

Biomathematics
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Lecture No. # 21
Statistics.

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Hello, welcome to this lecture on biomathematics. We have been discussing different things like calculus, vectors and all that that are relevant to various biological problems to understand various physical phenomena. That might be relevant to biological systems or that are relevant to biological systems.

Today we will go different section or different area called statistics as you all may you all might be aware statistics has a very important role in biology. So, that in the coming few lecture in the few lectures that in the next few lectures we will try and understand how I can use what is statistics and how one can use statistics in biology and why do we need statistics and those kind of things we will try and understand.

So, today's lecture is statistics that is the title of today's lecture and we let us think about asking why as typically we do always let us think about asking about the question why do we need statics.

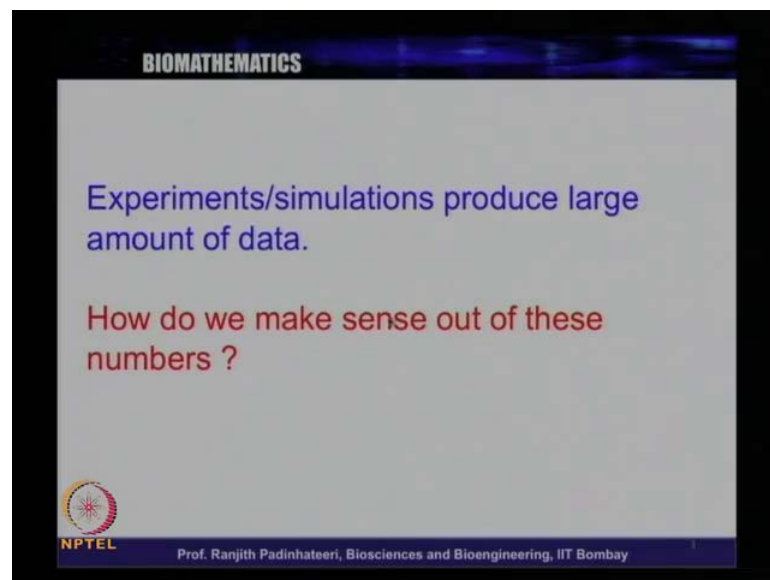
Why should we learn all this and all. So, one of the there are many reason we will come and as the end of this use many sections you will be clear why we need to learn statics but. Biology is a physical science and the understanding of systems biological systems are mainly obtained through experiments. So, we do many experiments to understand in the biological systems and when we do typically experiments we get lot of numbers lot of data. So, which are like a set of numbers and how do we make some sense out of this numbers like just be a number wouldn't mean anything.

So, one way of. So, some this idea of to understand some ideas from statistics absolutely necessary to make sense some sense out of the data that you are getting and even to understand like whether the data is correct or like how should we do the experiment. So,

that we get something meaningful it is to plan even plan the experiments properly, one need to understand statistics in principle.

So, that is the aim like we will see what kind of experiments one should plan. So, that the data that we get out of this experiments are have some significance statistical significant or it has some meaning. So, even before doing an experiment, one should think about how to plan this experiment keeping statistics in mind. So, let us what we use statistics in our daily life.

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So, basically what said we saying is that experimental simulations experiments or simulations or and simulations produce large amount of data and how do we make sense out of this numbers. So, that we get large numbers. But before going to experiment even in real life, we often use statistic knowingly or unknowingly.

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BIOMATHEMATICS

We often say:

"It takes 30 minutes to travel to my college, from my home."

What do we mean here? (and what we do not mean?!)

We do not mean that, each and every day, we take precisely 30 minutes.

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So, let us take some simple statements that we make in our daily life and let us think about what we mean by that. So, let us see like one statement we make is this. So, we often say for example, that it takes 30 minutes to travel to my college from my home.

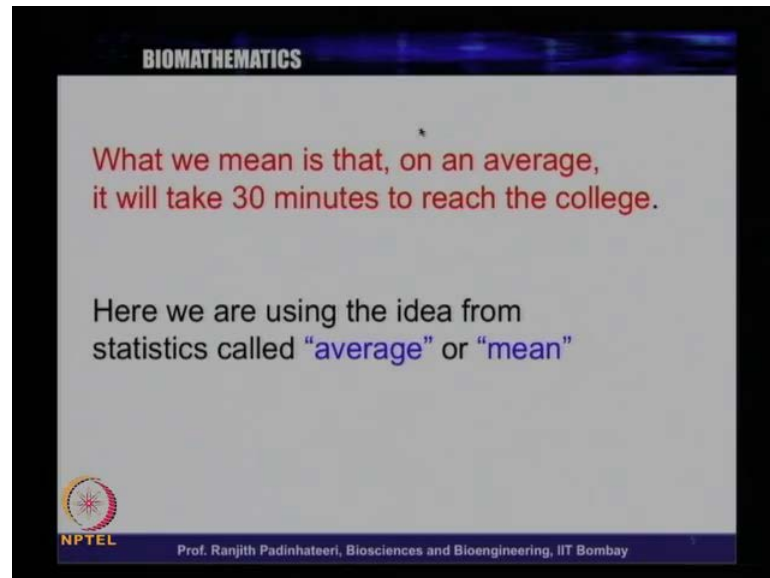
So, that like all of you might have said told to somebody that it takes like this many minutes it takes 30 minutes to reach my college or it takes 30 minutes to go from here to there it takes 30 minutes to go to the nearest movie theater. So, you say such kinds of statements you always make in day today life and what exactly we mean here, let us think about it when we say it takes 30 minutes to reach my college.

What exactly do we mean here and what we do not mean. So, look at here what do we mean and what do we do not mean and that each and every day we take precisely 30 minutes this is not true. In some days you might take like 40 minutes like let's say, one day the traffic is very heavy traffic like and the buses are going very slow.

Then you might take more than the 30 minutes some days roads could be empty and you might reach there in like 20 minutes some days you might take 35 minutes some days you might take 25 minutes like even if you walking let's say you are walking not you are not going by bus. So, the traffic do not influence you let's say you are walking some day you might have some physical you might not be that fit to walk very fast. So, we will walk slowly. So, you will take like 40 minutes some day you might be very your feeling good and you walk pretty fast and then you will be reaching in 20 minutes.

