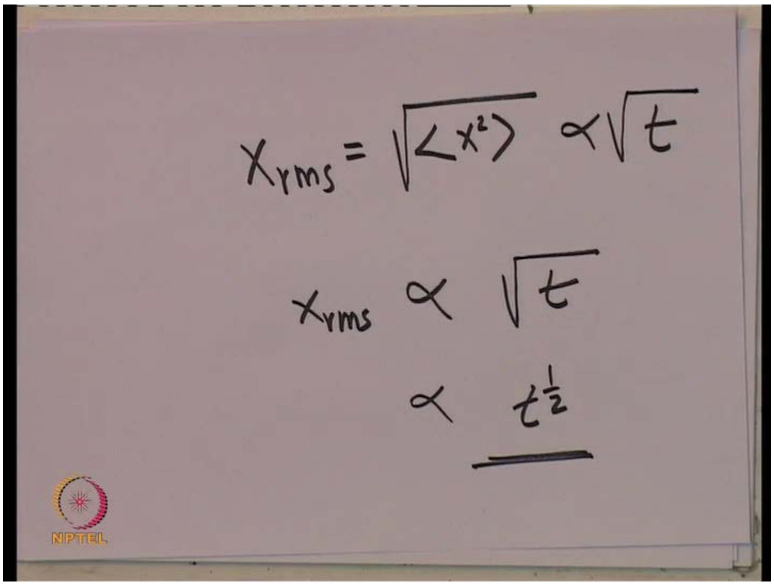

So, this is x rms is proportional to the square root of time. So, this is the interesting relation that we found and this is so the square root of time or you can write this t power half. So, this is this t power half is a kind of synonymous to diffusion on would say like it is like something moving like t power half, something moving square root of time, that is called diffusive motion. So, this is an important relation as far as diffusion is concerned, now we will calculate x average something which we want to learn, so the question is next question is what was x average? So, that is the question that we want to address next what is x average?

So, we found that x in the previous lecture that x square average is 2Dt, we discuss this relation as we said this is an important relation, which says that how does the rms distance, there is a root of X rms and how does the X rms is related to time. So, the X rms is, if you take square root on both sides the X rms as we wrote previously look at here, X rms is root of x square average, this is proportional to time. The square root of time, in other words X rms I proportional to square root of time or proportional to t power half.

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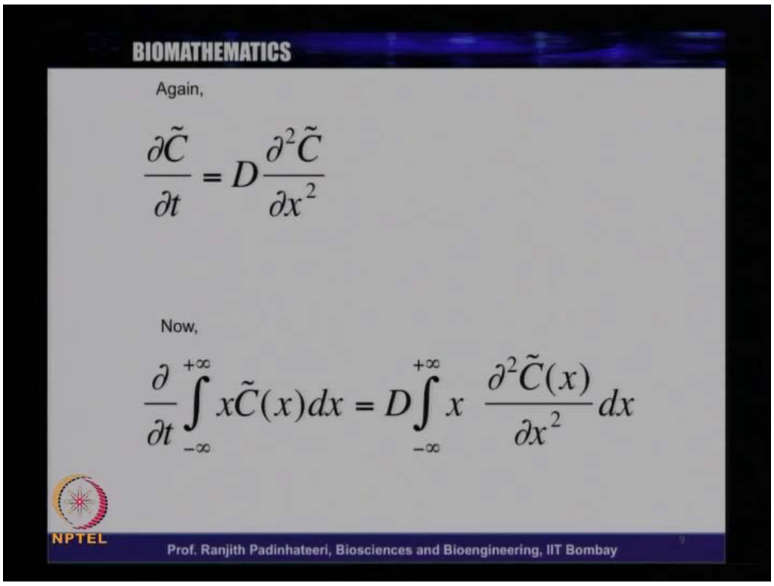

So, now, the next question is what is x average? We will calculate the x average is same way as we calculated the x square average, how did we do that? We took this equation, which is the diffusion equation, we multiplied both sides by x, we are multiplying both side with the x and integrating; so, you multiply here with x and integrating, so this becomes integral x d C d x with del by del t outside.

And here also we have the right hand side also we are multiplying with the x and integrating, so integral x del x square C by del x square d x, so with multiplied both sides of this diffusion equation with x and integrated from minus infinity to infinity and what do we get? So, the left hand side, as we defined earlier is integral x C tilde x d x is nothing but x average, so this is the left hand side.

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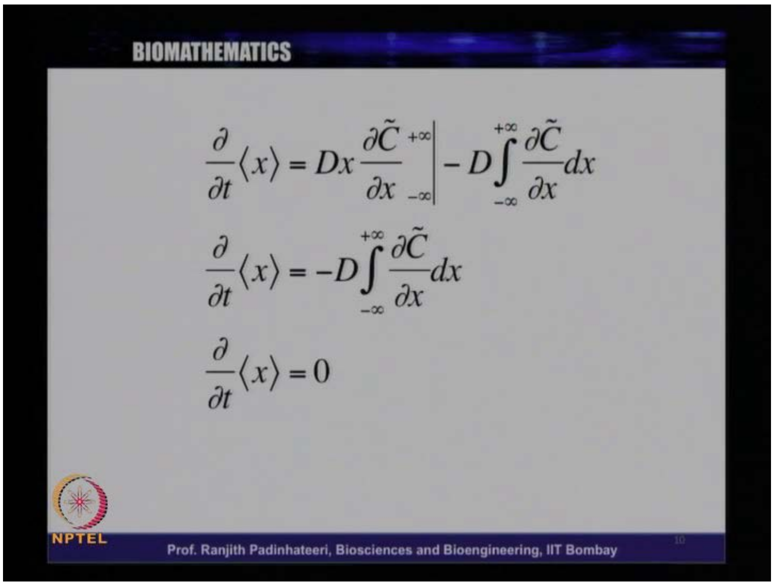

So, what do you get you get on the left hand side, so let us look at what we get.

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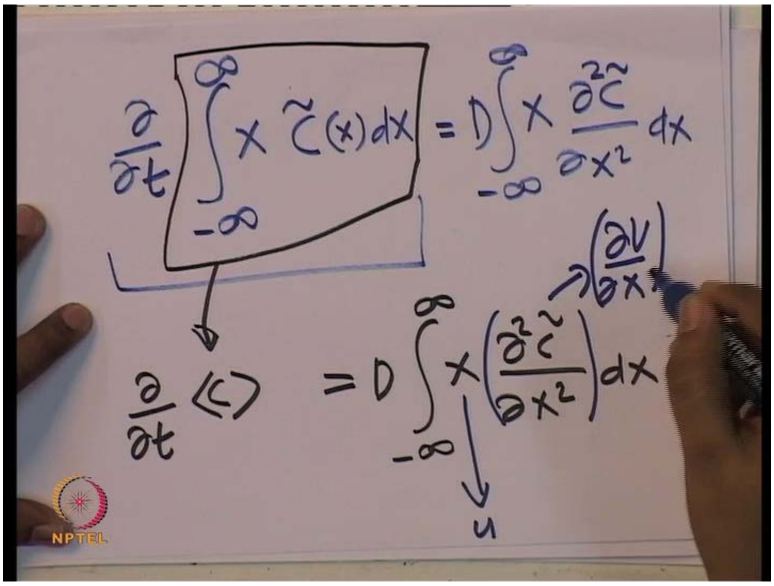

So, what we have is, del by del t of integral minus infinity to infinity x C tilde of x d x is equal to D into integral x del square C by del x square C tilde D x, now this part as we said last time is nothing but, del by del t of, so del by del t. And this is integral x C tilde x d x is nothing but, c average, so this is c average, so this one this one, this part which is I writing it in a box, this part can be written as C average. So, this is equal to D into integral minus infinity to infinity x del square C tilde a by del x square d x. Now, just like, we discussed last time this can be written as integral and you can take x as u and this term you can take as del V by del x, so d V by d x, so this is integral u d V by d x, so this is you can do this rule in calculus called integration by parts.

