Biomathematics

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Lecture No. # 27

Fourier series

Hello! Welcome to this lecture on Biomathematics. In today's lecture, we will start discussingnew section on Fourier series and Fourier transform. What is this Fourier series and Fourier transform? I will explain this. And, as we go ahead, later we will understand why we need to learn this.

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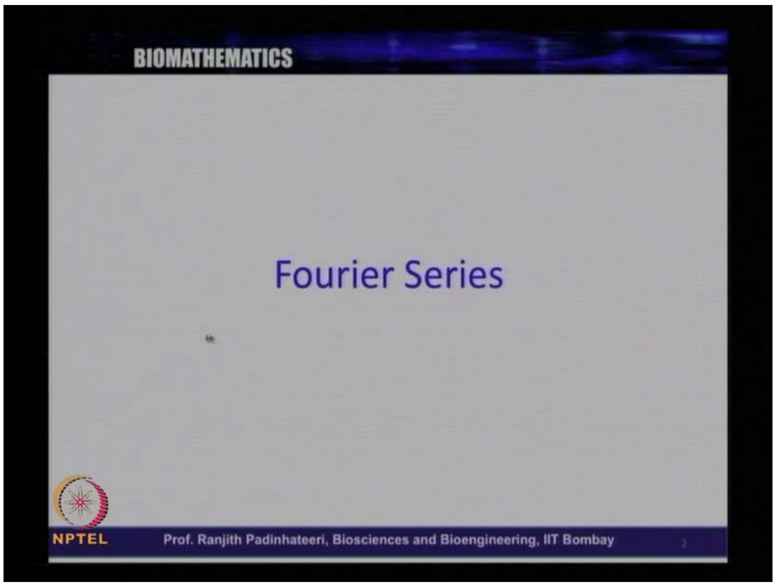

So, in today's lecture, the title is actually Fourier series. So, I will discuss something about Fourier series. So, first let us think about why we need Fourier series or Fourier transformation. So, to begin with, let us think about Fourier series.

So, we, at the beginning of this lecture, we said that Mathematics is like a language and you want to describe everything that we see in nature with some equations. We can, we can precisely write a few things with equations of mathematical equations,so that we can describe things using equations. That is the idea of Mathematics. That is how, that is what Mathematics does; describes things in nature in equations. Now, we have also said that, many of thisphenomenonthat we see… we also already saw that many things that we see, we can describe with some equations like we use various functions.

So, we said that, we use some function to describe that something we see in nature. But, and we already discuss like straight line, square, parabola, hyperbola, sin x, cos x, e power x and various functions, we discussed. And, we saw that these functions can describe some of those things that we see now. Let us think about something which is not that easily, that can be described with this, any of these known functions; let say some complicated curve, which is very difficult to describe with. Thus, then we can write it in the combination of few functions.

So, then we can write something, more complicated as a combination of x, x square or whatever. Combination of few functions will give rise to some other function, which is not that easy to describe by, just there is no one particular function excess to describea particular line or graph. But, you can use the combination of functions. So, by extending this idea, it is argued that any periodic functions let us say you have a periodic function, which is, that is a function that is repeating. We discussed some periodic functions like sine and cos and you also said that, why we need to learn about periodic functions.

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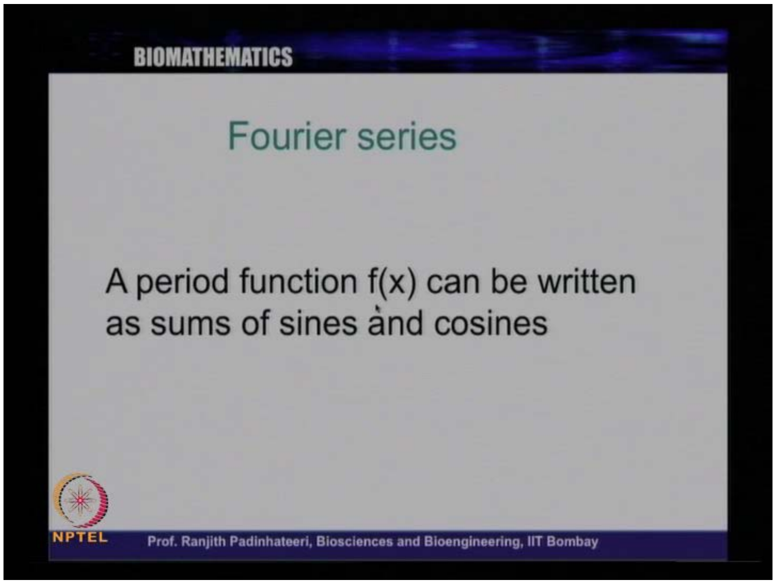

Let us say there is some periodic signals like, periodically varying things. It is argued that, any periodic functions f of x. So, that is what shown here.Any periodic functions f of x can be written as sum of sines and cosines. So, using sines and cosines, we can write a periodic function f of x. So, that is what is we do in Fourier series. In Fourier series, what we do is that we takeperiodic function and that you write, we write in terms of sine and cos, something that we know.So, what kind of periodic function you can think of.

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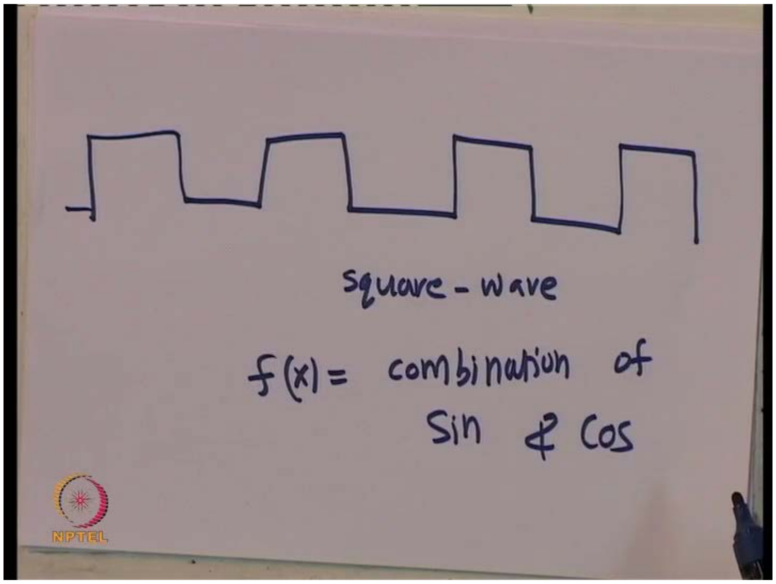

So,let us think of some functions like, so let us say you have something which is repeatinglike this, if you want you can call it as square-wave. So, this is something which just keeps repeating periodically. But, we do not know how to write an equation for this. It is not easy to write equation for this. But, the claim is that, this particular kind of thing that we might haveseen in nature or we might have seen somewhere, something like this, such things can be written as a function and some combination of sine and cos. So, some combination of sine and cos can be used to represent this kind of periodic function. This is the claim.

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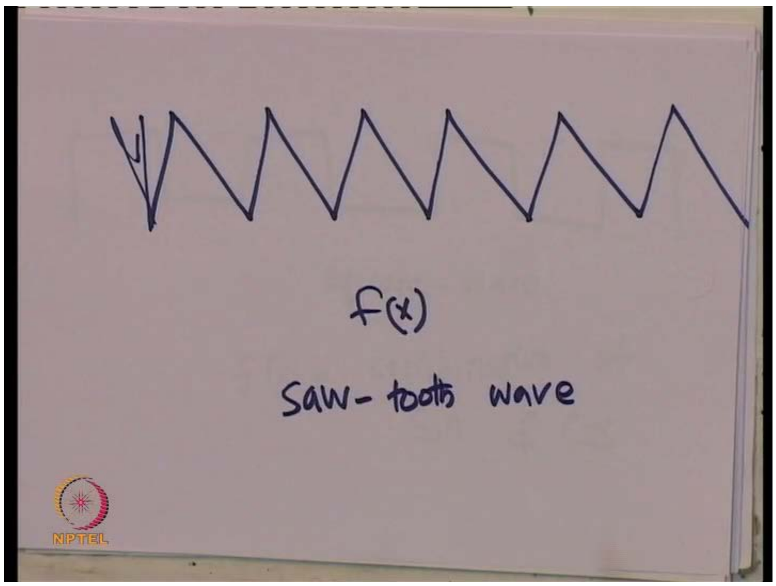

Can you think of some other periodic function? So, we can think of many kinds of waves. So, there is something like saw tooth wave, which is… So, if you have a wave like this, this can be described also by some function f of x, which is a combination of sine and cos. So, this is, we can call thisassaw tooth wave. It is like a saw-tooth.

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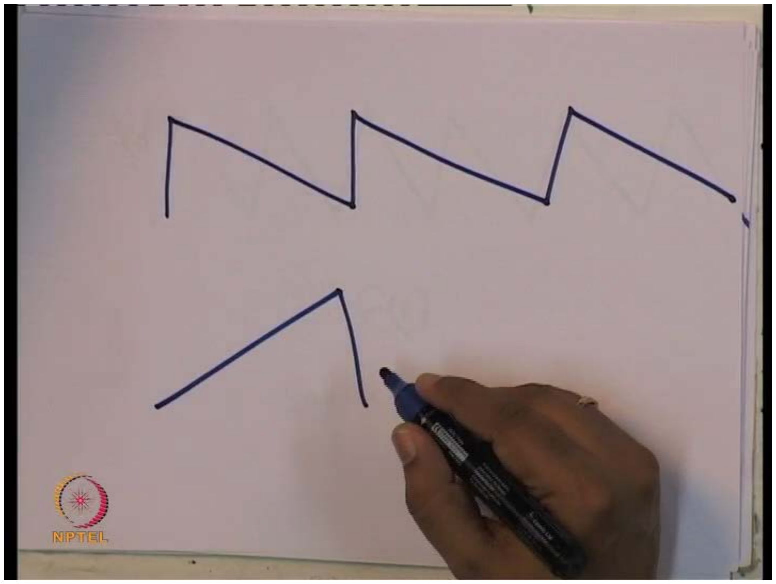

If you are little more dramatic, you can draw this in a vertical way like this. So, let us say it increases and decreases slowly. This is one way or we can also draw like this. It increases like this.