So, depending on various conditions it can fluctuate, around 30 minutes and when we say it takes 30 minute, what we typically means is that on average it will take 30 minutes that is what we are meaning, we on an average it takes 30 minutes.

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So, that is what its written here, when we say that it takes 30 minutes what we are saying is that it will take 30 minutes to reach the college on an average. On an average it will take 30 minutes to reach.

So, when we say such things we are using the idea from statistics called average or mean knowingly or unknowingly use this idea called statistics and you without even if a lay man who has not learnt any statistics has intuitional feeling about average or even lay man when you he would say or she would say, that it takes 30 minutes to reach from here to there.

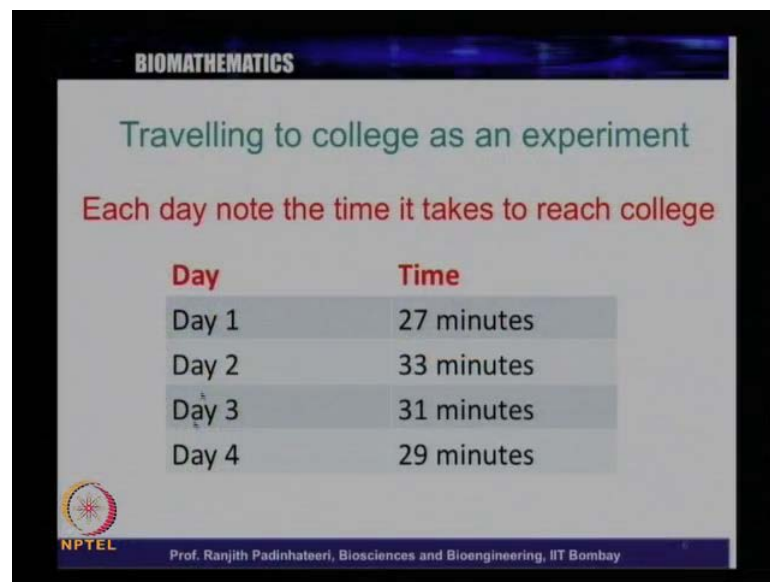
When the person says this she does not clearly mean it precisely in the 30 minutes she surely means that on an average it will take 30 minutes. So, this average is like in our day to day activities when day to day life this idea of average is actually built in intuitional it is we have this intuitional feeling that what an average means.

But, when we are in science like when we do science we have to be more precise especially when we do mathematics when we more view when we write things very precise manner is this is science is like the better the more precise you say it is more

reproducible and more scientific in some sense it is. So, we like things to be stated in very precise manner. So, when we say average we have to tell what exactly this average is. So, let us take an example. So, let us imagine that you are act of going to college from home to college think of it has an experiment.

So, travelling to college is an experiment. So, you start at a particular time you look your watch when you start and when you reach the college you will look the watch and note down the time. Let us say every day you note down the time the time it takes to reach the college and also you have the table day one it took 35 minutes day two it took like 25 minutes day 3 it took some other minutes.

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The slide displays a table with the following data:

Day	Time
Day 1	27 minutes
Day 2	33 minutes
Day 3	31 minutes
Day 4	29 minutes

The slide also includes the NPTEL logo and the text 'Prof. Ranjith Padinhateeri, Biosciences and Bioengineering, IIT Bombay' at the bottom.

So, you can essentially we will get a table. So, have a look at here this is some typical example of a table if you wish.

So, we are thinking travelling to college that is an experiment and each day we are noting down the time takes to reach the college and day 1 let us say it took 27 minutes day 2 it took 33 minutes day 3 it took 31 minutes day 4 it took 29 minutes. So, none of this days if you look we have not taken precisely 30 minutes like somewhere around 30 some days 27 33 31 29.

Still when we ask we will say it is like 30 minutes. So, what do we mean by that. So, that is the definition of the average the average is defined in the following way. So, the average or mean it is also called mean.

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BIOMATHEMATICS

Average/mean time to reach college

Day	Time (min)
Day 1	27
Day 2	33
Day 3	31
Day 4	29

$$\langle t \rangle = \frac{27 + 33 + 31 + 29}{4} = 30$$

Average over many experiments!

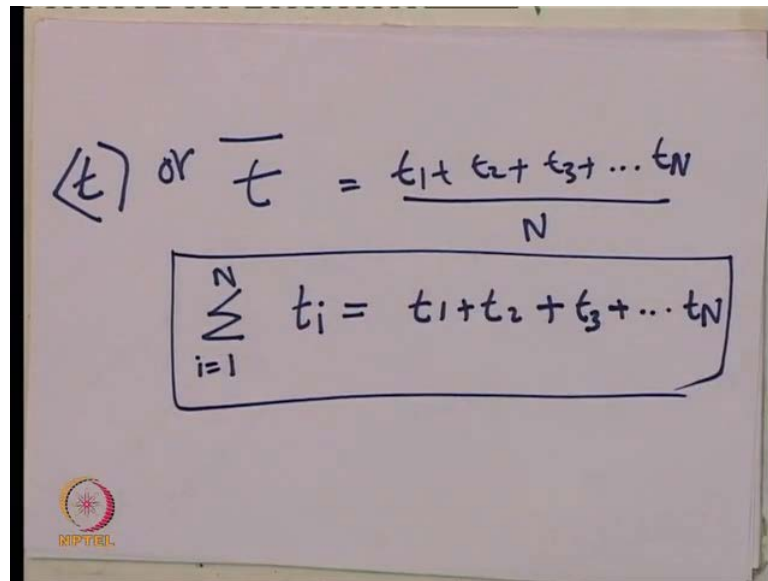
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So, the mean time to reach the college. So, the mean time it takes to reach college can be calculated in the following way. So, let us look at here. So, the mean time.

So, here we have here . So, the mean time to reach the college is 27 plus 33 plus 31 plus 29 divided by four is on an average we take 30 minutes that is what this average the definition of the average is sum over all this and divided by the number of given. So, what number of experiments? So, we have like four experiments here day one experiment day two another experiments day three another experiment day four another experiment that you can add the data that you get all the experiments and divide by this. So, you get the average.

So, this is minutes this is minutes this is minutes this is minutes. So, and this is the number this is the number of days number of experiments. So, this is the answer here is actually in minutes. So, this will be minutes. So, we can say that we did found. So, this angular bracket around t means that average. So, I will typically denote average by this angular bracket some people denote this by bar.

