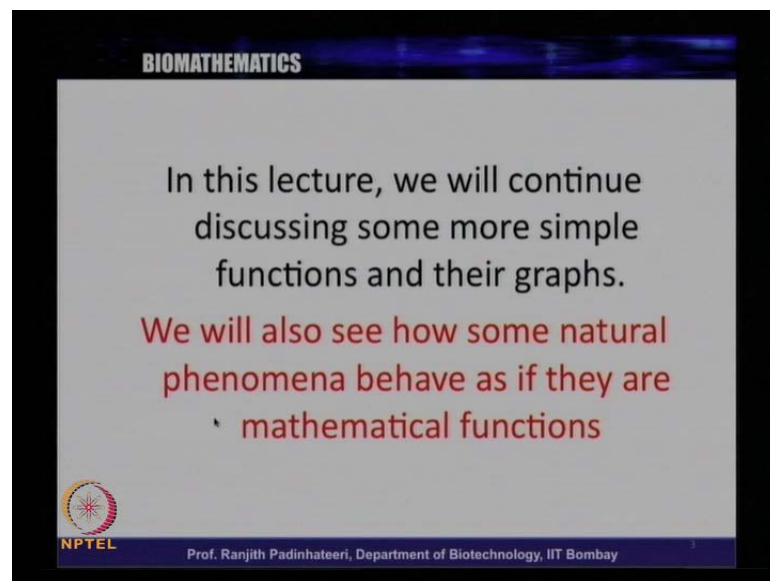


Biomathematics
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Lecture No. # 03
Graph & Function – II.

Hi all. Welcome to the third lecture on Biomathematics. Today, we will continue discussing graphs and functions. So, title of today's lecturer is Graphs and functions part 2. In this lecture, we will continue discussing, some more graphs and some more functions. Last time, we discussed different functions, linear function, quadratic function, etcetera, and in this lecture, we will discuss some more functions, and how to plot them as graphs.

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So, last time, in the last lecture, we plotted different functions and today, we will see some more functions, how to plot them and so on and so forth. We will also see, how some natural phenomena, that we see in, either **in biological**, in biological situation, some situations in biology, or in some other context and how this natural phenomena, somewhat, appear as if, they behave as if, they are like some mathematical functions. So, or, in other words, this phenomena, this behave in a similar way, as this mathematical functions behave. So, we will, we will see many examples today. So, let us just

remember ourselves that, what is a function; just remind ourselves that, what is a function.

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BIOMATHEMATICS

Function

- To plot a graph, we need X values and Y values.
- The relation between quantities that we plot in X axis and Y axis is called a **"Function"**

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So, we, last time, we said that, function is a relation between two things that we plot, in the X axis and the Y axis. We said that, to plot any graph, you need an X axis and a Y axis, the simplest graph and we will plot. We will take points from X axis, from a table, given x and y, and then, we will put on the X Y plane and the relation between the quantities that we plot in the X axis and the quantities that we plot in the Y axis is called a function.

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BIOMATHEMATICS

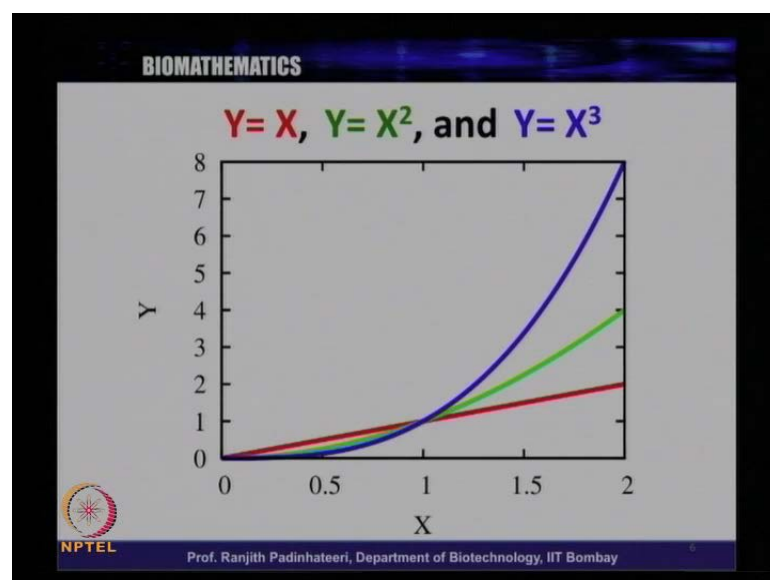
We learned :

- Linear function : $Y = mX + C$
- Quadratic function : $Y = kX^2$
- Cubic function : $Y = kX^3$

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Now, we saw many functions, like linear function Y equal to mX plus C ; we saw quadratic functions, Y is equal to kX square; we saw energy in a spring or the surface area. There are quadratic functions of, for example, surface area, surface area of an organism is a quadratic function of its radius. You also saw, cubic functions, Y is equal to some constant k times X cube.

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So, these are the three functions, in general, Y is equal to mX plus C , Y is equal to kX square and Y is equal to kX cube. And, we saw how we plot them in the next, see slide,

as we see, we saw this plot last time, that, Y equal to X is a line, seen in the red, here; it is a straight line; Y equal to X square is a quadratic function which is shown here in green and Y equal to X cube is this function, this curve seen in blue here. So, these three functions...

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BIOMATHEMATICS

Combination of x , x^2 , x^3 etc

Combination of x , x^2 , x^3 etc can give us new functions, like:

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \dots$$

Notation: $e^x = \exp(x)$

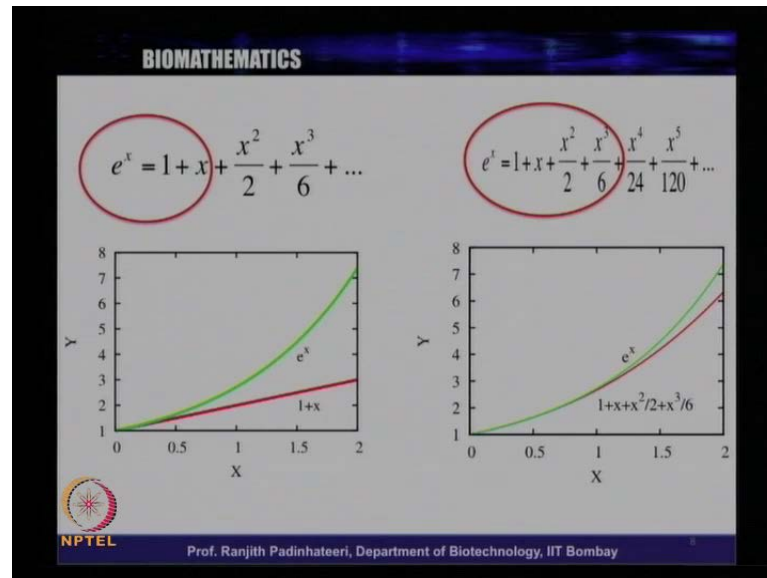
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We must understand these three functions. We also said that a combination of these functions X , X square, X cube, X power 4, X power 5, etcetera, can, will give some other function. For example, X plus X square will give some other function; X minus X square will give some other function. So, any combination of these functions, will give rise to some other function and this combination of these simple functions like X , X square, X cube, X power 4, etcetera etcetera, is used, this combination is very useful to, in describing many natural phenomena, as we will see today. We, last time, we said a very simple function, very well known function called exponential function and we expressed it as a combination of X , X power, X power 2, X power 3, etcetera etcetera. So, let us just remind ourselves like, what we wrote last time.