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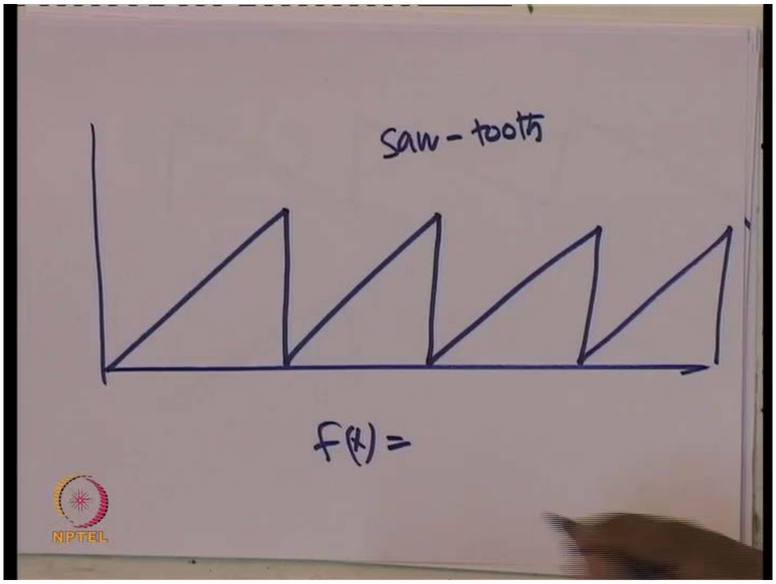

Then to be more precise like, you can draw like this, if you want. So, let us sayit increases like this, then suddenly decreases, then increases like this, suddenly decreases like this. So, this looks like a saw, like a tooth of a saw. Right. So, this is, we can call this saw tooth function or saw tooth wave. It is like a wave. So, the claim is, even this can be represented in terms of sine and cos, some combination of sine and cos. So, anything that you can think of, like, you can think of a little, anything more complicated. Then, even any other function you can think of, that can be represented by this combination of sine and cos. That is this claim.

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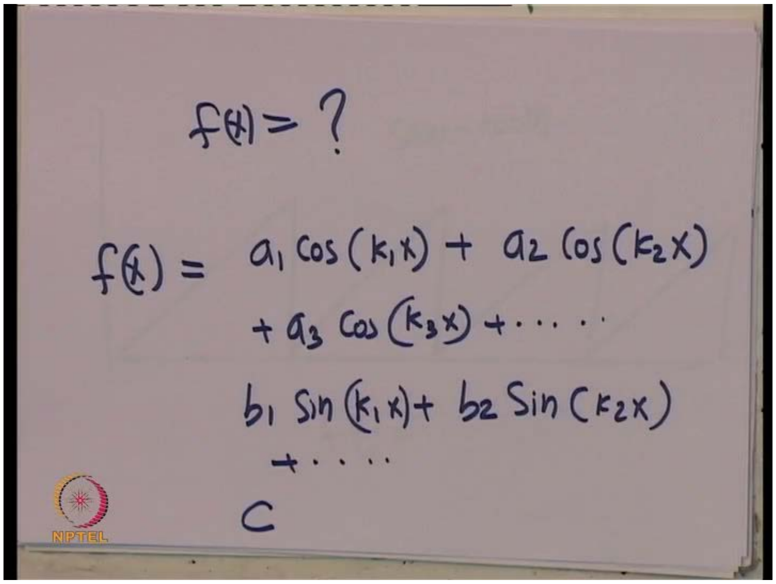

So, how do we represent, how do you write as f of x? What is this f of x? So, it turns out that f of x can be written as something like, now combination of sine and cos. So, let us say a 1 cos k 1 x plus, a 2 cos k 2 x plus, a 3 cos k 3 x plus, so on. We can say b 1 sink 1 x plus, b 2 sin k 2 xplus, dot, dot, dot, plus some constant c. This can be the most general combination of sine and cos you can think of. These are the various general combinations of sine and cos that you can think of.

So, the claim is that, if you take some value for a 1, k 1, a 2, k 2, a 3, k 3 and all that and c a 1, a I, a 1, a 2, b 1, b 2, b 3, c and all that, if you take some particular values, the function can look like, either like a square waveor a saw tooth wave. Or, any periodic function can be represented something like this. So, that is the claim. So, what is that? Let me repeat it once more.

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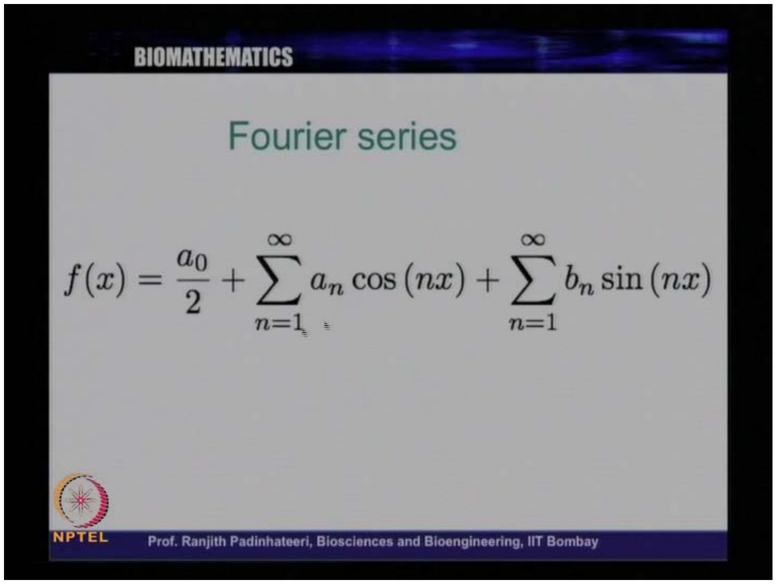

So that, in general it is written something like this. Any function f of x can be written as a 0 by 2 which is just a constant. a, this by 2, we can remove this. You call this a 0; just the definition of a 0 changes as plus sum over n is equal to 1 to infinity a n cos n x plus sum over n is equal to 1 to infinity b n sin n x.

So, this is a claim that any function f of x can be written as a combination of sine and cos; some combination of sine and cos. We do not know what is a n. I mean a 1, a 2, a 3, a 4. We do not knowthose coefficients. We do not know b n, the coefficients of b n. We do not know what a 0 is. So, our aim is that, if you find out a 0, a n and b n, we can write this function. So, the idea is that, if you can somehow know this a 1, a 0, a n and b n, we can get this function. That is the claim.

So, how do we get this function? How do we get this a 0, a 1 and a n. It turns out that the way to calculate this a 0, a n, b n, etcetera, is somewhat similar to what we did. And, there is some analogy related to regarding the vector, in the case of vectors as we did.

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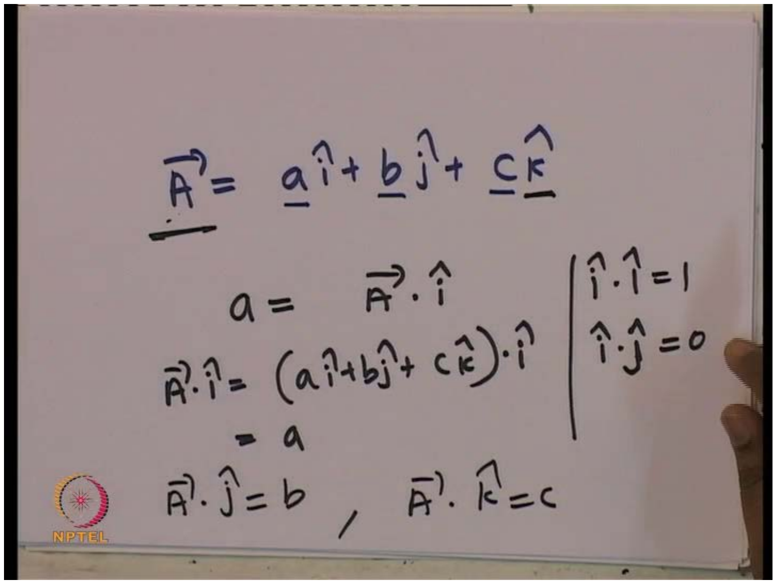

In the case of vector we wrote that, A is a i plus b j plus c k. So, we wrote in the vector that A is a i plus b j plus c k. So, we wrote any vector. Now, a, b and c are some coefficients. And, how do we get this a, b and c? So, in this vector case, we can write a is equal to A dot i.

You know that A dot i is nothing but this dot I; that means a i plus b j plus c k, whole thing dot i. And, we know that i dot i is 1. So, this isa and i dot j is 0. So, we use the property that, i dot i is 1, i dot j is 0. So, using this property i dot j is 0, i dot i is 1. Using this property, we can write a dot i is a. Similarly, we can write a dot j is b. Similarly, a dot k is c. So, you multiply this vector with this. That is what you did. And then, you get the coefficient.

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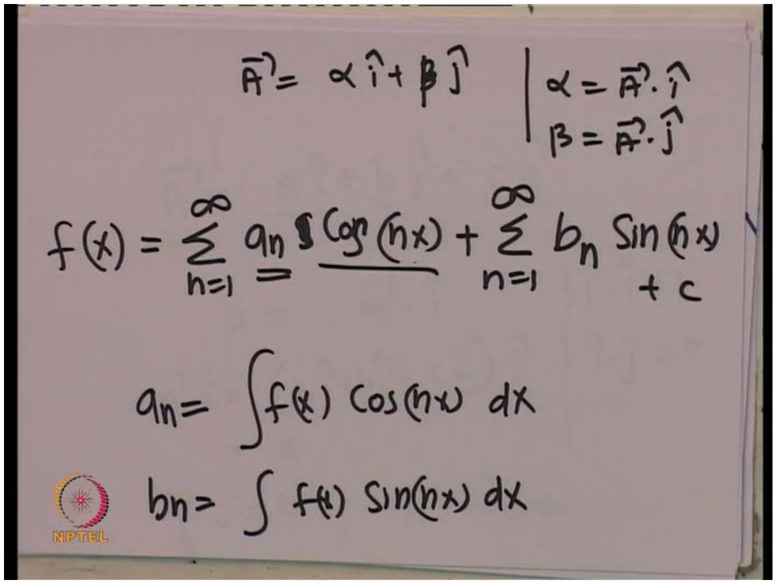

Now, let us think about in a similar fashion here. So, let us, now write in to a 2 d vector here, for simplicity. So, we wrote A as… let me write A as alpha i plus beta j. And, we know that alpha is A dot i and beta is A dot j. So, similarly we can write f of x as sum over n is equal to 1 to infinity a n cos n x plus sum over n is equal to 1 to infinity b n sin n x plus some constant c. So, in fact, actually if we you wish like this c can be written as… here itself we can write this 0 to infinity. So, it will be a 0 here. But, it does not matter. Let me write like this.

So, analogous to this; the way of finding a n is turns out is that a n is equal to f of x. We must find a dot product with i. a dot i gave you alpha. Similarly, you multiply with cos and integrate you get a. Similarly, multiply with sine and integrate, you get b. Similarly, so the way of finding a and b are like this. So, look here.