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$$\langle t \rangle \text{ or } \bar{t} = \frac{t_1 + t_2 + t_3 + \dots + t_N}{N}$$
$$\sum_{i=1}^N t_i = t_1 + t_2 + t_3 + \dots + t_N$$

So, some people denote t by average by \bar{t} . So, like \bar{t} is t_1 plus t_2 plus t_3 plus... So, you can take many n experiments and divided by n . So, some people write this is \bar{t} . So, both mean the same t or \bar{t} angular bracket or \bar{t} this is some typical notations for t average time and this is the answer.

So, we can generalize this as t_1 plus t_2 plus t_3 plus dot; that means, t_4 t_5 t_6 up to n if you do n experiment. So, the verses four in the previous case. So, it was just t_1 plus t_2 plus t_3 plus t_4 and divided by n .

This can be written as a in mathematically as using this symbol called sum sigma. So, this is capital sigma. So, sigma i equal to one to n means you do this sum t_i i will vary from one i will vary from one to n . So, somebody tells if you the when you say sum over i is equal to 1 to n t_i this means the first i is equal to 1. So, t_1 then i is equal to 2 t_2 . So, sigma means sum. So, this is sum then sum by putting i is equal to three then you have to just sum till i is equal to n .

So, the sum over i t_i sigma i t_i means this. So, when somebody says when you see this symbol sigma this what it precisely means. So, mathematically this notation sum over i this sigma notation capital sigma is typically used to represent summation.

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BIOMATHEMATICS

Average/mean

$$\langle t \rangle = \frac{t_1 + t_2 + t_3 + \dots + t_N}{N}$$
$$\langle t \rangle = \frac{1}{N} \sum_{i=1}^N t_i$$

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So, the average t average is nothing but sum over I t i one over n. So, this is what it is now let us take another example.

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BIOMATHEMATICS

Average mark

What do we mean when we say :

I have got 70% mark for my class 12 exam

We mean, average of all subjects is 70 out of 100

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Let us you might have also said for example, my mark is 70 percentage. So, let us take this example let us say we often say that I have got 70 percentage of mark for my class 12 exam. So, this is something that we often say we often say that I have got 70 percentage of mark for my class 12 exam what do we mean we mean that average of all

subjects is 70 out of 100. So, if I find the average of all subjects I will get 70 and total mark is 100. So, 70 by 100 is 70 percent.

So, let us try and understand this a bit more such a situation you might have you might have said it. So, let us take one case for this.

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BIOMATHEMATICS

Average mark

Case 2

Biology: 95
Physics: 40
Chemistry: 75

Average=(95+40+75)/3=70

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So, let us case this particular case. So, in this case I am discussing three subjects biology physics chemistry and biology has 75 marks physics has 65 marks chemistry is 70 marks. So, you got 75 for biology 65 for physics and 30 for chemistry and the average is 70. So, even in this case you will say that ok I got average marks 70 for by class 12 final exam.

You are correct, but your friend will come and tell you I also got the average 70 percent marks what are the marks he got he or she got like your friend got 95 for biology 40 for physics 75 for chemistry and the average of this 95 plus 40 plus 75 divided by 3 is 70. So, your friend also got average 70 percent marks you also got 70 percent mark.

So, what is the difference is there a difference are they the same? So, we found this statistical quantity called the average and we got the same thing is 70, but in the marks seems to be very different.

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BIOMATHEMATICS

What is the difference ?

Biology: 75	Biology: 95
Physics: 65	Physics: 40
Chemistry: 70	Chemistry: 75

Since average is the same for both the cases,
how do we differentiate the two cases ?

Answer: "standard deviation"

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So, let us compare these marks and then think about what is the difference. So, biology you have got 75 your friend got 95 physics you have got 65 and your friend has got only 40 which is much less than this on the other hand biology your friend has got much very high mark.

Chemistry you got 70 your friend got 75. So, by looking at this what would you say? you say that when your marks is seen you're pretty much good in everything you have got 75 60. So, above 60 or close to 70 in everything. So, you are equally good in all subjects roughly, but your friend is very bad in physics and very good in biology.

So, biology is like 95 and physics is very 40. So, there is some difference between this two, but the average is same. So, even though the data is different the average is the same.

So, by just telling somebody that I'll got average 70 marks that doesn't mean a lot it only means that it doesn't tell you much more inform it hides the lot of information just by checking average alone will hide lot of information it is not telling you that complete story look at this. So, if you look at here we get if you tell this full mark you will see that.

This gives you the full information this marks, but just by saying you both got average 70 you are hiding the fact that you are equally good in everything and your friend is has

some extremes like very bad in physics and very good in biology. So, this kind of extreme marks, but we are hiding this fact by just saying that you have average 70. So, how do we convey this little more information than the average. So, any this two for now when we say about mark the same thing you can thing about average experiment.

Let say you are measuring the length of something or you are measuring the amount of concentration of something. So, the concentration or in 10 experiments in one experiment it could be like instead of marks you can put like mille micro molar or nano molar and see like all this experiments you could have let me say that average concentration is 70 micro molar that is it mean that all the experiments had very close to 70 that is not mean that only means that.

The sum of all this concentrations you measured in one experiment in all the experiments divided by the number of experiments 70 micro molar, but that doesn't mean that in one experiment you might have taken 40 micro molar in some other experiments you might have taken 95 micro molar. So, whatever we have here in the in term of mark as we presented here can be even equally adapted to the concentrations. So, some instead of this you can say experiment 1 75 micro molar experiment 2 65 micro molar experiments 3 is 70 micro molar and 95 40 and 70 some other experiments. So, even here there is some difference in the way you are done experiments and it is not draw it is not basically reflecting when we just say average. So, what is the thing you should say? So, the answer is standard you have to know something about standard idea of standard deviation in statistics. So, when we say about standard deviation.

When we when we state standard deviation that gives you little more information than just the average. So, when you say standard deviation there is some deviation. So, what is deviation? Deviation from what? So, the answer is deviation from the average. So, we already got know about average and we have to think how much it deviates from the average. So, let us look at here.

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BIOMATHEMATICS

Deviation from the average
Average marks = 70

Mark	Deviation= Mark-average
75	5
65	-5
70	0

Mark	Deviation= Mark-average
95	25
40	-30
75	5

Sum of the deviations is zero!