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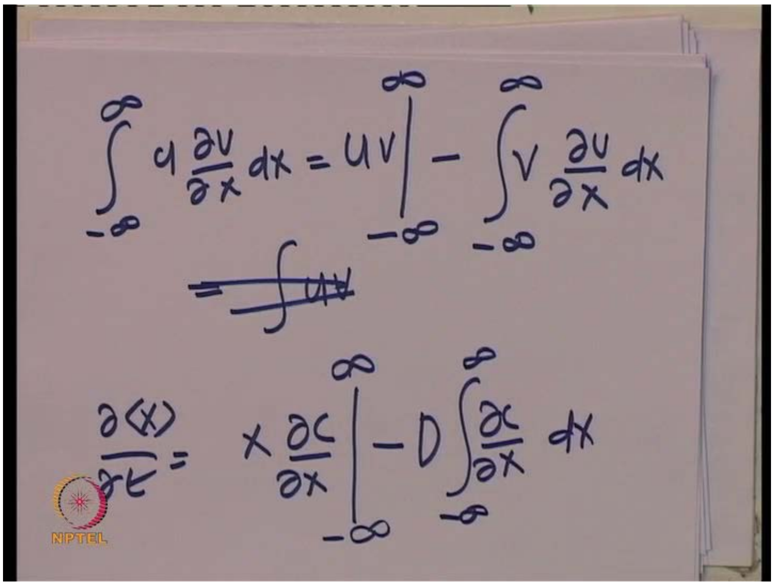

So, integral as we just discussed some time ago, integral as we just discussed in the last lecture, integral u d V by d x d x can be written as integral u v sorry it can be written as u v in the limits minus integral V del u by del x d x. So, we can use this formula, which is the standard formula in calculus, which we in the last class we figured out how this formula is coming. And let say, let us use this formula now, so what did we said is that, we just said have a look at here, so what did we just said that del C del by del t or C equal to minus this and we call this as u and this as del V by del x.

So, now, we can apply this formula there, so if you apply this formula there, if we apply this formula there what do we get, so we have x as u and del square C by del x square as del V by del x. So, what you would get essentially is this, if we apply this formula what you would get is that, del x average by del t is equal to u is x, so x and V is del C by del x in the limits minus D into integral V d u by d x.

So, now V is del C by del x d u by d x is 1, so this is what you will get, so let us this is what precisely, I have written what you would get is that D in to x d C tilde a y d x in the limits minus D into del C by del x del C tilde a by del x d x, so this is what you would get. Now, if you apply these limits, just by arguing there are plus infinity and minus infinity the derivative is, so by just arguing that this term, at you apply this del C by del x at plus infinity and minus infinity and calculate this term you will get this equal to 0.

So, then what here remaining with is just, so at this limit this is 0, so what you have is just minus D integral del C by del x d x, so we have just this term. So, now, let us see what is this term what term gives? So, let us think about that term a bit.

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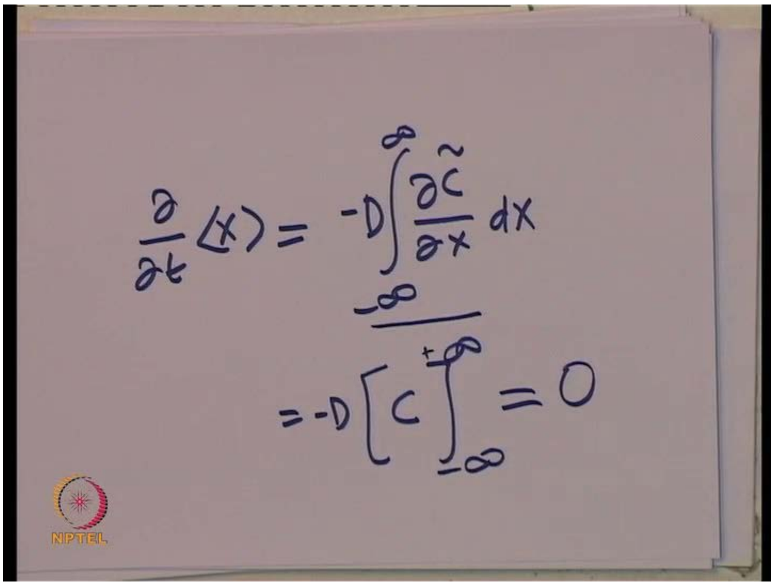

So, what end up essentially is that del by del t of x average is minus D del minus infinity to infinity del C tilde by del x d x, so what is this? So, this is derivative, so this is essentially minus D into C at the limits.

If you take minus infinity and infinity plus infinity at the limits, if you calculate the C, C at infinity and minus infinity is 0, so the answer that this integral is essentially 0, because if you do this derivative integral of a derivative is just the function itself, so you have C, C at infinity and C at minus infinity they are 0. So, essentially this derivative this integral is 0, so you will end up with this relation that del by del t of x average is 0, which means that x average is 0.

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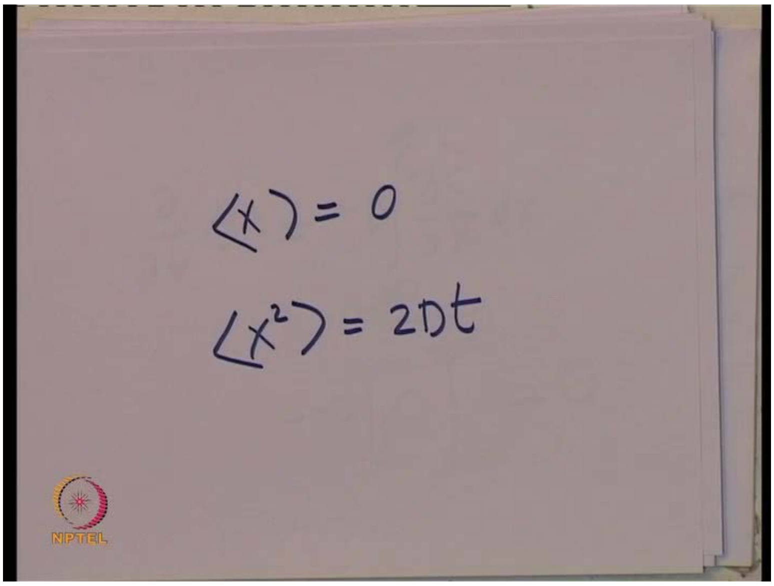

So, what we got essentially is that x average is 0, we had found that x square average is 2Dt. So, this is two interesting relations that we get that x average is 0, x square average is 2Dt. Now, what does this mean to say that x average is 0, physically what does that mean, so let us think about this. So, let us think about this diffusing in a pipe example that we thought.

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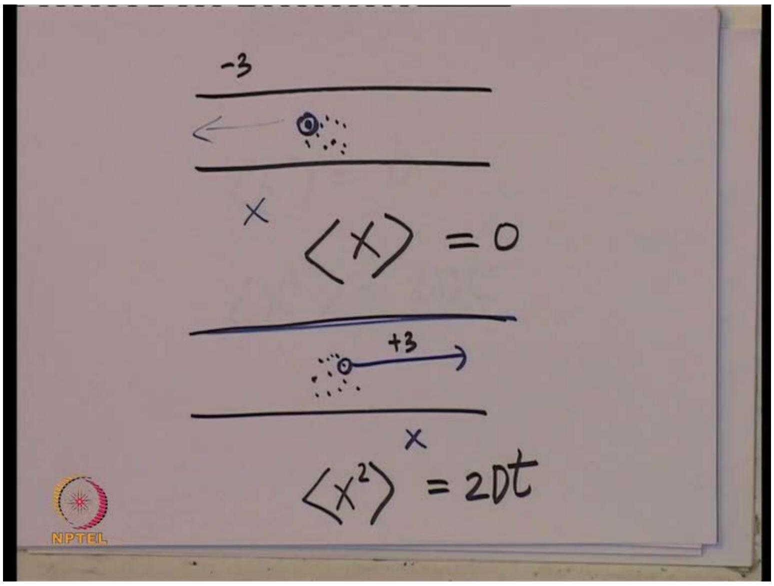

So, we let us take this pipe and to begin with you have some particles here, and let us say there is one blue particle here and one particle which I am circling here, which is in blue color, so let say there is one blue particle here and all other black particles. Now, if you look at this blue particle and in one experiment you might have see this blue particle is going this way, so it will diffuse some distance x in this way in one experiment. Let us say, you are doing the same experiment let us say, you are doing the same experiment in a different day or you repeating this experiment, now you have again you start with the same condition.

