Biomathematics

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Lecture No. # 31
Simple Mathematical Models to Understand Evolution

Hello, welcome to this lecture on biomathematics. Today in this lecture, we will discuss some of the ideas that we learnt in the previous lectures, to understand some very simple models on evolution. So, whenever we think about evolution as you all know is a very important a very important idea in biology and people have tried to model mathematic making simple mathematical models to explain various evolutionary processes.

So, today we will discuss one of the very simple models. So, the title of today's lecture is simple mathematical models to understand evolution. So, when we say about evolution, the first thing comes to our mind is basically things that change from one generation to other generation. So, thing as the generations passed by something is changing. So, that is evolution. Now, we will think of in that context; what is the simplest thing that can change when the as the generation goes and how do we think of it mathematically. And one of the simplest model for this kind of from this kind of thought is something called Wright-fisher model. So, the first today we will be discussing first we will be discussing the Wright- fisher-model.

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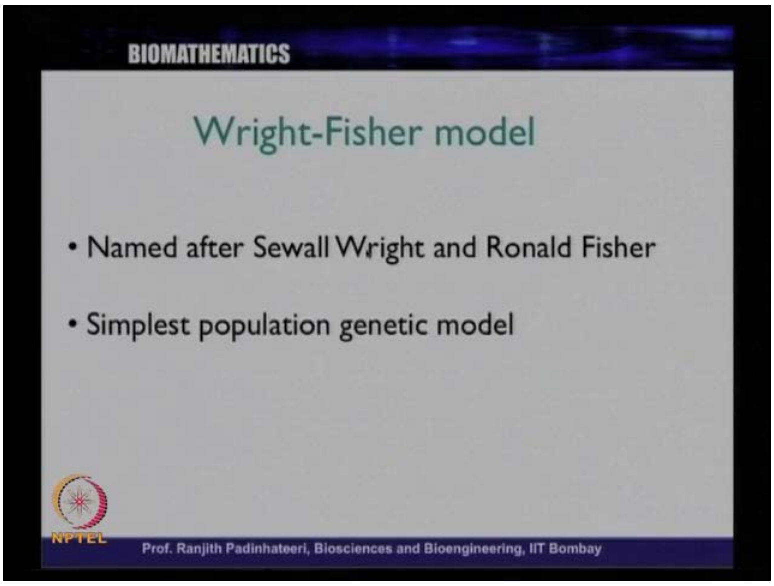

So, this Wright-fisher model is named after two individuals Sewall Wright and Ronald fisher. This is that's Wright from Sewall Wright's two persons two people like seawall Wright and Ronald fisher. And this is the simplest population genetic model. So, whenever we say think of models whatever mathematics we learnt and we want to apply it to various situations as we also said before what we should think is that what is the simplest of all things that I can think of and can I write it in a in a mathematical form. And, if we can write the simplest of all things then we can add a bit complexity and then write a mathematical equation for little more complex thing, or explain and I do mathematical analysis of a thing which is little more complex.

So, always even it is bit ideal. Even, if it is far from reality in modeling typically always we will start with the simplest of all models. Simplest of all we will make this simplest assumptions make the system very simple and we will start. So, today also we will do that. So, given that things happening after generation after generation what can we think of the simplest thing.

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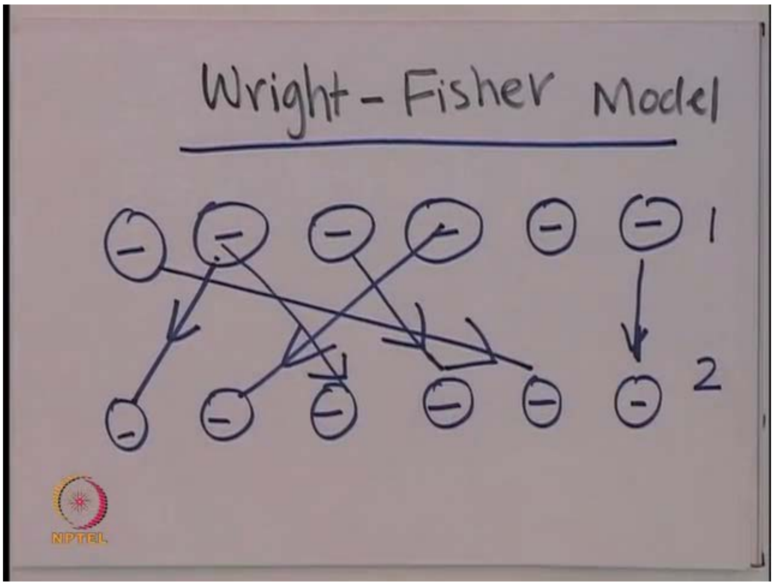

So, let us imagine so what we are going to discuss is Wright-fisher model Wright-fisher model. So, what is the simplest thing you can think of think of so think of there are N individuals. So, in our case this is 1, 2, 3, 4, 5, 6 let us see there are six individuals and let us assume that each of them have just one chromosome for simplicity. So, such things are called haploid individuals. So this is for simplicity. Let us assume N individuals. So, there are six individuals each of them have single chromosomes. So, this is the first generation in the next generation again let us assume that there are again six individuals this is the offspring.

So, this is generation one and this is generation two then so this is the first thing that we consider that there are N haploid individuals in our cases is 6 and assume that in the next generation we have again N see equal number of population the N individuals again. Now, this is basically offspring of this generation. So, who is the parent from here. So, you randomly choose the parent. So, each child will have the random parent. So, this basically this offspring individual will have a parent this is randomly chosen.

So, this is the random you choose randomly. So, this is the model you choose randomly. So, this is the randomly choose this and this will we have this is the parent of this particular offspring. Now, this offspring you have various options again randomly choose somewhere. So, this is the parent of this particular offspring individual. So, this is the next offspring individual again among this six of them you choose randomly. So, if you choose randomly sometime it can happen that you will end up with this individual with this particular parent twice, that can happen because you're choosing randomly, what for you know you have to do this. So, basically you close your eyes blindly put your finger on some of this one of this let us say you get this then this is your next individual next parent for this particular offspring individual.

Now, what is the parent of this so you again randomly choose it could be this. So, you randomly choose this and you this you got this as the individual. Now, for this you randomly chosen let us say you got this. So, each of this individual in the next generation as a parent and the parent is chosen randomly. So, this idea is called the Wright-fisher model. So, whatever we described now graphically is the essence of Wright-Fisher. This simplest of Wright-fisher model. So, let us see what all we did randomly.

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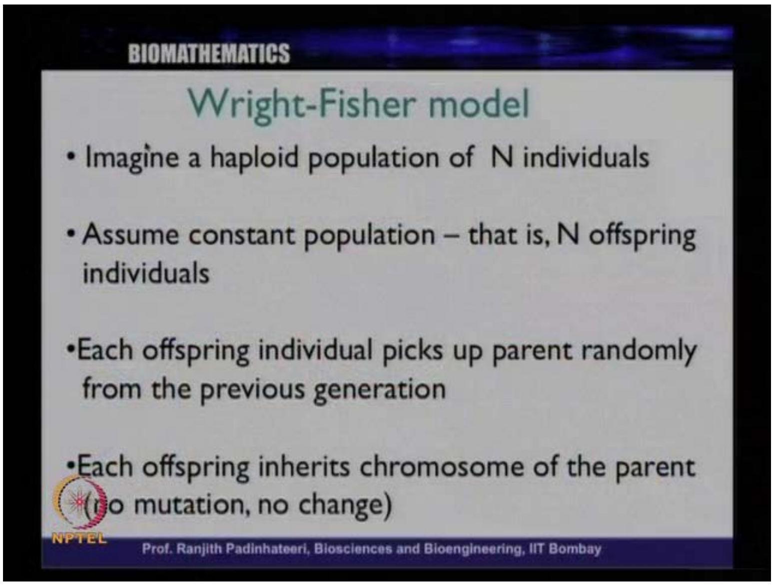

So, let us look here what did we did. We imagine that there is a haploid population of N individuals, haploid means in single DNA ah sorry there this only one chromosome one set and then assume that there is a constant population, that is there are N offspring individuals also, we consider here in our case, if you look at the paper here, there are if you look at here there are six individuals 1 2 3 4 5 6 and; six off springs 1 2 3 4 5 6.