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So, we had 75 marks and the average marks average is 70. So, when we say 75 the deviation that is the mark minus the average that is 75 minus 70 is five. So, the deviation is 5 here the deviation is minus 5 because mark minus average 65 minus 70 is minus 5 and here the deviation is zero because average is same as the mark it self.

So, what is the sum of deviations here 5 plus minus 5 is a sum of the deviations and the sum of the deviation is zero now look at here.

Here the average is 70. So, deviation is mark minus average that means 95 minus averages is 25.

40 minus 30 is minus 30 and 70 minus 75 minus 70 is plus 5. So, here also the deviation is 25 minus 30 and 5 and this sum of this deviation is zero.

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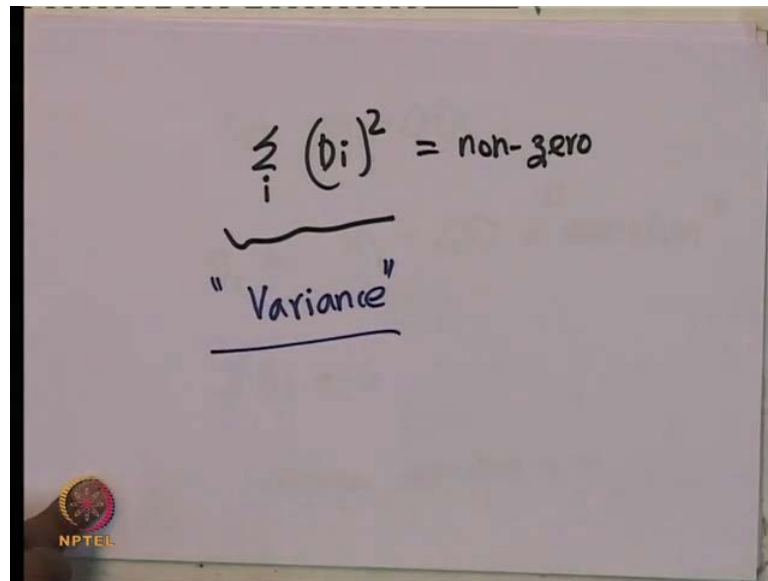
The image shows a whiteboard with handwritten mathematical equations. The first equation is $D_i = m_i - \langle m \rangle$. The second equation is $D_i = x_i - \langle x \rangle = \text{"deviation"}$. The third equation is $\sum_i D_i = 0$. The fourth equation is $\text{average deviation} = 0$. In the bottom left corner of the whiteboard, there is a small circular logo with the text "NPTEL" below it.

So, you find that the sum of the deviation. So, when we define deviation as the deviation from the average. So, let us define deviation D_i as the mark we got in i th subjects minus m average. So, or if any way well if you measuring some other thing length or any quantity x this is can be written as D_i this is deviation. So, we can call this deviation and we found that the sum of deviation is zero. So, what do we do when the sum of the deviation is zero. So, this average of the deviation is zero. So, sum of zero. So, average deviation or the mean deviation is zero because mean deviation is 1 over n by zero is zero.

So, something is zero mean something in average is zero. So, you cannot talk about average deviation because that is meaningless because it is always zero whatever the example you take you will always find that if you find define deviation in this particular way the average deviation will be zero you can see that 5 plus one minus 5 is zero 25 plus minus 30 plus 5 is also zero. So, you find sum as zero if sum if any case if the sum is zero the next thing you do is find the square of the sum find the square and then sum.

What does that mean? So, we had we had D_i is equal to m_i minus m average we had D_i is equal to m_i minus m average and sum over $I D_i$ was zero.

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So, instead of finding sum over i D_i can find sum over i D_i square because sum over i D_i is 0 it makes no sense. So, the next thing to do is D_i square since it is square and this will be always positive. So, this will be non 0. So, the first non 0 value you can get simplest non 0 value we will get is by squaring and summing. So, let us look and do that.

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A slide titled "BIOMATHEMATICS" with the subtitle "Deviation from the average". It contains a table and two calculations. The table has two columns: "Mark" and "(Deviation)²". The rows are: 75 with (5)²=25, 65 with (-5)²=25, and 70 with (0)²=0. Below the table, the text reads: "Mean of deviation square $\langle D^2 \rangle = \frac{25 + 25 + 0}{3} = 16.667$ ". Below that, it reads: "Standard deviation $\sigma = \sqrt{\langle D^2 \rangle} = 4.08$ ". In the bottom left corner, there is a circular logo with "NPTEL" below it. At the bottom center, it says "Prof. Ranjith Padinhateeri, Biosciences and Bioengineering, IIT Bombay".

Mark	(Deviation) ²
75	(5) ² =25
65	(-5) ² =25
70	(0) ² =0

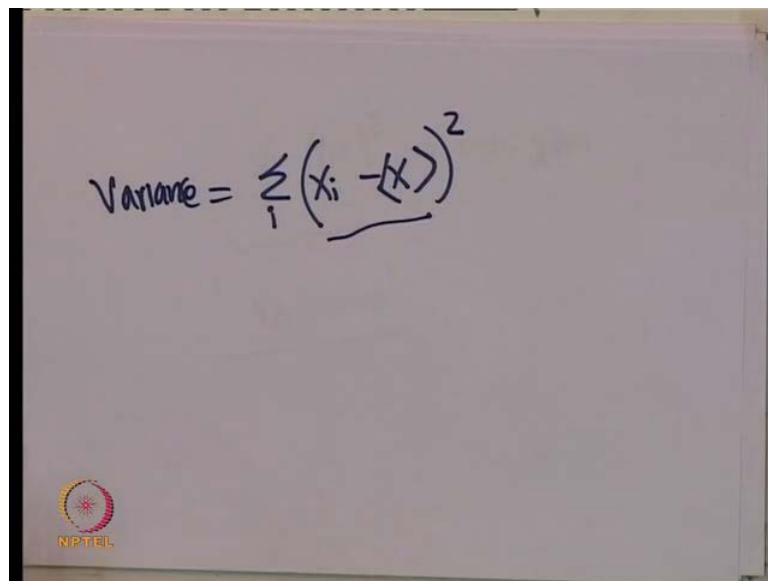
Mean of deviation square $\langle D^2 \rangle = \frac{25 + 25 + 0}{3} = 16.667$

Standard deviation $\sigma = \sqrt{\langle D^2 \rangle} = 4.08$

So, 75 mark means the deviation is 5 70 minus 75 minus 5 75 minus 70 is 5 and 5 square deviation square is 25 when is 65 the deviation square is minus 5 square is 25 and zero deviation square is zero. So, here deviation was minus 5 and plus 5 deviation square is

25 and 25. So, now, deviation sum over deviation square that is 25 plus 25 is 50 plus zero is 50, 50 divided by three is very close to 16.667. So, the mean square deviation is sum positive quantity. So, the mean square deviation mean deviation square that is this quantity is known as variance. So, this is very often used in statistics. So, this is called the variance. So, this is another term in statistics variance is nothing but sum over $\sum (x_i - \bar{x})^2$ this is your deviation you square it. So, this is your deviation and deviation square and sum and this is called variance.