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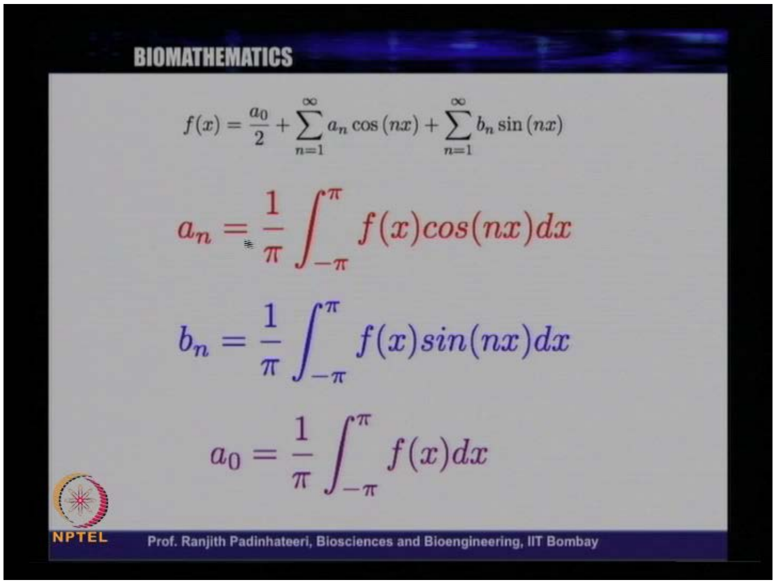

So, a n is, actually to be more precisely defined as 1 over pi minus pi to plus pi f of x cos n x d x. So, you can see that a n is a coefficient of cos. So, you multiple this like this and then you get this and b n is this and a 0 is this. Why a 0 is this? Just here, you put n equals to 0 in the a n. If you put n equals to 0, simply you will get the same thing. So, essentially a, this is nothing but this itself is written in a different way. But, that is fine. So a n and b n can be defined in this particularly way. So, how do we get a n and b n in this particular way? It is very simple. You take this f of x, multiply with cos x and see what we get.

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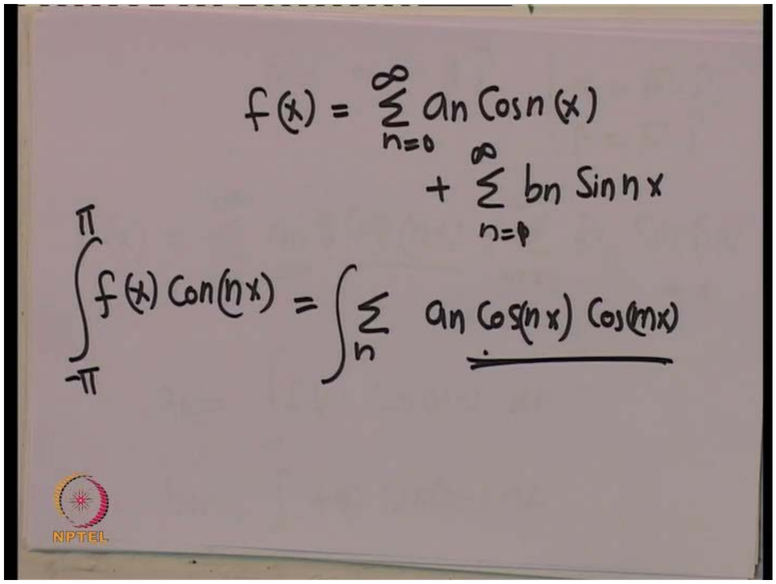

So, let us do this. Look here for it; f of x is sum over n a n cos n x plus sum over nb n sin n x. So, this is 1 to infinity; we can write this 0 to infinity, if you want, if you like. Now, what do we get? Now, you, let us multiply with f of x cos n x and integrate. What is minus pi to plus pi? So, what do you get here is, equal to sum over na n cos n x and you multiple with cos n x here and you integrate.

So, this is what you do. And, so what happens? What you get? You have an integral of cos n… So, for a particular value of n, so you have to do this properly. Actually, you have to write this particular value of m if you like; because just not to confuse. So, it turns out that, we have to use this particular property that sin m x sin n x integral.

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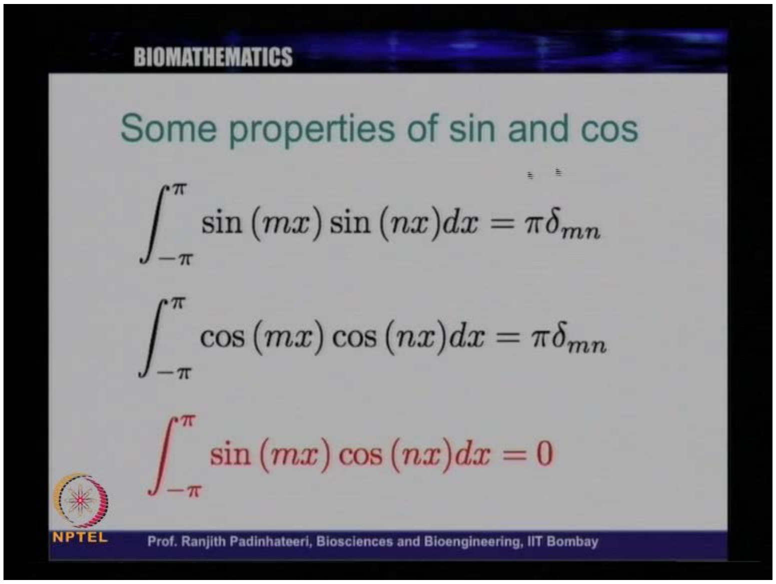

So, if you look here, if you use the property, some properties of sine and cos, if you remember that is integral sin m x sin n x is equal to pi into delta m n, what does this mean? This means is that if you take sine of some of constant x n, sine of some other constants of x, if thisconstants are not equal, if m and n are not equal, this is 0.

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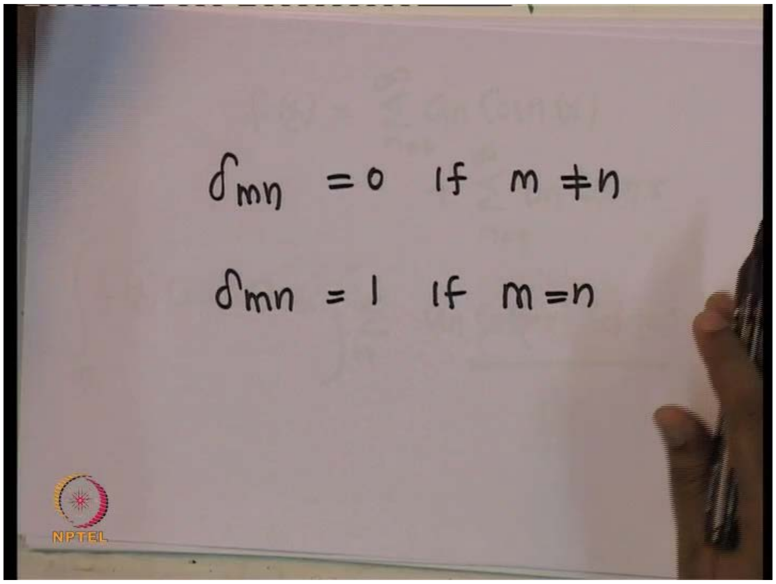

So, delta m n is called conical delta. So, delta m n is 0; if m not equal to n. Delta m n is equal to 1; if m equal to n. So, if m and n are equal, you get 1. If m and n are not equal, you get 0.

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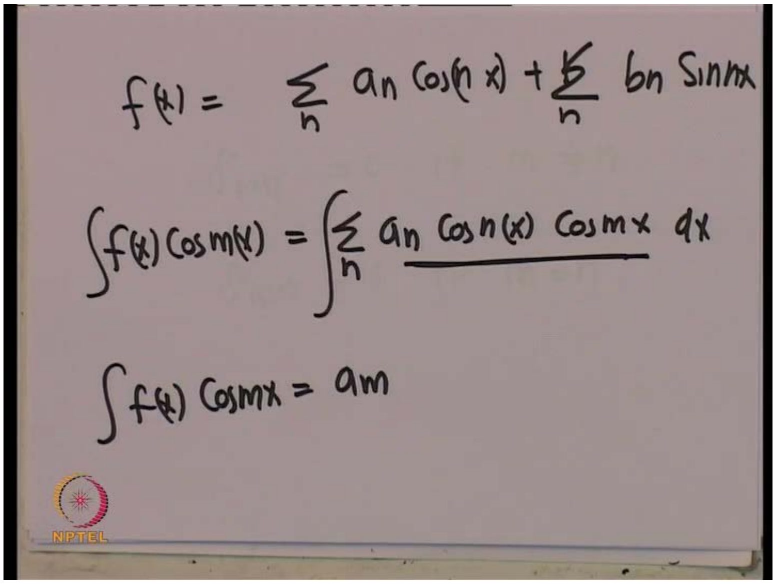

So, if you take f of x as sum over n a n cos n x plus sum over n b n sin n x, then you multiply f of x with some particular function m. Let say, let me multiply this cos m x and integrate. What we get? Sum over na n cos n x cos m x integral d x.

So, this is for various values of m. This will be 0 for all values of n, except this particular, where n is equal to m. So, where n is equal to m, this is 1. So, you will get a m. a m is equal to integral f of x cos m x. This is what you will get. a m equals to integral f of x. You expand this, if you really expand this and use this particular property that cos of m x in to cos of n x is equal to 0, if m is not equal to n and is equal to 1 by m equal to n. So, if you use that you will get this. So, then you get what you have written here. You get that integral f of x cos m x is equal to a m. Similarly, if you multiply with sine; now, let us look at sine.

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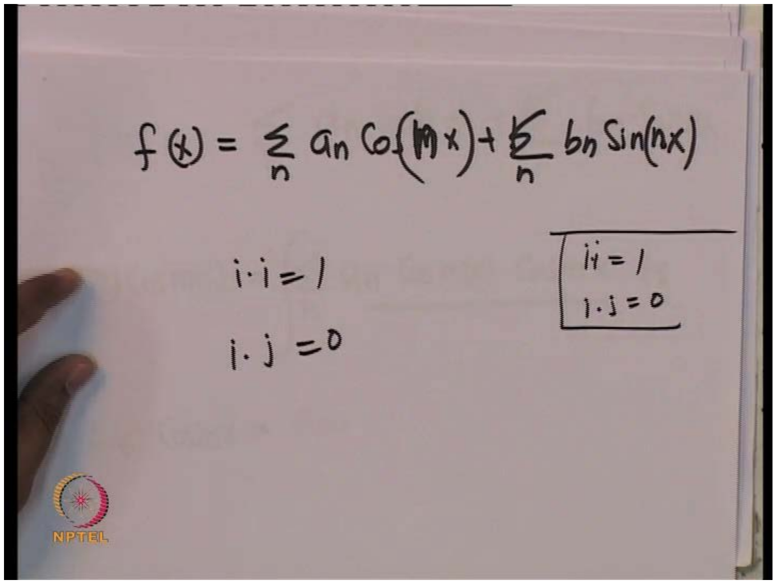

So, similarly, if your f of x is equal to sum over n a n cos n x plus sum over nb n sin n x. So, in the previous case, you have to just say one more thing. We also used this property.

