

Biomathematics
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Lecture No. # 32
Tutorial and Discussion

Hello, welcome to this lecture of, on biomathematics. In this lecture, we will try to do some kind of question answering, like that what do I mean is, some kind of tutorial session. Many of the we learned, many things and this will be like we will be discussing couple of few questions and will try to answer them and later I will also provide you a set of questions which you can try to solve yourself.

So as you know, mathematics is a basically like you will learn it only by doing it. Like somewhat like swimming, or just like, this is true for any other language in some sense, like as we said very in the beginning mathematics is some kind of a language to write and to express things very quantitatively. Anything that we see needs are can could be described using mathematics and this is some kind of a language and this is true with any language. That only by using it constantly, you will learn it.

So, the only way of learning mathematic is basically trying to solve problems, trying to frame problems. So there are two kinds of things in solving problems one is you are given some equation and ask to do something like fine derivative. Typically this is the kind of question that typically exam will see like find there derivative, find to ask you to do some operation that you have learnt, that is the easiest part of it which is your thought to do some kind of a operation, differentiation or even in schools you are taught to do multiplication and all that. I think, you might be asked to do some questions like that way. For a, if you think of in a more advance level, what you would need is, you will be described the phenomena, some phenomena that we see in nature and you will have to think of how to write a mathematic expression for that phenomena **mathematical expression for that phenomena**, how do you describe? How do you write down that phenomenological information that you got into an equation?

That is the bigger challenge, if we can write down that phenomenological information that you got into an equation. Then you are successful then you can solve it. So here we will do both we will do some exercises, where you will be asked to do some simple operation that you have already learnt. You will be also given some description, you will also have some description and from the description you will have to figure out, what is going on? and do write down some equations or plot that information presented it in a quantitative way.

So that is what we have to do, so first we will take a question where, through this I will describe to your phenomenon, which is a very well-known biological phenomenon but, some of you might be hearing for the first time. Hearing that phenomenon, description, first you have to present it in a quantitative way. So what is the simplest way of presenting something qualitatively to make a plot? So, you have to make a rough plot of whatever the description you see.

So, when I say description that will be in English, right? it is a language. I describe it, then when you convert that into a graph, you are doing some kind of transformation. From converting that qualitative description to a quantitative information, which we can write down as an equation later. So and from that, we can do various other operations, and get many other useful information.

So, let us go to the first question. Here the aim is basically, so first there will be description of a phenomena, you listen that description carefully, understand them and then you have to plot whatever it is described. So I will do that, I will do the description and I will do the plotting and I will take you through, how I do this?

And later you will have other examples, where you have to do it and then we will also do some very **convention**, very typical questions like you have to do some particular plot some function or do some particular operations, like find different integral, find derivative or so on and so forth. So let us first look at this description and let see how to plot that. So let us go head, so today's section is tutorial and discussion where will be discussing questions as I said,

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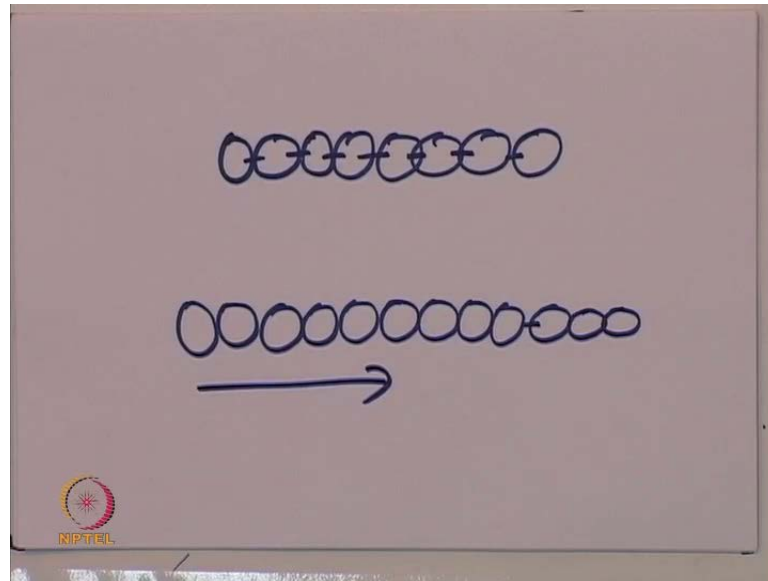
The slide is titled "BIOMATHEMATICS" and "Q.1 : Dynamic instability". The text describes the process of dynamic instability in microtubules, where tubulin monomers polymerize to form a filament of a certain length, which then suddenly depolymerizes and shrinks to a very small filament, and this cycle repeats. The slide also includes the NPTEL logo and the name of the professor, Prof. Ranjith Padinhateeri, from IIT Bombay.

So, the first question is basically, is on dynamic instability. So, here is a description, so we will slowly go through the description, so I will read it. We will read this carefully and let see what we understand from the description, so imagine that somebody is telling you some phenomena, that is been seen in biology or this is that we learn from our lab experiment, let us say somebody is telling you this description.

So let us go through the carefully, so microtubules exhibit a phenomena called dynamic instability. So microtubules are a particular filaments in biology, in that you see in cells actually, so they exhibit a phenomenon called dynamical instability. In this, that is, in this phenomenon, first tubulin monomers polymerize and grow with some speed to a microtubule filament of certain length. So first what happens is, tubulin monomers polymerize and grow to a polymer and the polymer will have a certain length.

Then suddenly, depolymerize and shrink to a very small filament. Then again starts growing and then shrink again. This process of growth followed by sudden shrinkage is repeated and this is called dynamic instability. So let us understand this once more, so there is microtubules, so this is the phenomenon exhibited by set of filaments in this cell called microtubules and the name of the phenomena is dynamic instability. And microtubule is a polymer, is a long polymer, it is the very stiff polymer. It is like a rigid rod, so almost like a sleeker rod, like this pen. We will imagine that this kind of a pen is made of many sub units. And for example, like I would just draw the picture here.

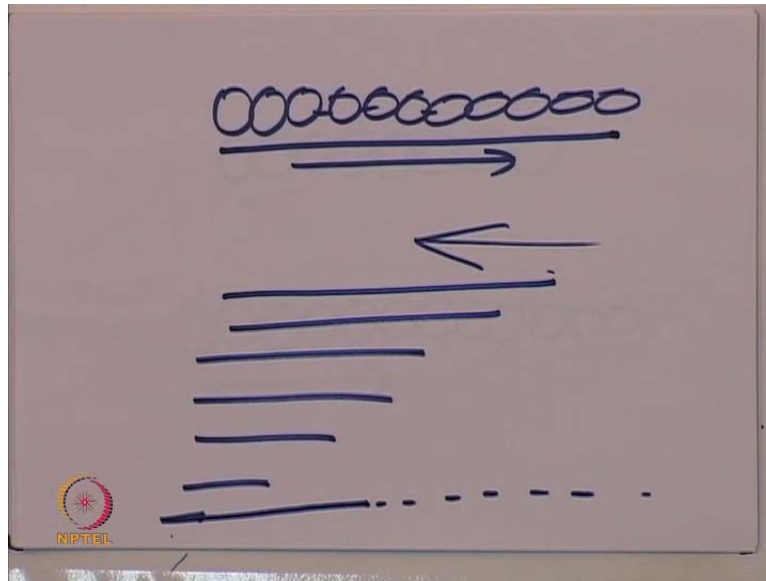
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So you could imagine that, many subunits joining together to form a filament with we have connections with like this. So they are similar, it is an imagine microtubule something like this for simplicity it is more complex. In that there are 13 proto-filaments so on and so forth. But, for the moment you all know that it is a polymer and they polymerize, so here tubulin monomers polymerize and grow with some speed to a micro tubule filament of certain length. So what happens here is that, the filament you start with one monomer, then they polymerize 2 3 4 so in 1 second certain number of monomers are added and then they polymerize and then grow, so they just keep growing like this.