We wrote, $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \dots$. This dot dot dot means, this series continues till infinity. So, this is an infinite series and the sum of this infinite series or the series having infinite terms, will give you this value e^x . Just a comment on a notation, e^x is also written as $\exp(x)$. So, when you

write, somebody, at some places when you see e^x p bracket x , it is same as e power x ; it is same as this series. So, once we know X , X square, X cube, X power 4, X power 5, etcetera, we can sum all of them in this particular fashion, you have to divide X square by 2, divide X cube by 6, divide X power 4 by 24 and so on and so forth, and sum all of them and with 1, you will get e power X .

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We also saw that, e power x can be approximated as something. For example, like, let us see, look at the, have a look at those slides again. When, here again, 1 plus, e power x is shown in green here and X plus X is shown as a red line here. When X value is very close to 0 , these lines are same; that means, e power x value same as 1 plus X value, when X is very close to 0 ; here, when you say 1 plus X ,

this 1 plus X is only 2 terms, in this series having infinite terms. So, we have seen the terms like 1 plus X plus X square by 2 plus X cube by 6; if we neglect all these terms, and take only, e power x equal to 1 plus X , this is true, this is ok, when X is very close to 0 . If we take more terms, like X square by 2 and X cube by 6 also, that is, we assume that, e power x is 1 plus X plus X square by 2 plus X cube by 6; that will be ok, again below 1, but, much, in a bigger range compared to this. So, the more terms you take in this expansion, this curve is closer to e power x . So, the red curve in this second graph is 1 plus X plus X square by 2 plus X cube by 6 and the green curve is e power x . So, the lesson here is that, some functions like a e power x can be written as an infinite series, a

series of, a power series like, X power something, plus X power some other thing, plus X some other thing, plus, plus, plus, plus. And, once we have this, if we take a few of those terms, that will still represent this function, but in a limited range. To get this function at any point, for any value of X, one has to take all this, all this terms. We **(())** sum this infinite series. So, this is, all these things, we learnt last time. So, now, let us look a couple of examples of e power x. So, where have you heard exponential? You have heard exponential, for example, in the context of,

exponentially growing something. So, you have heard microbial growth. Microbial growth, we learn that, it is a, some phase is growing exponentially. You might have heard of Arrhenius equation, the rate of a reaction is a e power something.

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BIOMATHEMATICS

Example in biology: e^x

- Microbial growth : when rapidly multiplying
- Arrhenius equation $r = A \exp\left(\frac{-E_a}{RT}\right)$

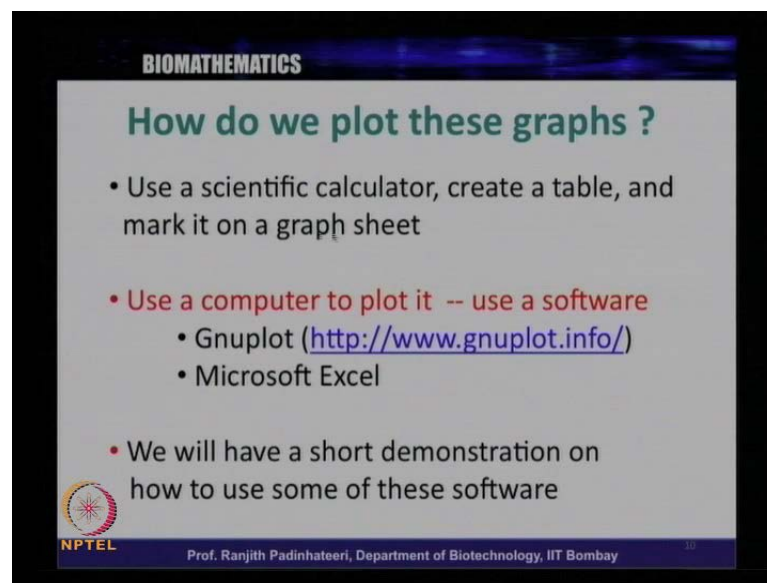
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So, let us have a look at this examples. So, when rapidly multiplying microbial bacterial cells, for example, or e cells, we call it exponential phase or exponential growth. Sometimes, it is also called the log phase; in the log phase, things grow exponentially. And, why we call it log phase, we will come to that later. But, in the log phase, the number of microbes is growing like an e power x, the Arrhenius equation, we have seen r equal to A e power minus activation energy

E A divided by R T, where T is temperature and R is universal gas constant. So, this is another example, where exponential function is seen in Biology. So, think about it, for example, this equation, a bit. The rate of a reaction behave, as if it is a mathematical

function. So, this is a natural process; natural process behave, as if it is like a mathematical function. What does it mean? It means that, we can describe this natural process of, or the natural, rate of a reaction using an exponential function. Now, let us, seeing this, one can have...So, we saw many graphs so far, and we could have a, you could have a question, how do we plot all these graphs? Like, it is a very complicated curves, sometimes, like e^x curve, X^2 , X^3 , etcetera, how do we plot?

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BIOMATHEMATICS

How do we plot these graphs ?

- Use a scientific calculator, create a table, and mark it on a graph sheet
- Use a computer to plot it -- use a software
 - Gnuplot (<http://www.gnuplot.info/>)
 - Microsoft Excel
- We will have a short demonstration on how to use some of these software

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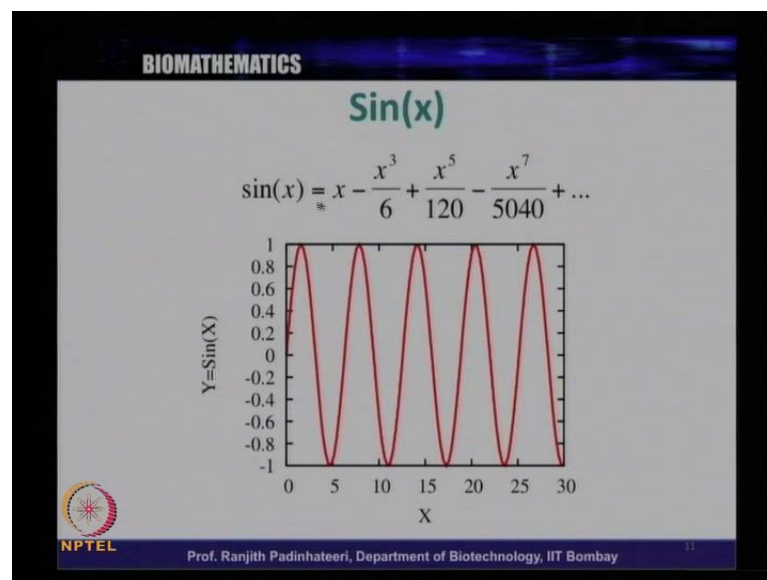
So, the conventional way of plotting is that, use a calculator; create a table and mark it on a graph sheet. So, all of you would have seen scientific calculator having exponential function and many functions that we use, is there in the in a scientific calculator. So, you could use a scientific calculator, create a table, like what we did last time, and mark it on a graph sheet and that it will give you a graph. But it is a lot of work. To get a smooth curve, one has to make many points and it is time consuming. Today, we can use a computer to plot it. You can use some software to plot it. For example, Gnuplot is a freely available software on the web. So, if you go to this site, one can download Gnuplot and you can use this

to plot any function. You can also plot this in Microsoft Excel, or example. There could be many other softwares, once you Google search, you could get many many softwares, using which, you can plot all these functions. Later in this lecture, in this lecture series, not in this, today's lecture, but in some other part of the series, may be in the next lecture

itself, we could have a short demonstration on how to use some of this software. For example, how to use Excel and plot this functions; or how to use G n u plot and plot this functions. We will have a short demonstration in one of the lectures. So, as of now, just, please understand that, in principle, we can use a calculator and plot all this functions and, or, use a software and

plot this functions and there is, we will learn how to do this. So, we understand X, X square, X cube, etcetera, and exponential function as a combination of this. Now, today, let us go ahead and see, a few more functions and see how you can express those functions as a combination of X, X square, X cube, X power 4, etcetera, etcetera. And, one of the familiar function is called Sin function.