So, in the previous case, where we had here, when you multiply with this cos; so, sine and cos will be multiplied here. And, it turns out that the sine, cos product is as you see in this slide. Here, it is sin m x cos n x integral is 0. So, we used two properties that cos m x cos n x is delta m n and sin m x cos n x is 0. If we use this particular property, these two properties are like those two properties that we used like i dot i is equal to 1 and i dot j is equal to 0. We use these two properties. So, this is just like, we used i dot i is equal to 1 and i dot j is equal to 0. We use these two properties that cos m, cos n are same, if it is 1. If all of them are 1 or same, m and n are same; otherwise, if sine and cos it is 0, so such things are called orthogonal function.

But, we would go into the detail of this. But, if you use this property and write it, essentially what you would get? This particular result that, a n is equal to this and b n is equal to this and a 0 is this. So, we might(( )) if it is needed, one can directly substitute, multiply this f of x with cos x and expand it and see and use this particular property and then you will get this. Use this particular property and then you get this result. You can verify this result by substituting this f of x. This particular formula for f of x here and use these particular properties.

If you use these two properties, you will get two properties. First, you will use this property and then this property, then this property and this property. So, these three properties we have to use to get these results. So, do this yourself. Substitute here, take this particular result. Look here, and if you take this particular result, substitute the right hand side of this, instead of this f of x here and expand it and use these three properties and you will end up with… You will see that you will get a n, b n and a 0.

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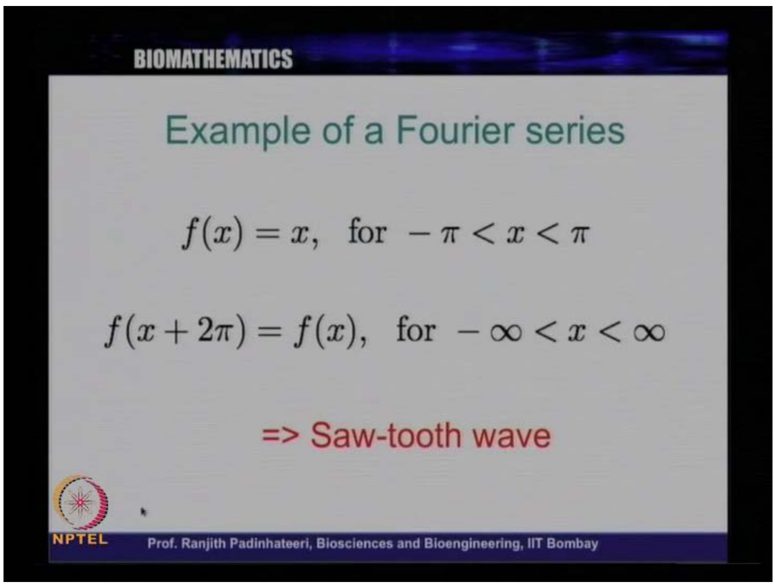

So, what is? How do we now know…? Now, can we do an example? So, we have these results and can we do an example. So, let us do an example of a Fourier series. So, what is example of… let us first take this simple example. This is the example of a saw-tooth wave, something we discussed.

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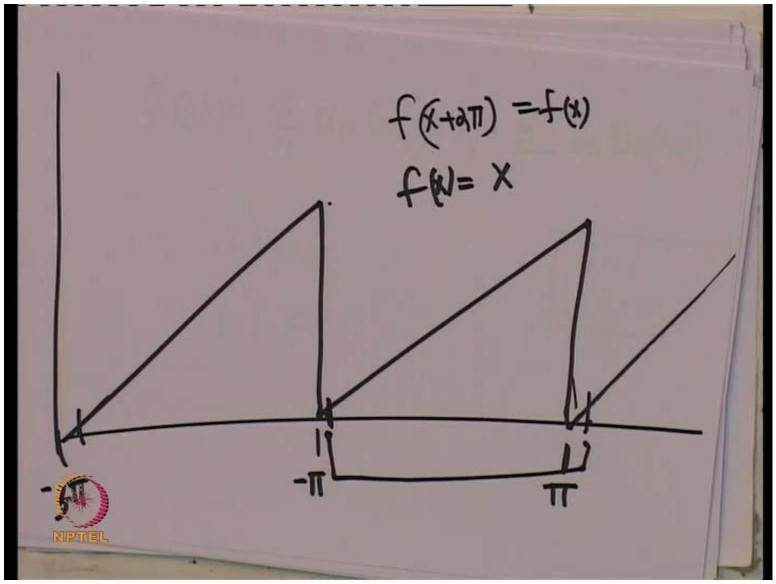

So, how do we represent this saw-tooth wave? Saw-tooth, as we say like this, so we can represent the saw-tooth wave something like this. So, if you take these two intervals like between this, let us call minus pi and plus pi as the two particular points.

So, between this minus pi and plus pi, our function goes like a line. So, this f of x, when between minus pi and 2 and plus pi, then plus pi suddenly, it falls down. Again, it behaves like a…, it just starts from here. So, between these two intervals, interval of 2 pi minus 3 pi is this. So, between minus 3 pi and pi, it is like a line. Again, between minus pi and pi it is like a line. But, this point and this point are same. So, this point, this point and this point are same. So, whatever is here is whatever is, same as here. So, the difference between these 2 points is like 2 pi.

So, when f of x plus 2 pi, the function is the same f of x. So, here it is f of x which is a line, and here, it is repeating. So, this can be mathematically represented in this particular way. It can be represented as f of x is equal to x for, when x is between minus pi and plus pi. When you go to x plus 2 pi, you get f of x itself; so, for all other values, if f of x for every other interval is f of x itself, plus line itself; for all x plus 2 is pi. So, it is x itself for x plus 2 pi. So, that is the saw-tooth wave we saw.

Now, how do we write this in terms of sine and cos? So, we learned in the previous case that, if we have this function a, b and c; a, b and a 0 can be calculated in this particular way. So, what is a 0? a 0 is integral f of x d x.

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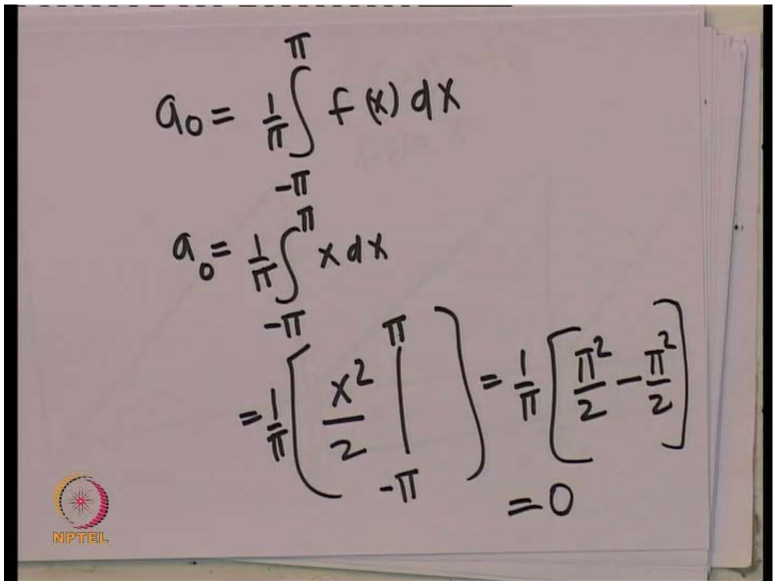

So, if you look here, a 0 is integral f of x d x. So, that is what, shown here. a 0 is integral f of x d x minus pi to plus pi. Now, what was the f of x here? In our case, f of x was x in the between minus pi and plus pi. f of x was x; so that is what we said in the previous case. That is what we said here that, f of x is x between minus pi and plus pi. So, what do we need?

So, look at here, a 0 between minus pi and plus pi that is the 1 over pi; this is the definition. a 0 is integral f of x is x d x between minus pi and plus pi. Now, what is this? This is times 1 over pi. Now, what is this integral x of x d x between minus pi and plus pi? So, you can do this, which is integral x d x is x squared by 2 minus pi to plus pi, the whole thing divided by 1 over pi.

So, this is 1 over pi. If you apply this limits, instead of x I have to put like pi square by 2 minus, minus pi x is equal to minus, when x is equal to minus pi, whole in to minus pi, which is pi square by 2 itself. So, pi square by 2 minus pi square by 2, which is 0. So, turns out that a 0 is 0. And, that is what we get. We get a 0 is 0. So, that is what I have written here.

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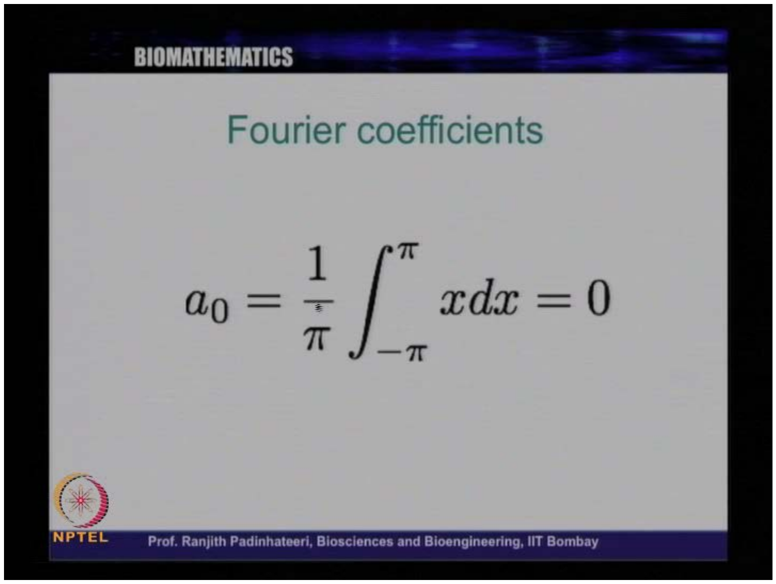

a 0 is the one of the first Fourier coefficients, which, by just 1 over pi to integral minus pi to plus pi x d x. And, this turns out to be 0.

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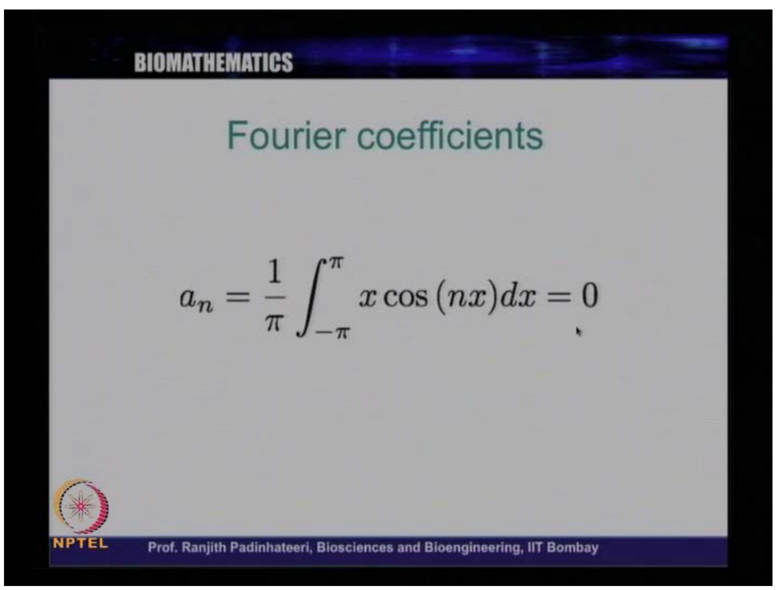

Now, what is the next one? Which is a n? a n is 1 over pi, minus pi to plus pi x cos n x d x. This also turns out to be 0. But, let us see how? Why this is 0? How this function is 0? Let us have a look at it. So, what do we have? We have… What we want to calculate?

