Biomathematics

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Lecture No. # 30

Master Equation: Polymerization Dynamics, Molecular Motor Motion

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Hello, Welcome to this lecture of biomathematics. In this lecture, we will discuss something some application of few things that we have learnt so far. So, something we learnt in probability and something we learnt in transforms. We learnt about Fourier transforms and zee transforms.

So, how do we apply this transform, as well as some ideas from probability together to understand some of the biological phenomenon. So, that is, that will be the aim of this lecture. In the probability theory, we said that probability of finding thumbs in f v b we wrote we also learnt something about differential equations.

So, all of them, many things that we learnt so far will come together and we apply that to our problems in simple problems in biology and then we will see how we will use these ideas basically to solve this problem.

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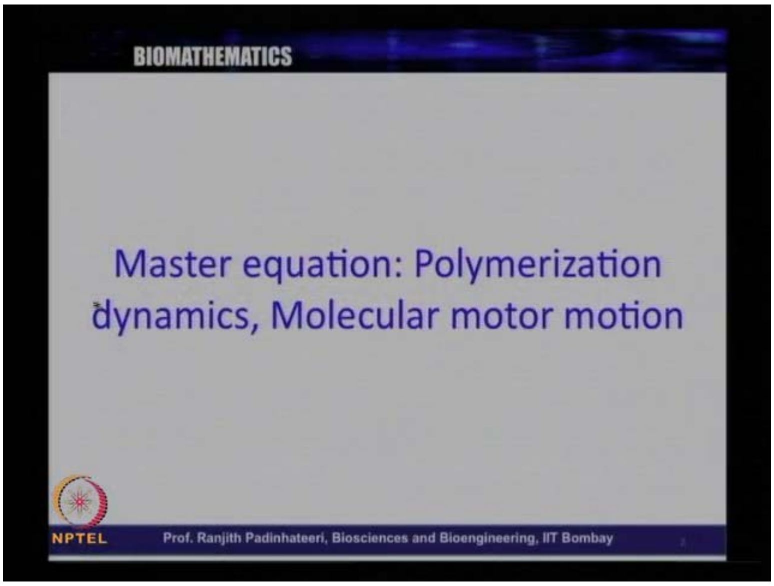

So, let us take some one of the examples that we learnt we came through before. So, let us take this example. So, what we will, the title of this lecture is master equation, polymerization dynamics, molecular motor motion. So, we will write an equation for understanding either polymerization or molecular motor motion and we will try to solve this, and while doing this we will use ideas from probability and differential equations and also from transforms.

One of the, some of the transforms that we learnt will be used here. So, this is also an example of how, when we this kind of transforms that we discussed were somewhere f of x is converted to some other f of k or g of k, can be used to solve differential equations.

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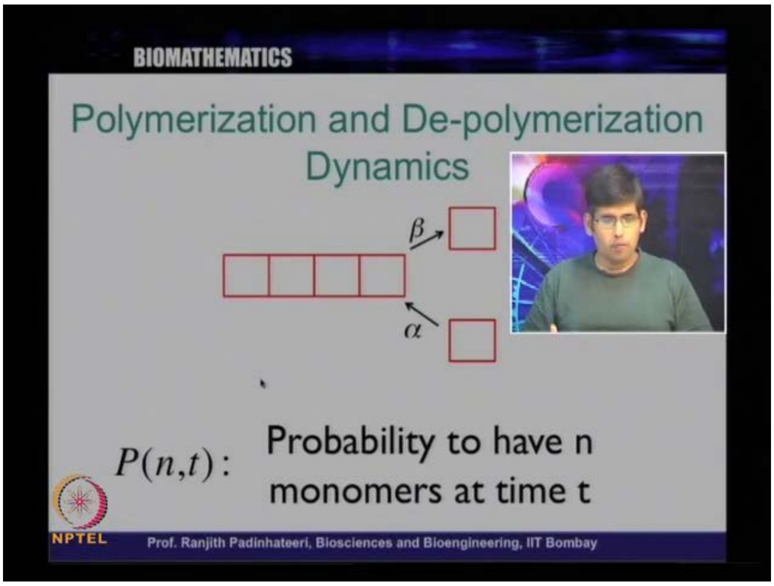

So, let us take this example of polymerization and de polymerization dynamics. So, let us say with some rate alpha. So, you have a you have a filament here and with some rate alpha you can polymerize, and with some other rate beta it can de polymerize. So, this is the simplest case one can think of polymerization and de polymerization. And let us define something about p and t where p and t is the probability to have n monomers on the filament.

So, you can have 1, 2, 3, 4 here. So, this 4 can increase to 5 or it can decrease to 3. So, P n t is the probability to have 4 monomers at any time t. What is its probability? So, this is that is what is defined by this P n t. So, how, how do we define this probability? So, one can think of basically.

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So, let us say you do an experiment, where you think the length of the polymer as a function of time. So, you start with, let us say some seed, than for some time then it suddenly increases, it increases remain same it can increase decrease it can.

Something like this can happen. It can polymerize like this and go. So, essentially length verses time can increase depending on the if the polymerization rate is more, but it can just keep changing with time. So, now, you do many experiments.

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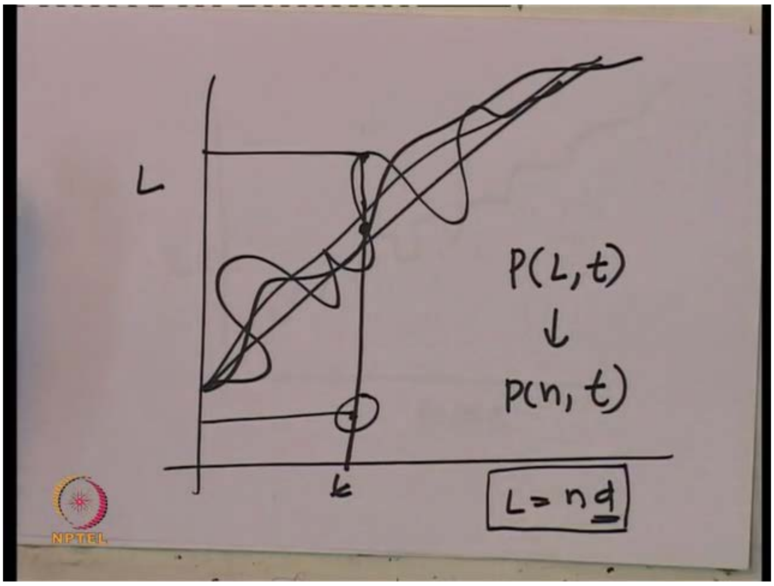

So, you can think of one doing many experiments. So, let us say one experiment have length as a function of time like this. Another experiment has length as a function of time like this, another experiment has length as a function of time like this. All of them on an average, same slope same increase, but you can imagine that the length, the length as a function of time can have different curves.

So, now, you can ask a question after a time t, what is the probability that you will find at a particular time t? What is the probability that you will find the length as exactly, let us say this.

So, this particular length, the probability here is very small; because all of this have only this. This is the probability here the most of the length is somewhere here. So, the probability to find this length, only one configuration here you can see that having this particular length. You can ask the question what is the length which is this. The probability to find this length is? In this particular case 0 because none of the configurations have this particular length and this particular time.

So, you can always imagine writing, probability that you will find a length L at a particular time t. So, this is same as P of n t, in other words; what is the probability we have n monomers at a particular time t. So, L is n times d, where d is the size of the monomer. So, if d is the size of monomer, L is n d and so you can instead of P of L t, you can also think of P n t, where probability to have n monomers at time t

So, you can see, you can, you can think of getting different values. Similarly same idea can be extended to molecular motors. So, let us say you have a molecular motor walking on micro tubules.

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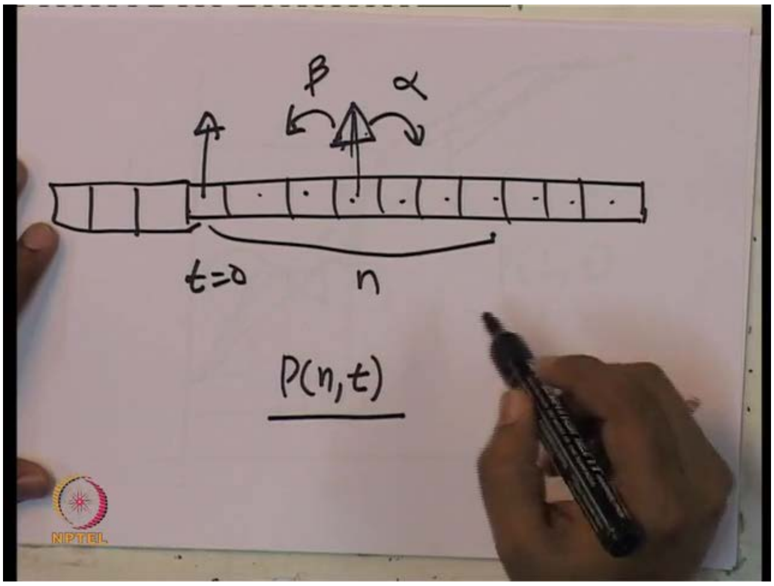

So, so let us say this is typically as you know like motors like kinesinmoto they walk on micro tubules. So, let us say this is micro tubules like a track. So, let us say you have a molecular motor here, a particular motor here. This can go here or go here. So, let us say you start from somewhere here at t is equal to 0. So, you have you start from here at t equal to 0 and you can ask the ask the question, what is the probability that it moved n distance and it moved n sides it moved n sides at a particular time t.