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The image shows a whiteboard with the formula for variance written in black marker. The formula is
$$\text{Variance} = \sum_i (x_i - \bar{x})^2$$
 The word 'Variance' is written on the left. The summation symbol \sum has a subscript i . The term $(x_i - \bar{x})^2$ is enclosed in large parentheses, with a horizontal line under the \bar{x} term. A small NPTEL logo is visible in the bottom left corner of the whiteboard.

Ok, so what are the variance here? So, here the variance is 16 and the square root of the variance is called standard deviation. So, whatever the square root of 16. Something is four point something. So, square root of sixteen is four point something this standard deviation it is often denoted as sigma is the square average square root it is four. So, standard deviation is a positive number always because you are finding square and finding the square root of square. So, this is a positive number. So, in the first case where in case of your mark where you had roughly similar mark in all the subjects your standard deviation was around four, but now let us take your friend's case.

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BIOMATHEMATICS

Deviation from the average


Average marks = 70

Mark	(Deviation) ²
95	(25) ² =625
40	(-30) ² =900
75	(5) ² =25

Mean of deviation square

$$\langle D^2 \rangle = \frac{625 + 900 + 25}{3} = 516.667$$

Standard deviation $\sigma = \sqrt{\langle D^2 \rangle} = 22.73$

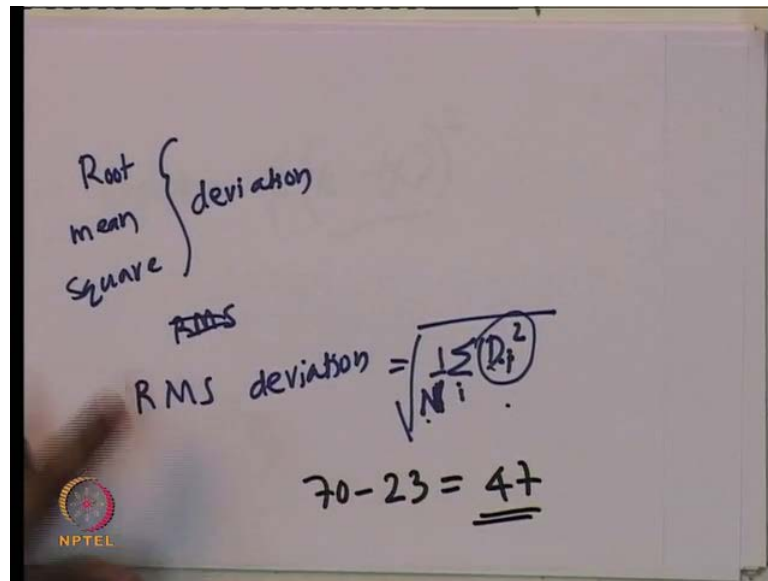


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In your friends case the marks were extreme very high mark for biology and very low mark for physics and then it is reasonable intermediate mark for and the next chemistry subject. So, now, let us look at the deviation square. So, the first one is 75 95. So, 95 minus 70 is 25 square. So, it is like 625. So, here it is 35 because 70 sorry 30 70 minus 40 is 30 40 minus 70 is 30. So, minus 30 square is 900. So, this is again positive and 75 and 70 is the difference is 5 square is 25.

So, the sum is 625 plus 900 plus 25 divided by 3 is 516, 516 is very close to 25 625 is the square of 25. So, square root of d square average is 22.73. So, this standard deviation is square root of deviation square average. So, such things are called root mean square. So, there is a square there is a mean and there is a root you might have.

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So, standard deviation can be also called as root means square deviation if you wish root mean square deviation root mean square deviation. So, we write R M S, R M S deviation; that means you find the square of the deviation average it and find the square root of it.

So, find the square first average it and find the square root. So, root then mean square the mean and this is the root. So, this is called root mean square deviation. So, might have heard in many places root means square.

You might have heard about root mean square velocity why do we do why do we calculate root mean square first place why cont we just square mean. So, as we just saw here the mean is zero. So, even in the case of gas molecule in this room you know that some molecules will go this way some molecules will go that way. So, the velocity as a direction. So, this direction velocity some other molecules direction. So, the average velocity will be zero some molecules will be the in this direction some other molecules will be coming in this direction. So, the average velocity of gas molecules in this room will be zero. So, then since the average is zero to make some sense out of this velocities you have to make the deviation from the average.

So, then that is essentially like a standard deviation and that is that is defined as a standard deviation or root means square deviation. So, that is the way deviation is defined.

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Case1	Case2
75	95
65	40
70	75

70 ± 4.08 70 ± 22.73

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So, now, let us go back to the data and let us look at it. So, if you look at it the case 1 we had the average was 70 and the standard deviation was 4. So, we can say that 70 plus or minus 4 because the deviation from the average is 4 the deviation from the mean. In the next case it was 70 then the deviation was 22.73 very close to 23 and this is very close to 4. So, 70 plus or minus 23 and here is 70 plus or minus 4.