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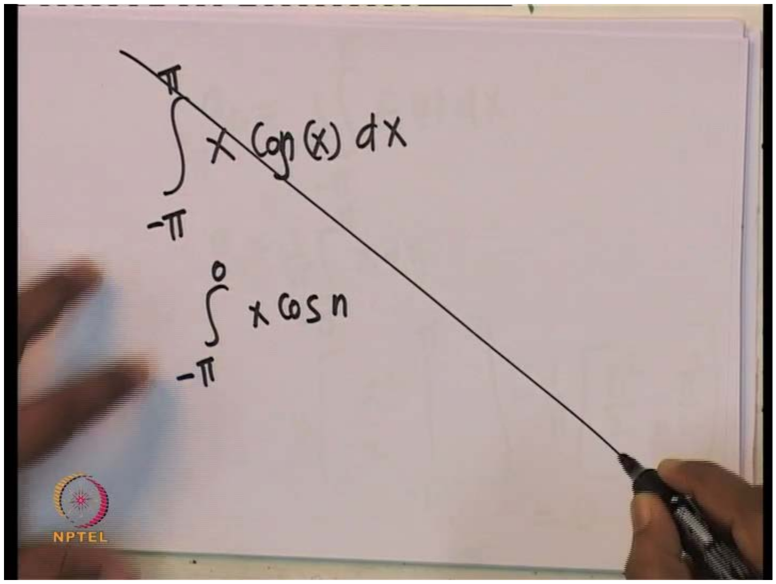

We want to calculate integral x cos n xd x minus pi to plus pi. Now, we were saying that any function; now, this can be written as…If you want this integral can be written as minus pi to 0. Sorry. This is cos and this, sorry. There is an error in this.

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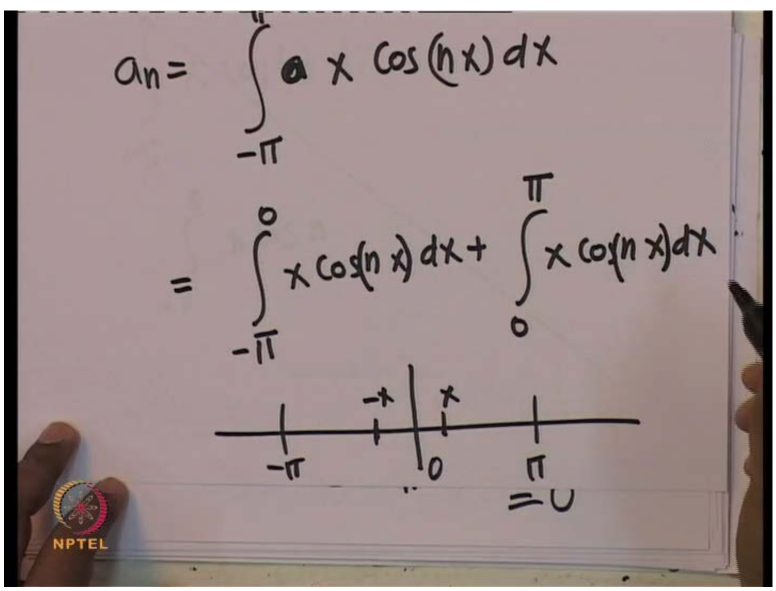

So, what we should actually write is that, a n. We want to calculate this minus pi to plus pi, x cos n x d x. So, this can be written as minus pi to 0 x cos n xd x plus 0 to pi x cos n x d x. So, it is like a sum. But, it just here, this is the negative part and this is the positive part. What we want to do? We want to make it a division at 0. And, this function above 0 this is pi; below 0 it is minus pi. So, this is 0.

So, now let us take a particular point here, x is… and let us compare with. So, this x and this minus some particular value of x; so, let us say x is equal to 1 and x is equal to minus 1.

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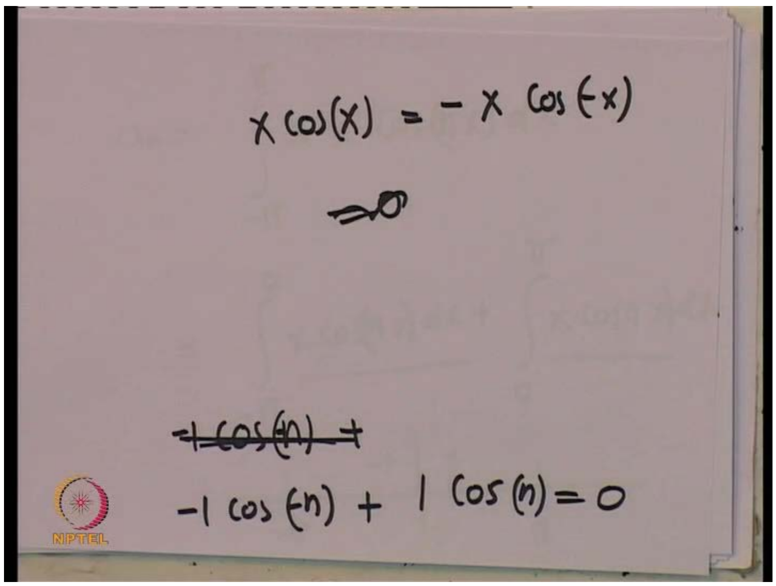

Now let us put here, x is equal to 1 and x equal to minus 1 in this equation. What do we get if you put x equal to 1 and x equal to minus 1? Term becomes 1 cos n x n is 1, sorry, n, x is 1, right. And, this becomes… sorry, this becomes minus 1. So, the negative part becomes minus 1 cos minus n.

So, the negative part becomes minus 1 cos minus n plus this 1 cos n. So, you know cos of minus 1 is cos n itself; because cos is always cos of negative, cos of minus, cos is the cos of minus n. So, this is like a cos of minus n. So, this is, basically this whole thing is 0. So, each and every term, whatever x you put the sum of this will become 0. So, for every x value of, so this you can call this in an asymmetric function; that means for all, this x cos x is equal to minus of x cos of minus x.

So, if you put x is negative value, you could exactly the same opposite. This is, sum of this is actually 0. So, if you do integral, since the function is asymmetric, that is x goes to minus x, there is a negative sign coming. You will see that, you can show that x cos n x integral is actually 0. So, there is an asymmetry in this function. If you plot this, you will see that for every value of positive x, there is exactly the same value with a minus sign for the negative x. So, if you sum of this, you will get the sum as 0.

So, you can see that, if you can do this integral you will see that this, indeed is 0. You can try doing this integral by integration by parts, like; let us try to do this as quickly if you want.

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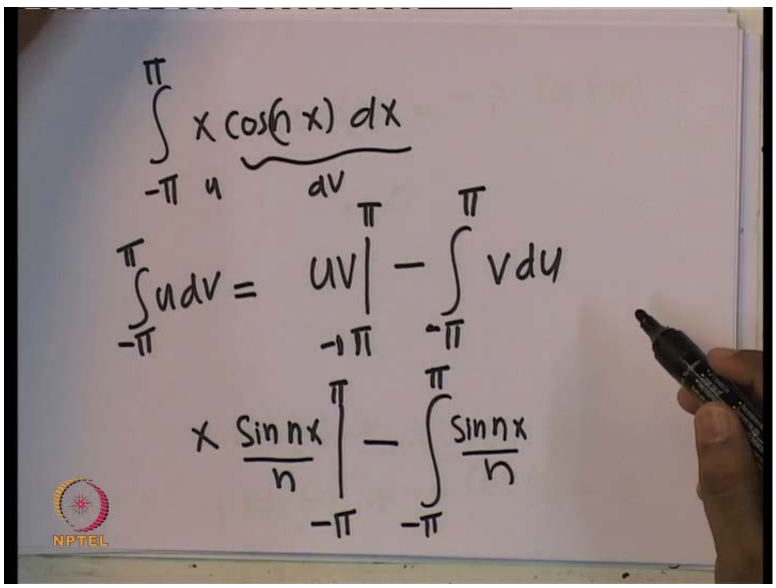

So, let us do this integral x cos n xd x. So, we can integrate this by minus pi to plus pi. You can write this as integral is u d v. So, this is u and you can write this as d v. So, you, we saw that integral u d v can be written as u v minus pi to plus pi. If you have and write u v in the limits minus pi to plus pi minus integral v d u, then from minus pi to plus pi, so you can do this.

So, if you apply this formula; so, this you can do integrate by parts. This is called integrating by parts. So, if you do this, our u is x and d v is cos n x. So, this is equal to u v in the limits. So, there is x and integral of cos n x is sin n x by nin the limit s minus pi to plus pi v d u. v is again sin n x by n into d u is 1. So this is what it is.

So, now if you apply this here, what do we get? So, you get, if you apply this limits here you will see that you get the first one, sin n pi. So, anyways 0; this term will go to 0 because if you substitute for x is of pi and minus pi, you anyway get this term 0. Here also, you know you have just sin n x by n x integral, you have to do.

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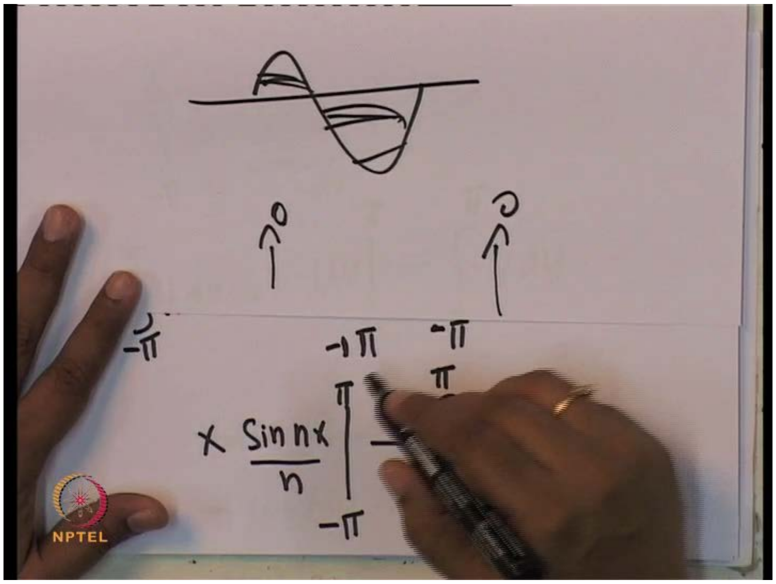

So, you know that minus pi to plus pi sine integral has to be 0; because sine function is any periodic function is like this, will have area under the curve same on the both sides. So, the sine of this is 0. So, you can see that this integral essentially will…, this will lead to 0. This will also lead to 0. So, both these terms, this will give you 0. The answer is 0. So, you can convince yourself by doing this. The answer is 0. So a n is also 0. Now, what we have to calculate is b n.