What is the probability that it moved at distance of n sub units at a particular time t? So, this is also the same way as polymerization de polymerization. At any point, it can go to the next one with some rate alpha or it can go to the previous monomer. It can move, it is moving along the monomer basically the molecular motor.

So, even for molecular motor we can define something called P n t, which is the probability that it would have moved at distance of n monomers in a time t. So, both for polymerization and de polymerization or motion of a molecular motor, one can define P of n t.

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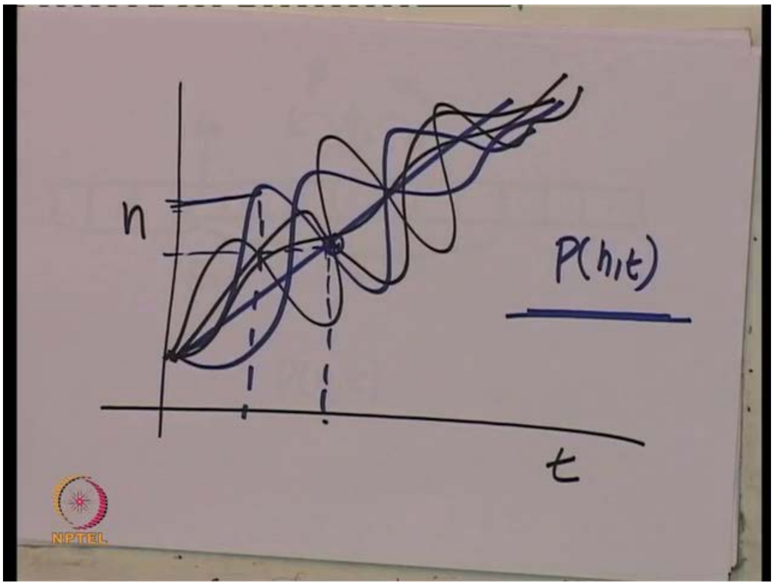

The same way as we have said, if you look at, if you ask the, if you plot how far the number of monomers it moved in a time, as a function of time. If you do one of these curve, you might get something like this. Another one experiment, if you do, one experiment let us say, you do, you fix a particular concentration of ATP, if a particular concentration of everything and then look at 1 molecular motor or 1 polymer or one filament and then you look at, how does the length change with time, or how does the number of the distances moved changes with time. It might get, in one experiment you might get this, in another experiment you might get something like this, in a different experiment you might get something like this, in a completely different experiment you might get something like this, in another experiment you might get like this. Even though an average you get this, but each time you might get a different path. So, what does that mean; that means, at any particular n and at a particular time the probability differs. So, here only 1 configuration has this particular n at this particular time. On the other hand here there are many configurations. At this particular time and at this particular length there are many configurations.

So, you can ask the question how many, how many times it gets this particular length at this particular time. So, from that you can calculate probability P of n comma t. So, in principle, one can think of P of n comma t, which is the probability to have n in the case of polymer is n monomers at a time t. There is a molecular motor probability such that the molecular motor has moved at distance of n sub units at time t.

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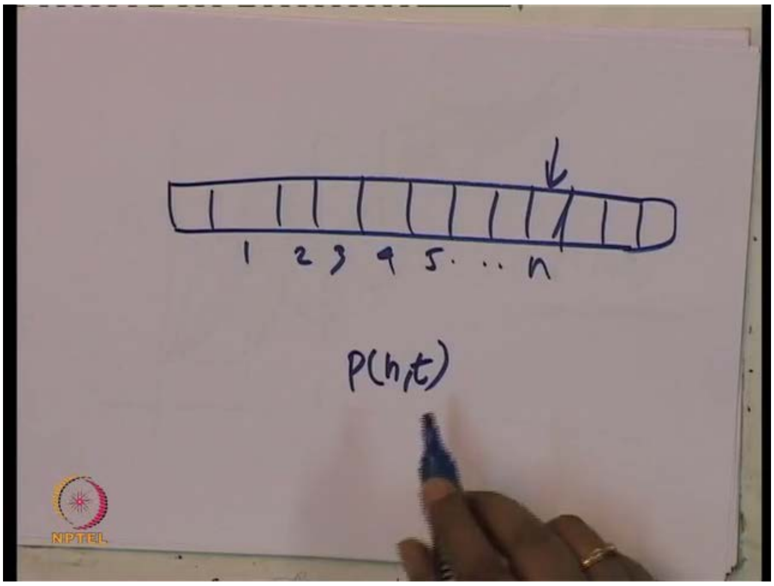

This is or you can the probability that you find molecular, you can also define a probability that you find molecular motors in the nth location. So, you can, you can think of this 1 2 3 4 5, like you can define, again this might be your n. So, what is the probability that you find molecular motor here at a particular time t.

So, that is P n t. So, we will mostly, so, just now how how the P n t does change? How does the P n t changes.

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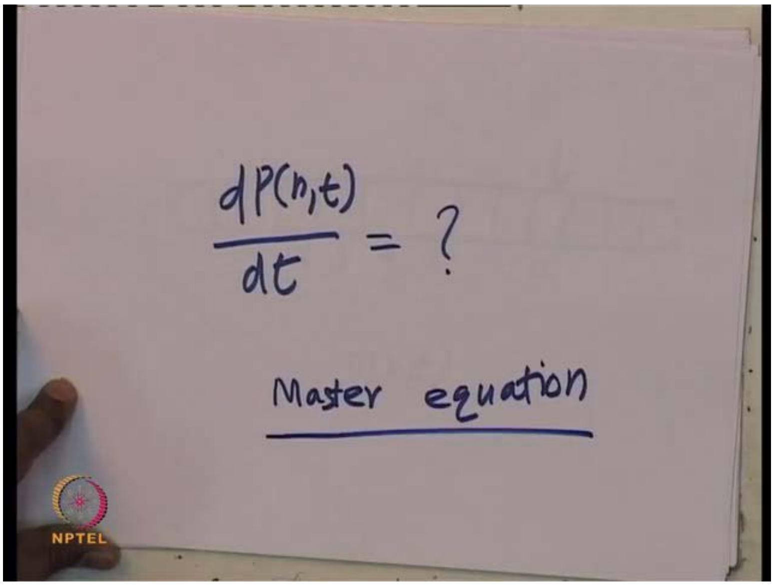

So, you can write something like d P n t by d t, the change in probability. So, the probability can change how howhow does, what is this, d P n how does the probability change with time. So, this equation for change in probability d P by d t, this is called master equation. So, if you write the equations of a probability, typically it is called master equation.

So, this is what we will discuss master equation. Master equations are typically equations for probabilities. So, we will write how this probability change with time. So, now, let us think of the first case for the for the polymer. If you look at here.

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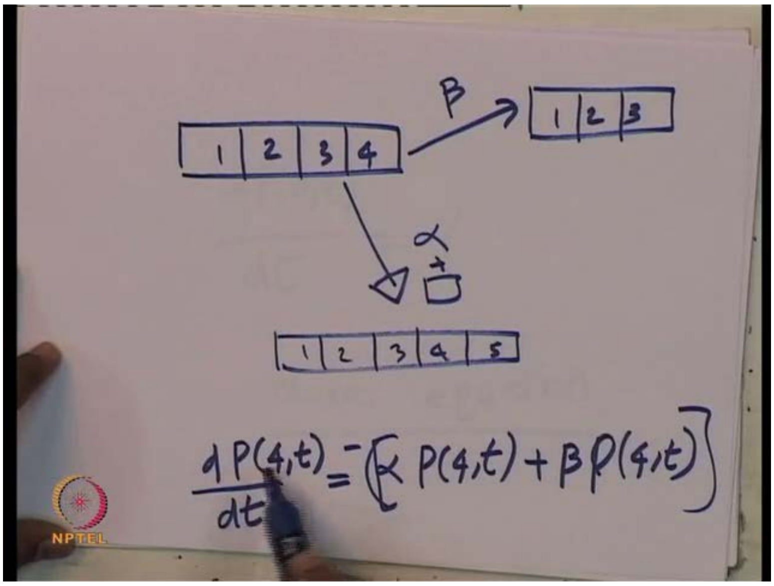

So, let us say you had 4 monomers 1 2 3 4. From this it can go to 5 monomers, 1 2 3 4 5 by adding a monomer. So, this is the rate alpha, it can change, it can go away, it can go to this, it can go to 3 monomers, 1 2 3, with beta; because where what you are doing is removing a monomer. So, this is you are adding a monomer. If you remove a monomer with a rate beta you can go to this kind of a filament

So, by adding and removing this P n t will change. So, if we had 4 of them at a particular time t this will change through alpha. So, by doing an alpha, this probability changes. With beta also the probability changes, but both of the, in both of the cases what happens is that, it is the probability decreases; because it is going away, from going away from this, 4 it is going to 5 here it is going. So, the P for t decreases. So, I have to put a minus sign here. So, in this case, in this both the cases we discussed here.