So that is the first part, so microtubules in this first tubulin monomers, polymerize and grow with some speed to a microtubule filament of certain length. They suddenly depolymerize and shrink to a very small filament.

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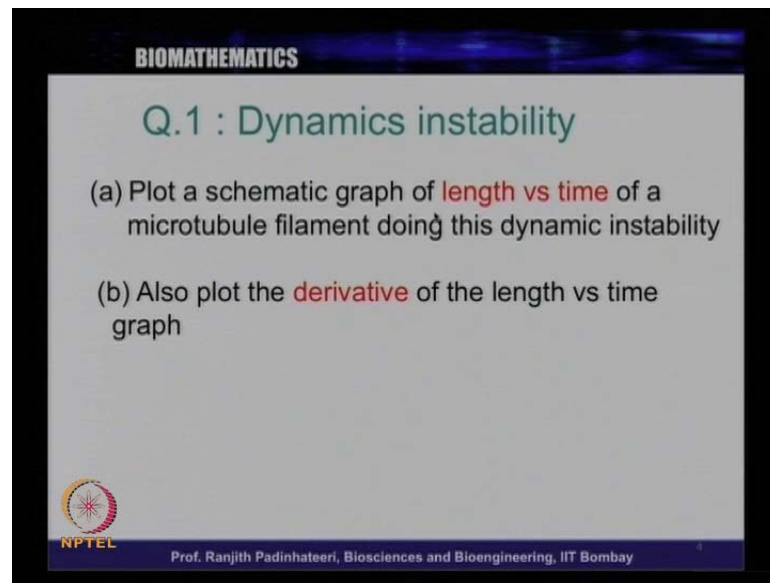


So what we saw, is basically polymerize and grow. So basically we saw, growing to a small filament like this, then suddenly, so this grew like this, then suddenly, at some point its starts to shrink, so that means, it starts to grow this way. So suddenly it will be so, you have a filament of length this much, suddenly it will become a filament of length this, it will just start shrinking, the length will just decrease. That is the second point, suddenly depolymerize and shrink to a very small filament, then again starts growing and after something like is again starts growing. So here it may again start growing and then shrink again and grow again and shrink again and grow again. So if we look at this, it is the filament, it will just grow, shrink. So you can think of this way, you can just grow, shrink.

So the filament will grow, shrink. So this phenomenon of growth and shrinkage, this is called dynamic instability. There are couple of points, one is, to note here that, it grows with some speed and then suddenly depolymerize and shrink then again start growing, and then shrink again, so this process of slow growth followed by sudden shrinkage, so that a growth is slow, but the shrinkage is sudden.

So this is another important point here, followed by slow growth, and sudden shrinkage, and this is repeated and this is called dynamic instability. So we understand the phenomenon now.

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BIOMATHEMATICS

Q.1 : Dynamics instability

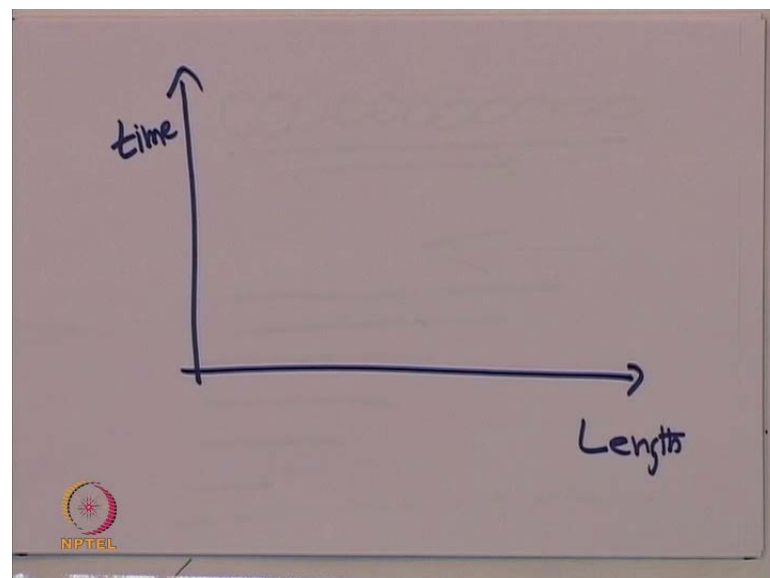
(a) Plot a schematic graph of **length vs time** of a microtubule filament doing this dynamic instability

(b) Also plot the **derivative** of the length vs time graph

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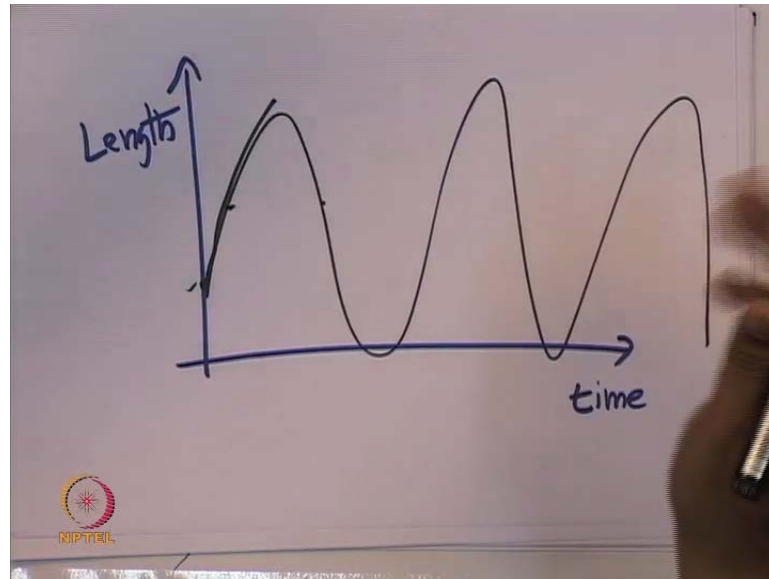
Now let us see what are the questions? The first question is basically, plot a schematic graph of length versus time of a microtubule filament, doing this dynamic instability, so roughly, you can plot a rough graph actually. You plot a rough graph of whatever this description you heard, from this description can you plot length versus time of strain of microtubule. So from this what we say here you know, that the length increases that decreases, length increases, decreases, so it has to.

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So, what we need is to plot length versus time, so when we want plot length versus time. Sorry here what we should have is, the axis the other way.

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So, we should plot time in the x axis and length in the y axis and at t equal zero, it will have some length and we know that, this is going to be growing and shrinking but there could be, I could plot many ways of growing and shrinking like something is increasing, decreasing, increasing, decreasing.