So, now you have the blue particle you put here, when you put the blue particles becomes here and this time it might move in this way, so it might move the distance x in this particular way. So, if you just keep repeating this experiment of. The experiment that we discussed, that is adding some amount of proteins at the middle of the tube and looking at where is it going, which way this is going, so at one experiment you might see that, this the basically what is this is like imagine that you have just one particle, that we can detect let say this it is fluorescing or it has some different color.

So, you will see that, in one experiment this particular particle might be going this way, in another experiment this particle might be going this way, in some other experiment it might be going this way. So, if you do many experiments and average over all this, so some time it would go in the plus direction, some time it will go in the minus direction, so minus direction plus direction finally, the average you will get 0.

So, the first experiments it might have moved minus 3 centimeter, in the next experiment it might have moved plus 3 centimeter. So, if you just repeat this experiment many times on an average, if you find the average of this, you would have, you will get 0 that is what it precisely means, it means that if you look, so if you in a in a diffusion experiment, if you look the position of one particle or many many experiments, the average over all experiments will give you the average position as 0.

On the other hand, if you calculate this square average minus 3 square is 9, 3 square is also 9, so there is no way this x square can average out to 0, so you will get some quantity which is 2Dt. So, the meaningful since, the x average is 0 the meaningful quantity is x square average, this is the meaningful quantity that we can that, we should know or that is useful, physically useful quantity is x square average in other words the rms distance.

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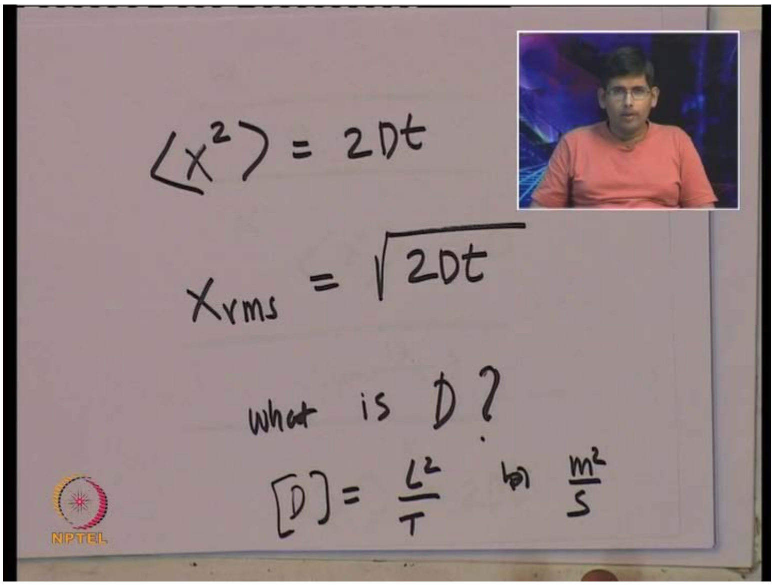

So, the important formula as far as the diffusion diffusion motion is concerned, its x square average is 2Dt or X rms is equal to root of 2Dt, so it is a square this is the important formula, this the root mean square distance that a particle would travel in a time t. If you do average over many many experiments, this is the root means square average things would travel in a time t. Now, what is this d, so that is the question what is D? We have been discussing we have been having this thing called D for a long time, so from this, we found that D as a D as a unit, if you dimension of D is length square by time length square by time, so it will have a unit meter square per second.

So, now, what is this D? D is the diffusion coefficient, what is that mean diffusion coefficient mean? Diffusion coefficient essentially that contains the property of the medium that in which you are putting this protein, if you are doing in water it contains the property of the water like viscosity of the water. It also contains the temperature, so you can imagine that, in the temperature is very large things will diffuse out very fast higher temperature, higher diffusion.

So, the information about the temperature viscosity all the property of the medium is put into this one quantity called D. So, the D contains the property of the medium, so now, how do you find out how does the D depends on the property of the medium, if the viscosity is more how does the D change, if the temperature is more how does the D change, how do we find it out.

So, just by learning, just by knowing, what you learnt in mathematics so for and with some intuition with some with some simple thinking, one can figure it out. Actually this was discovered by none other than Albert Einstein in 1905 in 1905. So, this is this is what we are going to discuss, so the relation of the D.

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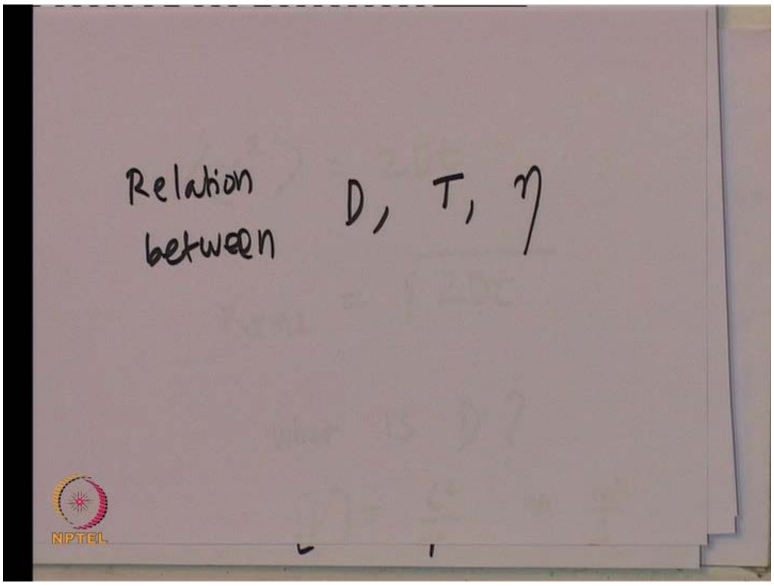

So, the relation between the relation between the relation between D temperature and viscosity let me call this eta as the viscosity, so this relation is called Einstein relation.

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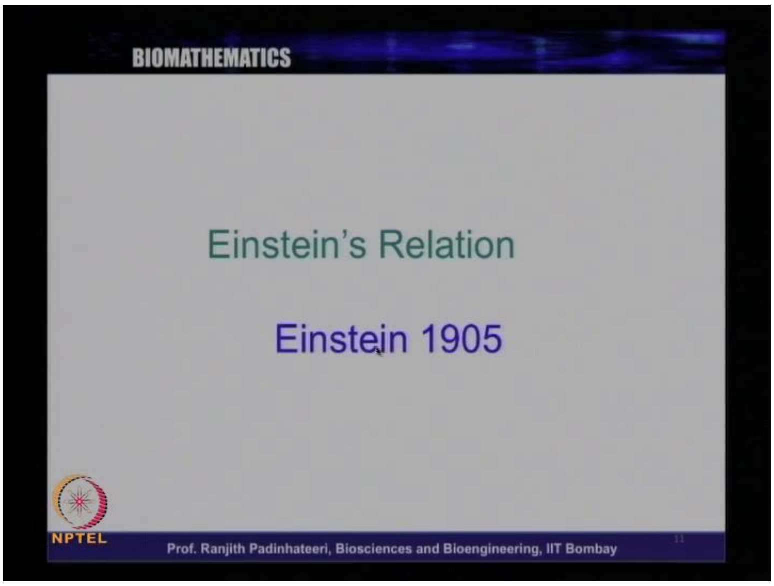

So, that is what we are going to discuss now, we will discuss this relation between diffusion coefficient and temperature and viscosity as and this relation is known as the Einstein relation, this was discovered by Albert Einstein in 1905. Albert Einstein, for his PhD, he was studying about Brownian motion of particles and he discovered this relation. I will tell you in a simple way how do we calculate, how do we derive roughly what Einstein did about 100 plus years ago, it about 100 and 106 years ago.

Einstein derived this relations, this as you might have also heard this 1905 is a very famous year for Einstein like, he wrote three very famous papers: one paper is related to this Einstein’s relation, it became very famous and this relation became one of the most popular relations like very highly slighter relations in science because, this is application on biology, in chemical engineering, in chemistry, in physics, in all sorts of fields.

Einstein‘s relation related to diffusion is used in environmental sciences in, you can think of any field, which virtually virtually any field and this relation will be or is this relation is being used. Then, he discovered the, or he explained the photo electric effect and he also explained, he also he also had his famous paper on relativity, so this three paper made him world famous like all this papers. So, one of the paper even got him noble prize, so this is miracle here as for the Einstein his concerned and the world of science is concerned.