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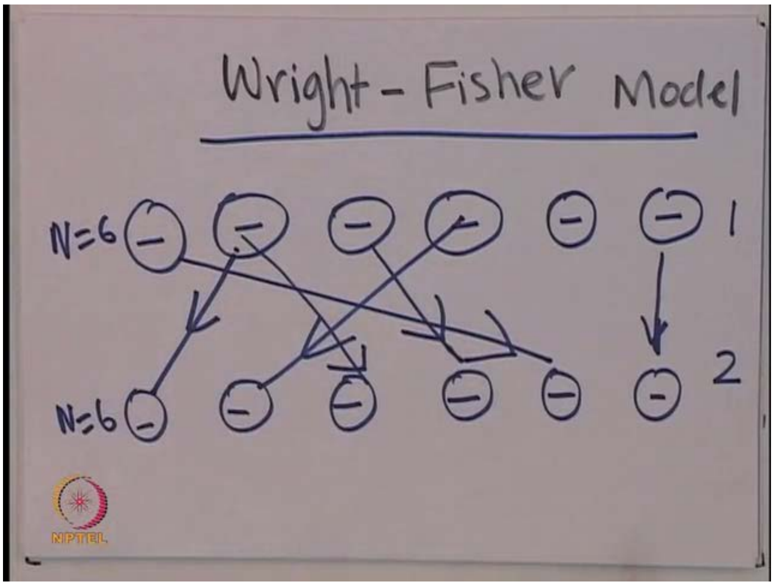

So, here N is equal to 6. So, this is the in both generations. There are equal numbers of individuals, the population is a constant it is not dying or there is no dying here. So, this is basically that is the second point here assume that constant population that is N offspring individuals; and assume that each offspring individual picks up parent randomly from the previous generation and that is what we again we did here in this paper that we said parent of this is offspring is randomly chosen again of this is randomly chosen.

It could be any of this; you could have drew you could draw another set of the same thing. So, you randomly choose this and the last assumption is that each offspring inherits chromosomes of the parent without any mutation without any change. So, there is no change that is the we should in reality of course, there will be mutations and all that but, for simplicity to have a simplest of all models we assume that there is no change.

So, with this assumptions can we learn something can we write down it is a mathematical formula for using mathematics can we predict something can we learn something that is the why we are choosing this very simple model because, even for this simple model can we make some predictions using the mathematical ideas that we learnt, if we can it is useful and if we cannot then there is no there is no hope for a complex thing when if we cannot even make a prediction for a simple model like this we have no hope of predicting the real complex models.

So, it turns out that we can do very pretty good prediction way very various interesting things for this particular model; and this model is very useful. So, let us look at this assumptions that we made for this model once more. So, first we imagined that there is a haploid populations of N individuals and we assume that is a constant population that is there is N offspring individual individuals also, that is N a parent; and N offspring; N parent individuals; N offspring individuals and each offspring individual picks up parent randomly from the previous generation and each offspring inherits no chromosome with the parent without any change, there is no mutation no change at all, So, whatever be the DNA here whatever be the chromosome of this particular parent it will be exactly transferred to this offspring whatever chromosome of this parent it will be exactly transferred.

But, look at here this particular parent has no child; this parent has two children or two offspring individuals. So, the chromosome of this increased in this generation this twice. So, whatever genes on this became twice here but, whatever genes on this is not here at all some genes are being increased; some genes are lost. So, this making some of the phenomenon that often is happening in evolution as we as the population change.

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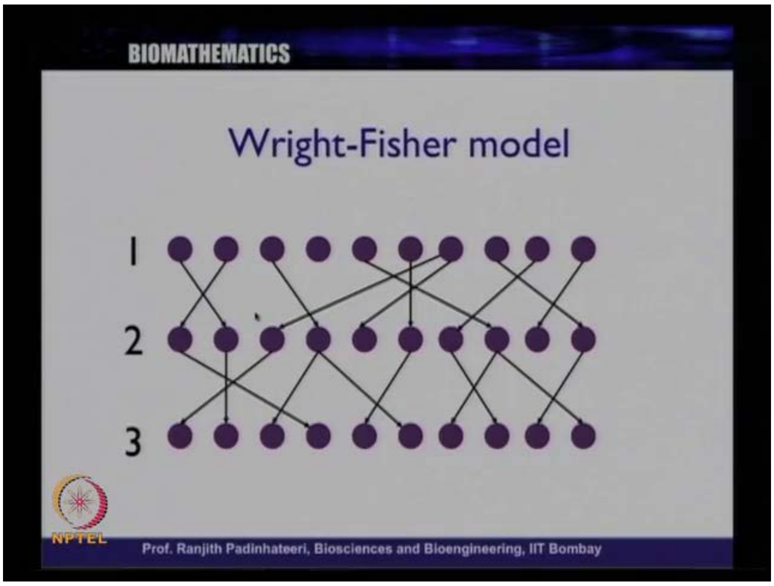

So, now this is what is drawn here what we drew the first generation and second generation. So, you can see here for example, this particular individual has no offspring at all. So, whatever genes whatever chromosome whatever genes was there on in this particular individual, would have lost on the other hand this particular individual as two offspring individuals; in the next generation there are two offspring 1 this and one this. So, the genetic material of this is doubled in this next generation.

So, such things are happening. Now, let us go to the if you want to continue this and you can go the next generation. So, you can have this is the 1 2 3.Three generation again you choose one individual randomly draw an arrow from a choose the parent randomly this choose parent randomly this individual choose the parent randomly his individual has this parent again random this individual has this parent random this individual has this particular parent again randomly chosen.

This individual chooses parent randomly this. So, in this particular exactly in this the way we came from 1 to 2; we can go from 2 to 3 and so on and. So, forth. Now so, this is the Wright-fisher model; this simplest of Wright-fisher model. And we will see now what can we learn from this and once we understand the few things from this, we can add complexity we can say we did haploid now what will happen for diploid; what will happen if there is mutation what will happen; what all things we can say and what all things we can learn from this That is what we want to see now and then we add complexity. Now, given this rules now let us think a bit about genes in this particular model.

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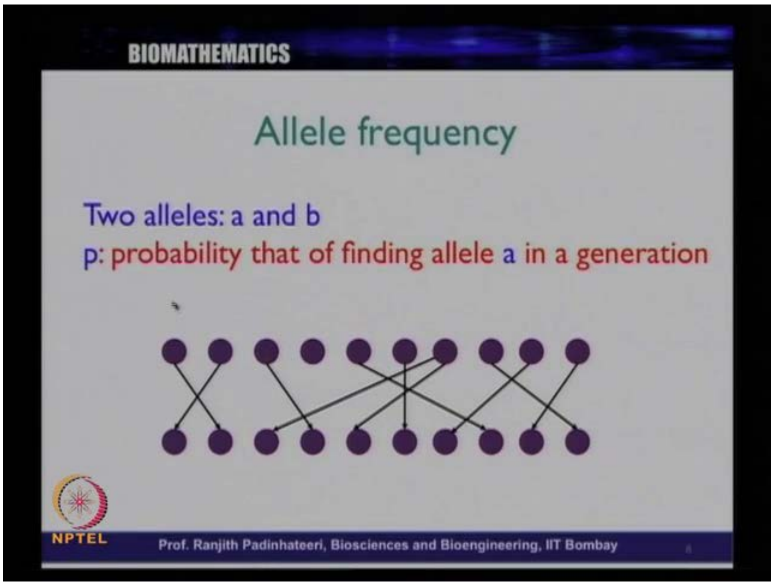

So, look at here. So, imagine that in this particular generation the first generation as two alleles a and b. So, allele a and allele b and P is the probability that of finding a in this generation. So, in this parent generation how many of this alleles a will be there; and how many of the allele b will be there and let P be the probability of finding allele a in this generation then the question is that what is the if you know this much.

So, if there are two alleles again in reality there will be many alleles but, for simplicity let us again think of only two alleles a and b. So, there is some fraction let us say there are ten individuals and let us say six of the individual has allele a; and four of this individuals have allele b; or gene b. You can think of that way or 3 of them have gene a allele a and have 6 7 of them has gene b.

So, in this particular way one if you think of it you can ask the ask the following question.