The P 4, the probability to find 4 decreases because here the probability from it goes from 4 to 3, here it goes from 4 to 5. So, in both these examples we discussed here this P 4 the probability to P 4 decreases because, if it becomes, if it polymerizes the P 4 decreases if it depolymerizes also the P 4 decreases.

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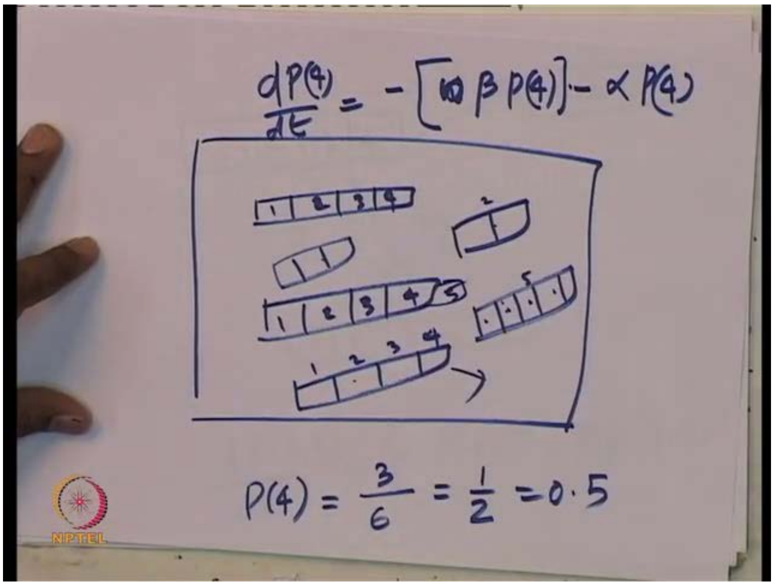

So, let us say in other words you can think of this particular way. You have a test tube which has like many filaments.

So, let us say there are here like many filaments having length. So, you can, you can think of like, this is 1 2 3 4, 1 2 3 4, 1 2 3 4. So, these are 3 filaments with 4 monomers. Now let us say there are some with 2, some with 3 some with 5, 1 2 3 4 5.

So, this is a 5, this is 2. So, there are all kinds of, all kinds of filaments having different length. So, there are, let us think of there is 1 total number of filaments is 1 2 3 4 5 6. So, there are total 6 filaments, out of that 3 is having 4.So, 3 of them have 4 monomers.

So, the P of 4 is 3 by 6, which is basically half. So, this is point 5. The probability in this particular example, the probability to have 4 monomers is actually half of them have 4 monomers. Now let us say some de polymerization happens in any of them. So, then let us say this de polymerizes, then what happens this becomes 3. If this becomes three. So, there is only 2 out of 6 you will have 3 4.

If the number of filaments having 4 monomers will decrease. So, then the probability becomes P 4 becomes less. So, essentially d P 4 by d t can decrease, if there is a de polymerization. So, which is beta and if you have to have 4 of them if, it is proportional to P 4; because if only there is P 4 probable with filament P 4, you can go away from it, you can also go away, you can also go away by alpha.

That means, if you have, if you polymerize any of these, numbers of filaments with 4 monomers will decrease because suddenly become 5, if this suddenly becomes 5, only 2 of them have 4 monomers. So, d P 4 by d 4 d t will decrease. The d P 4, essentially P 4 will decrease.

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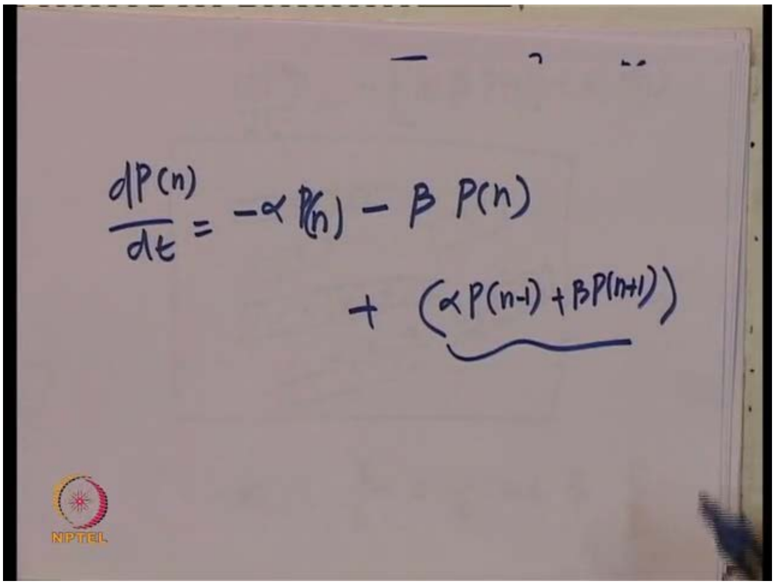

So, the master equation that we think of d P n by d t will decrease with alpha, will decrease with beta, plus there is few more things going to happen. So, what is this now? Let us say, imagine that there are, imagine that you have a case.

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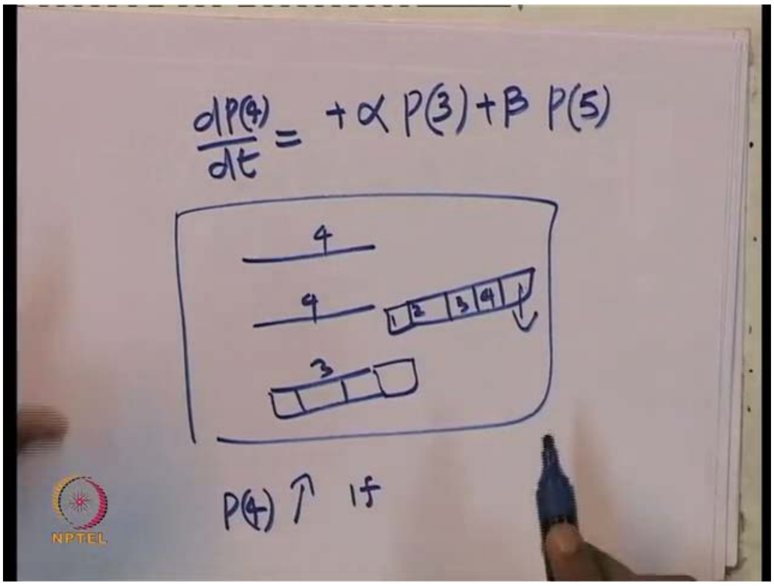

So, this has 4 monomers. This has 4 monomers, this has 3 monomers, this has 5 monomers.

So, if there are 3, 1 if there is a polymerization happens on a 3 monomer filament, this will become a 4 monomer filament. So, P 4 can increase if there is a polymerization on a filament having length three.

So, d P 4 by, can increase, it can increase plus; if there is a polymerization, that is alpha on a filament with 3 monomers. Similarly, let us say there are 5 monomer filament. If this de polymerizes, this will become a filament with 4 monomers. If there is a de polymerization happens on a filament having 5 monomers, the 5 will become four. So, this will be this will become a filament with 4 monomers. So, if at all if there is a de polymerization with a rate beta happens on a filament with 5 monomers, that also can increase P 4. Then if this happens also the probability to have 4 monomers will increase.

So, here the n can decrease with alpha and beta and it can increase given that a 3 comes to 4 or a 5 comes to four. So, this is n, this is 3 is n, minus 1 and this is n plus one. So, whatever here is alpha into P n minus 1 plus beta into P n plus 1. So, these are the only things can happen. It can happen, polymerization from n minus 1 if there is a. So, let me write this little more neatly.

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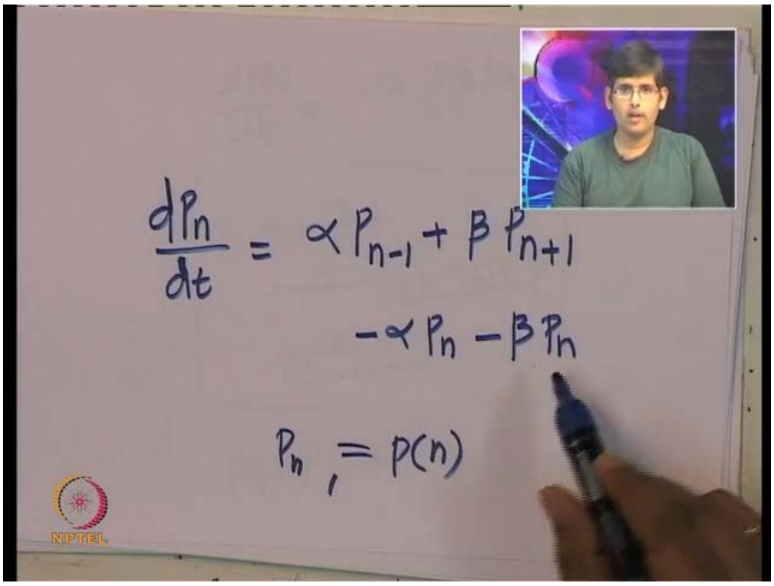

So, if you write this little more in a better way d P n by d t is equal to alpha into P n minus 1, plus beta into P n plus 1, minus alpha into P n minus beta into P n.