So, we will discuss one of his contributions in that year 1905. It is interesting that, just by understanding this simple mathematics we can derive this relation. It is very similar to what we did for Nernst equation. So, we will go in the same line as we did went for Nernst to understand Nernst equation. So, let us go ahead and think about Einstein’s relation.

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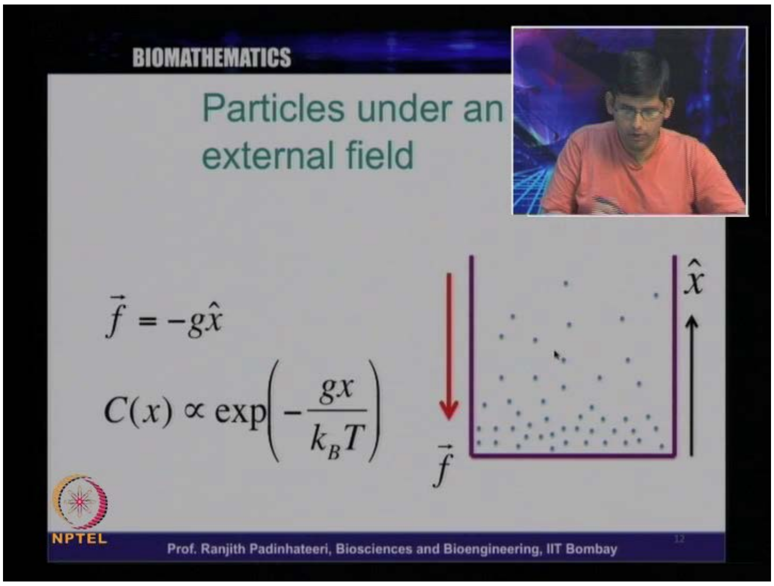

So, Einstein thought about the following example, so he thought there are some particles in water in a beaker, and this particle is subjected to some external field. So, let us say there is gravity downwards, so if there is gravity on all this particles with some mass, they will be forced to come down, because of the external force gravity for example, it could be either gravity or it could be if they are charged particle even electric field. So, you could think of this is electrophoresis, if you wish. So, basically this are you can even think of this as charged particles and then there is some force exerted on this charged particles due to electric field, this could be like some protein molecules under electric field.

Now, let this force be minus g times x mathematically, this force is minus g times x, where x is the distance from bottom to top, so x is the distance starting from the bottom to the top, so x cap has this particular direction and the force has this particular direction; so, force is acting down words and the distance is going upwards.

So, the f and x are having opposite direction, so that is why this minus sign, so g is the amount of force. So, g could be the amount of electric field, electro force due to electric field, it could be amount of force due to gravity whatever you wish, but g is some force and the magnitude of the force is g; and f is the force of the vector force, so let f is equal to minus g x. So, we can say that the energy, so for every particle if you if you want to this particle to go up a distance x it has to spend an energy f dot x, so it if you have a force f.

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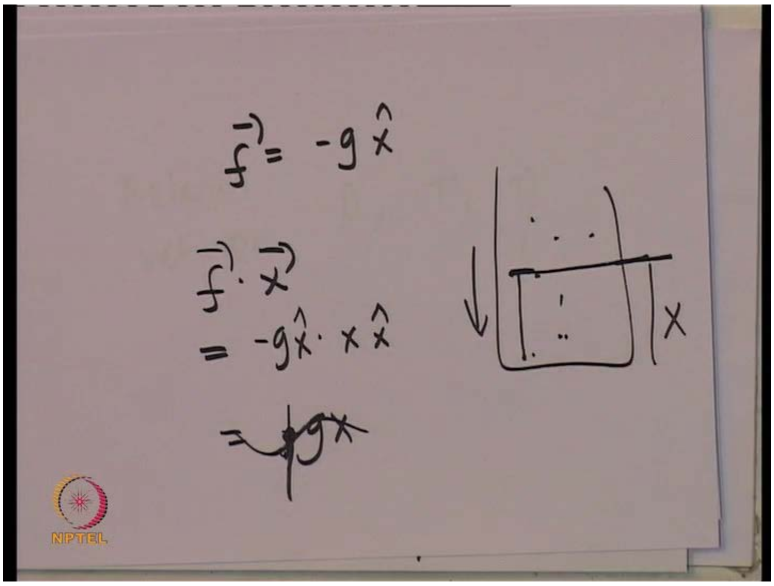

So, look at here the force f, so which is let us say, let us say it is g x cap and let us say that the energy, so if you have such particles and each of this particle is experiencing a force, so if this particle wants to go up a distance. If it wants to reach a distance x from the bottom, it has to spend an energy f with f dot x. So, which is nothing but, f is minus g x cap dot x, so this is minus g x. So, this has to spend this much energy, so it is not favorable, so the magnitude of the energy is g x essentially sorry the energy is g x.

So, it is not favorable to go up here, because the force is in this way, so it has to spend an energy, so most of the particle you will find at the bottom, because there is a force acting and there is an energy cost to go here. So, if you look at the concentration, if you think intuitively, the concentration will be more at the bottom and less at the top, you can think of any particle, if you put something in to water, you could think it of as sedimentation something will fall down to the bottom of the beaker right.

If you put some something, which is some objects onto water they will fall down, because the gravity is attracting it down, so the concentration if of anything will be more at the bottom and less at the top.

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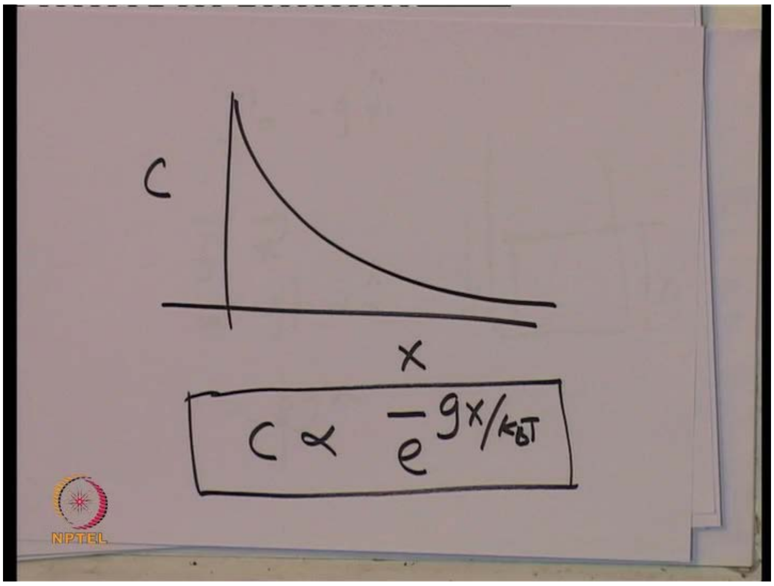

So, if you plot, if you wish if you plot, the concentration as a function of the distance from the bottom, you will see some exponential relation, there is some reason why it is exponential, but let us intuitively assume that, C is C is proportional to e power minus g x by K B T, so g x is energy and it has to be divided by another energy K B T to make it dimensionless. So, the concentration decreases as you go along x, this is what it means.

So, if you know this relation, we can derive the Einstein's relation. So, this is the one ingredient that we need to know, that the concentration will exponentially decrease as we go to the bottom, how do we get this relation? That we will discuss later, but for the moment just take this relation for granted, which is intuitively clear to you that the concentration will decrease as we go along x and knowing this we will derive Einstein's relation.