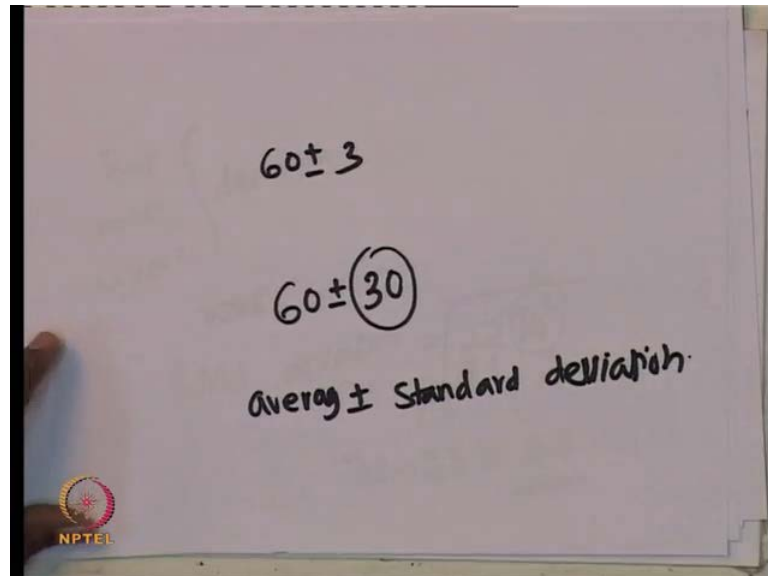
So, if you look at here the extreme values are not very different from 70 plus or minus 4. So, 70 plus 4 minus 4 is 70 4 plus something and the extreme value is 75 here the extreme value is 70 which is 65 minus 4 something. So, these extreme values are not very different from the once you take the plus or minus values.

Even here the average is 70 the y extreme value is 95 which is so and the standard deviation we got was 22.73 which is like 23. So, if 70 plus 23 is 93 which is very close to this value and 70 minus 23 is around 67 and as since 70 minus 23 could be. So, what we have is here 70 minus 23. So, this is 47 and 47 is not very far from 40. So, this is another extreme value here. So, basically you have two cases and the two cases the average is same seventy, but the standard deviations are very different.

Here the standard deviation is 20 close to 23 and here the standard deviation is only 4. So, much smaller standard deviation here and much big standard deviation here. So, by just telling the standard deviation also we are giving much more information that the even though the average is 70 the deviation is very large here and the deviation is small

here. So, the mark for this case can be very different from 70 the deviation from the average mark is very large and deviation in the average mark is very small.

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So, this gives you much more information. So, with the moment you see the moment tell somebody tells you that the mark is 60 plus or minus 3 then you can immediately guess that the mark is around 60.

And the extreme marks somewhere around the 60 is it can be very different it is like around 60 only like 63 and 50 seven if you take the extremes the extremes are very close to this 63 and 57, but if somebody tells you my average is 60 plus or minus 30; that means, the marks are extreme some marks are very low and some marks are very high.

So, this quantity the standard deviation. So, when we this is the basically the average plus or minus standard deviation that's what we just discussed. So, now before this. So, this average plus or minus standard deviation now let us once more check the way we calculate the standard deviation.

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The image shows a handwritten note on a whiteboard and a corresponding slide. The whiteboard has the following text:

60 ± 3

$60 \pm (30)$

Average \pm Standard deviation.

The slide, titled "BIOMATHEMATICS", contains the following mathematical derivation for variance:

$$\text{Variance} = \frac{1}{N} \sum_i (x_i - \langle x \rangle)^2$$
$$= \frac{1}{N} \sum_i [x_i^2 + \langle x \rangle^2 - 2x_i \langle x \rangle]$$
$$= \langle x^2 \rangle + \langle x \rangle^2 - 2\langle x \rangle \frac{1}{N} \sum_i x_i$$

where, $\langle x \rangle = \frac{1}{N} \sum_i x_i$

The slide also features the NPTEL logo and the text: Prof. Ranjith Padinhateeri, Biosciences and Bioengineering, IIT Bombay.

So, the way we calculated is define variance as each mark or each measurement and subtract its average. So, you have to first find the average and subtract the average and find the square and the sum very simple. So, we have x_i minus x average whole square now what does this will give. So, let us check.

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The image shows a whiteboard with handwritten mathematical derivations for variance. The first line is
$$v = \frac{1}{N} \sum_i (x_i - \langle x \rangle)^2$$
. The second line is
$$= \frac{1}{N} \sum_{i=1}^N [x_i^2 + \langle x \rangle^2 - 2x_i \langle x \rangle]$$
. The third line is
$$= \frac{1}{N} \sum_{i=1}^N x_i^2 + \frac{1}{N} \sum_{i=1}^N \langle x \rangle^2 - \frac{1}{N} \sum_{i=1}^N 2x_i \langle x \rangle$$
. The first term in the third line is circled. A NIPTEL logo is visible in the bottom left corner of the whiteboard.

So, what we have is variance and we call it v for variance x_i minus x average we know how to calculate x average and square this and sum over i and this one over n . So, what is this sum over i one over n sum over i x this is like $(a - b)^2$. So, this is $a^2 - 2ab + b^2$.

So, a^2 is x_i^2 plus b^2 is $\langle x \rangle^2$ minus $2ab$ is $2x_i \langle x \rangle$. So, this is what it is. So, now, I can take this inside. So, one over n in sum over i x_i^2 and the first term plus one over n sum over i $\langle x \rangle^2$. So, this is $\langle x^2 \rangle$ plus $\langle x \rangle^2$ minus $2\langle x \rangle \langle x \rangle$. So, $\langle x^2 \rangle + \langle x \rangle^2 - 2\langle x \rangle^2 = \langle x^2 \rangle - \langle x \rangle^2$. So, now, what is this sum over i one to n x_i^2 average this by one over n this term this term that we mark here. So, if we wish this particular term here.

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$$\begin{aligned}
 &= \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2 \\
 &= \frac{1}{N} \sum_{i=1}^N x_i^2 + \frac{1}{N} \sum_{i=1}^N \bar{x}^2 - \frac{1}{N} \sum_{i=1}^N 2x_i \bar{x} \\
 &\quad \downarrow \qquad \qquad \downarrow \\
 &\langle x^2 \rangle + \frac{N \bar{x}^2}{N} - 2 \bar{x} \frac{1}{N} \sum_{i=1}^N x_i \\
 &\langle x^2 \rangle - \langle x \rangle^2 - 2 \langle x \rangle \langle x \rangle
 \end{aligned}$$


So, this particular term here can be written as x square average because this is x one square plus x two square plus x three square whole thing divided by n this is x square average now let us take this term this is one over n times x square average n times you sum. So, when you x square is the constant like 70 it does not change with respect to y. So, if you multiply n in this 70. So, this is like if you sum some constant 70 n times you will get n x square average.