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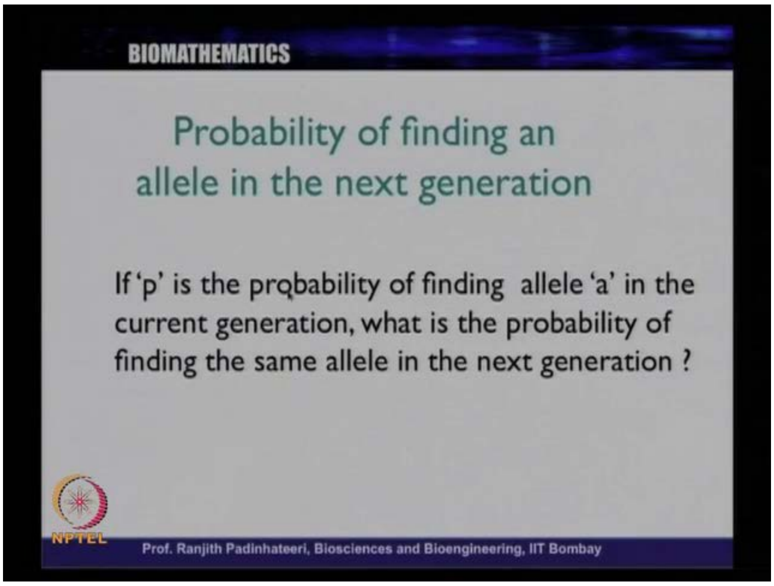

If, P is the probability of finding an allele a in the current generation; what is the probability of finding the same allele in the next generation. So, we have allele a and allele b and that can ask the question how likely that you will find an allele the same allele a. In the next generation and the answer is something very simple idea that we learnt already. So, we will discuss that allele but, let us understand all this what is this probability and all this that little more carefully.

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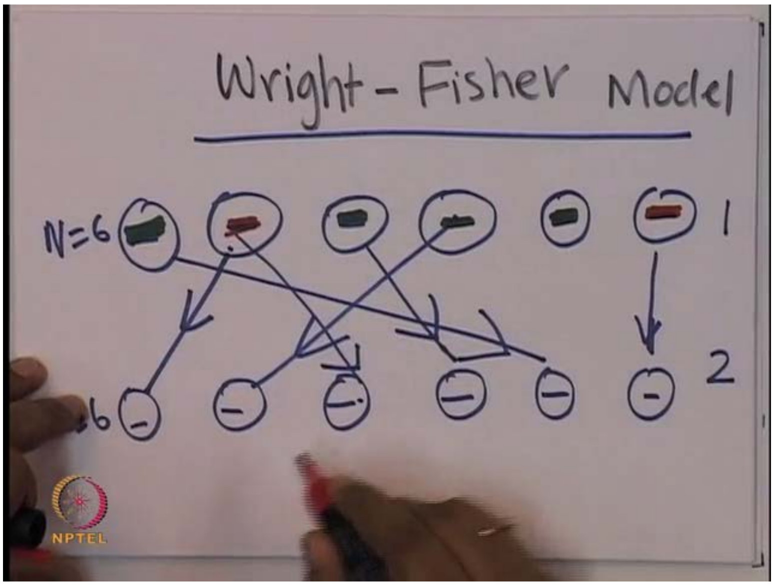

So, let us look at here so let us draw this little more interestingly. So, let us color this; so, let us color this genetic material green. So; this is green so 3 of them out of this 6 4 of them I colored green; and 2 of them I coloring red. So, the green let me let us draw this little more carefully.

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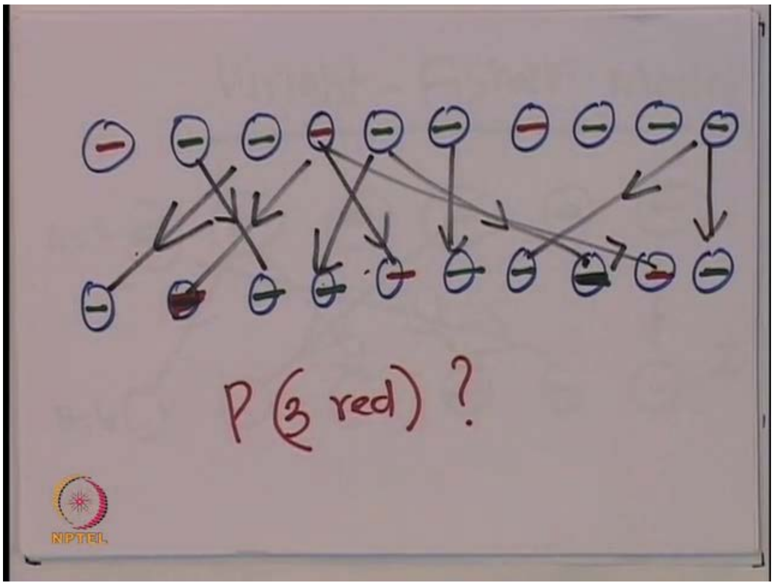

So, let us have again let us say 10 individuals again 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 individuals; and 10 offspring individuals 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. And let us say 3 individuals randomly I'm coloring red; and other 7 will have green chromosomes.

Then you again randomly choose a parent for this individual. So, let us say you choose parent for this particular way sorry for this way for this way for this particular I choose this randomly for this particular I choose this randomly for this I choose this way for this I choose this for this individual I choose this one for this individual I choose randomly this one and so for this individual I choose randomly this one.

So, now let us look at this one if inheriting a green one so let me draw a green here; so this is inheriting; from a green one so green this is inheriting from a red one so this is basically a red sorry the red it is a red one and this is inheriting from a green one so I draw a green this is inheriting from a green, so this I draw green; this is inheriting from a red; so I draw a red here this is inheriting from a green this is inheriting from a green inheriting from a red.

So, I draw a red here this is inheriting from sorry this was inheriting from a green actually. So, I draw green here this is from red; and this is from green. So, again we had 3 reds here. So, you can ask the question, if there are 3 reds in this now you did one in one particular population. You can do many such experiments if somebody if you were to draw the same thing; you will draw in a different manner the arrows randomly then you can ask on an average or what is the likely wood of finding what is the probability if you do this many many times what is the probability of finding let us say 3 red in the next generation; probability of finding 3 red in the next generation, you can ask this question and the answer is basically statistical distribution that we learnt and this can be understood very easily.

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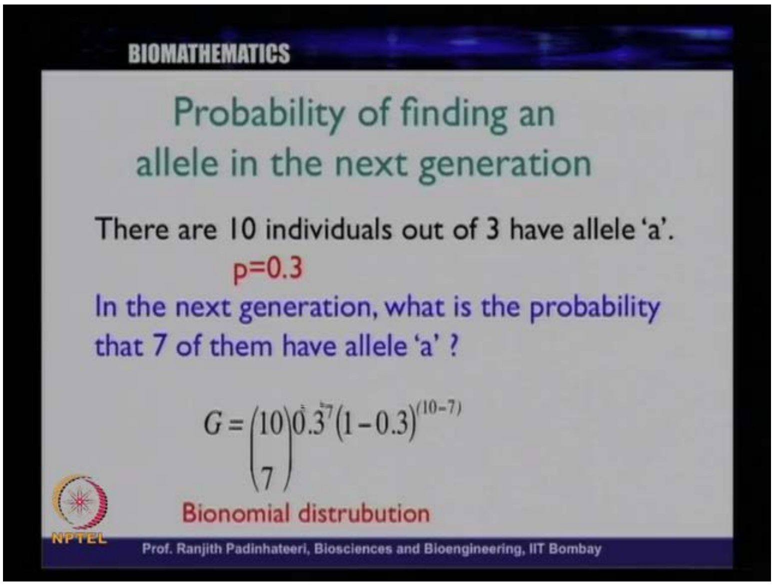

So, let us think of this is if you this what we said there are 10 individuals and there is 3 allele a for example, red. So, the probability in the parent generation is 0.3 there are 3 out of 10. So, in our case the probability is 3 out of 10, 0.3. Now, you ask the question in the next generation what is the probability that the 7 of them have allele a so in the our example, you can ask the question what is the probability there are 7 reds in the next generation; to have 7 red this have to have at least let us say this has this area had 2 off springs this as 2 offspring this has 2 and 2 4 and this had 3 then there will be 7.

So, the to have 7 reds in the next generation; each of this will have to have multiple off springs. Then only there will be 7 so you can ask the question what is the probability that in the next generation, if there are only 3 out of 10 red alleles what is the probability that there will be the 7 alleles red alleles in the next generation. So, this is the question if there is allele a with probability 0.3 what is the probability that there will be 7 of them will be allele a. So, there are 10 individuals out of 3 allele a that is P point probability is 3 out of 10 that is 3 by 10 which is 0.3 in the next generation what is the probability that 7 of them have allele a and the answer is something that we have learnt called binomial distribution.

So, the probability G is basically 10 c 7 this is combination; the way is written 10 c 7, 0.3 which is the probability power 7 into 1 minus 0.3 this is probability of not having that allele or the probability of having the green allele that is 1 minus 0.3 in 10 power 10 minus 7. So, this kinds of distribution is called binomial distribution. So, the answer is given by this so we know what this means so let us explain this.