So, what does this mean? So, this is the master equation for, how does the P n changes with time. Probability to have n monomers, sometime I will write as P n, sometimes I can write P also write P n. So, both are equal I might use both the notations, but here I use this notation for simplicity. So, what does this mean? This means that, if you have a monomer, if you have, if you have the proper, if there are lots of filaments with n minus 1 monomers, they can polymerize with a rate alpha and go to P n.

If there are lot of monomers with n plus 1 filament, sorry there are lot of filaments with n plus 1 monomers, they can de polymerize with a rate beta and then go to d P n by P n. If there are many filaments with n, they can polymerize and go to n plus one; that means, it can go it can go away from P n. So, P n will decrease. If there are lot of filaments with n some fractions of filaments with filament n it can also de polymerize and go away from P n. So, P n will decrease.

So, this is the master equation for polymerization de polymerization of filaments. Same thing can be said of molecular motors. If you find a molecular motor at a particular position n.

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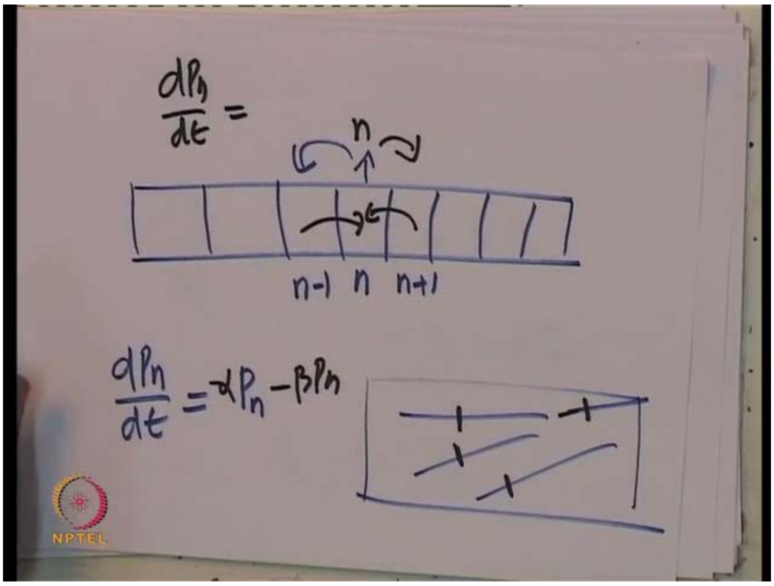

So, if you, so, this is n, this is n plus 1, this is n minus 1. So d P n by d t, if there is a molecular motor already here, that is, already probable P. It can go away and this P n probable P n will decrease.

So, if there are like imagine many many filaments or imagine same thing, like let us say there are many many many filaments. In all this filaments at a particular n. So, n let it be half, here if there is a molecular motor here. This is the position n on each of this filament. n could be like, like let us say tenth monomer.

So, if that a particular position ten, there is a, there is a monomer, there is a molecular motor this molecular motor can go away to n minus 1 or n plus one. So, the rate of going away could be. So, if it goes away then the P n decreases. If it moves here the P n decreases the probability to find a molecular motor at a position n decreases. If a molecular motors goes to this side also same thing happens. So, this is same as r minus alpha P n minus beta P n. Similarly, if you have a molecular motor here it can come here.

If there is a molecular motor, it can come here. So, this n can increase by a molecular motor moving from n minus 1 to n plus 1 or n plus 1 to n minus one. So, you can write d P n by d t is same, the same what we described here applies exactly, the same way to the molecular motor also. So, this is the master equation essentially.

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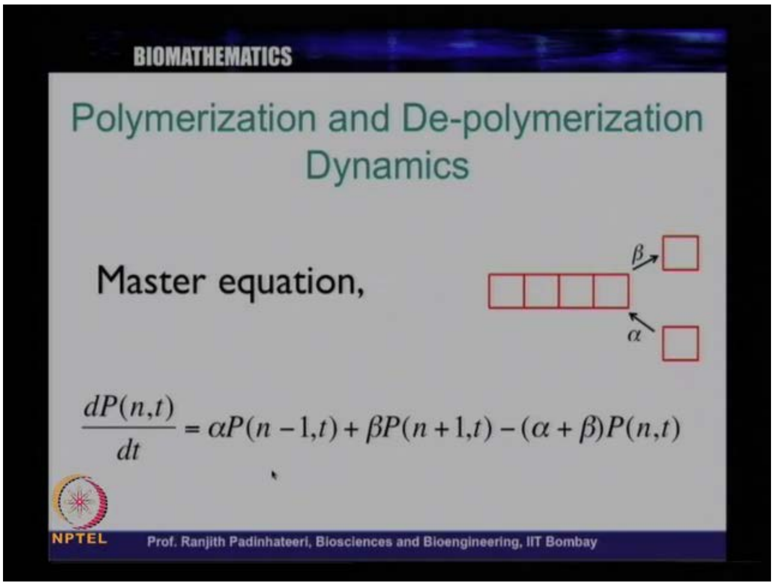

So, in the case of polymerization de polymerization, the master equation as you can see here is d P n by d t is alpha into P n minus 1 plus beta into n plus 1 P n plus 1 minus alpha plus beta P n.

So, this is the master equation. So, now, we have this master equation. Now the question is how do we solve it? How do we solve this equation? So, it is not that. So, the here is where we will make use of some of these transforms we learnt.

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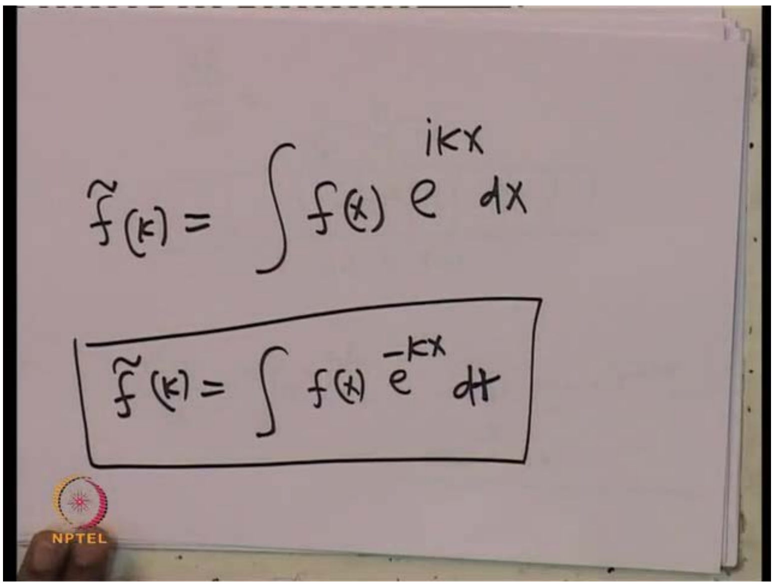

So, we defined Fourier transform in this particular manner, where we said, if you have f of x if you have a function f of x if you multiply with e power I k x and integrate over x and you get some other function.

Let me call this f tilde of k. So, I sometimes I call g of k f tilde. What does it only I means the some functions of k. Tilde only represents some different function f is different from f tilde. So, some different function of k you might get. We also said a zee transformation. You can also think of you can also think this writing in a different way. I can also write this as f of x e power minus k x d x. This is also, you can call this f tilde of k. Some other definition of transformation. The inverse transformation will be defined in a particular different ways for this there will be inverse transformation will different for this and this.

But we can define such transformation. So, today we will use 1 such transformation similar to this discrete version of this. So, what we will say is that.

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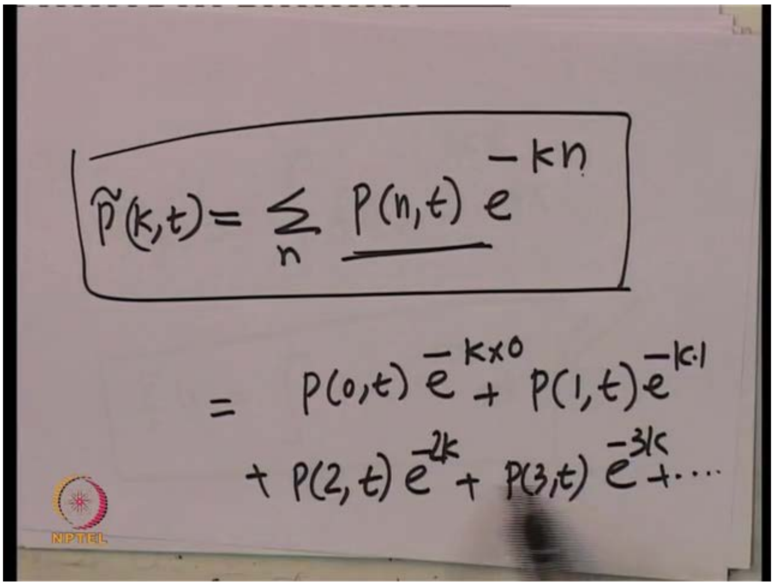

So, we have P n of t. So, we say that P n comma t e power minus k n, sum over n overall n like let us say minus infinity to plus infinity. Whatever be the n for the whole range. If you do the let us define this as P tilde of k comma t.