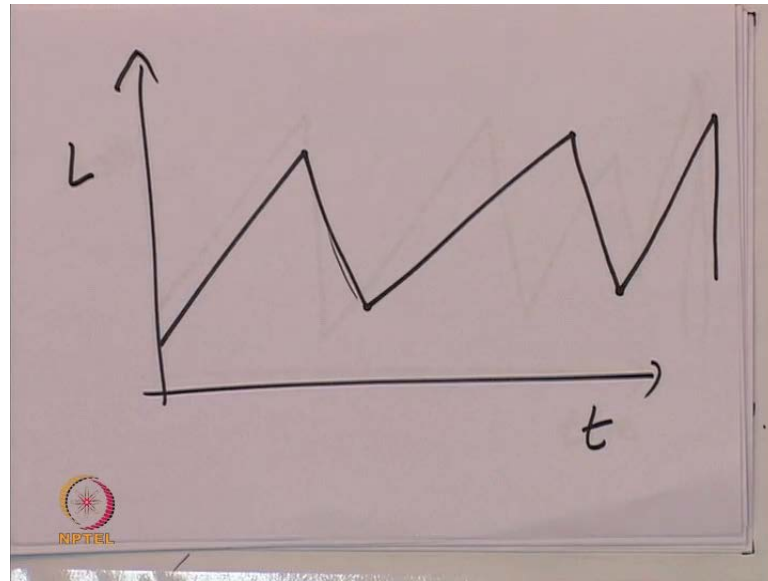
So one could plot something like this but, is this correct? So let us look at, the length is increasing, decreasing, increasing, decreasing, increasing, decreasing. So that much is correct. But, if you look at this question here, it said that there is slow growth and sudden shrinkage, so the growth is slow and shrinkage is sudden, so the speed of growth is slow, slow growth means this speed is very small, very low. So what is speed? Speed is basically how fast it grows. So if it grows like slowly, the shrinkage has to be faster than that. So this slope has to be larger than this, so it has to shrink faster.

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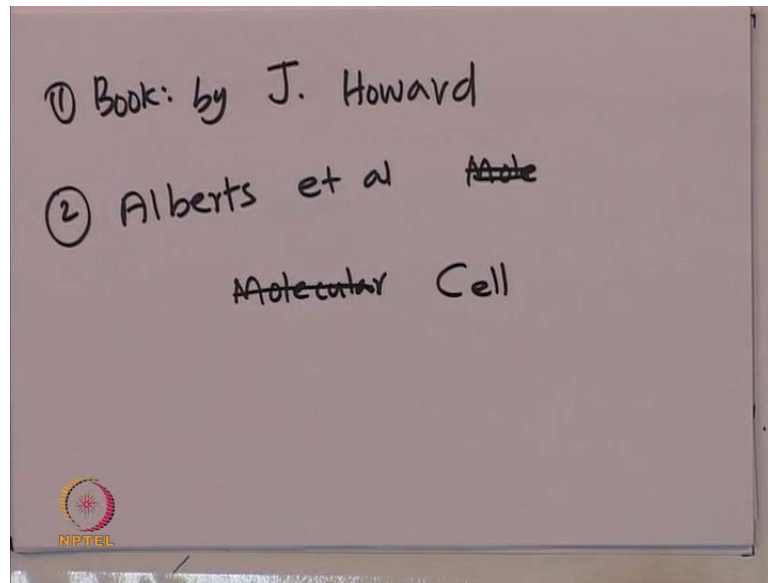
So taking this in to account, one can draw it slightly differently. So this is time and this is length and then you start from here and grow slowly. And suddenly come down, slow growth, sudden shrinkage, this should be at least perpendicular. So, this could be one way of drawing this graph, one could also draw a little more this, drop can be little a more faster, slower.

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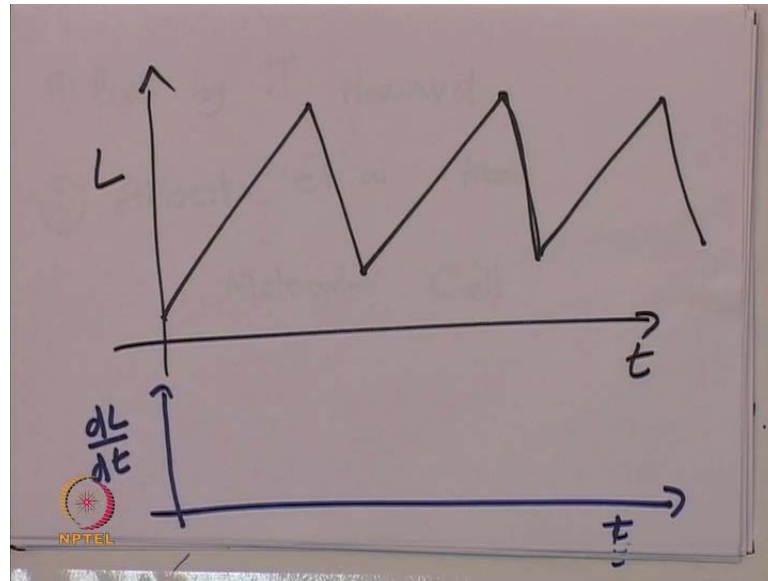
So one can draw, also this is time this is length so growth shrinkage, this particular way, there is a growth and shrinkage. So you can say, this shrinkage is faster than this growth. This is the slope, if you look at the slope here, this will be much larger than this slope. So that is the typical way, so typically if you look at various books for example, one book you should look refer for this.

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It is a book by J Howard so, this book on Moto-proteins and cytoskeleton filaments have a very description of dynamic instability. There are many books will have it of course, the Albert's book of molecular biology of the cell by Bruce Albert's et al is a very good book. To know the phenomenon or also, how Howard book will give you some kind of a mathematical description also, these 2 books you should look at, for this Albert's et al molecular biology of the cell, this is one book. Second book is Alberta al molecular biology of the cell, it is typically called cell. Typically it is called molecular biology of the cell so, this is the Albert's at this 2 books will have nice description of dynamic instability.

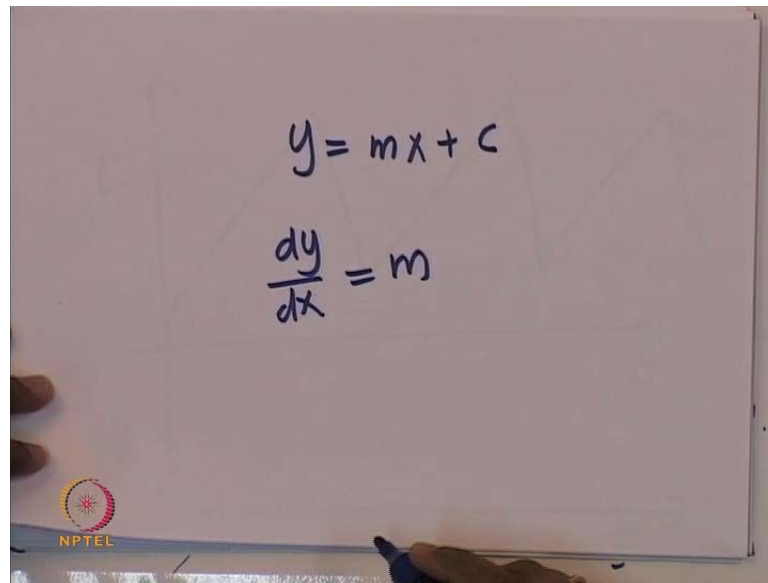
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So and you can also see in the video, there in Albert's book or you could even see it in you tube, and from those videos it will be clear for the you that, phenomenon is roughly something like this, very slow growth and nearly sudden shrinkage .

So now, this is length and this time. Now what do we need, what is the second question? The second question is also, plot the derivative of length verses time graph, you have to plot the derivative of the this. So what do we meant by this derivative? So derivative lets plot on the same graph here itself, derivative of this is dL by dt as the function of t . This is what we have to plot, dL by dt by t , now if you plot derivative of this, look at here, this is just a straight line. So what is the derivative of a straight line?