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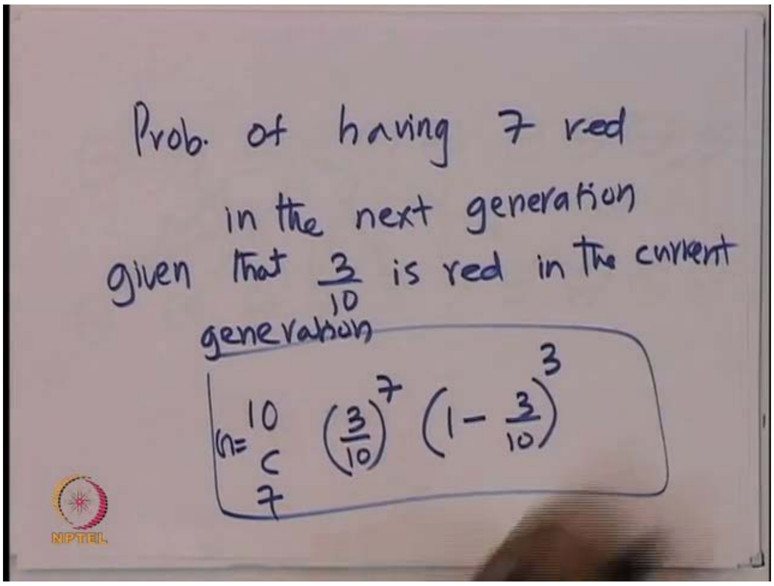

So, the answer we said; so what are we asking probability of having 7 red or 7 particular color in the next generation, given that 3 out of 10 is red in the current generation. And the answer to that is something, that we learned is called the binomial it is coming from the binomial distribution. So, this will this will have a binomial distribution that we learnt.

So, and the distribution is basically 10 c 7 3 by 10 that is the probability and the power 7 1 minus 3 by 10 whole power 10 minus 7 which is 3. So, this is the answer; this is the probability. So, let me call this G. So, this is the probability of having 7 red in the next generation given that 3 is 3 out of 7 is red in the current generation and this 7 out of red in this current generation is green.

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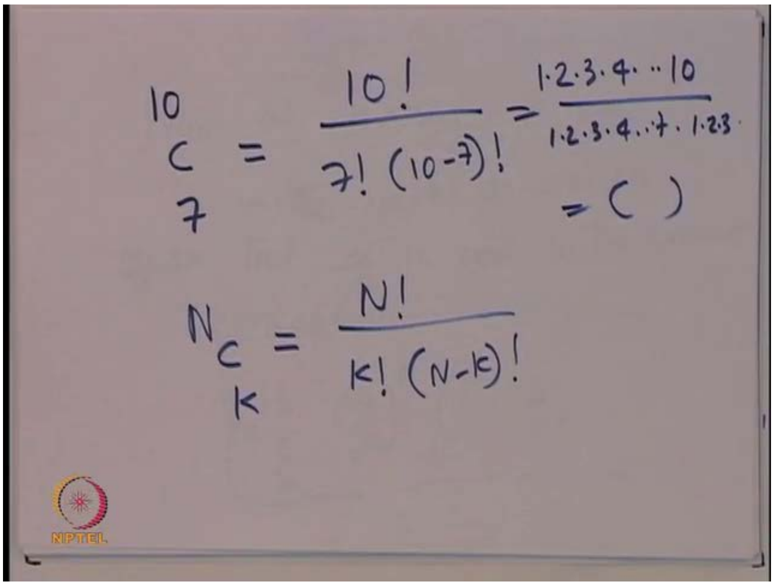

And now, what is 10 c 7 basically it turns out that the 10 c 7 10 c 7 is basically 10 factorial divided by 7 factorial into 10 minus 7 factorial. So, in general this is N c k is the combination combinatory so this combination or also called N choose k sometime.

So, it is basically N factorial divided by k k factorial into N minus k factorial so this is the this is the basic thing now this is the formula. So, we can based on this formula we can calculate this N c k so for example, this we can calculate, so 10 factorial is basically 1 into 2 into 3 into 4 into up to 10 divided by 7 factorial is 1 into 2 into 3 into 4 into up to 7 and 10 minus 7 factorial is 3 factorial, so this is 1 into 2 into 3.

So, with this is a number you will get a number you can calculate a number from this. So, this is basically that number times the 0.3 power 7 1 minus 3 so this is the probability will be some number which is between 0 and one now when we say binomial distribution it has some particular properties.

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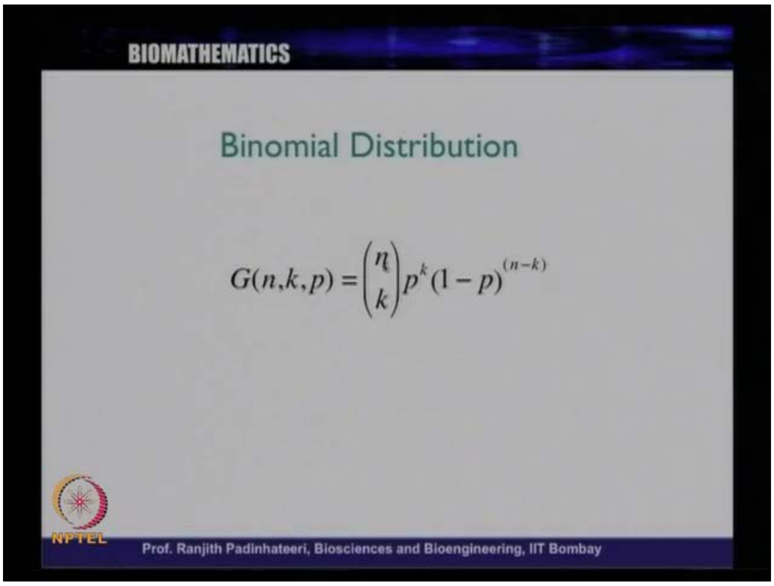

So, this is the in general binomial distribution is this n c k p power k 1 minus probability power n minus k. So, this is the general nature of binomial distribution just like we learnt various distributions.

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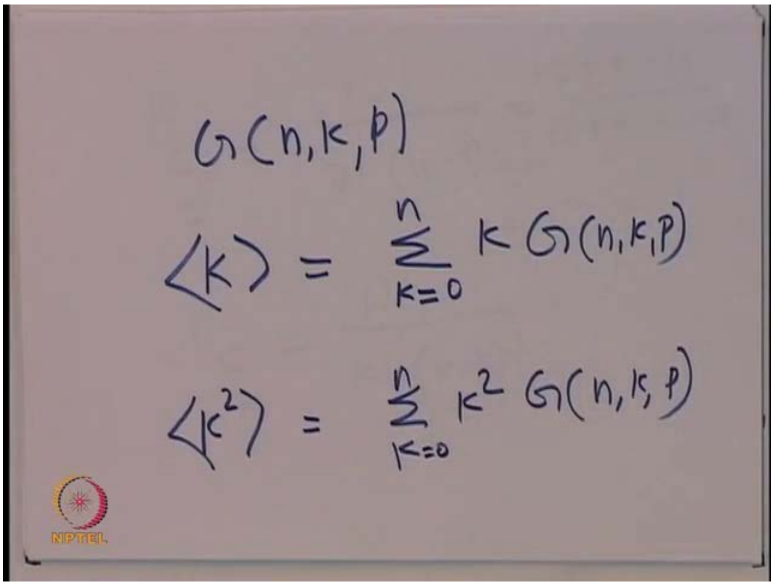

If, we have this binomial distribution as we said G of n comma k comma p where, if there are n what is the probability that k of them will have this will be this particular allele is a question and then we can calculate the k average just like we have been doing this is sum over k k G of n k p this will be the answer so this will be some function of n and p and you can also calculate k square average basically this will be sum over k k square G of n comma k comma p.

So, this case this runs from 0 to in principle infinity if you point or whatever there is there are n in this case rends of n so this is the similarly, here k 0 to right. So, this is we can we can calculate, if you have this we can calculate k average and k square average and then variance and all that.