So, this is the definition where, if we can define such a function; we can make use of this and solve the differential equation in an easy way. So, we want to define a new function which is P tilde of k of t. So, if we know P of n of t, we can always calculate P tilde of k of t by multiplying with e power minus k n and summing over n. So, this is, this is, if n for various values of n, you can calculate probability to have 1 at time t into e power minus k, probability to have 2 at time t into e power minus k into 2 plus so on and so forth. So, you can expand this and define some particular P tilde of k which is in this particular manner right you can think of this like. So, let us say probability to have 0 at time t into e power minus k into 0 plus probability you have 1 at a time t into e power minus k into 1, plus probability to have 2 monomers at a time t into e power minus k into 2 plus, probability to have 3 monomers at a time t into e power minus 3 into k plus dot dotdot

You can do this sum and calculate this P tilde. So, if we know this p's we can always calculate this P tilde by this particular definition. So, this is the definition of P tilde of k.

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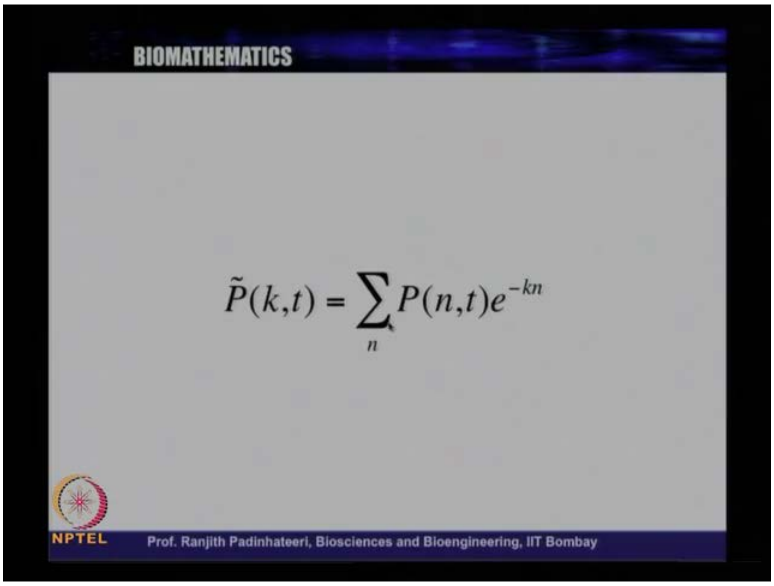

So, this what I have written here. P tilde of k is defined as sum over n P t e over minus k n. Now, what is our master equation? Our master equation is that

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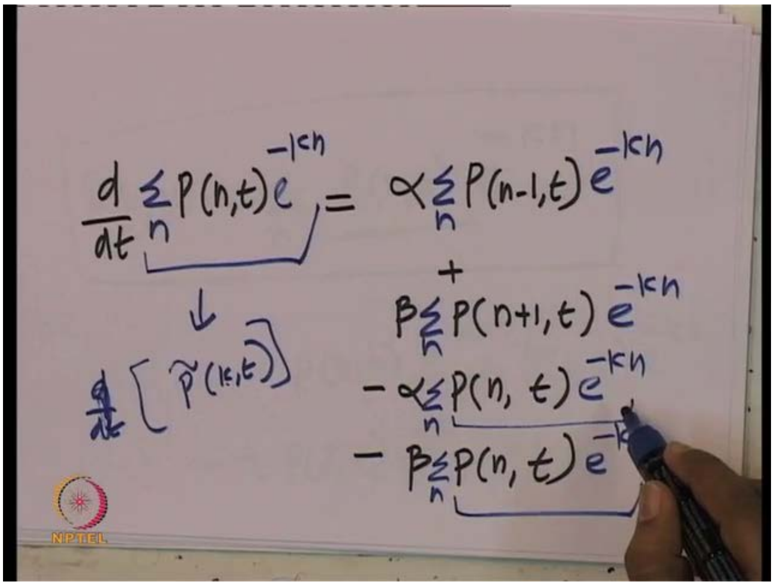

d by d t of P n comma t is equal to alpha into P n minus 1 comma t plus beta into P n plus 1 comma t minus alpha into P n comma t minus beta into P n comma t

So, this is the master equation that we wrote a minute ago. We can multiply throughout by e power minus. So, you can do a transformation for this, whole, whole thing. So, what you can do? We can multiply both everywhere by e power minus k n and sum over n. So, you can do sum over n e power minus k n sum over n e power minus k n sum over n e power minus k n sum over n e power minus k n. So, I do multiply everywhere with e power minus k n and sum over n both sides throughout the equation.

So, then what do we get? So, what is this? Sum over n P n t e power minus k n? So, as we defined here. So, if you look at the definition here sum over n P n e power minus k n is P tilde of k. So, sum over n P n e power minus k n here becomes P tilde of. So, this I can write as d by d t of this blue part alone. This part alone becomes P tilde of k comma t.

You and this becomes P tilde of k comma t. This also becomes P tilde of k comma t. So, these 3 terms. So, this is P tilde of k comma t, this is P tilde of k comma t, this also P tilde of k comma t. So, let us rewrite this a bit. So, here I write d if you look at here, here I will write. So, if lets carefully look at here. So, let us look at this equation little more carefully here.

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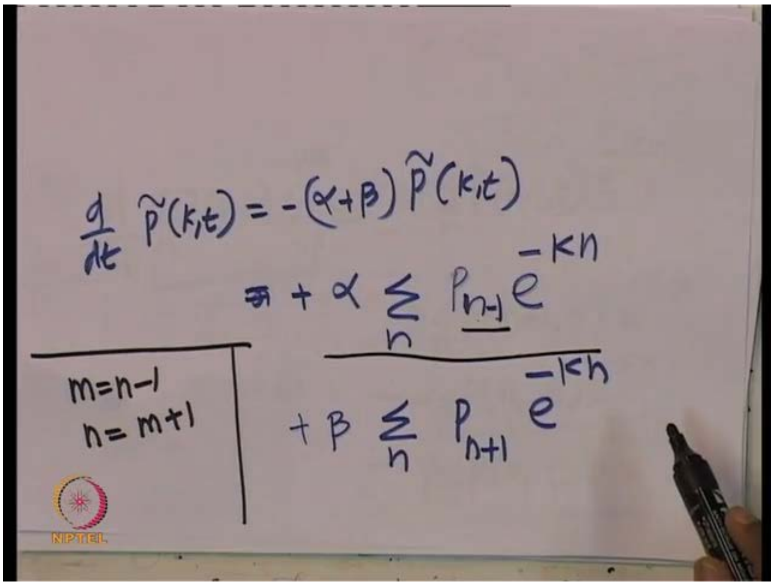

So, this term, I call this d by d t of, this is P tilde of k comma t. So, P tilde of k comma t. So, now, what is this term? Let us look at these 2 terms here. So, let us look at this term and this term. So, minus alpha into P tilde of k comma t.

So, that is what this term will give you and this will give you minus beta into P tilde of k comma t. So, there is P this and this is same. So, I can take it out in common and I can write minus of alpha plus beta. So, just look it carefully, you can write minus of alpha plus beta into P tilde of k comma t. So, this is these 2 terms. Now I have to do this 2 term, but thus those 2 terms. Let me write this particular way. So, this is this is plus alpha into sum over n P n e power n minus, sorry, P n minus 1 e power k n. Here it is plus beta sum over n P n plus 1 e power minus k n.

So, now this is. if this were n sum over n P n e power minus k n I could have defined it as P tilde of e power k minus n this is not n this is n minus one. So, I can do a small trick here. So, the trick is, let me define m is equal to n minus one. So, let me define n minus 1 as m. So, let me if I if we define. So, let us do a trick here. This is a small trick which we can use.

So, let us say if we define m is equal to n minus 1. What is n? n is equal to m plus 1. So, instead of. So, instead of wherever n, we I write m plus 1, wherever n minus 1 I can write m. So, this some I call it n minus 1. n minus 1 is say number anyway. So, I call this n minus 1 as m and. So, what happens to this term? So, let us look at this term only. If we look at this term carefully, what do we get is that; if we look at this carefully, what do we get is that, this term gives two. What does it give? Alpha sum over n.