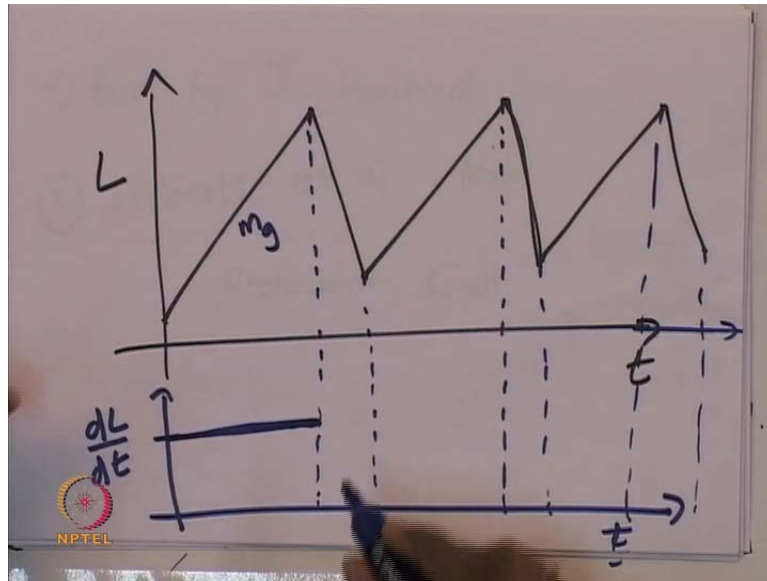
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$$y = mx + c$$
$$\frac{dy}{dx} = m$$

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So typically as you all know, straight line has equation which is y is equal to mX plus C . We discuss it and the derivative $d y$ by $d X$ is equal to m it is constant. So the slope of this part of a curve has to be a constant, and if you take this is a constant function.

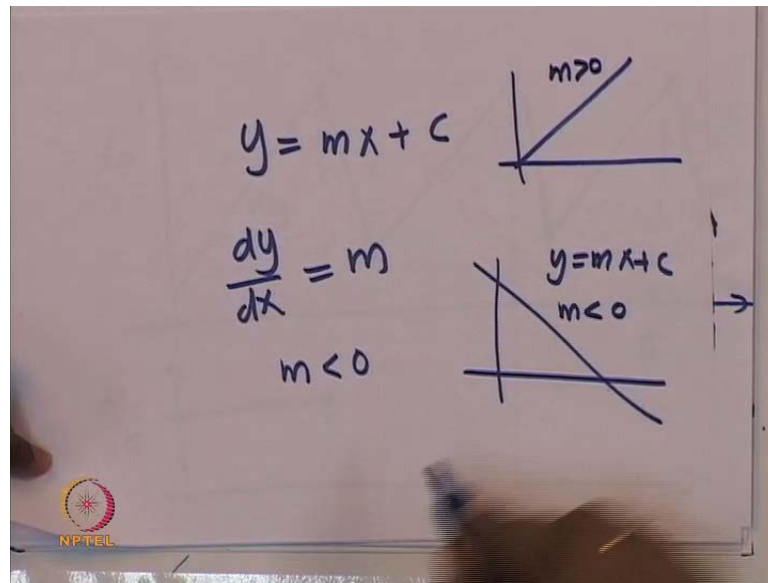
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So until this, **this is L**, so let us draw this points where this slope changes. So here till this is one line, then suddenly the slope changes in different line one line slope changes different line, by increasing, decreasing. So, these places where the slope changes, let's mark these points. These are the points where the slope changes, so now if he plots this, up to this, it is just one slope.

So let us call this slope m , g this is a growth, slope m g . So this will be the value of m g , so then it is just the same function till this, ok it is m g . And this, now the second part of this is a different slope and this is actually again another line but, aligned like this.

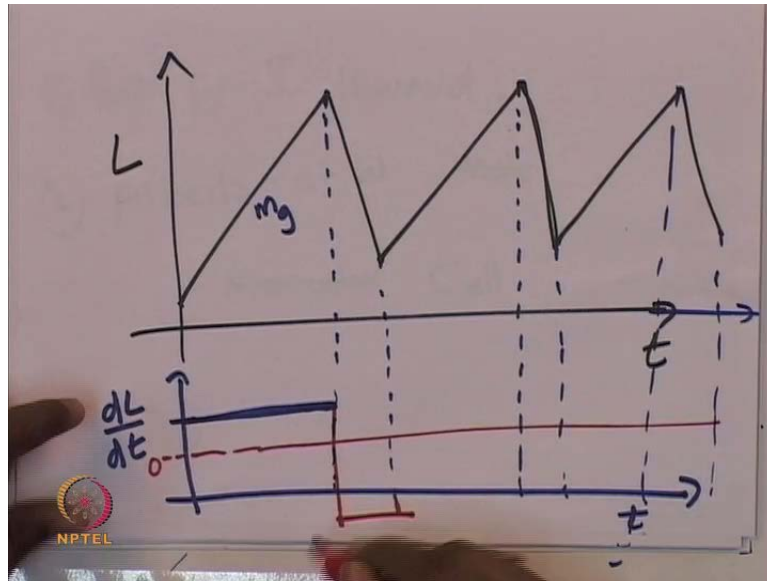
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So as you know, if you have m less than zero for a straight line. If the slope is negative, the line will look like this. So this is mX plus C with m negative, so this is y is equal to mX plus C with m less than zero.

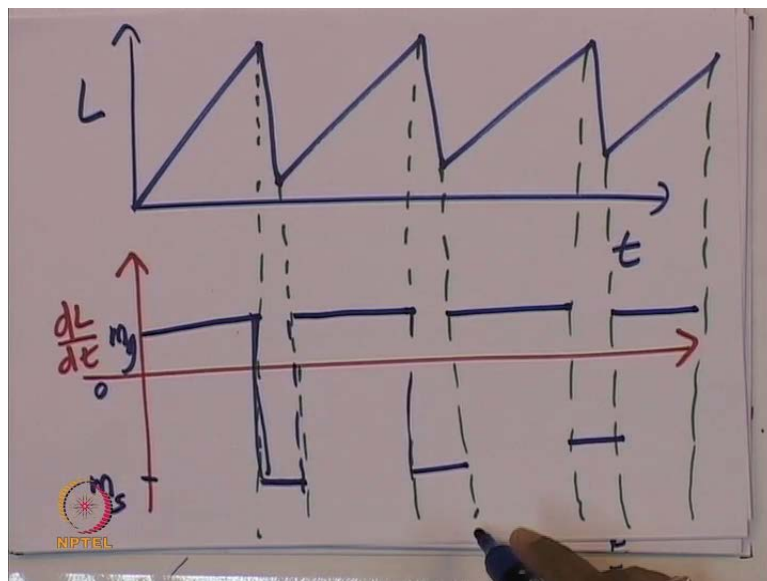
The slope less than zero, so this will look like the slope is like this, what does it mean if the X increases, y decreases. If y is equal to mX plus C its positive, means as this is m positive and this is m negative. So here the m is a negative value but, a large negative value. So we have to also make a negative access here.

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So let me make this as zero, so let me call this zero line, right? This is zero line. So this one has to be less than zero, much below and this has to be much larger than this. So somewhere very and somewhere here, so it will be much better to draw in a different paper. So it will be somewhere here much below this, so let me draw this in a different graph, so let us quickly draw it in a different graph here.