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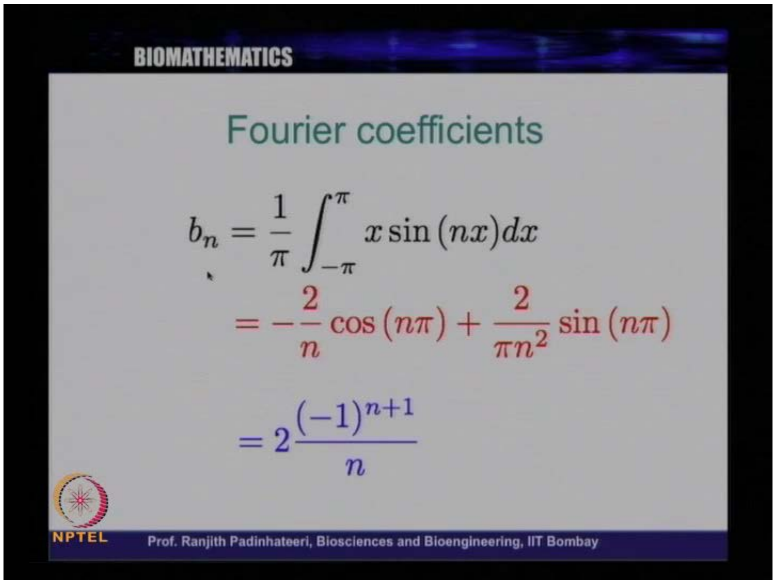

So, if you do this, it turns out that b n is not 0. So, let us quickly do this b n calculation.

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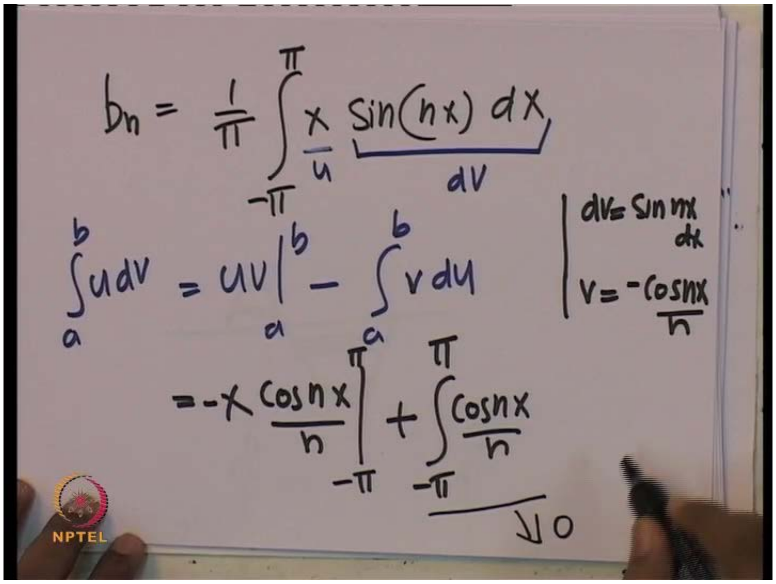

So, b n is again 1 over pi integral minus pi to plus pi x sin n x d x. So, again you can do this by integrating by parts. You can take this as u and take this as d v. And, integral u d v in the limits a to b is u v in the limits a to b minus integral a to b v d u.

This is the standard formula for doing such integral, which we have already discussed. So, if you apply this formula and take x as u and sin n x d x as d v, the answer is u into dv; u is x. if d v is sin n x d x, v has to be minus cos n x by n because the derivative of this is this. If you take the derivative of cos n x, you will get minus sin n x, if minus sin n x, it will be plus sin n x by n sorry… times n by n has to be the…, so, that is cancelled. So, this v has to be minus cos n x by n.

So, here, let us write cos n x by n with the minus sign in the limit minus pi to plus pi minus integral minus pi to plus pi. v is again minus cos n x is plus cos n x by n and d x is 1. So, cos n x by n, this is cos n x by n and this is d u and d u is 1. So, we have this. Now, what is this? So, this is, anyway this integral of cos of anything from minus pi to plus pi has to be 0. So, typically this will become 0.

So, now what you have is this particular term. You can do this term. Actually, if you want you can do this. You know the integral of cos n x will be sine and sine in the limit, sin n pi is anyway 0. So, this is again, we can see that this is going to 0, this will go to 0. Now, what we have is this particular term, which is minus x cos n x with limits. So, what you have is minus x cos n x in the limit minus pi to plus pi. So, what do you do? If you have first limit, if you apply, it is minus.

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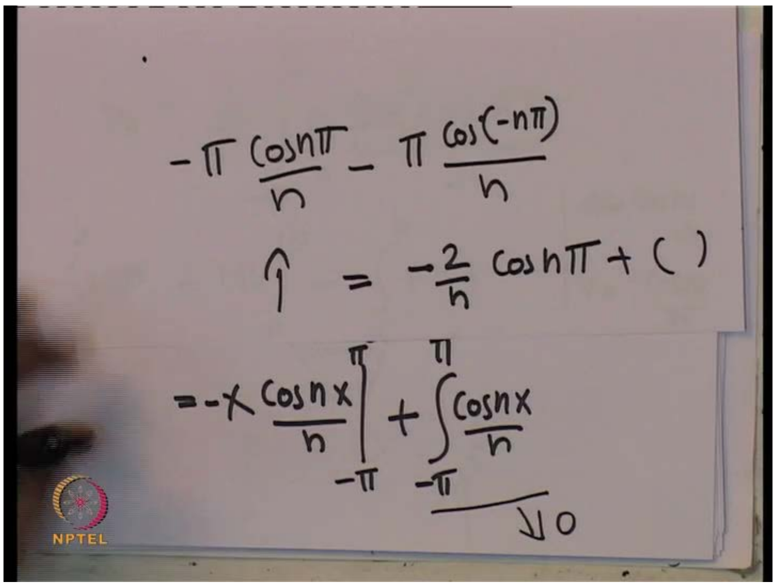

So, if you take this, the first limit is pi; that is substituted everywhere. Instead of x, you substitute pi. So, it is like minus pi cos n pi by n minus, you apply minus pi. So, when you apply minus x, x is minus pi. It will become plus. So, this is minus itself. So, this is again pi cos minus n pi divided by n. You know that cos minus n pi is same as cos n pi. So, this will become minus 2 by n cos n pi plus some term, which will essentially become 0. So, if you do this carefully, essentially what you get is, exactly what we wrote here. Look at here, what you get. If you look at this slide, what you will get is that b of b n is defined as in this particular way. And then, what you get is minus 2 by n cos n pi plus this sin n pi 0, this term is 0.

So, what you essentially get is minus 2 by n cos n pi. Now, this from n, n from 1 onwards only, so, n starts from 1 because that is the definition of b n normally; it starts from n equal to 1 only. So, this can be written as minus 1; whole power n plus 1 by n because you know we have… So, what the claim is that?

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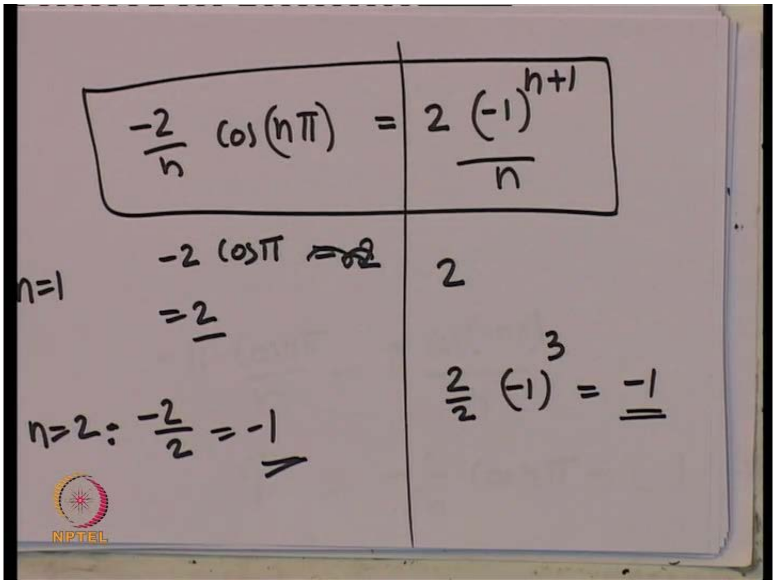

The claim is minus 2 by n cos n pi is same as 2 times minus 1 whole power n plus 1 by n. So, let us check for n equal to 1 first. For n equal to 1, what you get? Minus 2 by 1 into cos pi. What is cos pi? cos pi is… cos 0 is 1; cos pi is minus 1; so, what you get? Sorry. cos, so, yeah, cos pi is minus 1, so, what do we get here? So, what you get is minus 2 into minus 1, you get 2. So, you get this as 2. Now, what do we get from this side? This side, you get 2 into minus 1 and there is a hope by n here, yeah, so when n equal to 1, you get 2 into minus 1 whole power n plus 1, which is 1 plus 1 is 2. So, minus 1 square divided by n is 1. So, you get 2 here also.

So, you get both 2. Now, n is equal to 2 here. You get minus 2 by 2, which is minus 1 into cos 2 pi, cos 2 pi is 1. So, this you get minus 1. So, here what you get? Here, you get 2 by 2 into minus 1 whole power n plus 1, n is 2. So, this is 3. So, 2 by 2 is 1. So, this you get minus 1. So, this and this are exactly the same. So, you can write minus 2 by n cos n pi is 2 into minus 1 whole power n plus 1 by n, so cos, this is the way to write cos n pi. So, essentially what you get is that, b n; the only non-zero term is this. a is 0, a n is 0; b n is non-zero, which is this.

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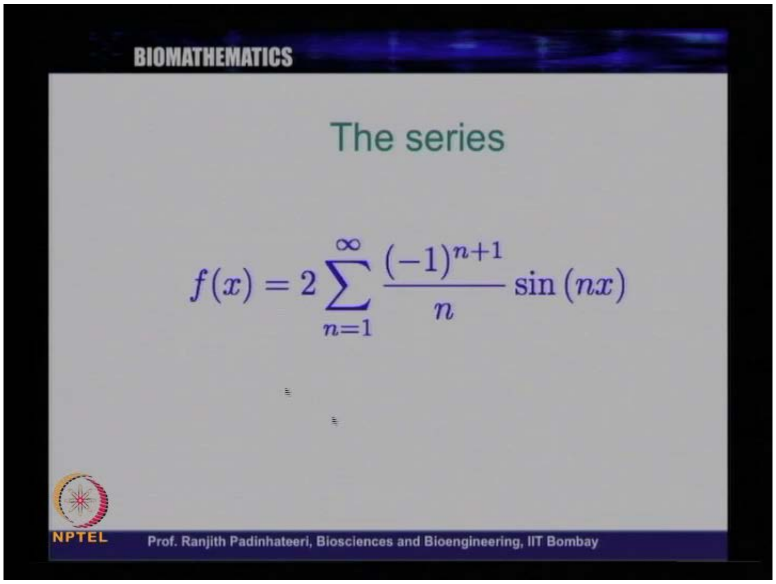

So, you substitute this back in the f of x.