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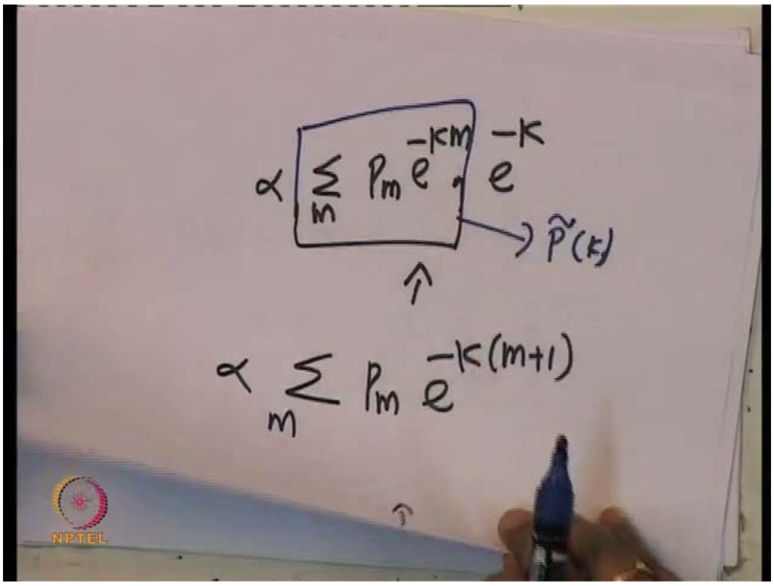

So, let us say n is now, this is become m, if n was infinity, it will become infinitive plus 1 or infinity minus 1

.So, infinity plus 1 or infinity minus 1 does not really mean anything, but infinity itself. So, this n and m it is just only by different by 1 and these limits do not change because they are infinities. So, then you have P of P of n minus 1 P of n minus 1 is m. So, let us write P of me power minus k n e power minus k n is m plus one. So, let me write m plus 1.

So, this term becomes alpha into sum over m P m e power minus k into m plus one. So, this can be written as alpha into sum over m. P m e power minus k m. P m e power minus k m into e power minus k. So, you can see this e power minus k m plus one. So, this just can be written as e power minus k. So, this can be written as this. So, this particular part, if you look at just this, this, this thing. This we can call again P tilde of k. So, this is our P tilde of k.

So, what you have? What you get there is alpha times P tilde of k into e power minus k. So, that is what, this term will end up. This term will end up as alpha into P tilde of k into e power minus k. If you do this, if you do this, if you do this trick of calling n minus

1 as m. since the limit does not change this is infinity and infinity plus 1 and infinity minus 1 does not change anything. This whole thing becomes P of P tilde of k e power minus k.

So, these whole things become P tilde of k e power minus k. So, similarly, I can also do a trick on this. So, the next term let us carefully look at it. So, the next term is b sum over. If you look at here the term is beta sum over n P n plus 1 e power minus k n.

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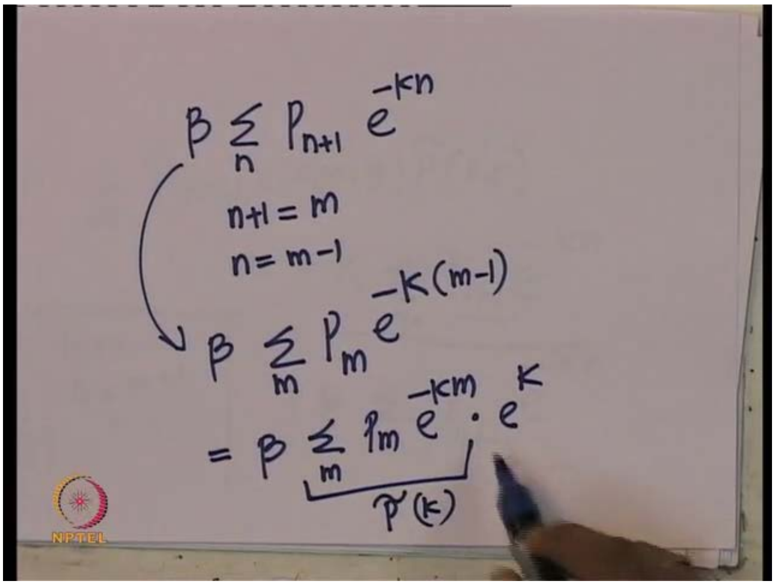

So, let me write that term here the term was beta sum over n P n plus 1 e power minus k n. So, here I use a trick, I will call n plus 1 equal to some m. I could call m or q or P or whatever another another integer. Then wherever I substitute n plus 1 I could put n and wherever; that means, n is equal to m minus one.

So, this whole thing becomes beta times sum over m P n plus 1 becomes P m e power minus k n becomes m minus one. So, this is m minus one. So, what is this? Beta times sum over m P m e power minus k m into e power plus k. So, this whole thing becomes P tilde of k again and this is e power k. So, this becomes P tilde of k e power k. Previously, we had that P tilde of k e power minus k. So, this time becomes P tilde of k e power k.

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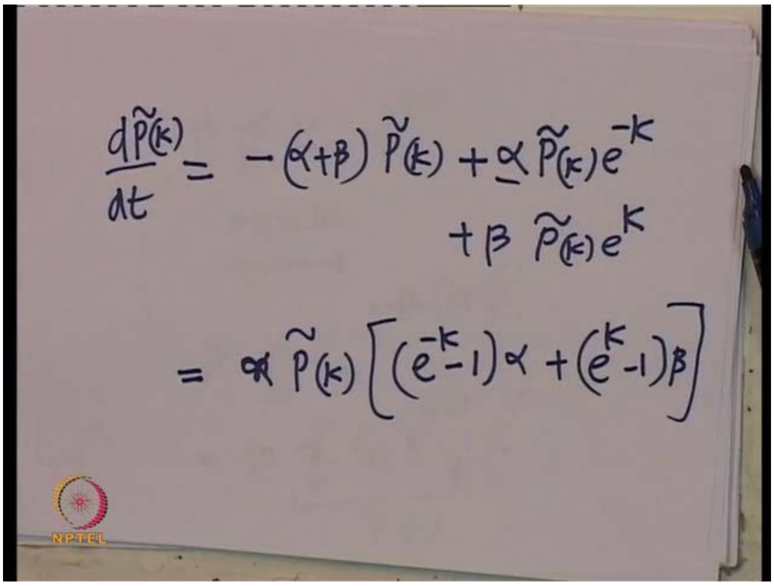

So, if you re write, if you consider whatever we have discussed so far and do it properly, take carefully and write everything; what you get is d P tilde by d t is equal to minus alpha plus beta into P tilde a of k. So, this is all this at a particular time t, plus alpha into P tilde of k e power minus k plus beta into P tilde of k e power k. So, this is what you will get. In other words I can combine this alpha term and this alpha term and write it as alpha sorry. I can write P tilde of k and there is e power k here and there is minus alpha e power k and there is minus alpha. So, I can write P tilde of k into e power minus k minus 1 into alpha plus e power k minus 1 into beta

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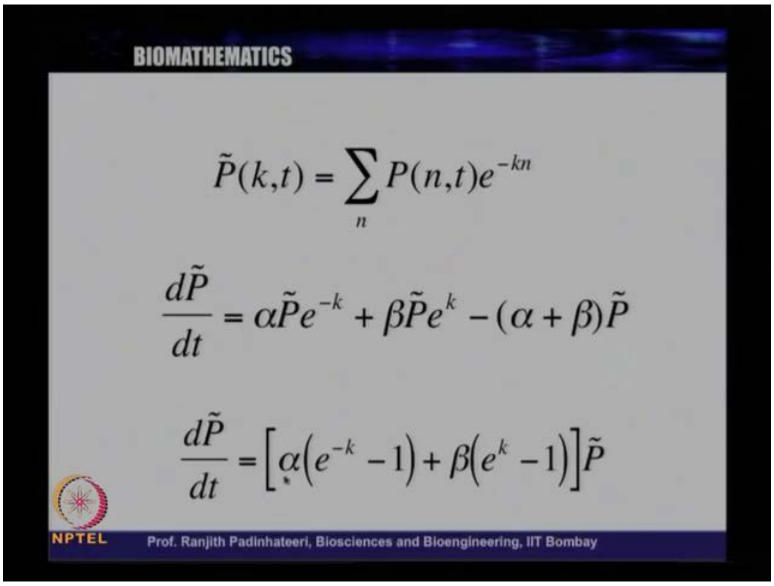

So, this is the equation you will essentially get. So, if you look at this carefully, if, if you carefully look at it, what you get is that. If you if you write this carefully what you get d P tilde by d t is alpha P tilde e power minus k plus beta P tilde e power k minus alpha plus beta P tilde of k. So, then as we just rightly wrote here this can be written in this particular manner. So, if you look at it what you essentially get is d P tilde by d t is some function of k times P tilde.