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So what we have here is first length versus time. So this is length versus this time and now what we want is the derivative, the derivative we want to plot. So we want to plot

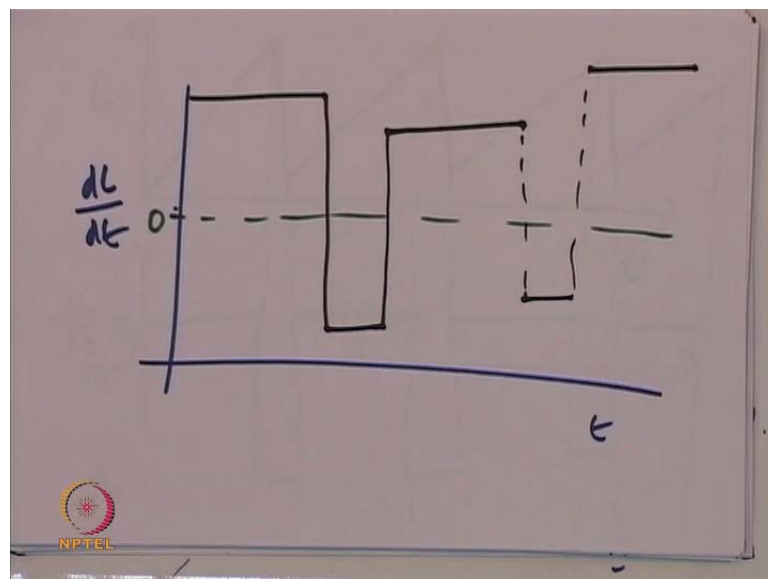
here the derivative, so we want to plot dL by dt and we will mark these points, I will just draw vertical lines so that you can mark this. So, now as we said this part it is straight line y is equal to mX plus C so, this is an positive number so this is something here this is one constant. And this value is as we said, can be let us say $m g$ some growth. Now this is shrinkage which is negative slope.

Slope is negative but, much larger than this. So it is like this, will be much larger than here somewhere. This distance and this distance, from here to here is zero, so this is zero. And how much negative it goes from zero, this is minus $m g$. We call this m shrinkage, m minus which is a negative value. So up to this, between this line and this line, what we have is something with large negative value.

So here is large negative value and here its again positive value, goes again same slope as this, then again shrinkage some slope here. So you could connect this two and here again. So what you will have is something like a square wave here if you wish. This is the growth rate or the speed of growth and this speed of shrinkage, so derivative of this will look like this. So this is length and this is derivative and you can now there are only two speeds, if there are many speeds, growth velocity there are many numbers, you will get slightly differently, you will get this here.

You could in principle also get, if the growth and shrinkage speeds differ in each attempt. You could also get something like this the following:

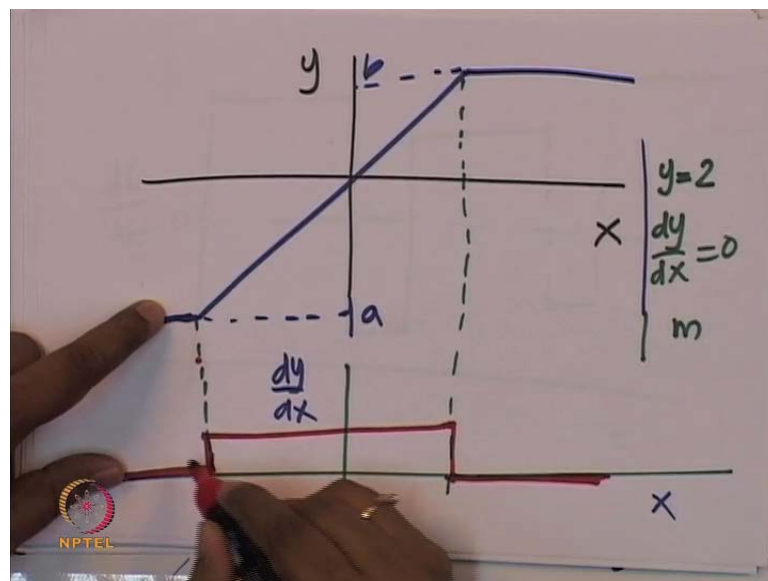
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You could also get, let us say dL by dt as t and this is your zero line. This is zero and you could also get something like to go to different value, this growth value here is different. Here the speed of growth is different then, this is different from this. So actually, I should draw all this vertical lines, dots, because they are not real. So this kind of way, you can draw. So you should but, you should think that the shrinkage time is low little, because shrinks very fast and then this is much larger than this. So this you could have this kind of dL by dt verses t graph basically, this means that the speed of growth in each attempt is different with the shrinking rate.

So essentially, this is one way of drawing this graph and from this you could learn many things about the rate of growth and so and so forth. And we could get many quantities but, what we learn here is, from a description how do we make a graph? So talking about this, drawing function and its derivative, you should have like very, one another interesting question?

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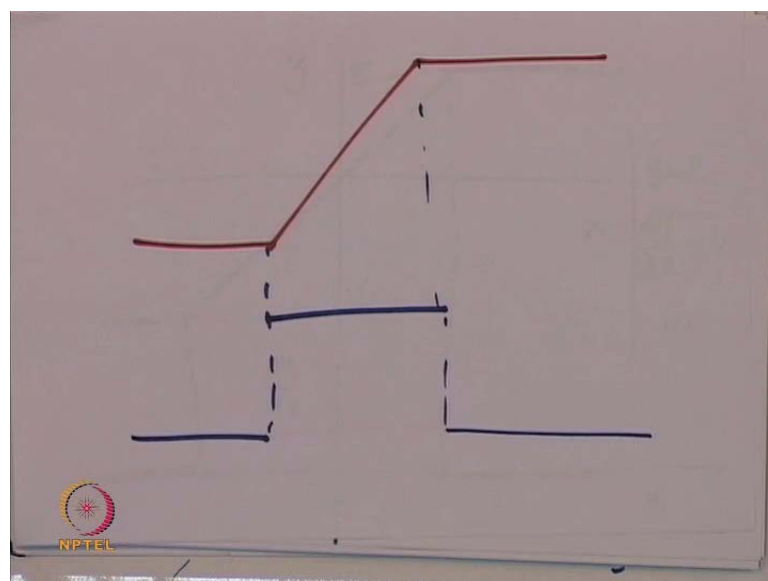
Which you all should know, is basically for example let us think of y and X and let me draw a function which is this particular way. So the function is, so let me call this function as some particular value, a here and some other particular value b here. It is constant values and between that, is a straight line and just below that here we have to draw the derivative of this.

So we have to draw here, dy by dX versus X so you can see here. So this curve has three paths, so here in this part it is like 'a' is a constant line, it is like y equals to a is a constant here. Here also it is a constant y is equal to b and in between this is a straight line. So you have a part, its three functions so the question to you is that, by looking at this without doing any calculation can you draw the derivative? So again to look at the graph here, so just by looking at it without doing any calculation, can you draw derivative here? So you see here three parts, for this so immediate literalize that for this part, it is a constant and when it is a constant, let us do from calculations in the margins here and write the first, is a constant. So this is, y is equal to 2 or y is equal to 3, it is like a constant and the derivative of that $\frac{dy}{dx}$ is zero for a constant.

So here, the slope has to be zero. Here it is a straight line, the slope is a constant. Some number m here again it is a constant, so the slope is zero. So zero constant zero, so it has to be so. Let us mark this, till this the derivative is zero. Till this the derivative between this 2, the derivative is a constant m and here again the derivative is zero after this. So let us try to draw this, here the derivative is zero, so let us draw zero. And here to here, the derivative is a constant.

So let me draw a constant line here and again it is zero so this is zero. This looks like a box like a function.