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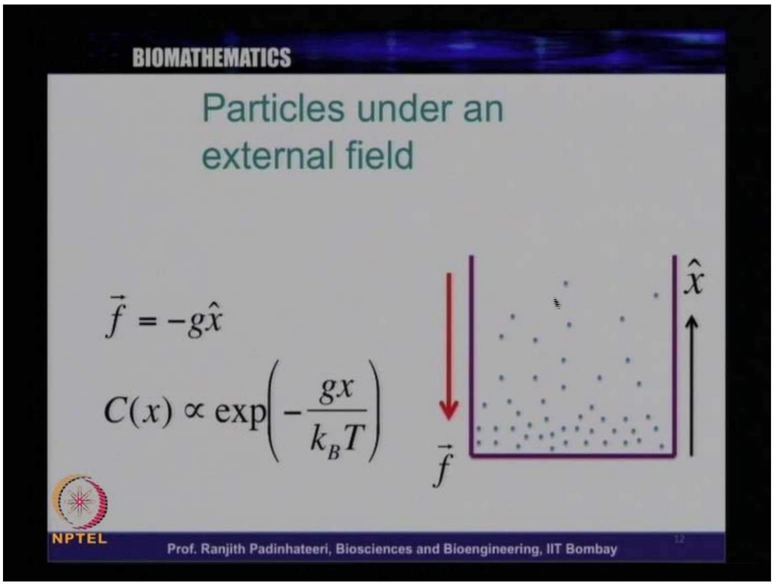

So, let us say the concentration will be more here and the concentration will be less here and this has this particular functional form.

Once we know this, we will follow roughly what we did for deriving Nernst equation, we said that in the case of Nernst equation, there is a current due to electric field or the force. Similarly, here there is a current due to this force, this gravitational force or electric force, which is pulling down these particles downwards, they will want this particle to flow down.

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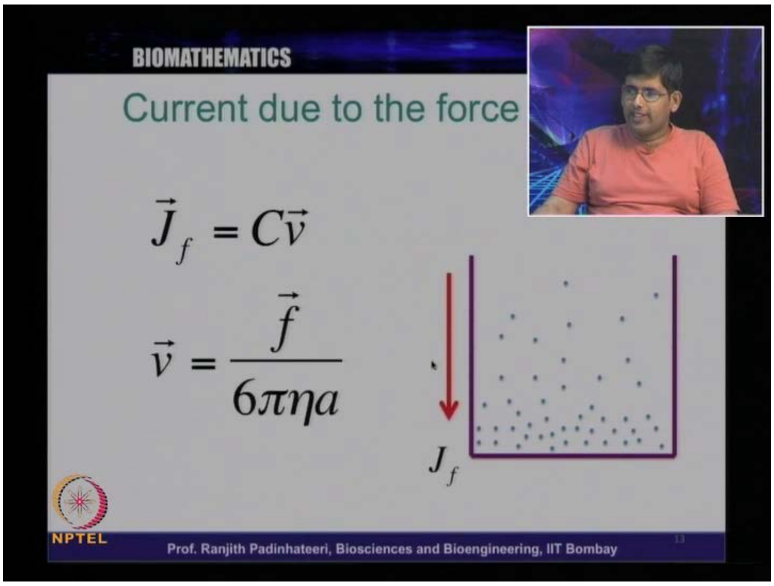

So, the flowing down happens in the velocity v and this v is related to the current or the flow j, as we discussed previously in the case of Nernst equation J f is C concentration times v, which is velocity. Now, any particle moving in water will have a velocity, which is given by f by 6 pi eta a, where f is a force acting on that particle, pi is the constant, eta is the viscosity of the water or the medium and a is the size of the particle. So, there are quantities, which you should remember eta is a viscosity, a is the size of the particle and f is the force acting on this particle; if you know this much the velocity is this and the flow is proportional to the velocity the more the velocity the more the flow is.

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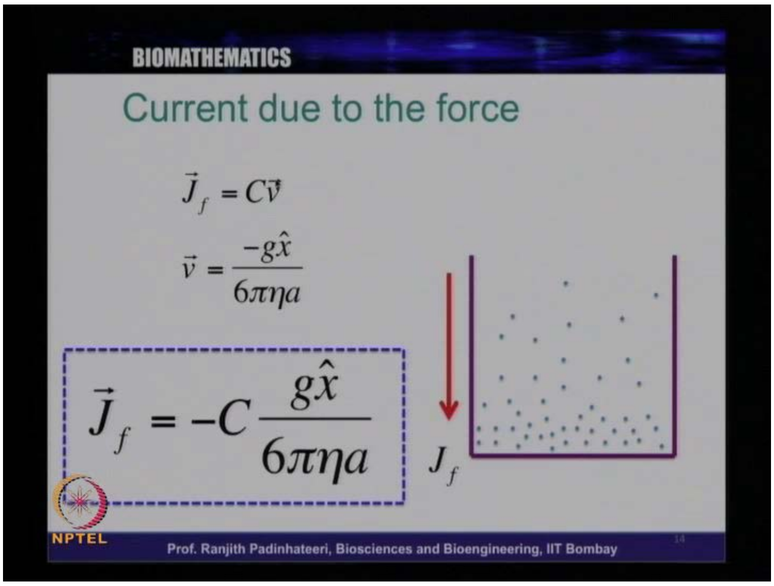

So, the flow downwards is C times v. So, this can be written in a different way the C times v and C v is forces can be written as minus g x cap, where x cap is this direction, so minus g x cap, so substituting this f is minus g x cap, you get J f that flow has minus C g x cap by 6 pi eta a. So, this is the current that is making, so this is the flow due to this attraction, this this force that it could be gravitational attraction downwards, it could be the attraction due to electro static forces or electric field could be the electric field down applying downwards, forcing the proteins to move in this particular direction.

So, this could be any force whatever be the force you wish, but that force will flow will lead to a current or a flow given by this particular formula, as we saw in Nernst equation we had similar similar flow. Now, this flow leads to one interesting thing, that the it makes the concentration more here and the concentration less here. If the concentration is more here and the concentration is less here.

Diffusion can happen because, diffusion is a flow from higher concentration to lower concentration, so in principle things can diffuse back from here to here, it can diffuse back from lower concentration to a higher concentration. So, here it is sorry you can diffuse back from higher concentration to a lower concentration, so here it is higher concentration here it is lower concentration, so from here to here you can think of you can imagine that, there can be some flow due to diffusion or flow due to concentration change, you could think of some kind of diffusional or diffusive flow. So, how much is that flow.

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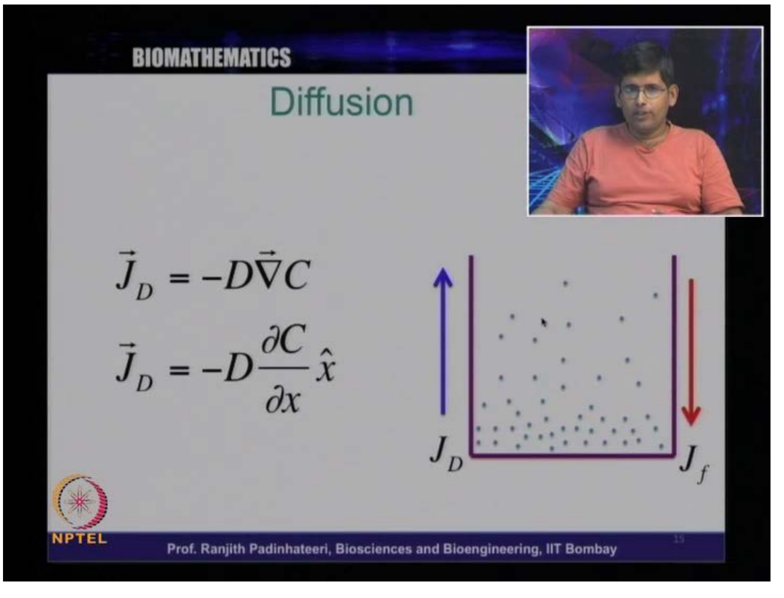

So, we said that due to diffusion there can be a current or a flow and that current J D is related proportional to derivative of the concentration, as we saw previously J D is proportional to del C by del x. And as we go along the x C decreases, so the del C by del x is negative.

So, with this minus sign the flow is actually along the x cap direction, which is in this direction shown by this blue arrow. So, we have a diffusion, which is basically taking this in this particular way, we have a flow which is in this particular way given by J D equal to minus D del C by del x.