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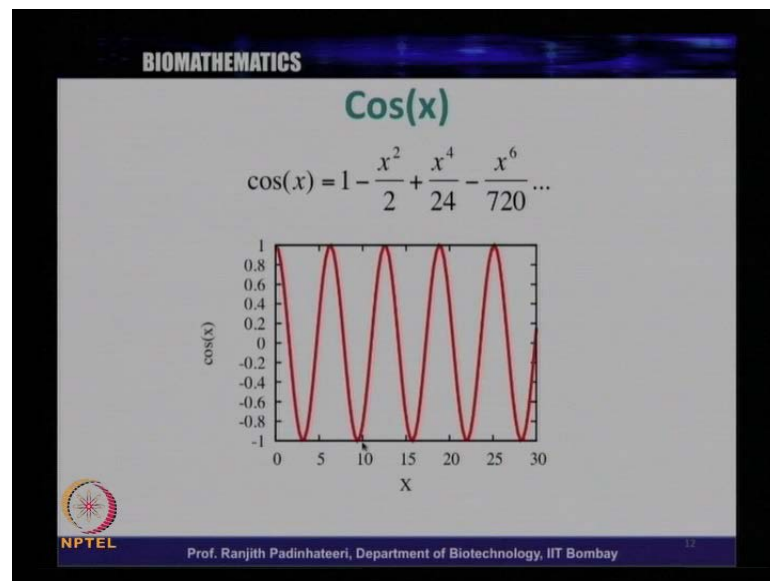


So, let us have a look at it. The Sin function, sin of x can be written as, X minus X cube by 6 plus X power 5 by 120 minus X power 7 by 5040, 5040 plus, so on and so forth. So, this is again, another infinite series. There are infinite terms and this, if you sum all this terms, you will get sin x. So, take any x value, calculate all this and sum up all this, you will get sin of x. So, and this sin of x is a periodic function; or, it is an oscillating function. So, have a look at this graph here. So, you can see that, when x is 0, sin 0 is 0; and, as the x increases, till some extent, the function increases; after that, it reaches the maximum value and this start to decrease; and at some point, it reaches the minimum

value, it starts to increase, this function, again, increases; then, again reaches the peak, then, decreases, again, increases, decreases, increases, decreases.

So, sin of x is a function, is a mathematical function, which behaves, which appear as if, it is oscillating. You might have seen many natural phenomenon. Think about a natural phenomenon that you have seen, that is oscillating; or, that is behaving in a periodic fashion, increasing and decreasing. So, it just keeps increasing and decreasing. So, there are many phenomena. We will see, we will see some of them today. But, if you want to represent them mathematically, if you want to represent those phenomena, or, if you want to talk about those phenomena, in mathematical language, one can use sin x as a function. Just like sin x, there is another function, called cosine of x or written as cos of x. So, let us look at the next slide, which is cos of x,

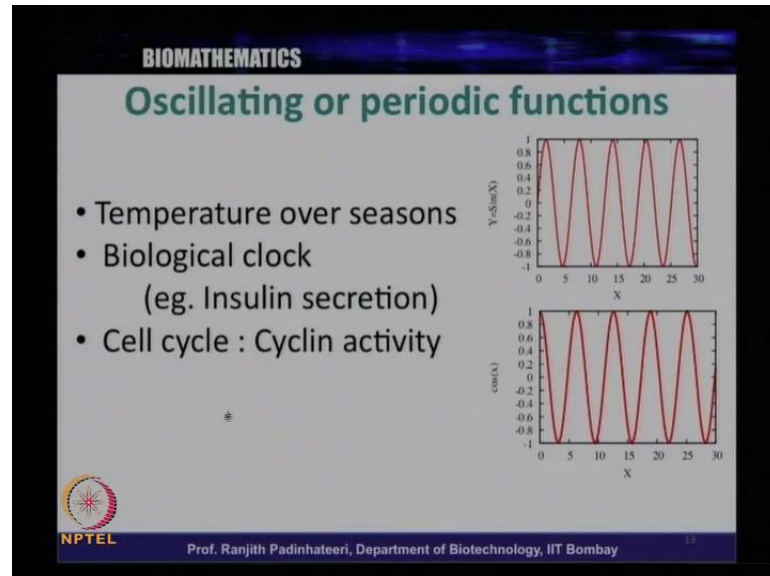
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which can be written as 1 minus X square by 2 plus X power 4 by 24 minus X power 6 by 720 and so on and so forth. So, again, cos of x can be written as combination of the known functions like X square, X power 4, X power 6, etcetera. And, if you sum all of them, you will get cos of x. Again, but, at, when x equal to 0, this will be 0, this term will be 0, this term will be 0; all terms will be 0, except 1. So, when x equal to 0, cosin of x or cos of x is 1. So, you can see. when x equal to 0, this is 1; then, it decreases, until minus 1 and then, increases, then, again decreases, increases. So, it is also a periodic function or

is an oscillating function and both \cos of x and \sin of x , as a , they oscillate between plus 1 and minus 1.

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So, you can see, that this are oscillating functions, periodic functions. As we said, you can think of many periodic phenomena, oscillating phenomena in natural, in day to day life or in Biology. And, one could write, any such phenomena as a combination of $\sin x$ and $\cos x$. So, let us think of a couple of phenomena that we know and they are oscillating. We said in the first lecture that, if you take seasons, like the winter, summer, rainy season; they all repeat; they are all periodically repeat. So, for example, the temperature. Let us have a look at this slide and as we said, temperature over seasons, they,

they periodically repeat; they increase and decrease. So, temperature in the summer is maximum and so, let us, let us have a look at it. So, if you plot temperature and then, season, time, you will see, the temperature in summer is maximum and as the rainy season, some temperature decreases; and, as the winter comes temperature reaches to a minimum value in January. Then, it again increases, in March, it has some intermediate value; again, summer comes, it is a maximum; and, then, decreases, increases, decreases, increases, decreases. So, temperature over seasons, in time, you have this periodic function. As you see, this is the peak month, this mostly like May, June. So, this like