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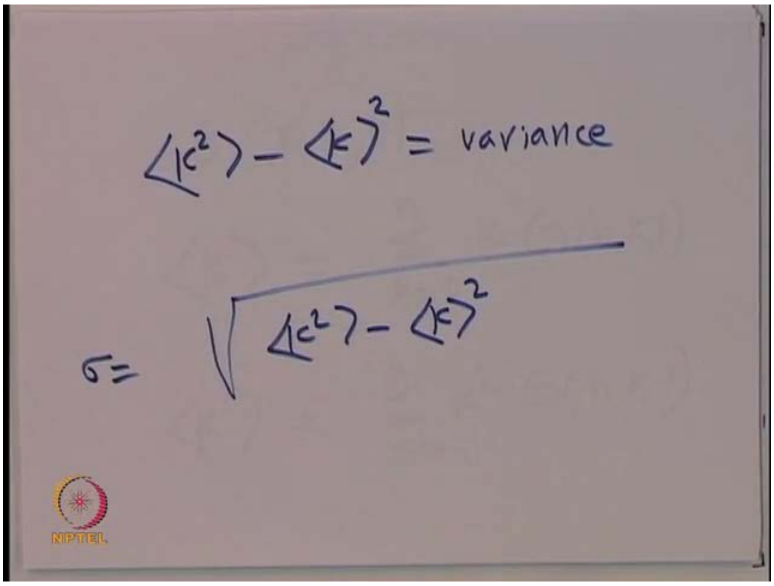

So, let us look how we calculate how we can then we can calculate k square average minus k average square this is the variance. So, this is so we can calculate variance then we can calculate standard deviation which is basically k square average minus k average square.

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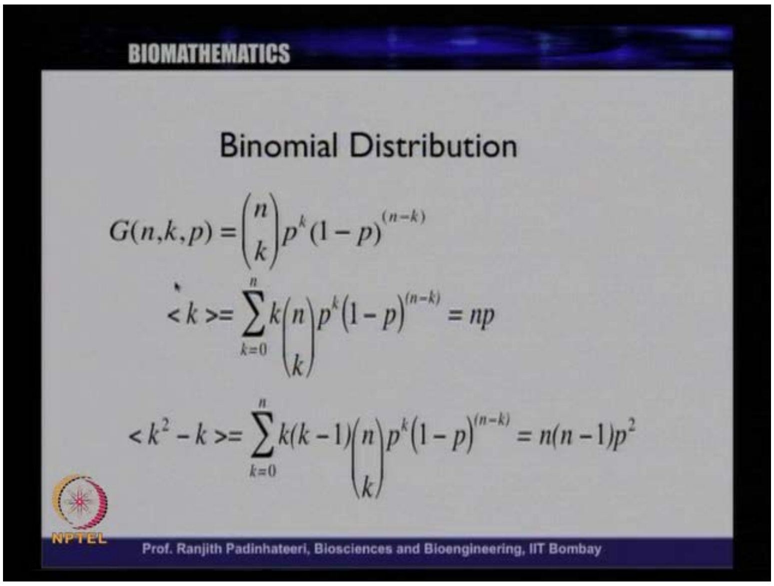

Let, us look at in this example; so that k average; so this is G n k and k average is k multiplied this and sum over k and it turns out that the answer is n p. So, this answer of this particular this if you do the answer is n p similarly, so are the variance

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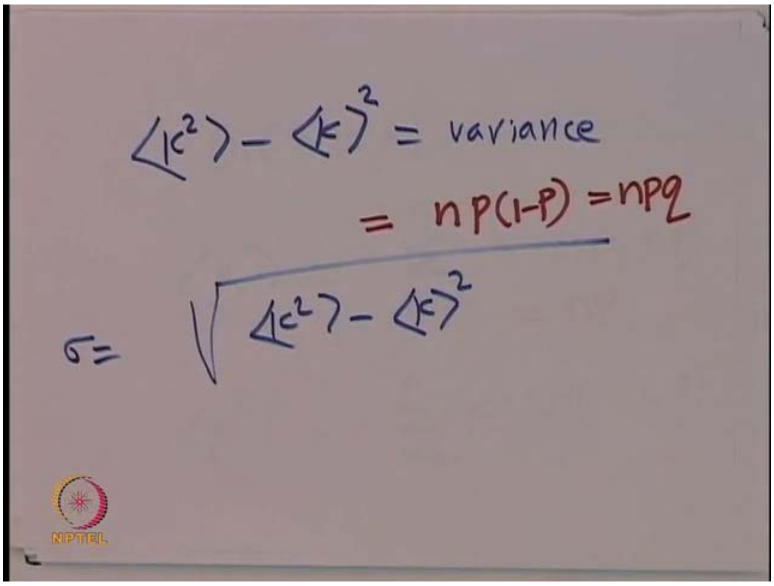

So, then we can calculate variance and the answer turns out to be for variance is n p into 1 minus p if you call 1 minus p as q this is n p q So this is the variance and it is square root of this will be standard deviation.

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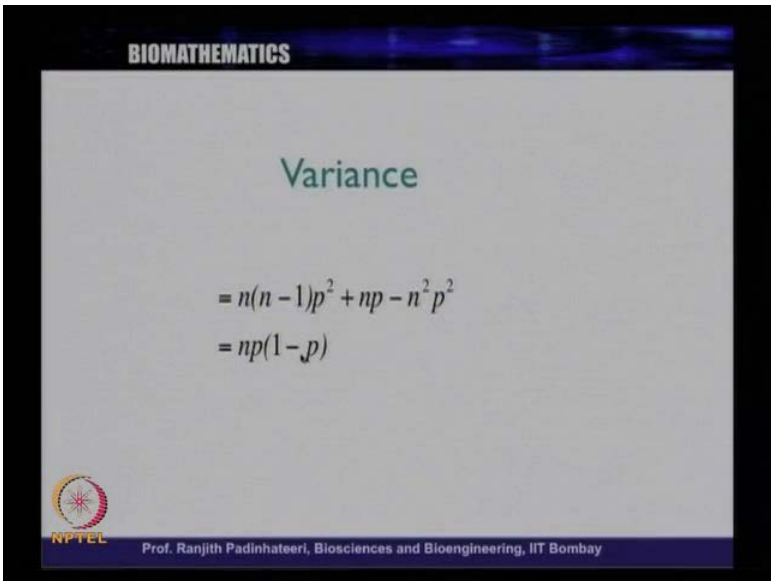

So, we know that on an average there will be n times p into alleles on an average and this is the way standard deviation; which is n p into 1 minus p this is the variance n p into 1 minus p. So, we learned some statistical properties of this distribution and we have now some idea that if we have such a rule we can basically predict if there is a particular allele in this generation density of this alleles, how many of the frequency of this alleles what is the probability that the allele frequency will be a given number in the next generation can be easily calculated.

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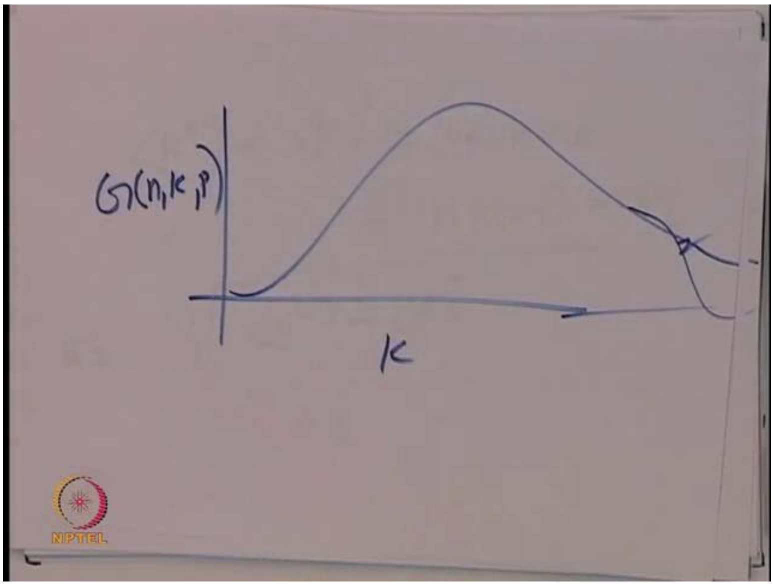

Now, this can be immediately put to use, if we now first of all we can plot this we can for example, we can plot G of n, k, p as a function of k and it'll now it'll look like some curve we do not know we have to we have to it'll have some particular shape whatever shape I just draw a random curve here but, we let us look some exactly exact curve.