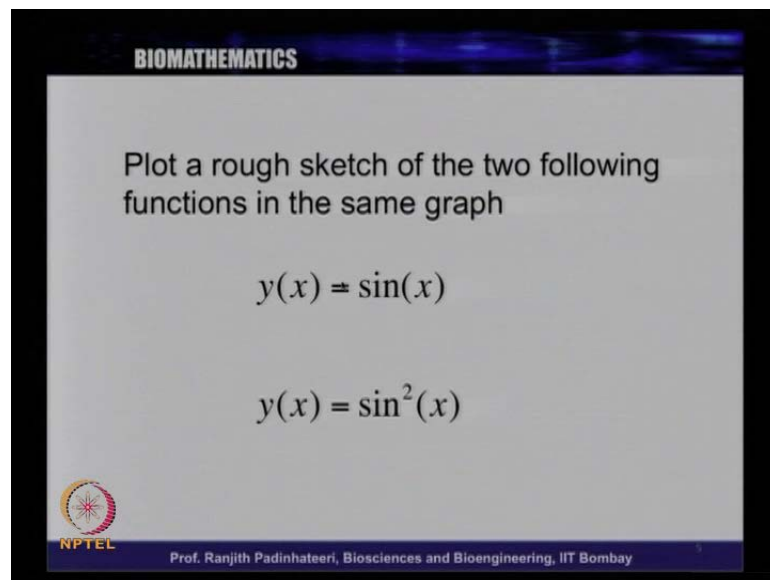
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So the answer looks like, if we have something like this, derivative of this is zero here then it is a constant till this and then again zero from there. So the answer, will look something like this, so this is your function and this is derivative. So just by looking at it, you should get training to plot the derivatives and this will help you immediately thinking about derivative without really doing the differentiation. By differentiation by hand, by knowing **the learning** just by physically looking it you could make a lot of sense out of many of the things that you see.

So this is the aim of this training, we will also see couple of some questions, which will basically say some other functions to plot quickly because, if you can think of functions as graphs that will surely help you.

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BIOMATHEMATICS

Plot a rough sketch of the two following functions in the same graph

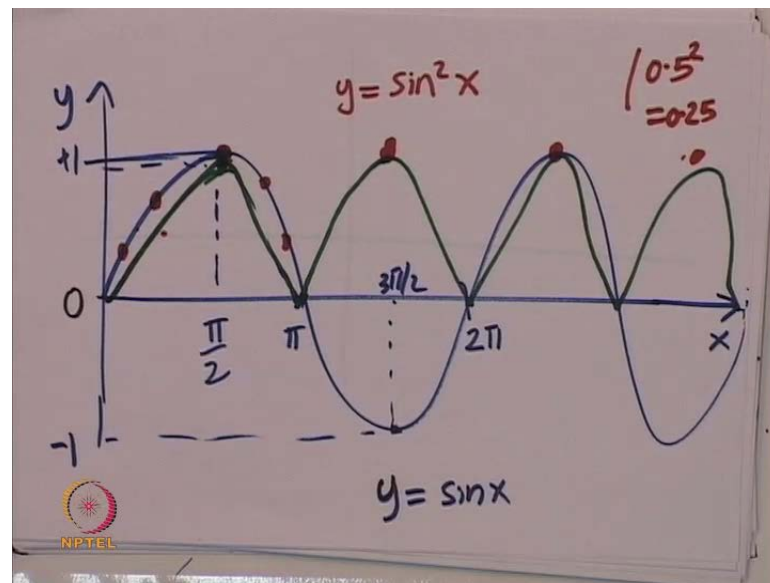
$$y(x) = \sin(x)$$
$$y(x) = \sin^2(x)$$

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So let us look here, the next question on the tutorial which is plot a rough sketch of the two following functions in the same graph, so y is equal to sin x, y is equal to sin square x, both of them we should plot in the same graph.

So we have two functions, y is equal to sin X and y is equal to sin square X. So when we say sin X, we immediately know how does this graph look like? So the sin X typically, so let me draw this little more carefully here, in a different plot.

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So $\sin X$ typically will look like, let me start from here, starting from zero is like a oscillating function. And so when you say plot, you know at zero is zero and again \sin , the maximum value of \sin has to be plus 1 and the minimum value here has to be minus 1. So let us mark this and here this becomes this point, is basically π by 2 and this is π . This is 3π by 2, this is 2π . So 2π 1 cycle is completed and reached back and then everything repeats, so you add π by 2 and just keep going this way. So this is, y is equal to $\sin X$, the blue curve. Now what I will show here in red is, y is equal to \sin square X , now this is our zero, $\sin X$ goes from minus 1 to plus 1.

Now this is, y is equal to the blue. What do we show here in blue is y is equal to $\sin X$ and this is our X and this is our y . Now when it is a \sin square X , so what we want plot is basically y is equal to \sin square X , so this is square. So if something is square, it always has to be positive.

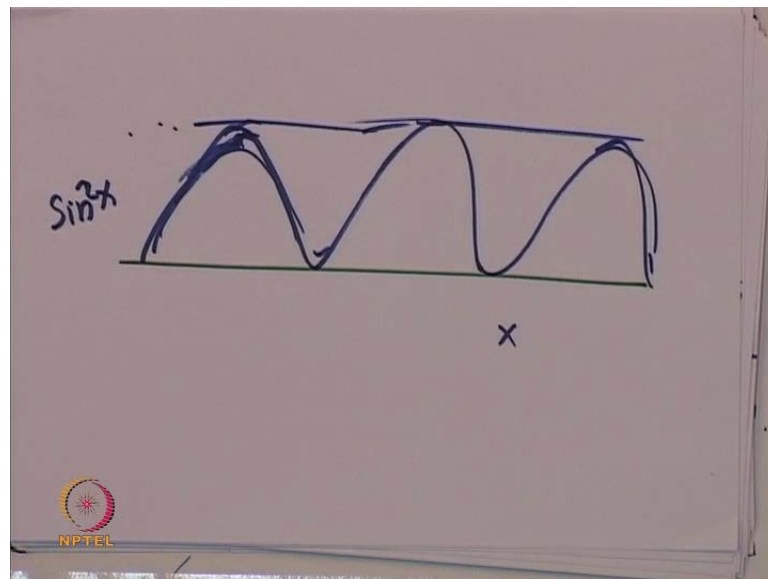
So whatever, $\sin X$ is a square of this. So whatever I want to plot, is only going to be above this because, it is square of something. \sin square of \sin square, this is only positive. Anything square is only positive even minus something square is positive, that is first thing we learn, it has to be above this. Second see here, the maximum values are plus 1 and minus 1, so the maximum values of the square, also has to be 1 because, 1 square is 1. Minus 1 square is also 1, so the maximum value has to be 1. But, all the

values here as you know, immediately we can say $\sin^2 X$ at this point will be the same because, 1 square is 1 and minus 1 square is also 1.

So these points, this is going to be same value here, this is plus 1 value is plus 1. Now here, for any other value for example, these values, the square of this, is basically less than 1. This is red spot here below 1, so square of this, let us say for example, let us take 0.5 and square of this is 0.25, which is less than 0.5. If this is 0.5, square of this will be below this because less than 0.5.

So anything less than 1 square, has to be go below this. So the curve all over here, has to go below this and hit here, and come below this zero, it should hit zero because zero square is zero, so it should hit here and come back. So it has to go smoothly below this and hit the curve, this is $\sin^2 X$ as roughly will look like this.

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So if you ask to plot, $\sin X$ and $\sin^2 X$, so the $\sin^2 X$ will roughly look like, to be one everywhere. So the height has to be same, this has to be one everywhere nicely, smoothly, it has to hit one, so smoothly between zero and 1, it will go. So the curve, looks like this. So do plot this yourself, so this is $\sin^2 X$ versus X so plot yourself and learn this how to plot it but, this is an interesting exercise that you can do.