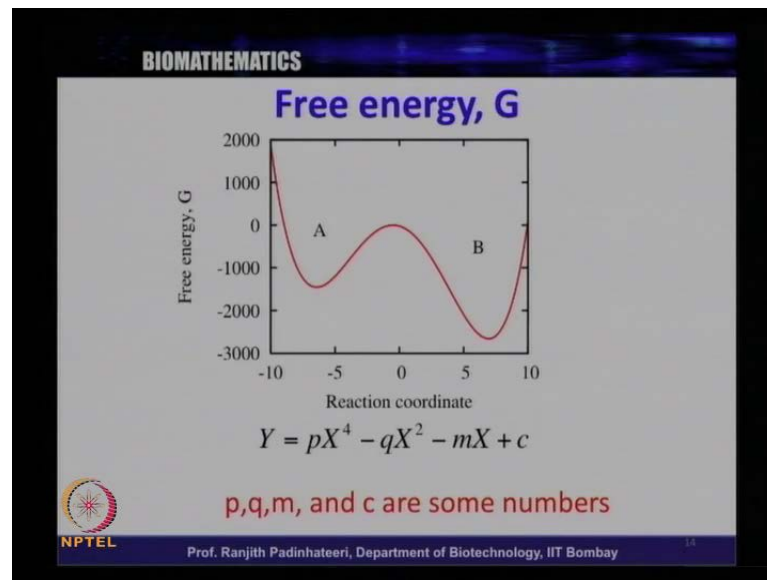
May in most part of India, in May, the summer is in its peak; and, it is the minimum, the minimum temperature is somewhere, sometimes in Jan, sometime in January, the summer, the temperature has its minimum value. So, temperature is a periodic function over seasons and if you want to represent this phenomena of temperature, the natural phenomena, you can say that, this can be represented using $\sin x$ and $\cos x$; a combination of $\sin x$ and $\cos x$ will give you, will represent this phenomena of temperature rise; increase and decrease in temperature can be represented using $\sin x$ and $\cos x$.

Let us think of another function; another natural phenomena is biological clock. Like, as we get, like, for example, the, we get hungry, when it is noon, we want to eat; when it is night, again, you want to eat; and, morning again, you want to eat. So, depending on the biological clock, as a, as the time goes, the biological clock has a cyclic behavior; has a, has a behavior, which, periodic behavior or an oscillating behavior. So, for example, insulin secretion is some protein expression, gene expression related to biological clock and this is a periodic function; it is like a oscillating function. So, again, if you want to represent this insulin secretion, one can use this mathematical functions of $\sin x$ and $\cos x$. The cyclin activity, the activity of this cyclin, is also a periodic function, an oscillating function.

In other words, any protein, for example, the expression of many proteins, if you take over cell cycle, it could be again a periodic, oscillating function. For example, if you have seen histone, look at histone expression, or any protein, you take, in some phase, for example, histone in s phase, during, just at the beginning of the s phase, lot of histone is expressed, because in the expression, d n a replications happens and chromatin assembly has to happen. So, if you take histone as an example, at some part of the cell cycle, histone is expressed a lot and in some other part of the cell cycle histone expression is very little. So, the histone expression again, has a periodic behavior. So, there are many, many examples in Biology, where you have phenomena, which happens like, increasing and decreasing expression of genes, or increasing and decreasing activity or a periodic activity or an oscillating activity. So, we will, strictly speaking, there is some difference between periodic activity, with, compared to any other repeating activity; so, any other oscillating activity. So, we will come to all this, strict definition of all this, what is periodic and all that later, but as of now, in this lecture, it is enough to

understand that, $\sin x$ and $\cos x$ represent periodic functions; and, any periodic activity, any activity that repeats, can be represented mathematically using these functions $\sin x$ and $\cos x$. Now, we understand $\sin x$ and $\cos x$, let us move to some other functions.

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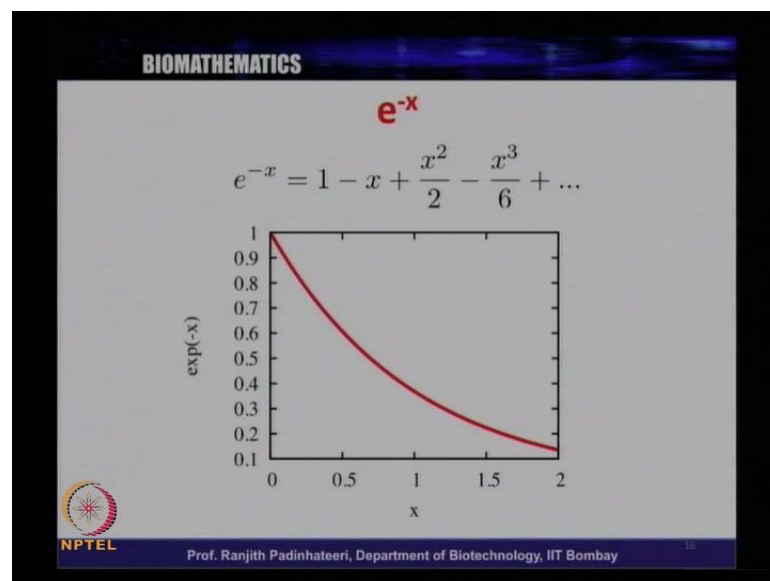
Some other, another example, which we see biologically, where you often, in Biology, we see this thing of free energy; in thermodynamics, we see free energy and any reaction, if you want to explain, we use this diagram - free energy diagram, as we see here. Let us...So, this is, has two minima and when we say, A going to B, so, let us...In any reaction, if you want to explain, let us say, A going B, we use this kind of a diagram. So, this is state A and this is state B. So, this protein having two shapes or a chemical A converting to B and back. So, all this can be represented and you see a free energy diagram, immediately associated with this, which has in particular shape. How do we get this?

This we can get by plotting a function, which is some constant p times X power 4, minus some other constant q times X power 2 minus, some other constant m times X, plus some other constant c. If we take, plot this function, we will get this. p, q, m and c are some numbers, here. So, take this different numbers for, take different numbers for p, q, m and c and try plotting this; and, you should be getting this kind of a curve, for some values of p, q, m and c. What we should learn from this is that, another curve, called free energy, that we see in Thermodynamics and in Biology, can be obtained, as a combination of X power 4, X square, X, etcetera. So, again, a combination of some of, or a combination of this functions, can give you another curve that you are familiar with.

Now, we saw many curves, let us also ask this... So, most of the curves we saw, e power x, for example, it is an increasing function, like, as x increases, e power x increases. Now, there could be some examples where things decrease. So, we saw that, e power x is 1 plus X plus X square by 2 plus so on and so forth. So, now, let us ask, get, let us get familiar with another function called e power minus x. So, let us have a look at it. This function e power minus x, which is essentially, exponential function itself, but instead of this x, wherever there is this x, put minus x there. So, if you, wherever there is x in this e power x, if you put minus x there, you will get e power minus x.