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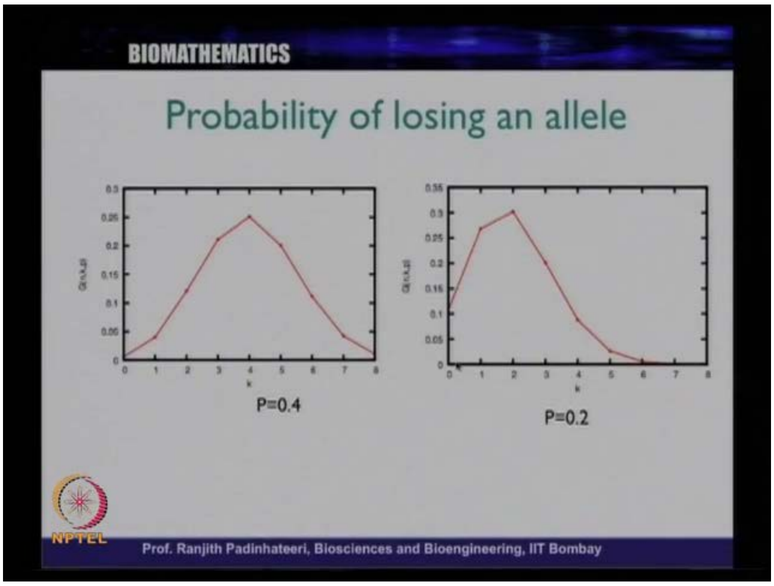

So, let here is a curve which is basically for P is equal to 0.4 this is the plot the red line here for various k is for a this is for 10 individuals. So, this is various k values. And what do we see here the probability you are having. So, the first P was 0.4 what is it mean? out of 10, 4 of them are allele red out of 10, 4 of them have allele red and in the next in the next generation again 4 of them a probability of having allele red 4 having allele red is very high probability of having everybody all the 10 allele red is zero is very small near zero.

Probability of having completely what is this mean? probability of losing the allele there are zero individuals having this particular allele completely losing the allele that is probabilities also very small. So, then we can ask the question, so that now we can vary the value of we can vary the value of P for example, we can get the value of P as 0.2 and then plot this G of n k p as a function of k and you see interestingly this as increased and this as here it has increased and here it as decreased. So, let us compare this two if we compare this two, look at here the probability of having zero allele in the next generation, the probability of having zero allele is this particular point and this is much far for 0.2 then an S probability is 0.4 and initial frequency is less; if the initial frequency is small the probability of losing that allele is very large.

So, here this is one prediction we can make. So, essentially we are we are predicting given this information what is the probability there in the next generation the allele will be lost, one particular gene will be lost; so, that can be predicted using this you can also predict what is the probability that a given allele will be lost after t generation. So, there is something called mean first passage time in statistical physics and in statistics in general.

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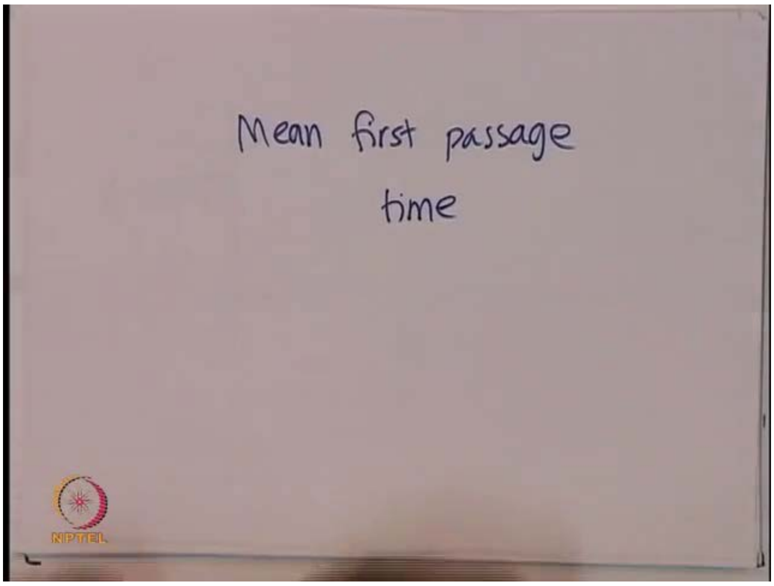

So, mean first passage time, so what is the probability; so for the first time an allele will be gene will be completely lost, the mean time this can be calculated. So, this is the another quantity that we can compute from this model so, far we took a very simple model and this model could predict what is the probability of losing an allele; what is the probability of this allele being a particular number what is the probability of losing this allele. After a few generations such questions we can ask.

So, and we can answer various interesting things but, then the question is what we did is basically for a haploid individual; what if what is it for a diploid individual if there are two genes two chromosomes in one individual how does this work can you have this rule.

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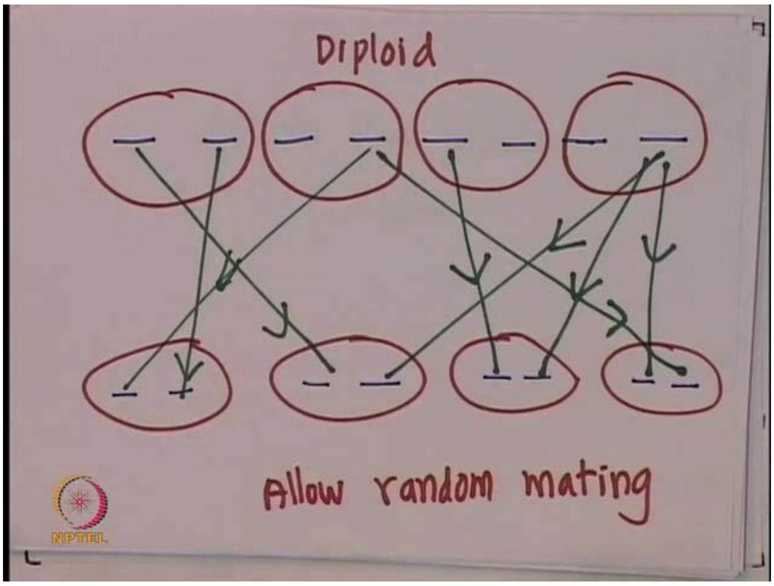

So, let us think of this so let us think of this so let us first draw similar thing that we had. So, let us draw 1 2 3 4, so these are chromosomes 1 2 3 4 5 6 7 8.

So, there are 8 chromosomes and if you are imagining a diploid 2 chromosomes will come from 1 parent these are sorry one individual will have 2 chromosomes. So, there are two individuals here and each individual has 2 chromosomes, so this is the diploid. Example, now this now we have off springs and again so again let us draw off springs so this is an offspring individual and this is an offspring individual this is an offspring individual offspring individual this offspring individual will have again 2 chromosomes.

Now. let us try to draw the arrow that we learnt the same way so let us forget for a moment this circles and choose this and randomly draw arrow from one of the above once. So, let us do this let us randomly draw the arrow I choose this I randomly drawn the arrow I choose this I randomly draw arrow I choose this I randomly draw arrow I choose this I randomly draw arrow I choose this I randomly draw arrow I choose this I randomly draw arrow I choose this I randomly draw arrow now what did we get we got we didn't do anything except we drew a circle but, I every otherwise everything looks the same as the Wrigh-fisher model that.

We discussed 4 diploids the only thing changed is that so let us took of individuals this 2 chromosome came from one from this and one from this so this two gave birth to this came from this and this came from this and this came from this and this so if you assume if you if you allow random if random mating or random mixing we can say that this is essentially the same as the Wright fisher model so if what we did if we can allow random randomly mate or randomly mixed then this exactly this whatever we did is exactly same as the Wright-fisher model we just choose one chromosome randomly decided the parent chromosome choose chromosome randomly decide the parent chromosome this and this mixed and this mixing can be random.

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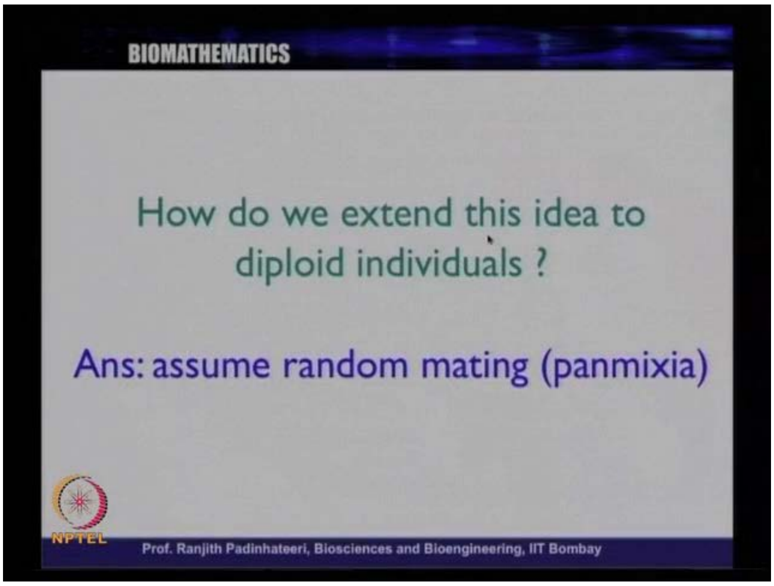

So, the only thing so the question we had is how do we extend this idea to a diploid individuals to diploid individuals and the answer is assume random mating if you assume random mating or there was the technically this is called panmixia. If you assume random mating this will turn out that is exactly the same as the Wright-fisher model. So, we can extent this idea to random diploids also. Now, what else we can learn so exactly all this whatever statistics we can we applied all the binomial distribution applies to diploids you can ask the question what is the probability that a particular chromosome particular gene will be lost again we can color we can draw various colors it can be green, red out of this 8 3 of them will be green color 4 of them will be 5 of them will be red color and so on and so forth.