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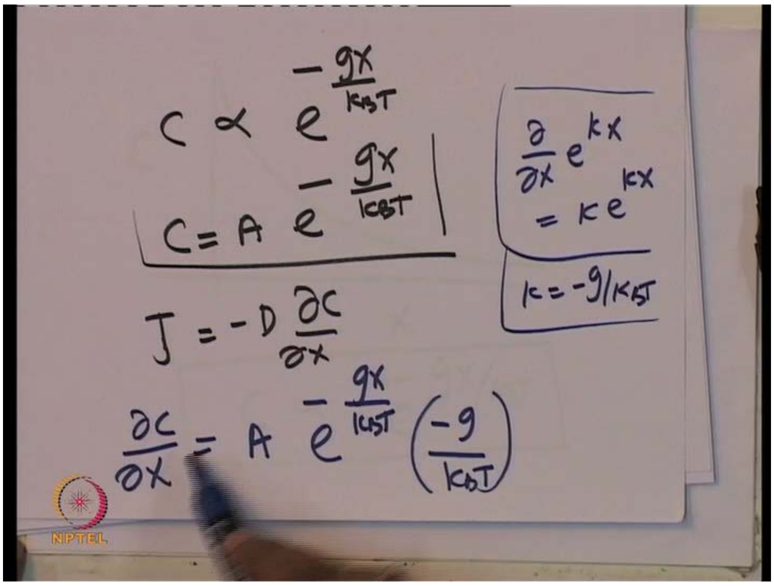

Now, what is C? We just saw that C is proportional to e power minus g, so we just said that C is proportional to e power minus g x by K B T, so that means, C is some constant A, it would be some constant times e power minus g x by K B T, now we also said that J is minus D del C by del x.

Now what is del C by del x? del C by del x of this will be, so let us find the derivative of this. So, what is the derivative of this? So, del C by del x will be A is there, derivative of e is e power minus g itself this itself, times the derivative of this, which is minus g by K B T. So, we said that derivative of e power K X is K e power K x, so we had a K, which is minus g by K B T. So, that is the K, which is coming here, so by using this relation that we learnt that derivative of e power a x is K e power K x, where our K here in our K was minus g by K B T. We have del t by del x is a e power minus g x by K B T into minus g by K B T.

Now look at here, what is this A e power minus g x by K B T, what is this part? This part is C itself, look at here this is C is a e power minus g t x, so del C A C is C e power minus g e x by K B T, so this is C itself.

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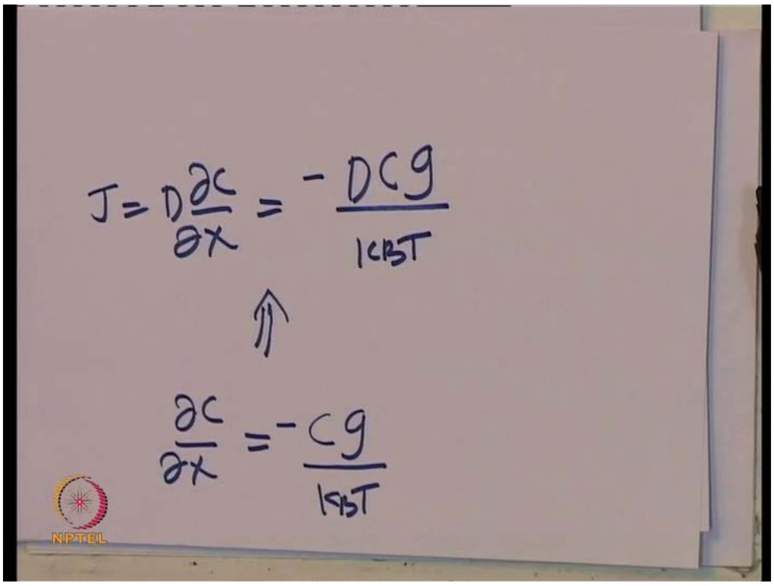

So, del C by del X del C by del X is nothing but, C itself times minus g by KBT minus c g by KBT, so what does this mean? This implies that, we had J which is D del C by del X is minus D times C times g by KBT.

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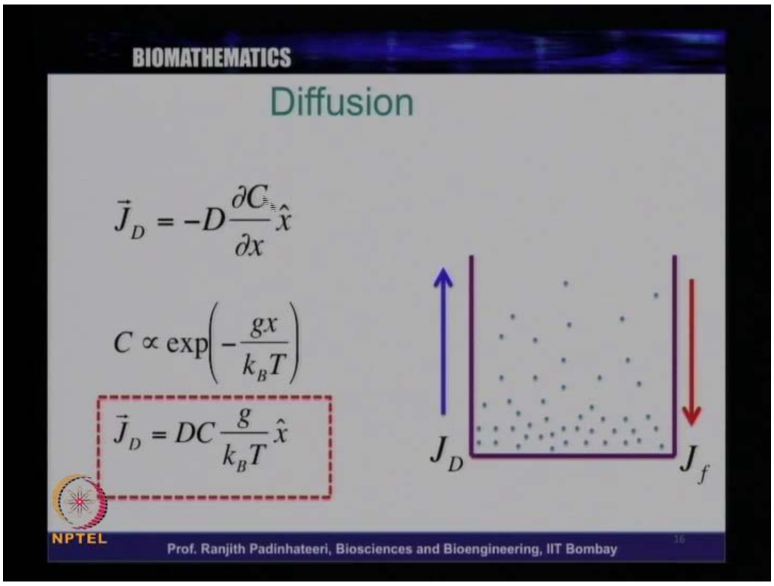

J is minus DCg by K B T, so that is what we have here, so J D is g is del C by del X and substituting for D, know the C is proportional to e power minus g x by KBT and substituting this, In this, we get J D is D C, there is a plus sign here I might have mistaken there is a type of this, so if you just substitute this minus, there is a minus sign here by taking this minus sign into A account we will get a plus sign here.

So, essentially you get this, what is shown in this square, in this rectangle here, what is marked here, the current upwards is DCg by KBT along the J f downwards and J D upwards. So, we had current downwards and the current upwards.

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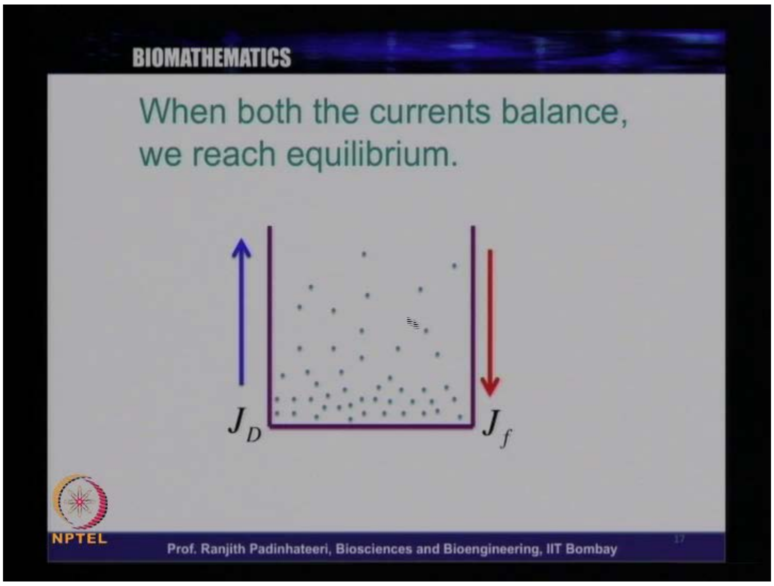

So, when you have currents in opposing direction, we said that when both this currents balance, that the current upwards and the current downwards when they are, when they balance, we reach equilibrium we call it equilibrium. For example, if you look at this particular point, there will be some current upward, there will be some current downwards and when this currents balance.