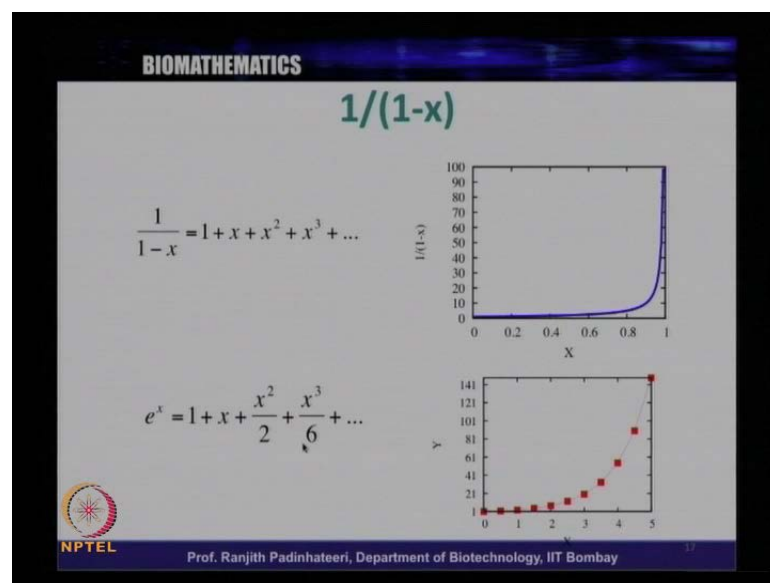
So, let us see here. So, what is shown here is, in the red, here, is wherever there was x, I put minus x there. Though, 1 plus minus X plus minus X whole square plus minus X whole power, whole cube and so on and so forth. So, if you do this, now, 1 plus minus X is 1 minus X; minus X square is plus itself, so, plus X square by 2; minus X cube is minus times X cube. So, this is minus X cube by 6. So, if you do this, you will get this. So, exponential of minus x is nothing, but 1 minus X plus X square by 2 minus X cube by 6 and so on and so forth. So, e power minus x is a mathematical function, which is again a combination of X, X, X square, X cube, etcetera, etcetera, etcetera. How does this, if you plot this graph, how does it look? Let us have a look at it. It looks like a decreasing function.

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So, e power minus x. So, we saw Arrhenius equation as A e power minus x. So, this is decreasing function. So, we saw some point of time that, r is A e power minus x minus E A by R T. So, the rate of a reaction, it decreases with, decreases with this factor, this; whatever in the bracket increases, whenever this thing in the bracket increases, this rate of reaction decreases. The higher the activation energy, the lower the rate of the reaction is. So, this is an exponentially decreasing function e power minus of something. So, similarly...So, this is same thing. So, we can see here, this curve, e power minus x decreases as the x increases. So, this is another example. Now, you could ask another question. So, we saw many combination. So, but one simple combination, we can think of, which is 1 plus X plus X square plus X cube plus X power 4, etcetera, etcetera, what does it, what does it give? This is simplest function; like, when we said e power x, we had some coefficients here, divided by something, divided by something; but without any of this, if we just write this, does this give anything? It turns out, that it does and so, let us have a look at this.

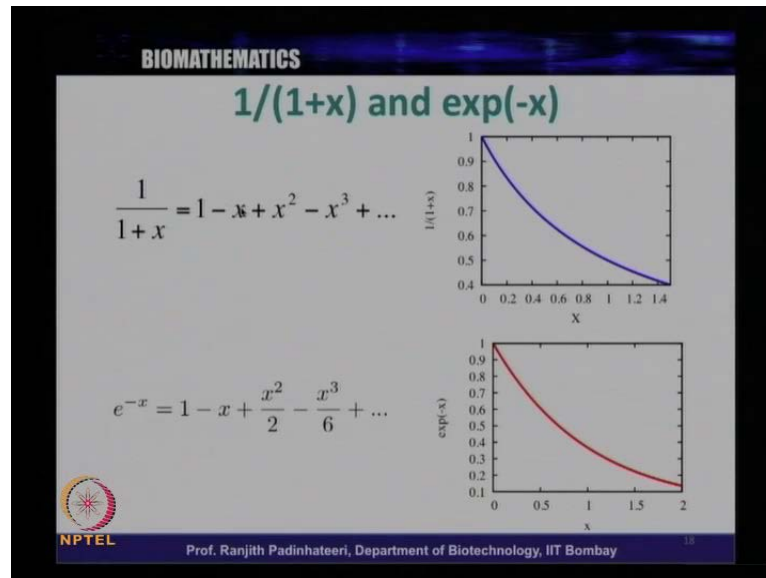
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So, this, if you add this 1 plus X plus X square plus X, X cube and so on and so forth, you get a function called 1 by 1 minus X. And, in the appearance, it is very similar to e power x. You can compare this two. Here, it is 1 plus X plus X square plus something, something; here, it is 1 plus X plus X square with some coefficient, X cube, with some other, with some other number, X cube with some coefficients. So, without this, this two functions looks similar and their behavior is also similar. Both of them are increasing

with x, very fast. So, again...So, this is a function, which increase with x. Similarly, you could also think of some function, which is 1 minus X plus X square. So, let us have look at the next slide.

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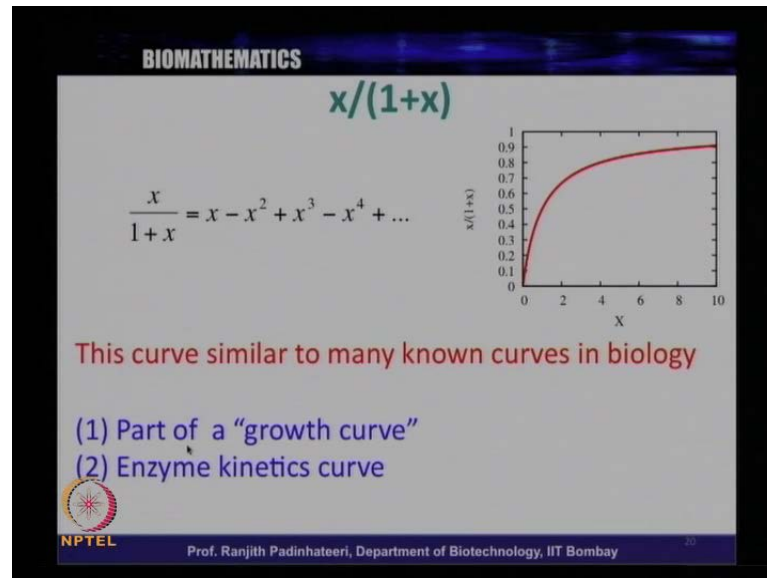


1 minus X plus X square minus X cube and so on and so forth. So, this function is called 1 by 1 plus X. This looks very much like an e power minus x. We said, e power minus x is 1 minus X plus X square minus X square by 2 minus X cube by 6 and so on and so forth. So, this two, if you compare, the only difference is, there are some coefficient of this e power minus x; otherwise, they look similar. And, their behavior is also similar. The both of them decrease, start from 1 and they decrease as we go along x. So, we saw that...So, these are another set of function, which look very similar. And, why am I coming to this? So, we saw, we have this function 1 by 1 minus X is something.

Now, if we multiply this with x, that is, if we write X by 1 minus X, what do we get? Or, in other words, let us, let us think about X by 1 plus X. So, let us look at the next slide which is...So, let us look, a slide here, which is X by 1 plus X. So, if we look at this function X by 1 plus X...So, let us have a look at this. 1 by 1 plus X can be written as 1 minus X plus X square minus X cube and so on and so forth; and, if we multiply both sides with X, you will get, X by 1 plus X; I do multiply numerator with X. So, you get X by 1 plus X; so, that means, I multiply all this terms with X. So, 1 times X is X; minus X times X is minus X square; X square times X is X cube. X cube times X is a X power 4.

So, then, you get this function, X minus X square plus and so on and so forth. And, when you plot this, this looks like this. Now, why is this important? This looks very much like some functions we have seen in Biology; that is why, why I came to this is that, this function X by $1 + X$ looks very much like function, which you saw in Biology and let us have a look at next slide.