Then whatever we did so far we can just repeat and then redo this again and you will ask this same thing will happen. So, for example, lets quickly do this. So, we can draw each individual with a red or a green color each chromosome.

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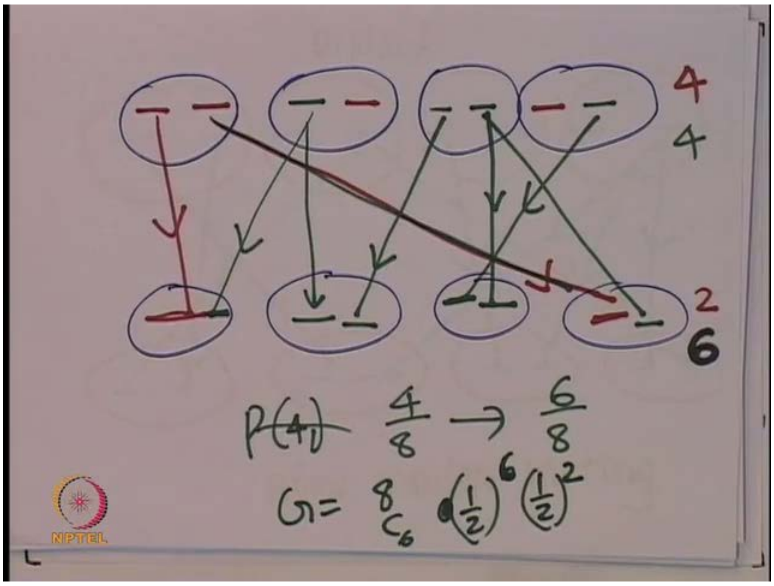

So, let’s say the first individual both is red and the second individual is one red and one green sorry one green and one red and the next individual will have both green second individual will have again one red and one green, let’s say now so this are the individuals so, there are 4 diploid individuals with so in the out of in this so we had lets have again 10 so let us so let us have it 8 only there are out of 8 there are 4 of them are red and 4 of them are green.

Now, we can ask the question in the next generation again there are four individuals now this four individuals in the offspring generation you have to choose the parent chromosomes. So, imagine that they randomly mate that is what our assumption is so let us imagine random mating so that this and this mate and this gives this red and this gives the green so the green comes here so this individual will have red chromosome or a green chromosome let us say this is the production of random mating of this and this.

So, this and this randomly mate and then this we randomly choose to mating partners and then you get this particular individual. So, let us assume that this gives its green chromosome; so let us give this to green this also has green so this also gives its green so you have this now let us imagine that this and this and this have a mating and then you get this so this gives again one of this green individual green and this gives also its green.

So again you have two greens now here let us imagine that this and this randomly mate. So, it has to have one sorry one red one red and let say one green from this so one green So, here we had 4 red and 4 green and here we have one only 2 red and 6 greens 6 green.

So, similar things could happen a different things and you can ask the same question. What is the probability of having if there are 4 green in the in the in the parent generation out of if there are 4 out of 8 greens in the parent generation what is the probability that 6 out of 8 green in the next generation and the answer is again a binomial distribution. So, the answer we said is basically again 8 c 6, 4 out of 8 is half power 6 into half power 2.

So, this the answer same thing is binomial distribution we said and we can calculate this questions. Now you can ask the question so this things we will enter this is more scope you can ask the question what is the probability that two individuals have the same parent in few generations ago.

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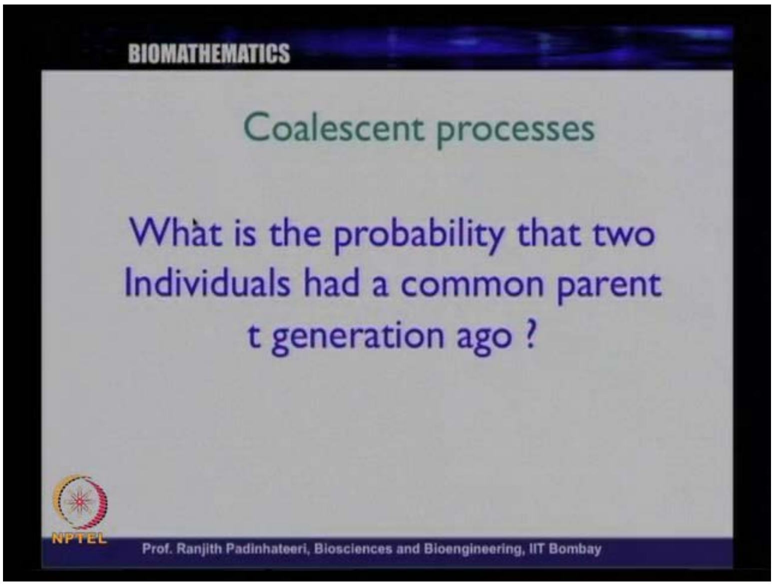

So, this is the question we are asking what is the probability that two individuals had a common parent t generations ago. So, if you draw many arrows like this many generations few generations ago two individuals might you've heard a common ancestors. So, what is the probability that t generations ago two individuals had common ancestors and the answer can be calculated from this model. So, let us imagine again a simple case so let us imagine that.

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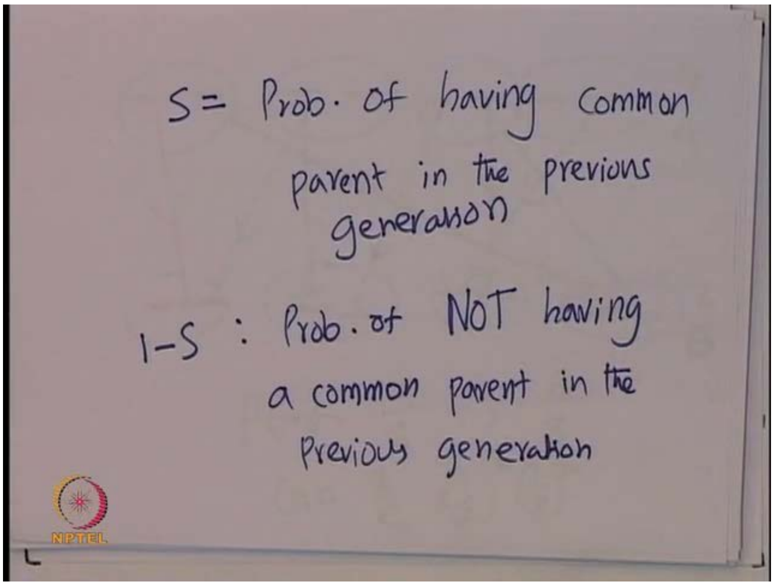

So, let us say S be the S is equal to S be the probability of having common parent in the previous generation probability of having common parent in the previous generation then 1 minus S is basically the probability not having a common parent in the previous generation.

So, S is the probability of having a common parent in the just previous generation and 1 minus S is the probability of not having a common parent in the previous generation and then we ask the question what is the probability at t generations ago.