And you have a divided by n. So, you cancel you essentially get an x square average. So, do this carefully by taking some example and minus if you do this x average is a constant two is a constant. So, I can two outside two x average outside and then what you have this sum over I x I one over n. So, this is nothing, but again x average. So, what do you have is x square average minus x this is x average square sorry its x square minus two x average into x average.

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BIOMATHEMATICS


$$\begin{aligned}\text{Variance} &= \langle x^2 \rangle + \langle x \rangle^2 - 2\langle x \rangle^2 \\ &= \langle x^2 \rangle - \langle x \rangle^2\end{aligned}$$
$$\text{Standard Deviation } \sigma = \sqrt{\text{Variance}}$$

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So, this is again x average square x average into x average. So, what we have here is that essentially x square average plus x average square minus this where some over x is x average is define in this particular way which can be written in this particular way x square average plus x average square minus two x average square. So, x average square minus two x average can be written as minus x average square because this is minus two times is a same thing. So, basically you get x square average minus x average square.

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$$\text{Variance} = \langle x^2 \rangle - \langle x \rangle^2$$
$$\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

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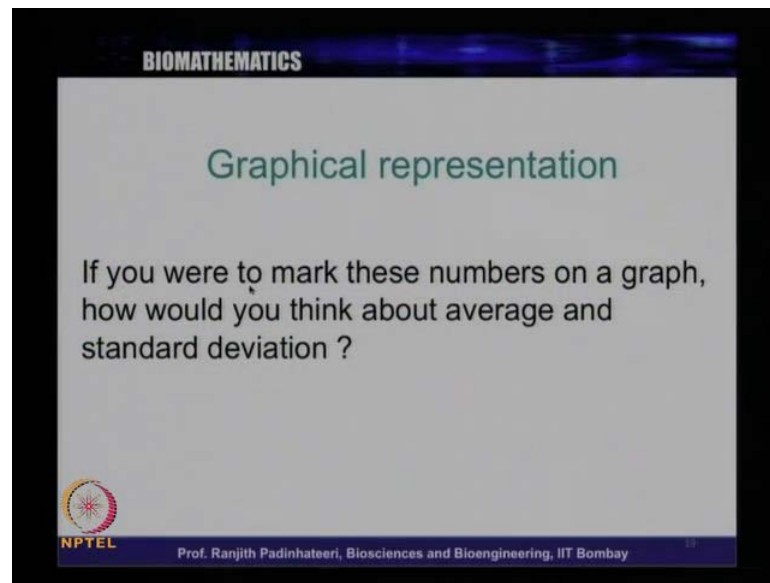
So, this is a very important formula that the important formula is that variants that is the square mean, mean square of deviation is nothing, but x^2 average minus x average square whenever I see this angular bracket; that means, average. So, first calculate x^2 and then calculate the average of it then subtract from that x average square and the square root of this variance is called standard deviation or sigma. So, sigma is a standard deviation this is square root of variance which is x^2 average minus x average square. So, this is.

This is the definition of the standard deviation. So, the standard deviation can be defined as sigma is equal to square root of variance. So, just square root of x^2 average minus x average square.

Ok, so this is. So, now, you can understand how to calculate the standard deviation in the beginning we said that we had some we said that anything that we say in a sentence can be some ideas can be written as equation and can be also shown us some graph you can plot it. So, we have been always using equations something what does it how do we say it in English or what do we mean by something how do we write it in mathematical equation how do we plot it how do we show it is a graph.

That is the we had 3, 3 parallel ways of representing the same thing equation was smaller just one line will tell you a lot of things graph you need like a big reasonably good plots and English you might be many sentences to say that in a plane language now how do we think about this variance average etcetera in a graphical manner.

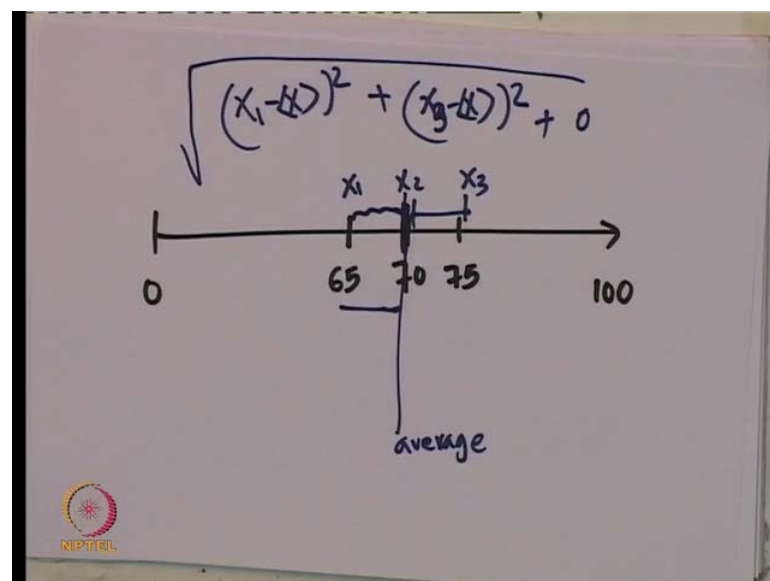
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So, how do we graphically represent this. So, if you were to mark this numbers on a graph how do you think about average and standard deviation?

So, I strongly urge all of you to think things in a graphical in a pictorial way. So, that you get much more idea. So, let us quickly look how do we do this in this pictorial manner. So, let us look here. So, what we have here is let us take the example of mark itself.