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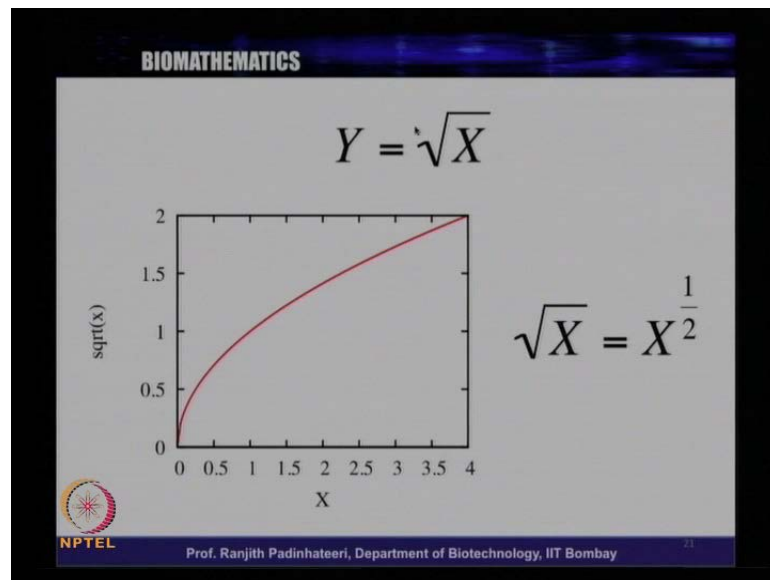


So, let us go back and remember, there is some function that we saw in Biology; you might have seen growth curve. Some part of a growth curve, or, also think about enzyme kinetics. So, how do they, how do they look like? How do those curves look like? So, let us, in this, look at, let us look at, look back to this slide. X by $1 + X$ is increasing and going to, appears like its saturating; its slowing down. Now, this growth curve does something like this. It is going to saturate. It is a part of a growth curve. Also, in enzyme kinetics, you might have seen Michel's menthe's reaction and so on and so forth; as such reactions and then, the questions for them look like X by $1 + X$ and the curve will look like this. So, what do you want it to convey you here is that, X by $1 + X$, again, can be written as a combination of X , X square, X cube, X power 4, etcetera, and this function appears, or, it may mix, many biological phenomena like growth, growth curve or enzyme kinetics.

So, we, by starting from X , X square, X cube, we came to a function, which $\left(\frac{x}{1+x}\right)$, which mimics a biological phenomena. Or, this biological phenomena of this particular enzyme

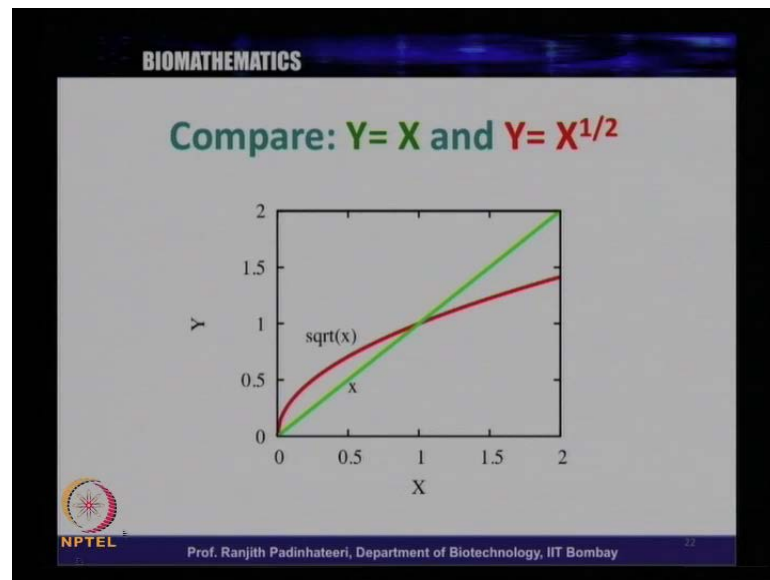
kinetics reaction, for example, behaves, as if it is a mathematical function. Think about it. It is a very intriguing thing that, a natural phenomena behaves like some function, that we learn in mathematics; it is intriguing. So, let us learn, now, more of this. This is one reason, why we need to learn all this function. Or, the advantage of learning of this function is that, this, by learning about this function, we can learn about this mathematical, this biological phenomena, or this natural phenomena. Once this phenomena, once we understand that, this biological phenomena or natural phenomena behaves, or this phenomena behaves, as if, like this mathematical function, we can understand...The more we understand the functions, the more we understand the phenomena.

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So, we will go and ((C)) and learn a few more functions. Another function, we know, we use this root x. Root x is also x power half. The root x looks like this, which goes very slowly.

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So, let us compare root x, square root of x with x. So, the green curve is y equal to x and the red curve is y equal to root x; and, if you compare them, you see that, you see that, root x goes slow, compared to y equal to x.

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BIOMATHEMATICS

Functions that decrease with X

- Height of a falling object decreases with time
- Concentration of nutrient in bacterial culture
- Free energy vs Temperature

$$\Delta G = \Delta H - T\Delta S$$

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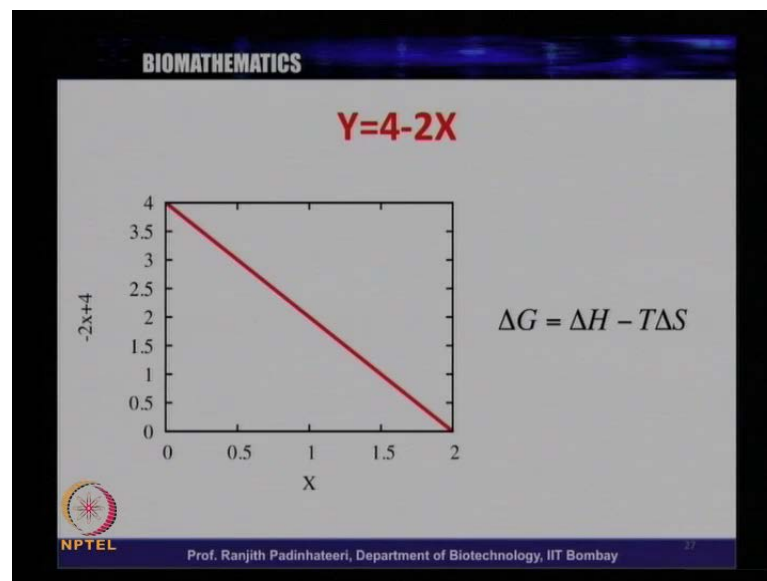
Prof. Ranjith Padinhateeri, Department of Biotechnology, IIT Bombay

So, this is another function that you use very often. So, you also saw some function that decreases with x, like e power minus x and so on. There are more function that decrease with x. So, think of height of, if you drop this object, from a height, the height of this falling object decreases with time. This is a function which decreases with time. You

could also think about growth; when you have this growth of, let us say, any microbes bacteria, yeast, and so on, the nutrient concentration is decreasing with time; another function, which is decreasing. You might have seen, delta G equal to delta H minus T delta S; this equation in Thermodynamics. The free energy is related to enthalpy and temperature and entropy etcetera. So, when you, more the temperature, that more, less than delta, the delta g decreases with temperature. So, let us...