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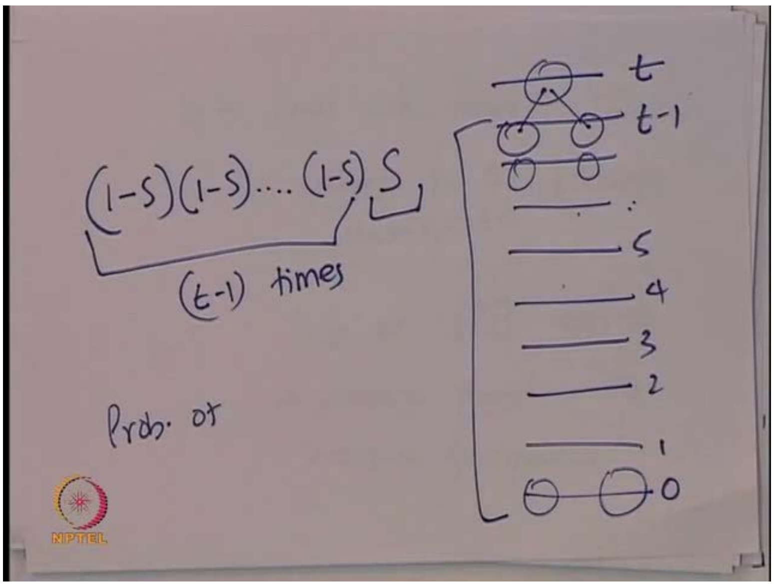

So, let us draw many many generations so this is generation 1 2 3 4 5 6 many generation. So, t generations ago so this is 0 1 2 3 4 5 similarly, t minus one and then this t so up to t generations two individuals should not have any common parent but, in the t th generation this gave birth to some this and then they all gave birth to various offsprings individuals and then you ended up here.

You, ask the question so up to t minus one generations you do not want to have a common individuals. So, if 1 minus S is the probability of not having a common parent in the next generation, 1 minus S square that is 2 times 1 minus will be that of not having in the 2 successive generations and there are t minus one successive generations so up to 1 minus S t minus one times, you multiply this. So, this is the probability of not having a common parent in the t minus one generation and then S is the probability of having a common parent in the previous so this the t minus generation you should not have a common parent and in the next t th generation you have to have a common parent so then this product essentially will give you the probability of having, so the what we discussed here probability of having probability that two individuals had a common parent in t generations ago, So, that is the answer this will give you the answer that two individuals had a common parent t generation ago.

So, such things we can calculate it's a very simple very simple calculation what so basically we can calculate what is the probability that two individuals had a common parent t generations ago. So, we learned this so again this is again so what where is the another the when we say about binomial distribution you should also remember that this is very analogous to tossing a biased coin like, if you had a biased coin if it is not if it is unbiased coin the probability will be half if it is a biased coin the probability could P and 1 minus P and this very much like a tossing up biased coin and you could think of this analogy and compare with this and whatever the Wright-fisher model is.

And then think about in that particular way. Also when I say this if it is not very clear to you what this product means you could also think of this example that tossing a coin like if you if you if you had let us say let us say there is an one of the let us say there is the cricket match happening and in cricket matches it is very common it is a you all know that there is a toss to decide who to field first or who to bat first, which team will field first or bat first. So, imagine that in each cricket match there will be a toss of coin and you can get a head or tail.

So, and what is the probability the two successive matches for example, a given team let us say India will get tosses like the toss that you head you want always let us say the Indian captain calls head and what is the probability that both the matches India will get head is half into half which is one product of half into half so to understand this.

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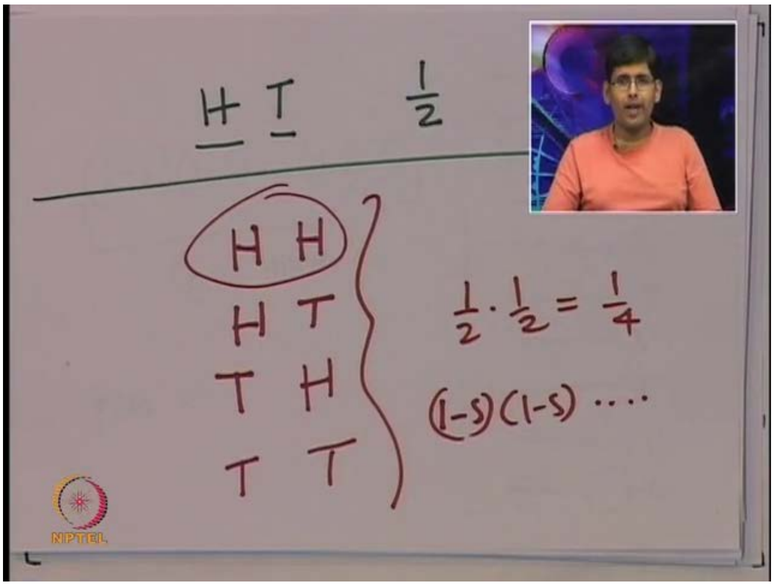

We can just quickly think of it there is two head and tail there are two options only. So, the probability of each of this is half because there are one after two options.

Now, there are 2 matches. So, it can happen that in the first match it can be so there are 2 2 matches so the first tossing you can get the head will fall in the second toss again head will fall this is one possibility so in the first toss there will be a head and in the second toss there will be tail this is the possibility in the first toss there will be a tail second toss there will be a head this is the another possibility in both there will be a tail for possibility 4th there are 4 possibilities and both getting both of this we getting head this just one of this 4 possibilities so this is half into half which is one by 4.

So, this is the same way we had 1 minus S 1 minus S up to now you can ask the question what is the probability that in the all the 5 matches that we have in a 5 series match against a given team that India will get toss this is half into half into half 5 times similarly, what is the probability of not having a common parent in the in t minus one generation is 1 minus S into 1 minus S into up to t minus 1 times so this is very analogous to this.

So, you should you think about this in this particular way. Now we had diploid individuals and we had we learned something about common nearest common ancestors, then we can ask the question that, how do we incorporate mutations there was no mutations. So, far how do we incorporate mutations it becomes it takes the problem a little more complex.

So, we can if you look back the rules we had we had this particular rule that each offspring inherits the chromosomes of the parent as a as it is without any change; without no mutation; without any mutation as it is each offspring inherits its chromosomes we can change this rule a bit and introduce that each offspring inherits its chromosome with some probability and the mutation happens with some other probability.

So, let us say with the probability m the mutation happens if the probability 1 minus m the mutation does not happen. So, we can change just one rule in this set of Wright fisher model and then you can incorporate mutation.

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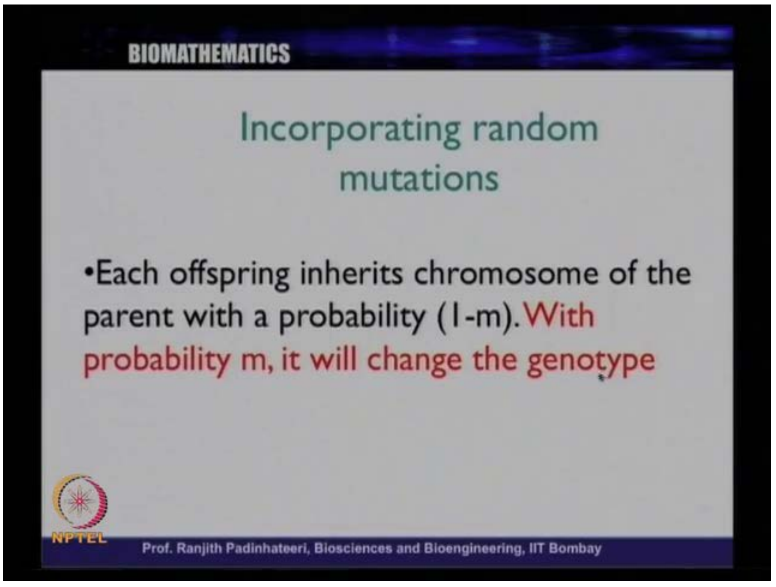

So, when we change this model what we end up is this so this is how you incorporate random mutations that each offspring inherits chromosome of the parent with a probability 1 minus m. And with probability m it will change the genotype. So, once you introduce this will become a little more complex than many things you may not able to get lot of things analytically.

Maybe some of the things we get analytically. So, I wouldn't discuss that in this class we will leave it at this that you can modify this rules and think about think of mathematical model.

So, one thing you should understand is that this simple idea of just exchanging individuals just by mutation or simple mutations or no mutation, itself can explain lot of phenomena and one think we haven't in incorporated at discussed at all is selection here but, we can also think of how do we get selection and do a mathematical model for it. It is again some things we learnt people use diffusive model, we learnt about diffusion. So, people learn use something called diffusive model for to incorporate selection or even mutation. So, the whatever the techniques that we learnt so far is indeed lot of them or being used in modeling evolution. I just gave you a flavor of what can be done for the simplest case and with this we will stop today's class so summarize, we can use different ideas we learned to make simple models of evolution. There is a error here it is not evolutions its evolution so simple models of evolution and with this I will stop today's lecture see you in the next class.