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So, you have a line representing marks. So, this is zero and this is let us say 100. So, the marks we discussed in the example here in this two examples one was 75 65 70. So, let

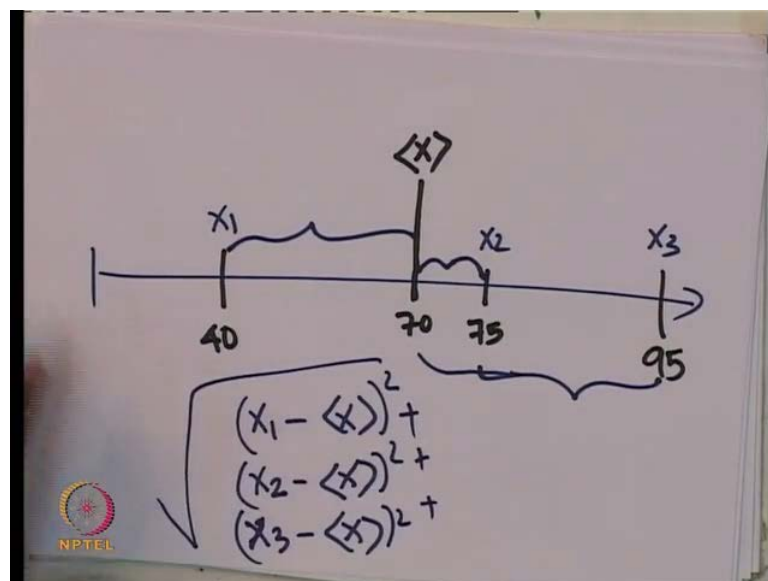
us take the case one here. So, the 65. So, let me mark this 65 and then there is a 70 is a 70 here. So, let us mark as 70 and then there is 75. So, there is 75.

So, now the average now when we say average we know that is the sum of all this, but when we say standard deviation what we are doing is if we mark this x_1 , x_2 and x_3 and the average as this particular here is the average. So, let us mark this as a average, average mark is 70 by doing standard deviation what we are calculating is this particular distance. So, x_3 minus x average square. So, we had to find out x_1 minus x average square. So, that is like finding this distance square x_1 minus x average will be like this distance. So, this distance square plus this distance square.

This distance. So, that is x_1 minus x_3 minus x average square plus x average minus x_2 x_3 minus x average is same is 0. So, that is 0 any way. So, this is the kind of when we think of graphically essentially we are calculating this distance. So, average distance between this. So, you should always think this standard and finding this square root of this is essentially like calculating the distance. So, from the average value how much how distant are other numbers. So, essentially we are calculating the distance in the graph.

So, let us look at the next two examples in more carefully here. So, you have you had this example look at here this is 95 40 75 this example.

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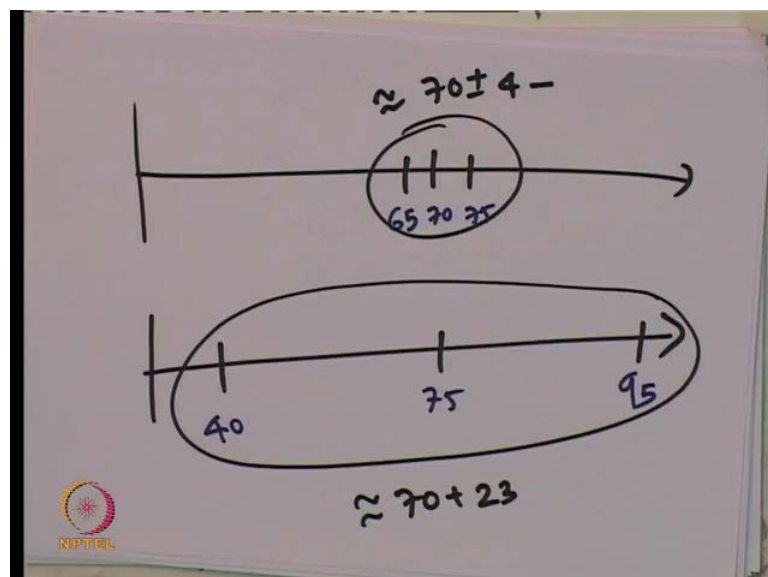
So, again let us draw line. So, we had this is 70 and this is the average mark 70. So, this is the x average.

Now, look at here the x 1 is 95 which is 95 can be marked here 95 and the other 1 is 40 this is like some where here 40 they are very far. So, the distance and the other one is 75. So, 75 is here pretty close. So, this is let me call this x 1 this is x 2 and this is x 3. So, when we say x one minus x average that is this distance roughly and this is exactly this is distance. So, this is distance. So, then we calculate x 2 minus x average that is this distance.

Then we calculate x 3 minus x average that is this distance. So, we calculate the sorry that is this distance from here distance from distance from the average. So, we are calculating the distance from the average in each of these cases and then squaring all this. So, this is distance square and then summing and finding this square root. So, essentially finding the r m s distance just like we discuss in the case of diffusion we the r m s distance it will diffuse here essentially we are calculating some way in r m s distance in the graph.

So, when we use a standard deviation we are essentially calculating some kind of how distant the other when we mark in a graph if the points are very far away we know that the standard deviation is very large. So, by just looking at the graph we can get some idea about standard deviation. So, by looking the two.

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So, in the two cases we had here the first case we had three marks very close 76 5 70 and 75 in the same second case we had marks like this. So, first this cases where 65 70 and 75 here I t was 40 75 and 90 five. So, this where very far and this is very close to each other. So, just by looking at it we can figure out that this is very close to each other. So, this is less standard deviation and this is very far. So, this is like a bigger standard deviation.

So, standard deviation is essentially a measure of how far this points in the graph are if they are very far the large standard deviation. So, here the standard deviation is was first 70 plus or minus 4 something here the standard deviation was 70 plus or minus 23 approximately. So, here it is very small standard deviation big standard deviation and essentially standard deviation is some measure of how far the points are. So, think of anything graphically.

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Summary

Average: $\langle x \rangle = \frac{1}{N} \sum_i x_i$

Standard deviation: $\sqrt{\langle x^2 \rangle - \langle x \rangle^2}$

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So, with this let us summarize today's lecture. So, what we discuss. So, far this two important points one is average which is defined as one over n sum over x I you should remember this and the next one is standard deviation often denoted by a num little sigma.

Which is x square average minus x average. So, these are the two points that you should remember and you should also remember what they what they represent if they are if the points or the marks or experimental data are far away from each other you will get very

large standard deviation and if they are close to each other you will get a small standard deviation.

So, this much if you understand you get some basic idea of statistics,, but it is very important to understand this very carefully you think about it do it many example. So, take many example from any experiment you have done and calculate the average and standard deviation calculate the standard deviation of your marks calculate the standard deviation of heights of your friends calculate the standard deviation of anything that you can think of that you will see in day to day life and see what we get and get a clear feeling for this. So, that is that is the thing that you should do. So, with this with these simple basic ideas of statistics we will stop today's lecture and we will continue discussing statistics in the coming lectures bye