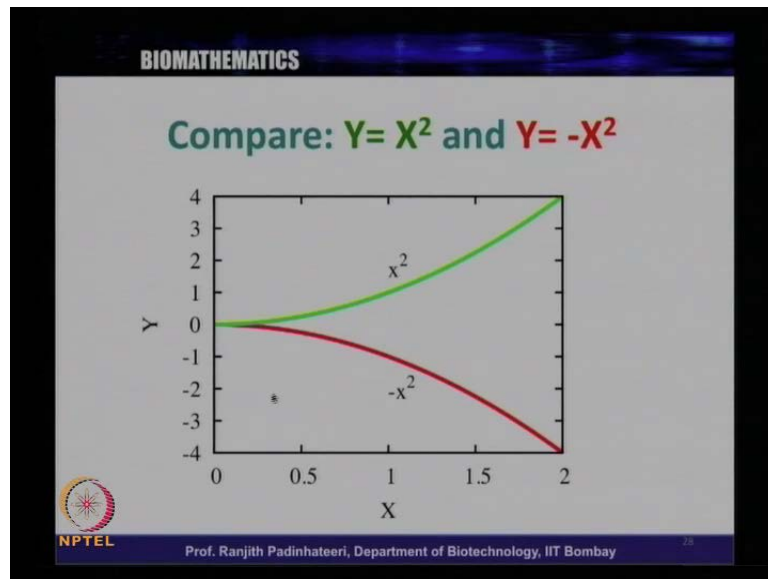
So, if you have such cases, we have decreasing function; the function that decreases with this parameter. So, let us learn a few functions like this. Let us get familiar with a few functions. Let us have a look at it. So, we learn y equal to X, which is a red line here; but if we take y equal to minus X, this is a decreasing function; this, as the X increases, y equal to minus X decreases. Similarly, y equal to minus 2 X. So, let us look at this, y equal to minus X is red curve; y equal to minus 2 X is another, the green curve, which is decreasing faster than minus X. So, let us look at another example, y equal to 4 minus 2 X.

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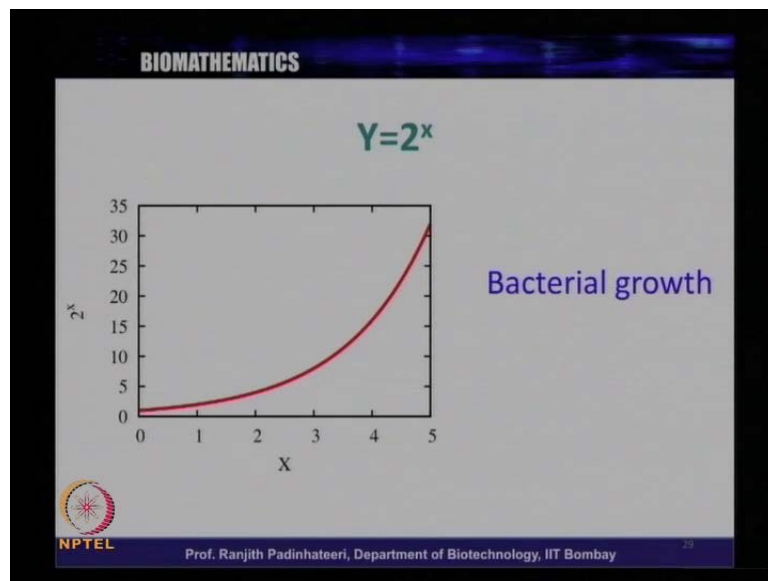
This example looks like delta H minus T delta S. So, we have some value for delta H, which is 4, and minus 2 X is like minus T delta S. So, if you plot this again, it decreases; it is a function, which decreases. So, we will... So, such functions are useful in different context and we will learn about this; we will learn about, more about this functions later.

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Another function is, Y equal to X Square and minus X square; if we compare, minus X square decreases with X. So, you have many such functions, which decrease in time.

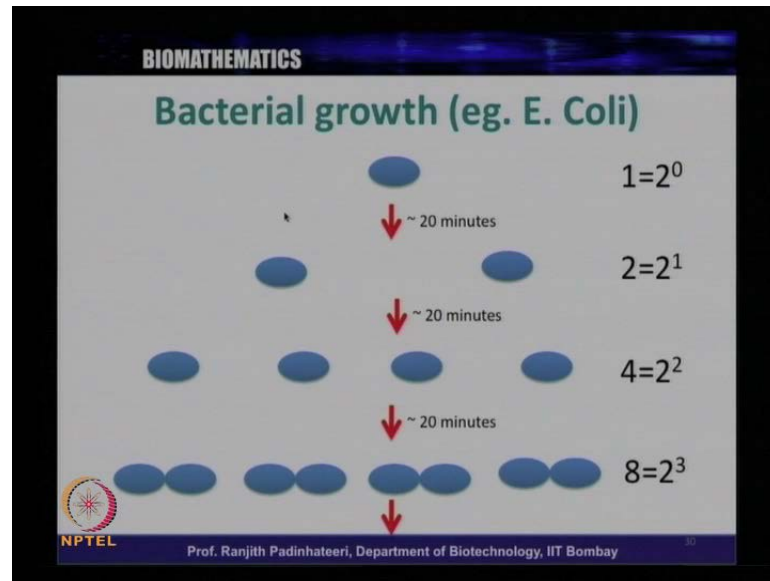
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So, now, let us look some more functions, which is, where you often seen in biological systems. So, let us have a look at function called 2 power. So, we have, what we have seen so far is, we saw e power x and we saw that, we told that, e is some number, 2 dot 2.7, something, something, something. So, e is a number and this number power, number power x is a function, that we saw. Now, let us have a look at another function. If this, if we can have a number power x, you could, you could imagine something like, 2 power x; 2 is a number and 2 power x. So, this is a function. So, y of x is equal to 2 power x. So,

let us have a look at this function. So, how, this slide shows y equal to 2 power x ; just like e power x , it increases. Now, why did I say this 2 power x function, because, very well known phenomenon of bacterial growth in Biology, behaves in, like...So, the number of bacteria in this bacterial growth, it is like 2 power x function. So, let us have a look at, how exactly this bacterial growth is 2 power x , similar to 2 power x .

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So, let us have a look at this slide. So, what happens in a bacterial growth? You have, for example...Let us discuss example of e coli here. So, let us say, you start with one cell of bacteria. So, you have number of bacteria one here. In about 20 minutes, they divide into two cells. So, we have, this is the first generation, this 2 power 1 . So, you have 2 cells. In again another 20 minutes, each of this cell, divide into 2. So, this get, becomes this 2 and this becomes, this 2. So, you have 4 cells. So, 2 power 2 , in the second generation, it is 2 power 2 , 4 cells. Again, in another 20 minutes, all of them, all of this, each of this cell divides and you get 8 cells. This gives 2; this gives 2; this gives 2; this gives 2. So, you have, total 8 cells here; 2, 4, 6, 8. So, in the third generation, you have 8 cells here. Now, this 8 is 2 power 3 . So, in each 20 minutes, it is just dividing, each of them is dividing by a factor of 2. So, 2 power something, gives you the number of cells in, after a time. So, in 60 minutes, you have 8 cells.

So, let us write it down here; let us write slowly and understand. So, in 20 minutes, you have 2 cells. So, at time equal to 0, you have just 1; 0 minute, you have just 1 cell. In 20

minutes, you have 2 cells. In 40 minutes, you have 4 cells; in 60 minutes, you have 8 cells. So, how do you describe this in, mathematically? The way to describe this mathematically, is seen in the next slide.