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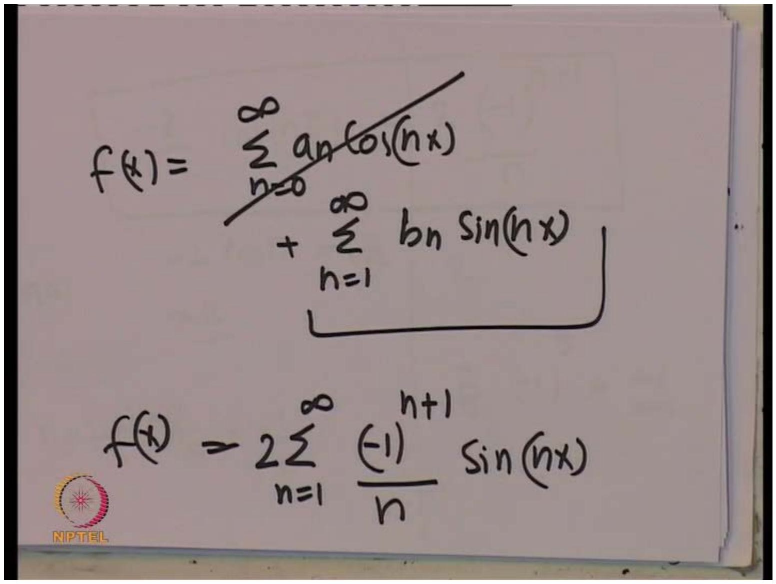

And, you get, we had already found that f of x can be written as sum over n a n cos n x plus sum over n equal to 1 to infinity. You can write this 0 to infinity, if you want. b n sin n x, we found that, a n was 0. So, the only remaining term is b n. And, b n was found as 1 to infinity. So, b n was found as minus 1 whole power n plus 1 divided by n times 2 and there is sin n x.

So, this is our f of x. So, f of x can be written as 2 into sum over sin n x minus 1 whole power n plus 1 by n. So, this is an infinite series. There are like, many terms. But, the claim is that, if you sum this series you will get this.

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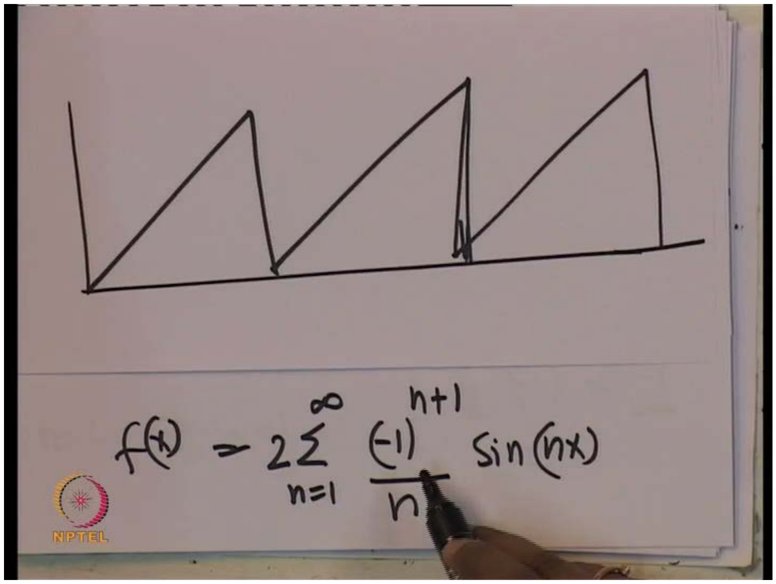

If you do this sum infinite times, you will get this saw-tooth like pattern which is like this. This is the claim. If you sum this, you will get this. This claim, kind of looks like, non trivial because sine, typically does not look sharp like this. Sine more looks like a very… very smooth curve. And, by summing as smooth curve infinite times, you get some, such saw-tooth like pattern. That is the claim. So, now let us examine this. Let us, plot this. So, now let us take only the first term. So, what is the first term? So, the first term here is, in this sum, just like only one term.

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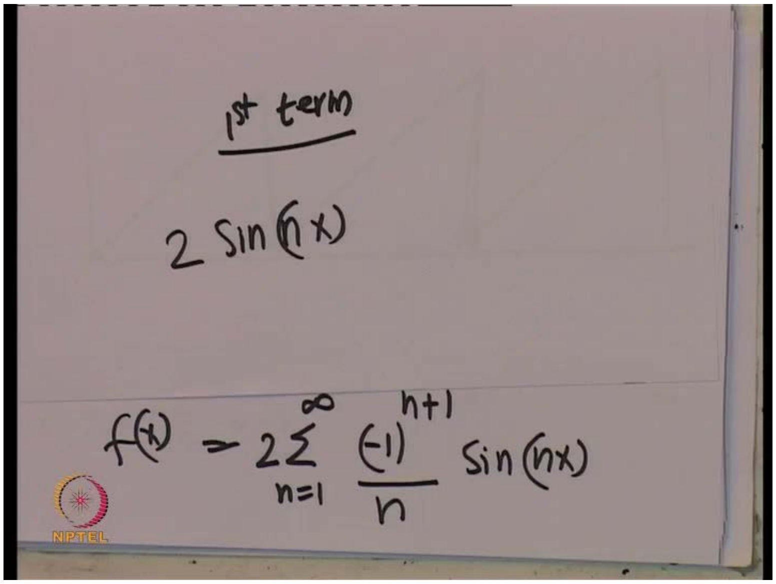

So, the first term is 2 times n is equal to 1. So, minus 1 power n plus 1, n is 1. So, n plus 1 is 2. So, minus 1 power 2 is 1. So, 1 by n is again 1. So, this is 2 sine n x. So, this is the first term. First term is 2 sine n x. So, let us plot this.

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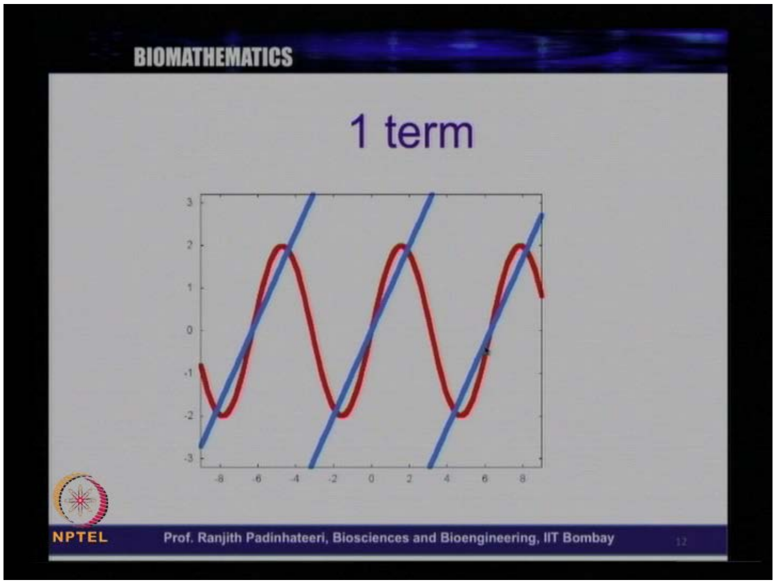

So, if you plot this 2 sin n x, so you will get something like a red curve. But, the saw tooth pattern will look like this and then again fall down suddenly, then again, this again fall down. So, this is the blue curve. Blue curve is our function, but just if you take one term alone, just first term alone, what you get is this. So, first term alone will not be able to represent these blue lines.

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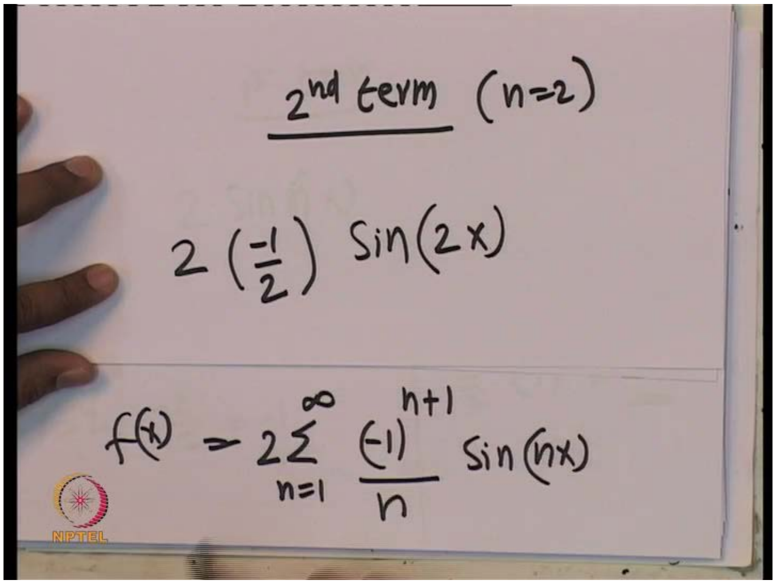

Now, let us take the second term. So, what is the second term? Second term was n equal to 2. So, n equal to 2 is your second term.

So, look at here, second term. What is second term? 2 into minus 1, n is 2.So, 2 plus 1 is 3. So, this is minus 1 power 3 is minus 1. So, this is minus 1 by n is 2. So, this is second term. Second term is n is equal to 2. If you do n equal to 2, you do minus 1 by 2 into sin 2 x. So, this is, so the first one is sin x, first one is 2 sin x, n is 1. So, it is just 2 sin x. Second term was minus 2 n; minus half goes like minus sin 2 x. So, the second term was minus sin 2 x. So, you add the first term and the second term.