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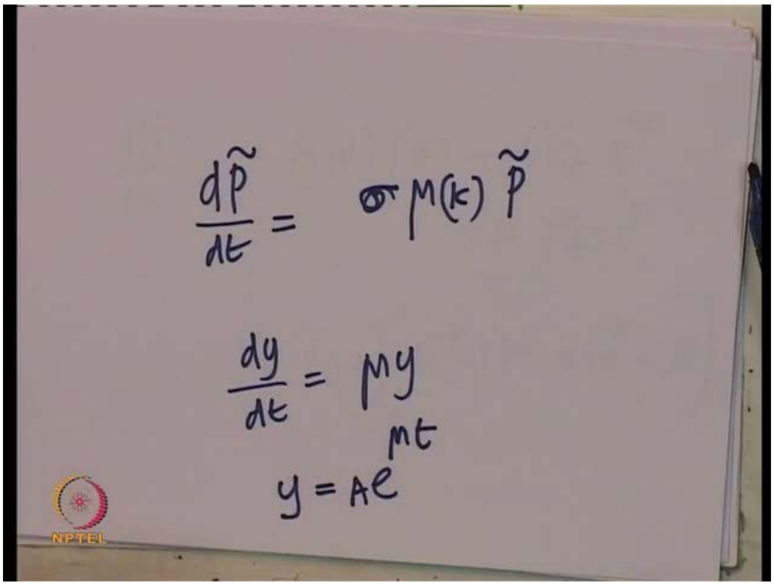

So, let me call this write this equation in a different way. So, d P tilde by d t is some function. So, let me call this some mu of k some function of k into P tilde. If you have such an equation, you can easily find the solution you know that if you have d y by d t is equal to some other mu times y, you know that this is the solution, which is y is equal to e power some y is equal to e power mu t with some constant a here. So, you can now get immediately this as a solution like this which we discussed some time ago in differential equations; because you can write this d y take this mu y down stairs, integrate both sides you will find this an exponential solution.

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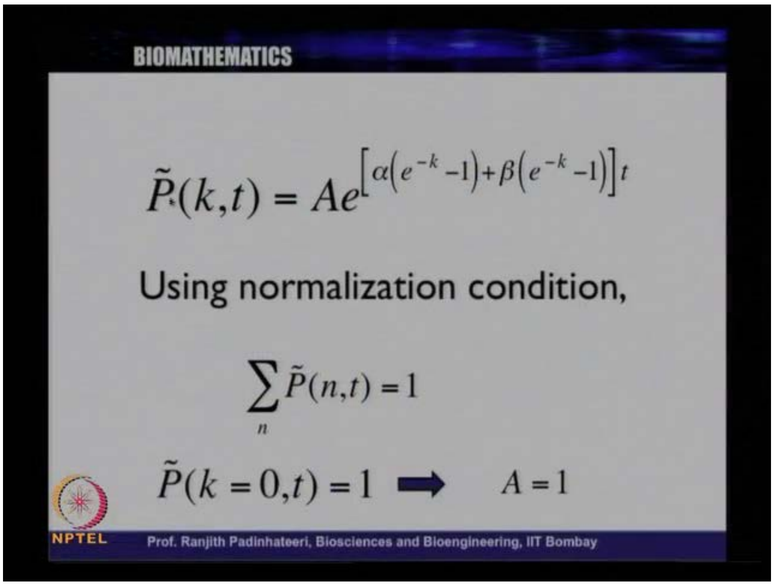

So, just similar to that here what you have here is just next 1 exponential solution P tilde of k t has an exponential solution e power this.

So, we could immediately solve this. It became a very simple equation. So, previously we had a complicated equation here; in terms of alphas and betas. That became a simple equation like this and it could easily find a solution. So, you find the solution which is P tilde of k of t is a e power a P tilde of k of t is equal to a e power alpha into e power minus k minus 1 plus beta e power minus k minus 1 times t there is a minus typing error this has to be plus k.

So, here it is e power plus k. Sorry here it is e power plus k and here it is e power minus k. So, this is what the solution you will get. Now what is A? How do we find out A? We can find out this normalization condition. The normalization condition is that if you sum over all probabilities you should get a one; that means, there is, now again another small typing error there is no tilde here it is sum over n P n has to be one.

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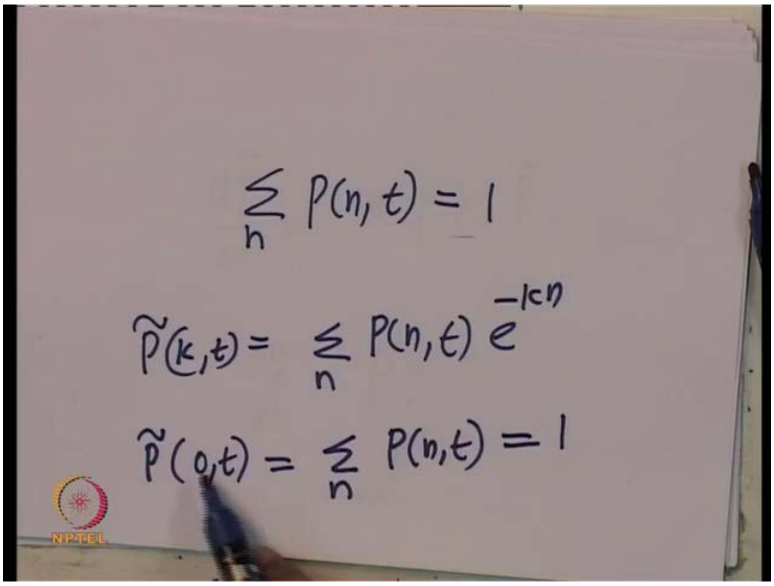

So, what we have is; sum over n P n at a particular time has to be 1, because at any particular time there will be probability to have 0 filament, plus 1 filament, plus 2 filament, probability to have 0 filament of 0 length, plus 1 length, plus 2 length, 3 length.

So, if the probability to have n monomers were any as at least the sum of all probabilities has to be one. So, if you use this. So, our definition of P tilde of k of t is sum over n P of n comma t e power minus k n. So, now, if you take k is equal zero. So, that is P tilde of 0 comma t what do you get you get sum over n P of n comma t e power minus k zero. So, this is one. So, this is just sum over n P and t and we know that sum over n P n t is one. So, this has to be one. So, P tilde of 0 comma t has to be 1 and we have a solution for P tilde of k comma t here

So, now if you put k is equal 0, this whole thing has to be one. So, if you put k zero. So, this is 1 minus 1 0 here also 1 minus 1 0. So, this is whole thing is 0 on that e power 0 you get. So, you get P tilde of k 0 comma t as a.

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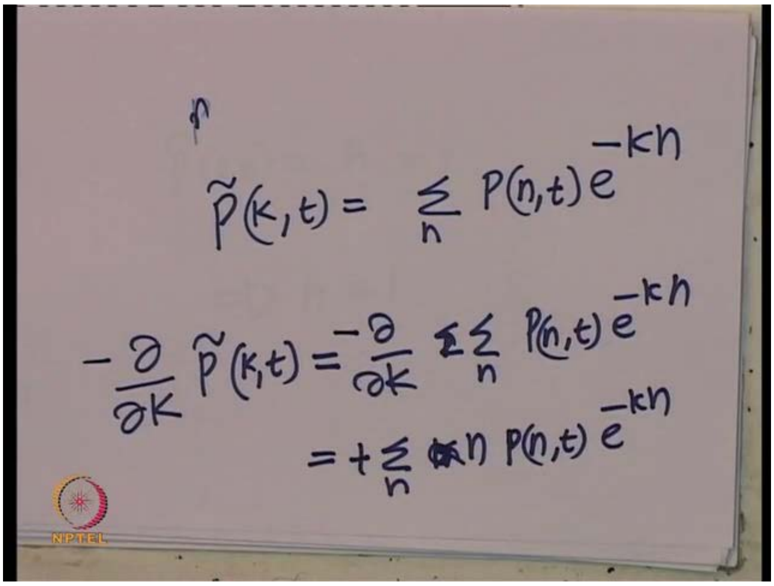

So, if you do this you calculation and P tilde of 0 comma t you will find that this is a and this has to be one. So, this implies that the normalization constant a is actually one.

So, that is what we find, that the normalization constants a here is actually one. So, we find this. So, we could easily solve it and we could find out this constant. Now if you really want to get P of n comma t, we have to do an inverse transformation of this we can do that in principle and get it. If it is an easy function you can easily do it. If it is difficult function one has to do or you can do numerically, but either way 1 can get an easy solution for this equation in the k form and you can always transform back this to the n form.

So, this is one idea of doing this transformation, but there is another interesting thing which you can see; is that if you have this definition of the, how did how we let us think of this definition of what we did. So, what we did was P of P tilde of k comma t was defined as sum over n P n comma t e power minus k n. Now, if I take a derivative on both sides with k. So, let us say I do a del by del k of P tilde of k comma t.

This is nothing, but del by del k of sum over n. So, sum over n I can take out side. Let me write here only sum over n P n comma t e power minus k n. I can take this outside. This can be written as sum over n and if you find the derivative of e power minus k n there is a minus k comes out. So, there is a minus there is a k sorry n will come out minus n. So, this is n P n t e power minus k n.

So, 1 derivative with respect to k will give you this. Now if you put a minus sign here you can put a minus sign here this will become plus. So, what you find is that minus del by del k P tilde of k comma t is sum over n P n t e power minus k n.