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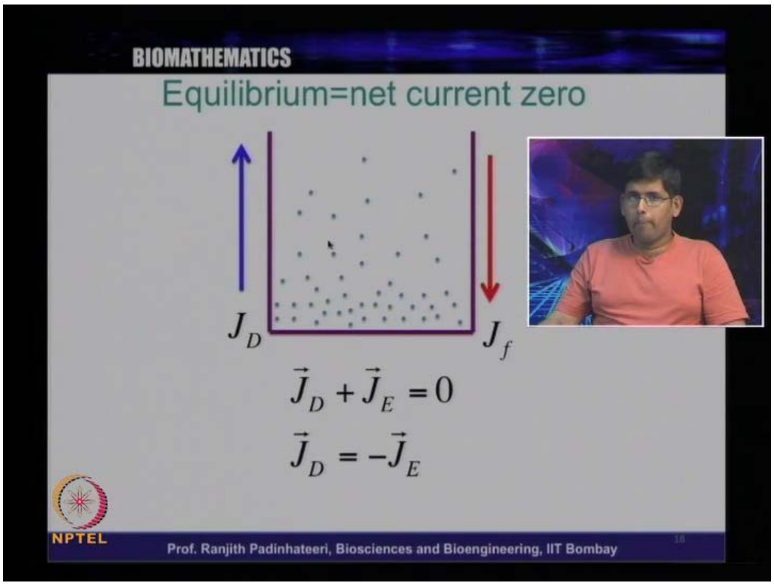

We have net current zero and we reach equilibrium. This is exactly the argument that, we discussed in Nernst, in the case of Nernst equation. So, what does that mean? Equilibrium means net current zero, net current is nil, what does it mean? The total current J D plus J E is 0, in other words J D is equal to minus J E, there is a what I meant here is J f, J this is typo here should be J f here.

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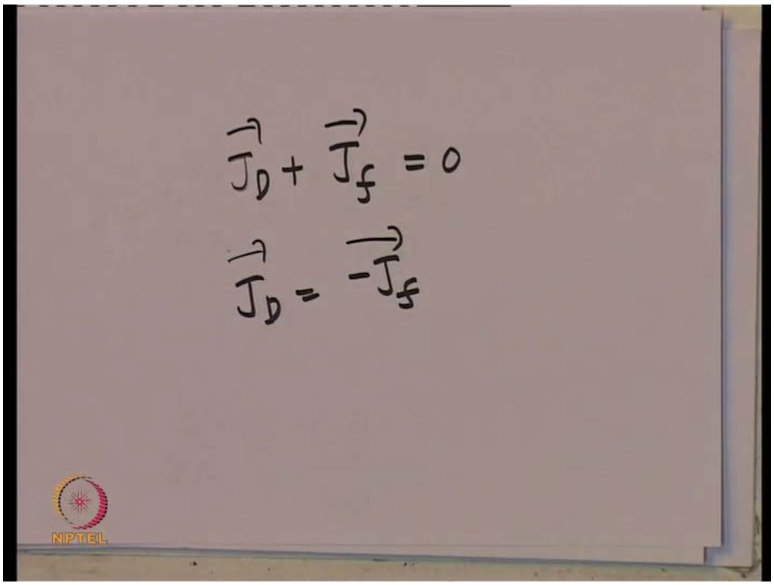

So, what I meant here is that, J D plus J f is 0, in other words J D is equal to minus J f. So, the current due to the force is equal and opposite is equal with the opposite sign, that of currents due to the diffusion, so this is what, this is this is the condition for equilibrium.

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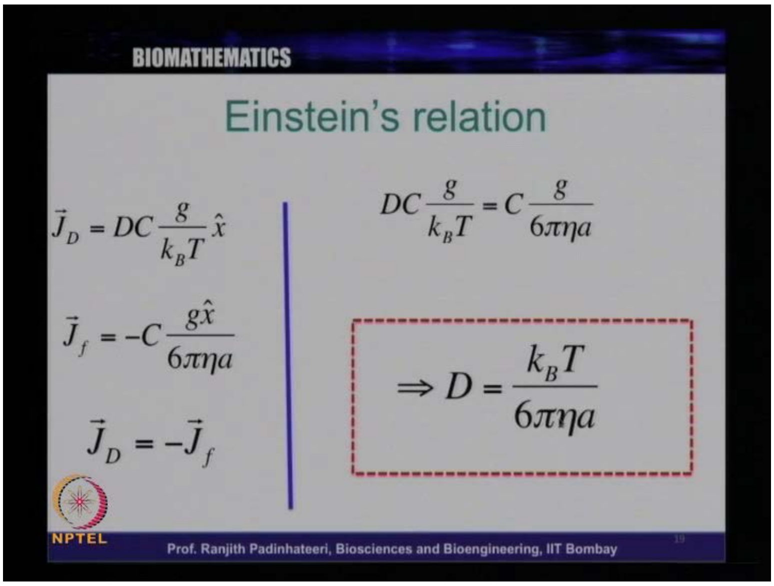

So, what do we how do we, let us try to do with this, so what we have is J D as DCg by KBT and J f as minus g x by 6 pi eta a and we want J D is equal to minus J f. And that is that means, DCg by K B T is equal to Cg by 6 pi eta a, so which implies I can take everything D alone keep this side and take everything else to the other side; and what you would get is that, D is equal to K B T by 6 pi eta a.

So, this is says that diffusion coefficient is equal to KBT by 6 pi eta by a, so this is the famous Einstein's relation that this relates the diffusion coefficient to Boltzmann constant, temperature viscosity and the size of the particle what does it say? The more the temperature the more the diffusion coefficient, the more the viscosity this is inversely proportional, so if viscosity is very large the diffusion will be less, which is intuitively clear, something might diffuse better at water and much less in honey, where honey has higher viscosity to compare water. So, the viscosity of something, which is highly viscous let say take example of tar or honey, you will it will be very difficult for things to diffuse in tar or a very highly viscous medium.

And if this either the particle is very large look, if you look at the a is the size of the particle, if you have proteins that are very huge, a very large they will not diffuse the diffusion coefficient of those objects will also be very small. So, this is the famous relation called Einstein's relation, which relates the diffusion coefficient to this quantities and the relation that we derive this relation by the way we derive Nernst equation by arguing.

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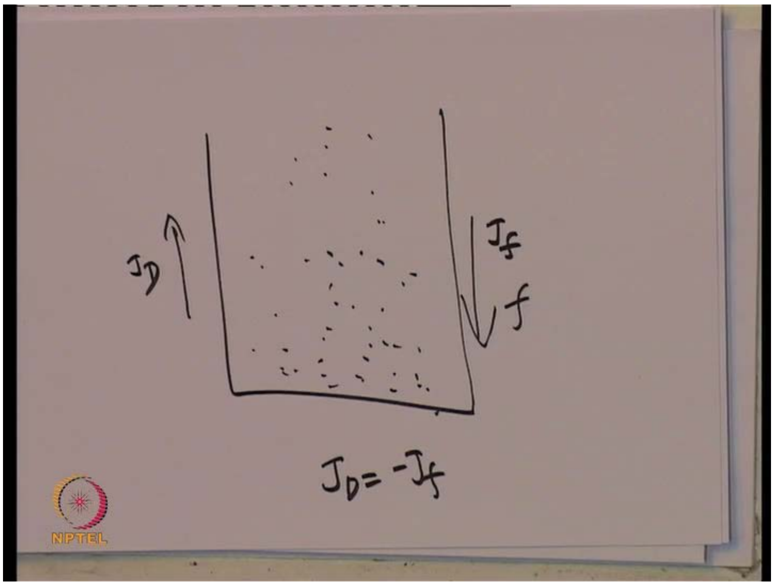

That, if you have an object if you have a beaker and particles under a force f, there will be two currents the concentration will be more here, because of this force is attracting in it downwards and very few particles up. So, since the concentration is less here in concentration is more here, there will be diffusion in this way, diffusive current and the current due to the force and when there are equal and opposite.

We get by arguing that they equal and opposite and equating them, we get this relation, which is D is equal to K B T by 6 pi eta a.

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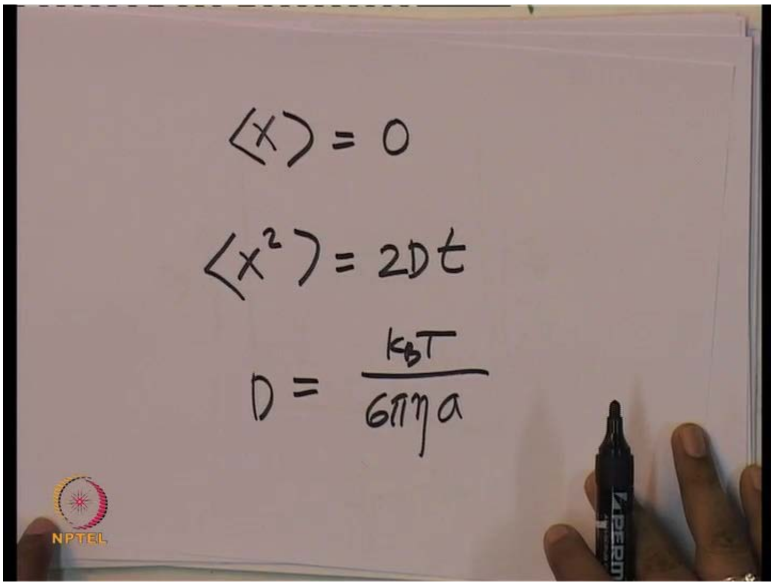

So, now, we learn two three things, we learn that x average is 0 in today's lecture. In the previous lecture, that we learn that x x square average is 2D t and we also learnt that D is K B T by 6 pi eta a. This is true for a spherical particles typically, if this is not a spherical particles, this 6 pi eta a could be something else, but however, let us let us strict limit ourselves to a case of spherical particles is proteins could be the thought of us spherical particles, so this is another relation, that we learn today.