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The slide is titled "BIOMATHEMATICS" and "Bacterial (eg. E. Coli) growth". It states that the number of bacteria at time t is given by the equation $N(t) = 2^{kt}$. It defines k as the rate of cell division, typically $k = \frac{1}{20\text{min}}$ for E-coli. An example calculation shows that when $t=60$ minutes, $N = 2^3 = 8$. The slide also features the NPTEL logo and the name of Prof. Ranjith Padinhateeri, Department of Biotechnology, IIT Bombay.

So, you can describe this, the number of cells...So, let us have a look at this equation. The number of cells N of, as a function of time, N of T goes like 2 power $k t$. So, what is 2 power k ? k is a rate of cell division. For e coli, typically, k is 1 per 20 minutes; every 20 minutes, one cell division happens. So, that is called rate of cell division. So, if k is 1 over 20 minutes, after, when T is equal to 60 minutes, 2 power $k t$ is 2 power 3 equal to 8 . So, let us understand this a bit more. So, when...So, we said, k is 1 over 20 minutes. So, 2 power $k T$ is equal to 2 power k is 1 over 20 and T is 60 minutes. So, this is 60 minutes. So, 60 minutes by 20 minutes, you get 2 power 3 . So, this is the way you got this 2 power 3 , here. So, after, when T equal to 60 minutes, you have 2 powers 3 , 8 cells. So, you can, you can extend this and you can ask this question. After many hours, let us say, after 10 hours...So, 10 hours later. So, 10 hours is 60 into 10 is 600 minutes. So, the number, after 10 hours will be, 2 power 600 minutes divided by 20 minutes. So, this 2 power some factor and this will give you, this, **this** is 30 . So, this is 2 power 30 . So, there will be this many bacteria, after, this is a 2 power 30 bacteria after 10 hours.

So, this is, this is a, this is how, this mathematical function is useful in understanding biological phenomena, for example, bacterial e coli growth, as we saw here. We will

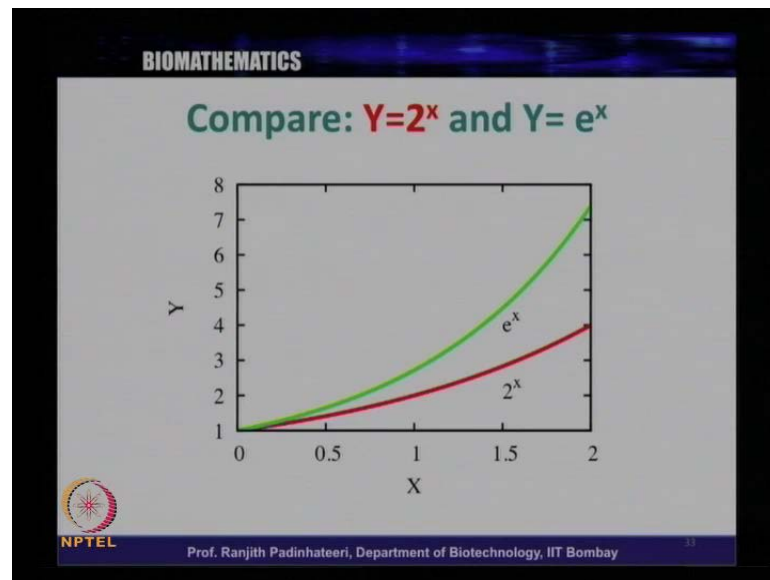
come and discuss this in great detail later, but again here, as a first part of this lecture, where we discussed functions, I just want to tell you that, the function 2 power x is useful in Biology and we should understand this later. Another example of 2 power x, as you can see here is, DNA in PCR cycle.

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The slide is titled "BIOMATHEMATICS" at the top. Below that, the main title is "DNA in PCR cycle". The text on the slide states: "Number of DNA molecules in a PCR cycle obey a similar equation". The equation $N(t) = 2^{kt}$ is displayed in the center. At the bottom left, there is an NPTEL logo. At the bottom right, the text reads "Prof. Ranjith Padinhateeri, Department of Biotechnology, IIT Bombay".

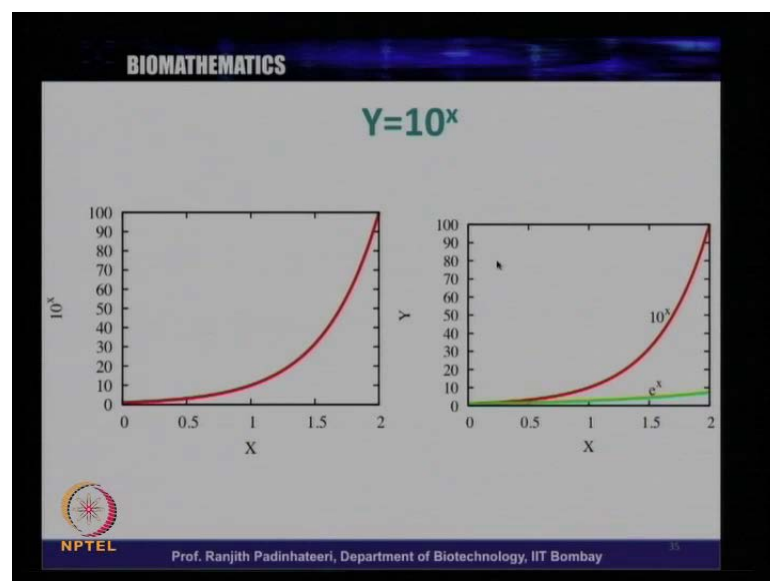
So, you all know what PCR is. PCR is Polymerized Chain Reaction and you can see, the DNA molecule is again, divided by each, when you replicate, you have DNA molecule increases. So, you have 2 DNA molecules. So, basically, you can again get 2 power k t, like this. Another example is this exponentially increasing DNA molecules. Now, let us have a look at another function.

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So, let us look at, how this 2 power x is compared. If you compare this 2 power x to e power x, how does, how do you get, how does it look like. So, e power x and 2 power x look like this. So, this red curve here is 2 power x, and the green curve is e power x. So, this is another example, where you have biological function, biological is, 2 power x is a function which is relevant in Biology and we will learn about this in great detail. So, and, you can see, now you can think of some other function 10 power x; y equal to 10 power x.

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You could have any number, 3 power x, 4 power x, 5 power x. So, how does, they all look like an increasing function and you see here, 10 power x is, this red curve and you compared with e power x is, grows much faster.

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BIOMATHEMATICS

Summary

- Periodic events can be represented using Sin(x), Cos(x)
- Functions that we see in biology can be written as combination of simple functions
- Many natural phenomena – Eg. bacterial growth – behave like mathematical functions

NPTEL Prof. Ranjith Padinhateeri, Department of Biotechnology, IIT Bombay

So, to summarize. So, let us summarize today's lecture. So, what we learnt today is, few things. We learnt many more functions. We learnt periodic functions like sin x and cos x and we said that, periodic events can be represented using sin x and cos x. And, functions that we see in Biology, can be written as a combination of simple functions. We saw X by 1 plus X written as some combination of X , X square, X cube, X power 4, etcetera. We saw exponential function. We saw few more other, few other functions like free energy written as combination of X power 4 and X power, etcetera. We also said, many natural phenomena like bacterial growth, behave like mathematical functions. Or, for example, rate of a reaction behave like a mathematical function. So, this is the summary of what we learnt today. And, in the next class, we will extend this for few more biological examples, and go ahead with new ideas from mathematics, that will be, that will be useful in, under, for understanding biological as well as other natural phenomena. Let us stop it here today.