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BIOMATHEMATICS

Find the derivative of the following series

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \dots$$

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So the next question basically, is to find the derivative of the following series. So we learn many series and for example, we learn e^x and so on and so forth. We even learned, $x \sin x$, $\sin x \cos x$ etcetera in terms of series. This is a series, $\log 1 + x$ and you have to find out the derivative of this and the derivative of this will appear, will turn out to be one of the series that you already learnt. So do this here yourself and just leaving question to you, to do this and try this, as an exercise and it will turn out that the answer is something that, you already learnt. One on the series that we discuss, so do this here yourself and we will just go through the next question and if there is time we will come back and do the answer for this.

But now, will go and look at another function, another important interesting function to plot, which is very much used in biology. And see this is again something, we learn this to schematically plot functions. So we will learn this function, so that we can plot it. We will learn to do an exercise; we will do a tutorial of plotting a function in a very systematic way, so that this will be an exercise for that.

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BIOMATHEMATICS

In Biology, typically, enzyme kinetics is described using Michaelis-Menten equation

$$v(S) = \frac{V_m S}{K_m + S}$$

Schematically plot the function

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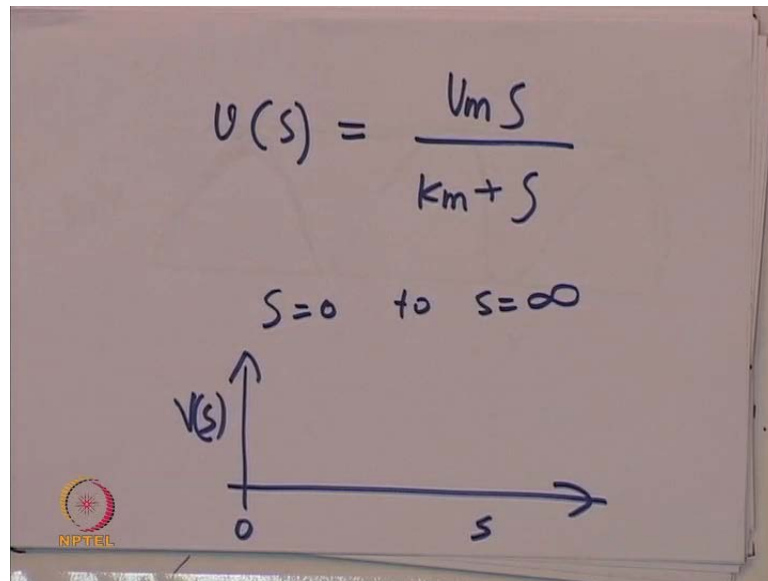
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So let us go to next question, so the question is the following in biology, typically, enzyme kinetics is described using the Michaelis-Menten equation.

So this kind of an equation most of you have seen, the Michaelis-Menten equation, which is V of S is $V_m S$ by $K_m + S$ and this is basically, the enzyme kinetic speed velocity and S is basically related to the substrate and V_m is a maximum speed and so on and so forth. Does not matter whether, you know phenomenon or not at this moment. But most of you know, most of you have learned bit of biology, would know this relation is a equation in enzyme kinetics, known as the Michaelis-Menten equation. And we have to schematically, systematically plot this function.

Most of you know, How to plot? How does it look like? But, let us use of all this, learn and then systematically plot this function. So we discuss the set of rules:

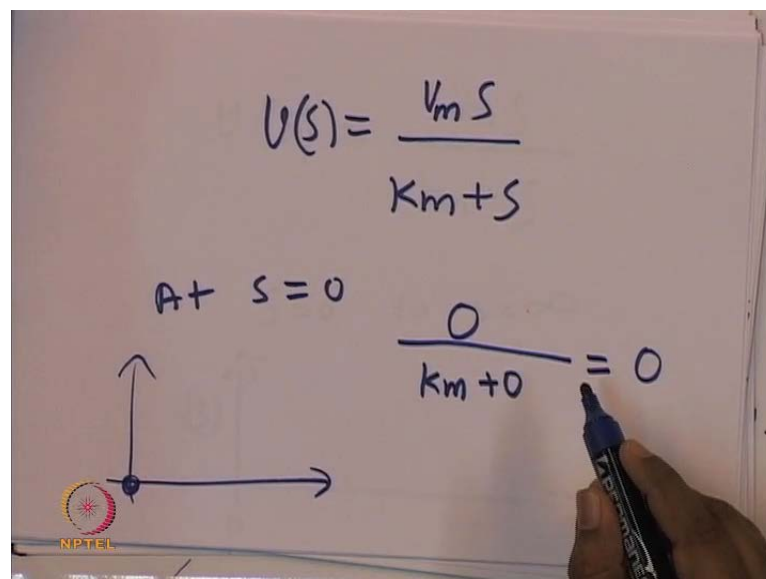
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So let us look at the function what we have, V of S V ms by K m plus S . So this is the function that we have and we want to plot it, so whenever we want to plot it, so typically S will go from zero to infinity. So zero to any large number, so S equals zero let us say, it goes to S is equal to infinity.

So that is, if you plot here this S from zero to infinity, we have to plot V of S . So the first thing we should know is, what is the value of this at S is equal to zero and at S is equal to infinity. So, let us look at the function at S equal to zero and S equal to infinity.

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So let us quickly look at this, so our function is V again. V of S is $V_m S$ divided by $K_m + S$ and at s is equal to zero, we have V_m into zero. So we have w_m into zero is zero in the numerator, plus K_m plus zero, which is K_m . Zero by a constant which is zero, so at S equal to zero, the function is zero. So in this plot here, it is zero. Now, let us look at far away, S is equal to infinity what happens?

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At $S = \infty$

$$U = \frac{V_m S}{K_m + S} = \frac{\infty}{\infty}$$

$V_m > 0, K_m > 0$

L' Hospital rule

Now, at S is equal to infinity or at as S goes to infinity. So let us look again here, we have V , $V_m S$ by $K_m + S$ K there is **infinity** large, if you just substitute S equal to infinity, this will be infinity by infinity. Because, this will be infinity here, this is also infinity plus K_m is also infinity and infinity into V_m is also infinity. Another important point, that you should know is that, w_m and K_m are positive constants for the moment. V_m was something zero greater than zero and K_m was something greater than zero.

So this is, something which is given to you and then we have infinity by infinity. Now whenever we have such a infinity to infinity, the rule is something which we discussed called L hospital rule or Lapthal rule, which is basically, find the derivative in numerator and denominator and then substitute the value of S . So if you find the derivative in the numerator, which is derivative of $V_m S$ with respect to s .

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The image shows a whiteboard with handwritten mathematical work. At the top, the derivative of velocity with respect to distance is given as $\frac{d}{ds}(V_m s) = V_m$. This is then simplified to $= \frac{V_m}{1} \rightarrow$. Below this, the derivative of the denominator is given as $\frac{d}{ds}(K_m + s) = 1$. At the bottom, the limit is stated as $\text{As } s \rightarrow \infty \quad v \rightarrow V_m$. An NPTEL logo is visible in the bottom left corner of the whiteboard.

What you would end up, is just derivative of $V_m s$ with respect to s , so $\frac{d}{ds}$ of $V_m s$, is just V_m , this is derivative of numerator.