And this are the three important relations as far as diffusion is concerned and using simple arguments from calculus and vector vectors, we could derive this relations and this has high, very high significance as far as as far as diffusion is concerned. Now so, one more interesting thing I just want to share with you is that, so in the beginning we graphically represented the diffusional profile, the profile of the concentration in this particular manner.

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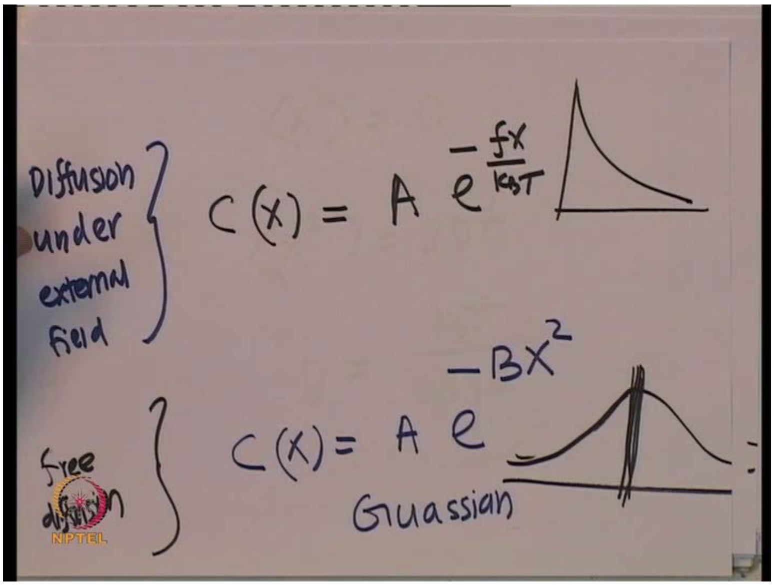

Now, what is this mathematical function that can represent this concentration? So, it turns out, that the mathematical function C of x is in the case of q at diffusion, that diffusion with no force this is not the case, so in the case of diffusion with force we found that, this is e power minus f x by K B T. So, this is diffusion under external field, but when there is no external field, that is diffusion under no external field or free diffusion C of x will be having some form which is A e power minus some B x square, so this function is called Gaussian function, this has a bell shape curve this is the this is the bell this is the bell shaped curve, if you plot this, so this is called a Gaussian function.

So, this is diffusion, this is free diffusion, there is no external, this is free diffusion and the free diffusion is, now if you plot this if plot this we saw that this is this particular form and if you plot this it will have some form which is symmetric like this, I am not properly drawing just go and see how the Gaussian function plot this function if you wish like putting some values A and B, plot it yourself and see how does it look like? This looks like a bell shape curve symmetric both sides to the x axis, nicely symmetric around the peak.

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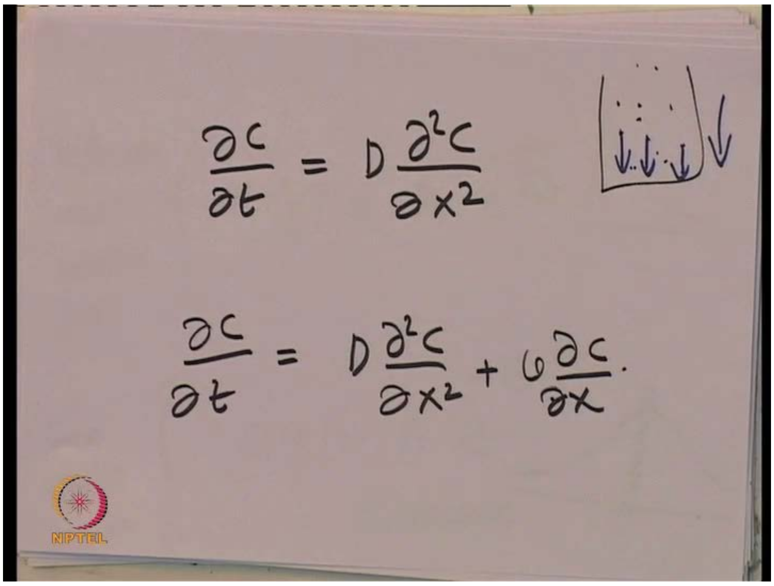

Now, the free diffusion is governed by this equation del C by del t is equal to D del square C by del x square, it turns out that the diffusion under some external force is govern by the following equation, so we have this equation plus, some effect due to the external force, so let us say the particle when they have an external force. So, let us if you have just no force, this will be the equation, but let us say each of this particle is experiencing some force, downwards or an particular direction.

Then each of this particles will get some velocity due to this external force, then the equation will be this (Refer Slide Time: 47:31). So, this is the diffusional equation under an external force and this is free diffusion, where there is no external force. So, this two are two equations and this equation this equation has a solution, which as of the form this and this equation has a solution of the form this.

So, we will we will see how do we get to the solution of this later, if we may see this how do we get this relation, but for this at this movement we would not go to solve this equation, we will just understand the there are such differential equations, when it comes to probability etcetera we might revisit this equations in a different context.

But at this movement, it is it safest to say that, this are very interesting equations in biology and there are two important relations, that as per as this equations from this equations, from this mathematics essentially you get two important, three important relations which you should all remember, that is the average distance that the particles moves. If you just mark a particle and ask the question, if you do many many experiments by marking a particle and ask the question on average where it will go, some time it will go to the plus 1 minus x, some other time it will go to plus x, so on an average it will not go anywhere and x average will be 0, that is what this means.

But, the x square average is 2Dt and the diffusion coefficient is K B T by 6 pi eta a, so by knowing this relations with this relations we will summarize todays lecture.

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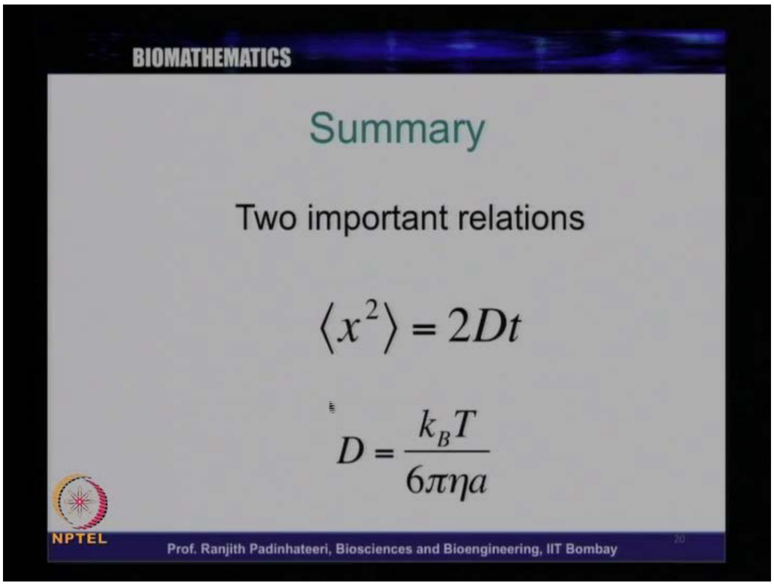

So, the important relations are x square average is equal to 2Dt and D is equal to K B T by 6 pi eta a, you will come across this relations many many times in your life at with this will stop today's lecture, the some remember this relations and with this we will kind of completing the section on diffusion. We will go ahead on learn new things in the coming lectures with this we are stopping today's lecture. Thank you.