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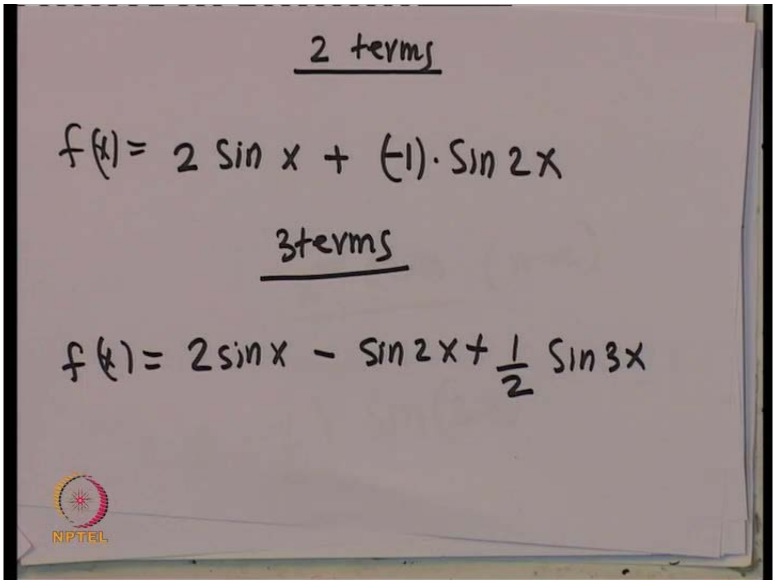

So, you add this term and this term. So, what you get? The first two terms; so, the first two terms are, so if you take the two terms, what you get? You get 2 sin x plus, minus sin 2 x; minus 1 into sin 2 x.

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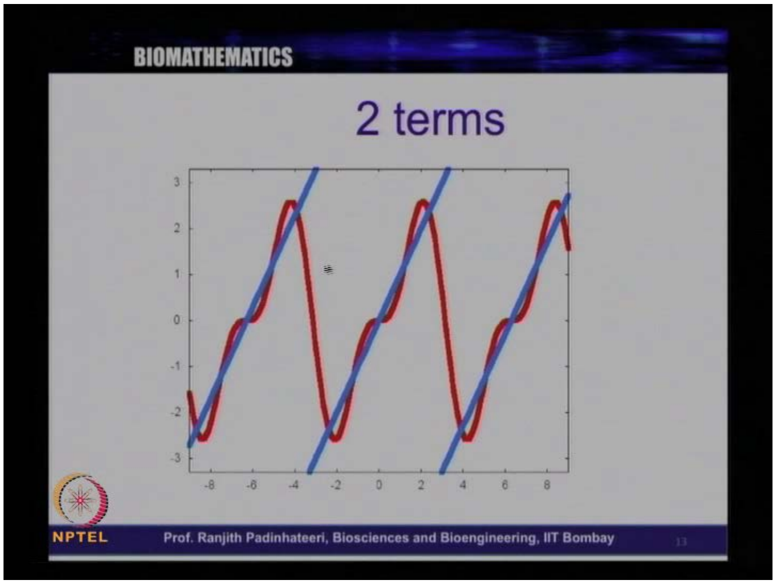

So, this is the first two terms in the series. If you expand the first two terms, let us see what we get? If you plot this particular function, if you plot, if you take this as your f of x, that is only two terms in this sum of series, what do we get? Let us plot this and see. We get this. Let us look at it. It is slightly becoming better. You get this.

So, it is, this is better than the previous one. This, so this is surely becoming better. Two terms is becoming better than one term. Now, what is the third term? So, let us look at the third term.

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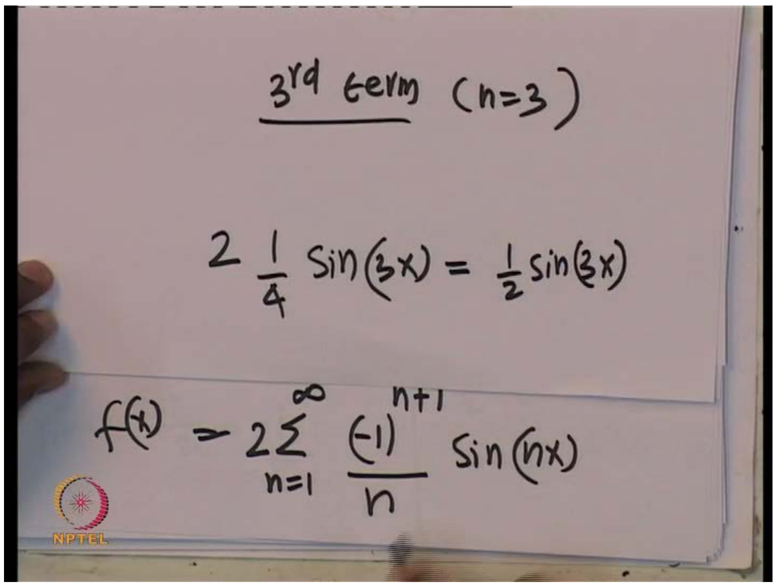

If you look at the third term, so third term means what? In the third term, n is equal to 3. So, if you take n equal to 3 here, what you get? You get f, the third term is 2 into n is 3. So, 3 plus 1 is 4. So, this is 1 by 4, 2 into 1 by 4 sin, n is 3. So, sin 3 x. So, this is half sin 3 x. This is our third term.

So, now we add three terms. Now, we add one more term to this. So, let us make it three terms. So, if you plot three terms, what is our f of x? 2 sin x minus sin 2 x plus the third term was half sin 3 x.

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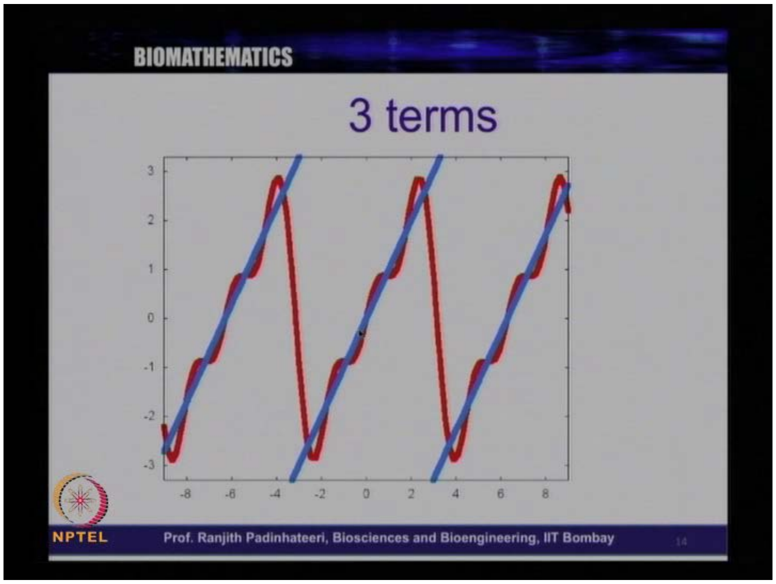

So, let us plot half. Let us add this. So, if you add these three terms and plot this as f of x, what you get is this. Look at here and this plot here, you get this. So, three terms is better than two terms; slightly better.

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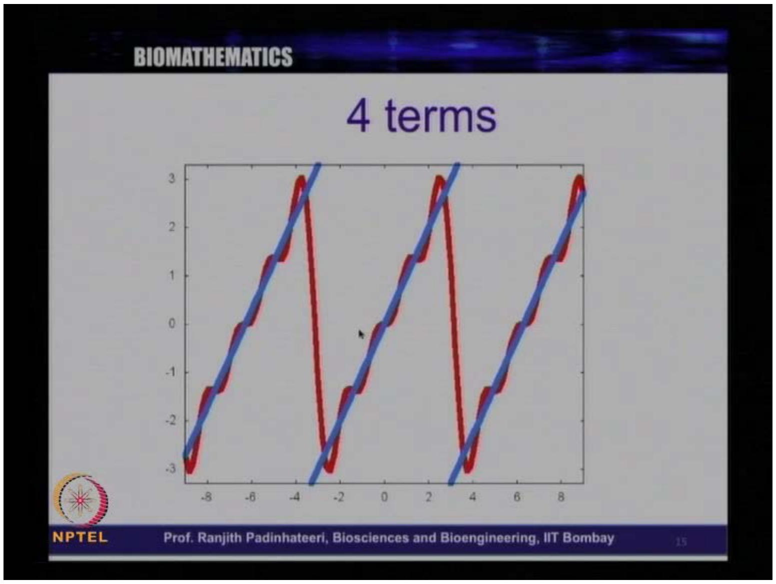

Similarly, if you add one more term, you get four terms. So, this four terms is further better.

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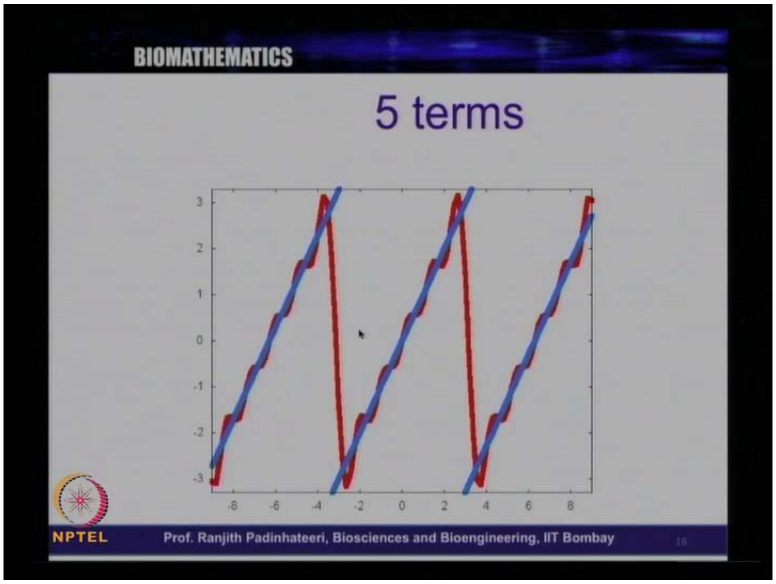

Five terms is much better. So, if the more and more you add, it was very different. One term was very different, second term became better, third term became matching little more with the blue curve. If you add one more term in the series, it becomes further matching with the blue curve. If you add another term, it pretty much perfectly, not exactly like, it is becoming much better.

So, if you just add by taking five terms, it is pretty close to the blue curve. Now, if you take infinite terms, it will be exactly like blue curve. So, it will be exactly like this blue curve here and then suddenly falling again in this blue curve, falling suddenly blue curve. So, you essentially, you take by adding sines, you can generate as saw-tooth pattern. So, let us look this is as a movie. So, look at, this is a movie here. So, one term after one term if you plot this, it is like a movie.

So, let us play this first term, second term, third term, fourth term and fifth term. Just play, just one term around two terms, three terms, four terms and five terms. So, if you add five terms, you are already getting somewhat good agreement with the blue curve.

So, what did we do now? We used sine alone and generated. And, sum of all sines in some particular fashion, generated something like a saw-tooth pattern. So, we could generate a saw-tooth pattern by sines alone. So, this is the essential idea.

This is the essential idea of Fourier series. That, by summing sines and cosines we could generate any function we want. So, we just discussed one example. In the next lecture, we will discuss couple of other examples more. So that, we will see how we can write down any kind of shape; be periodic shape that we want using sines and cosines because if we can write any shape into equations, we can imagine, we can have various applications. Let us say you have particular shape that you see in Biology, if you can write this in terms of sines and cosines, surely it is going to be useful. So, in the coming classes, we will discuss little more examples, a few more examples and make these points more clear. So, this is all for today’s lecture. Thank you.