And derivative of denominator, which is derivative of $K_m + s$ is basically, 1. Because, derivative of K_m is zero and derivative of s is 1, so derivative of this has to be 1. In this particular way, so we have now if you substitute here, the answer we get is basically V_m by 1, so the answer is going to be a constant which is V_m . So the answer is basically V_m , so at s is equal to infinity or s going to the infinity, v of s will go to V_m the maximum velocity.

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The image shows a whiteboard with handwritten mathematical expressions. At the top, the Michaelis-Menten equation is written as
$$v(s) = \frac{v_m s}{K_m + s}$$
. Below this, the text "At $s = 0$ " is written. To the left, a small coordinate system is drawn with a vertical axis labeled v_m and a horizontal axis labeled s . A dashed arrow points from the origin of this graph towards the right. To the right of the graph, the equation
$$\frac{0}{K_m + 0} = 0$$
 is written, indicating the value of the function at $s = 0$. In the bottom left corner of the whiteboard, there is a small circular logo with the text "NPTEL" below it.

So in this graph, that is we have to plot, here the value is for the large value of S . The function has to have some V_m starting from here, it has to end here. So it has to have some curve between this value and this value, and the only question is, is there any maxima or minima or not. So the next rule that we learnt, is to find out the first derivative and then see if there any maximum or minimum or a maxima. So what we need to find out a derivative of this function, so we have the rules that we learnt, we have to find out the derivative of this function.

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The image shows a whiteboard with handwritten mathematical expressions for the derivative of the Michaelis-Menten equation. At the top, the equation is written as
$$v(s) = \frac{v_m s}{K_m + s}$$
. Below this, the derivative is calculated using the quotient rule. The expression is written as
$$\frac{dv}{ds} = \left[\frac{d}{ds} (v_m s) \right] \cdot \frac{1}{(K_m + s)} + \frac{1}{(K_m + s)} \cdot \frac{d}{ds} (v_m s)$$
. The first term $\frac{d}{ds} (v_m s)$ is simplified to v_m . The second term $\frac{d}{ds} (v_m s)$ is simplified to v_m . The final expression is
$$\frac{dv}{ds} = \frac{v_m}{K_m + s} + \frac{v_m}{(K_m + s)^2}$$
. In the bottom left corner of the whiteboard, there is a small circular logo with the text "NPTEL" below it.

So we have this function, V of S is $V m S$ by $K m$ plus S . So let us find the derivative, which is $d V$ by $d s$ which is basically, d by $d s$ of the numerator, which is $V m S$ minus and you have $V m S$ into derivative of, actually you have plus $V m S$ into derivative of, so let us apply u by V rule.

This is like u and V , so you have to find the derivative. The numerator multiply this with 1 by $K m$ plus S , then 1 by $K m$ plus S then we have to multiply $V m$ into derivative of 1 by $K m$ plus S , this is what you have to do. So let me write this little carefully, so what we have this is the u by V rule.

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$$U(s) = \frac{VmS}{Km+S}$$

$$\frac{dU}{ds} = \frac{d}{ds} \left[(VmS) (Km+S)^{-1} \right]$$

$$= \left(\frac{dVmS}{ds} \right) (Km+S)^{-1} + (VmS) \frac{d}{ds} (Km+S)^{-1}$$

$$= \frac{Vm}{Km+S} - (VmS) \frac{1}{(Km+S)^2} = \frac{Vm}{Km+S} - \frac{VmS}{(Km+S)^2}$$

So we can you can write our function, which is V of S as $V m S$ by $K m$ plus S , $d V$ by $d s$, you can write as d by $d s$ of $V m s$ into $K m$ plus S of whole power minus 1, which is basically $V m$.

So now, let us find the derivative of this whole thing, which is basically derivative of this into $K m$ plus S whole power minus 1 plus $V m s$ into derivative of $K m$ plus S whole power minus 1. Now what is the derivative of this, which is a constant, which is basically $V m$ and we have 1 $K m$ plus S inverse, which is $K m$ plus S here and then there is the derivative of something power minus 1, will give a minus and there we have $V m S$ and this will have 1 by $K m$ plus S whole square, the minus 1 will have a derivative which is minus 2, which is 1 over 2 into derivative of this, which is 1.

So this is what will get and we want this, we want to find out for any value of S, will this be zero. If derivative is zero, that means there is a maximum or a minimum at least 1. So let us see, if the derivative is zero at all, so let us equate this to zero.

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$$1 = \frac{S}{km + S}$$
$$\frac{\cancel{vm}}{\cancel{km + S}} = \frac{\cancel{vm} S}{(km + S)^2}$$

So what we have here, is basically if you go from here to here, V m by K m plus S as V equal to V m S by K m plus S whole square. So there is K m plus S whole square. So, this 1 K m plus S here, 1 K m plus S cancels and V m 1 V m cancels, so you have 1 K m plus S cancelling here with 1 K m plus S and V m V m cancelling. So what we have here, in the left hand side is 1, so basically you have 1 equals to S by K m plus S.

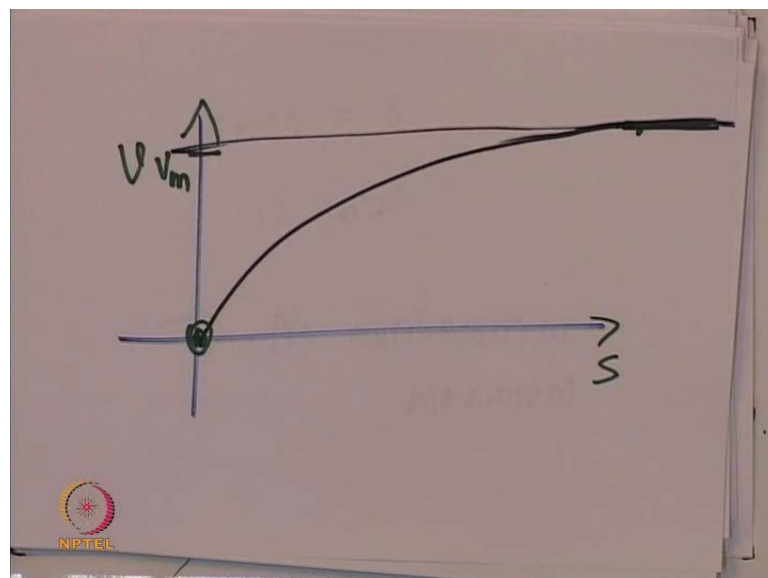
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$$k_m + S = S$$
$$\text{if } k_m > 0$$

\Rightarrow No minimum or maximum

In other words if you rewrite this, what does this mean is that $K_m + S$ equal to S , for any value if K_m is greater than zero, this will not be true at all. If K_m is zero which is given, this can be never be true therefore, there is no such minima exists. So this, dV by dS cannot be zero because, this cannot be true. We know that, K_m is greater than zero, and if dV by dS is zero this is true. And we know that, it cannot be true. This implies that no minimum or maximum. The derivative cannot be zero.

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So we had this function, which we saw had value here at zero. And for large value is S here and V here. And for small value of x , we got zero and for large value of x , we got V m. And the curve is a smooth curve, between these two things so just going and then saturating like this. So for S tending to infinity, this has to saturate to this value V m, which we have here and there is no minima or maxima. It is just curve like this, so you know that enzyme kinetics curve, looks roughly like this, so this is slowly increasing and saturating to a constant value, so this is what we saw.

So we had few questions and of different flavors. And we had discussion of, How to answer them? Now, as a separate file, I will provide some other questions, which you can answer but, in this tutorial we discussed a set of questions and we will have other tutorials with other topics, which we will discuss. So with this, we will stop today's lecture and we will continue with a discussion of another topic tomorrow, bye.