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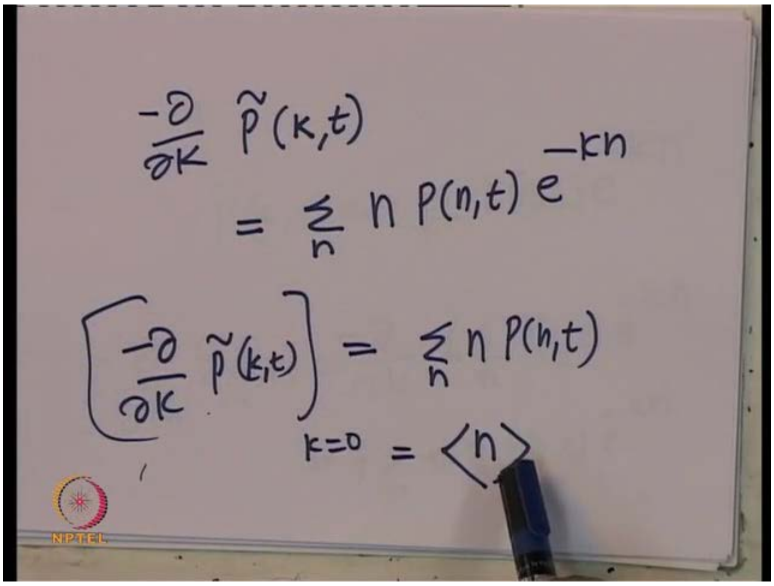

So, let me write this. This has a significance. So, what did we find? We found that minus del by del k of P tilde of k comma t is nothing, but sum over n, n P n t e power minus k n. n P of n comma t e power minus k n. Now if I put k equal zero in this. So, I calculate this at k equal zero. So, that is if I calculate del by del k of P tilde of k comma t and then put k equal to zero, apply k equal to zero. Evaluate this a t k equal to zero. So, then what you get is sum over n n P of n comma t and when k is zero, this is one. So, this is what you get. So, minus del by del k, if you find 1 derivative of this transformed version of the probability. If you find a derivative with respect to k, if the transformed version of the probability and you calculate this at k equal to 0 put k equal to 0. After doing this, what you get is sum over n n P n t. So, we have learnt that sum over x P x t is this is nothing, but n average. We learnt that in the time of when we learn probability, we learn that sum over n P n is n average sum over x P x this x average or integral as P x and d x is x average. This is definition of average n. So, this is nothing, but the average is number of monomers can be easily calculated by by finding the derivative of the newly found the transformed probability P tilde of k with respect to del k.

So, this we already found and by doing just 1 derivative we get the average number of monomers on the filament. Let us say this average number of the monomers can be depend on time, sometime it keeping the average can be small. Later after sometime the average can be high.

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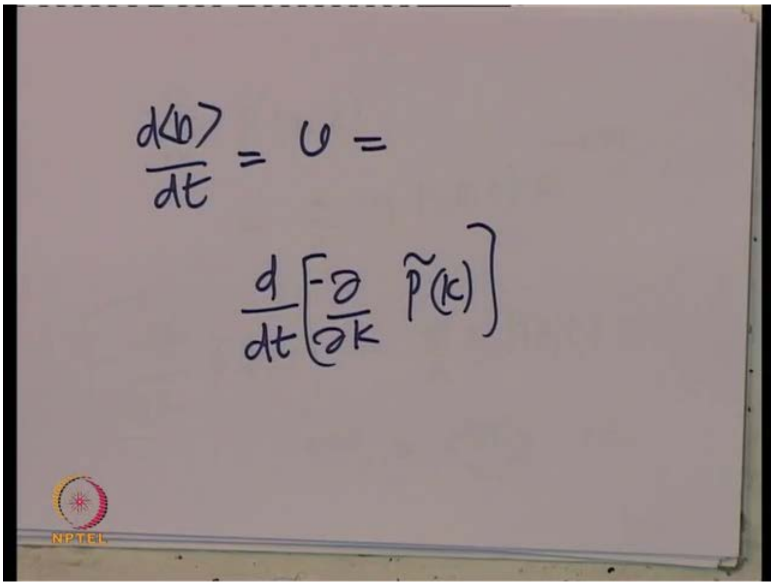

So, you can also find the derivative of with respect to time derivative. So, you can say d n average by d t is how the average number changes. So, we can call this is the velocity of growth and this can be found essentially shown as del by del k of minus del by del k of P tilde of k comma t.

So, in this way we can define the velocity. So, if we define the velocity in this particular way.

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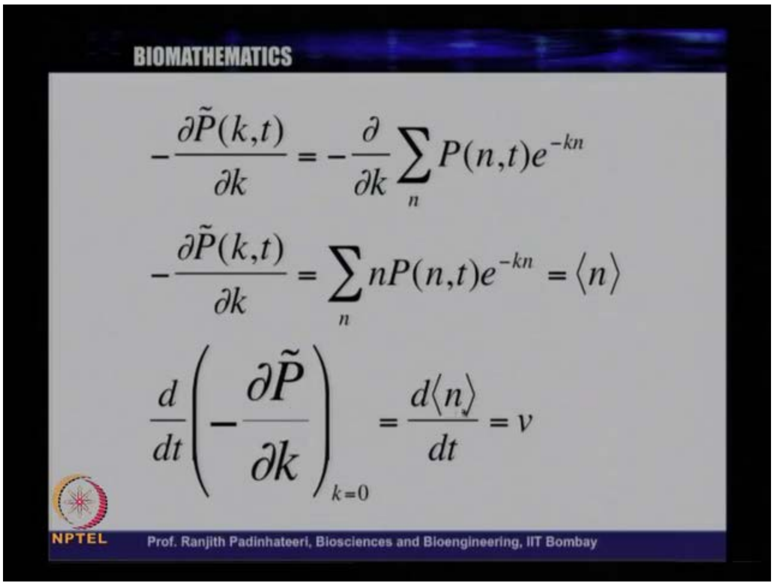

So, as we rightly as we just discussed now, we can write del d by d t of del P tilde by del at k equal to 0 as d n by d t and this will give you velocity. So, this is just we discussed. If you find the 1 derivative with respect to k, what you essentially get is n P n t e power minus k n and if you calculate this at k 0; there is again. This has to be evaluated at k equal to zero.

It will give you n average and then the this is what the correct statement is. My d y by d t of minus del P tilde by del k at k equal to 0 is average n average.

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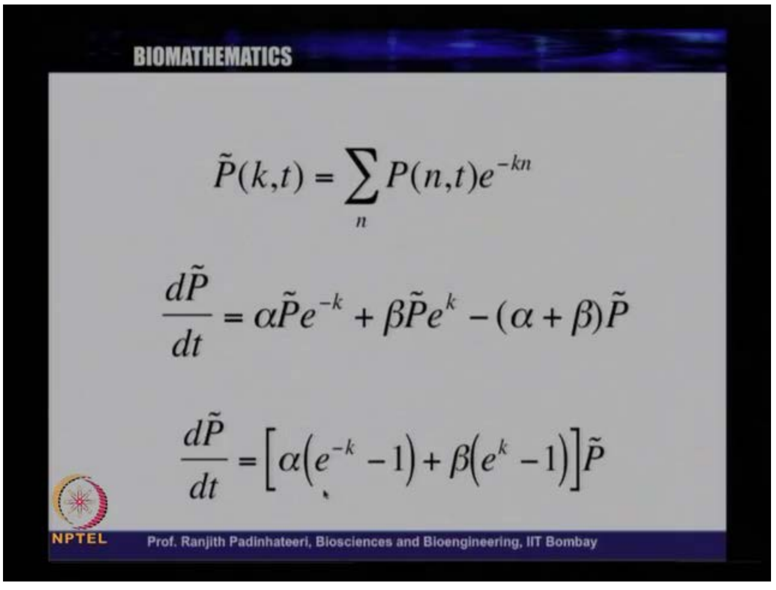

So, and we know this P tilde and then v can be calculated as d n by d t and d by d t of del P tilde by del k at k equal to 0. If we do that, we know that d P tilde by d t already from our equations and if we find del k by d t and evaluated at k equal to 0. We will find that v is alpha minus beta.

So, we can also calculate the velocity of growth or the rate the velocity of growth is basically polymerization rate minus de polymerization rate as you can see here. So, v you will end up finding that v is alpha minus beta. This is the rate of polymerization minus the rate of de polymerization. Similarly, on the case of molecular motor this is the rate of moving forward minus the rate of moving backward.

So, this is the velocity of molecular motor; which is the rate of moving forward minus the rate of moving backward. So, essentially what we quickly showed is that, if you use the idea that we learnt for probabilities and transforms; we can apply that for simple problems like for like v that the velocity of molecular motion of the molecular motors or the polymerization growth velocity, the average length of the filament, the average distance the molecular motors moved, etc can be easily calculated. So, we will learn more applications in the coming lectures. In this lecture we end up only with this. We will stop today’s lecture and we will discuss more in the coming lectures. Bye .