**Biomathematics**

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**Lecture No. # 33**

**Tutorial and Discussion**

Hello, welcome to this lecture on biomathematics. Today we will do a bunch of set of problems as it is like a tutorial, so we will discuss a set of problems related to what we have been learning so far, couple of points to make things, clear. And the main aim of this tutorial is just like we had before; one aim is given a description can you write down things in mathematically, but can you think how to write down the description that you hear in terms of mathematical quantity, and also to give you some feeling for numbers and also to train you to really solve some problems that is pretty much relevant to biology.

So, let us go through the questions, what we have today. And we will learn and try to answer one by one. And discuss if there is, anything need were something not. Some part of this question you will have to do it yourself. But I will discuss most of the answer to most part of all the question. So, let us start with the tutorial.

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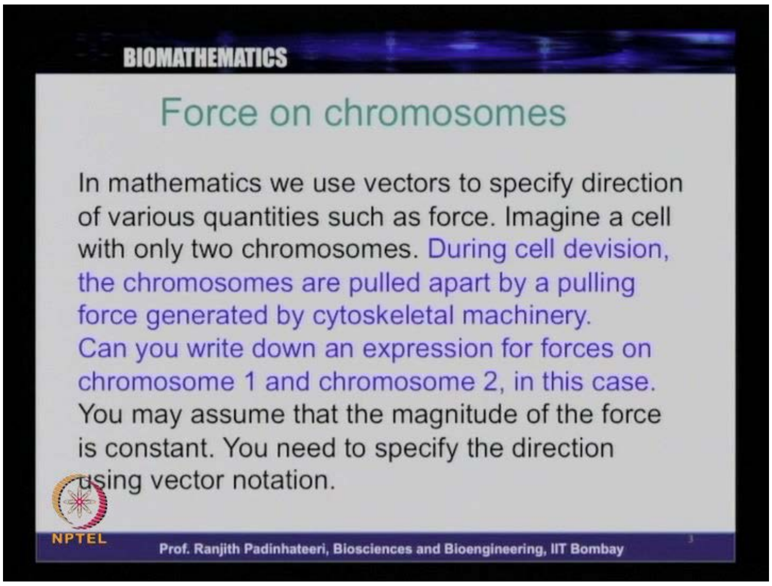

So, it is tutorial and discussion and let us go to the first question. So, again I will read the question in detail and then, we can go through each sentence. So, first let me read it in full, so in mathematics we use vectors to specify direction of various quantities such as force, imagine a cell with only 2 chromosomes during cell division the chromosomes are pulled apart by a pulling force generated by cytoskeleton machinery, can you write down an expression for forces on chromosome 1 and chromosome 2 in this case.

You may assume the magnitude of the force is constant, you need to specify the direction using vector notation, so basically this is a question just to help you think using vectors, it is a very simple question but, you have a physical situation that you know very well and you want to write. Take a little step like a baby's step, it is not a big problem like taking a baby step towards writing down, thinking about the situation and writing down the force in the vectorial notation. So it is a very very simple step. But it is important that you read it, figure out, how to do it and then do it. So let us read through this sentences carefully. So, the first sentence we say that is, in mathematics we use vectors to specify direction of various quantities, such as force so this is about vectors. And now imagine a cell with only 2 chromosomes.

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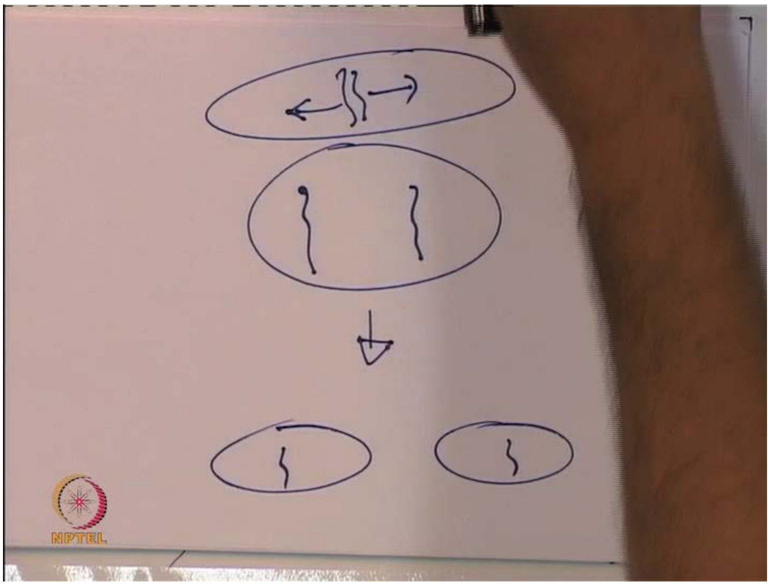

So, first let us draw a picture of a cell here and lets only draw chromosome. So, I will draw chromosome like this. So this is the chromosome, which actually does not matter what the shape is, I just drew this. Now after the cell division so lets say here to divide, make this 2 chromosomes forget the biological detail what we want is 2 cells with this chromosomes like this or we have to take this is what essentially happens but, for this chromosome to reach this 2 parts of the cell. So initially the chromosome will be like this and it has to be pulled apart to reach this kind of a situation and then the cell division happens. So we have 2 chromosomes, this have to be pulled in this direction and this has to be pulled in this direction. So that the chromosomes are pulled apart and then you get this, so this is the idea, this kind a of cell division we are thinking about you could have 2 chromosome becoming 4 and pulling 4 of the part but for simplicity let us not worry about all this we only want to think about the physical part of it.

So, let us look the question once more. During the cell division the chromosome are pulled apart so there are only 2 chromosomes and the chromosomes are pulled apart by a pulling force generated by cytoskeletal machinery. So, there is some cytoskeletal machinery to pull this chromosomes apart so typically it is like

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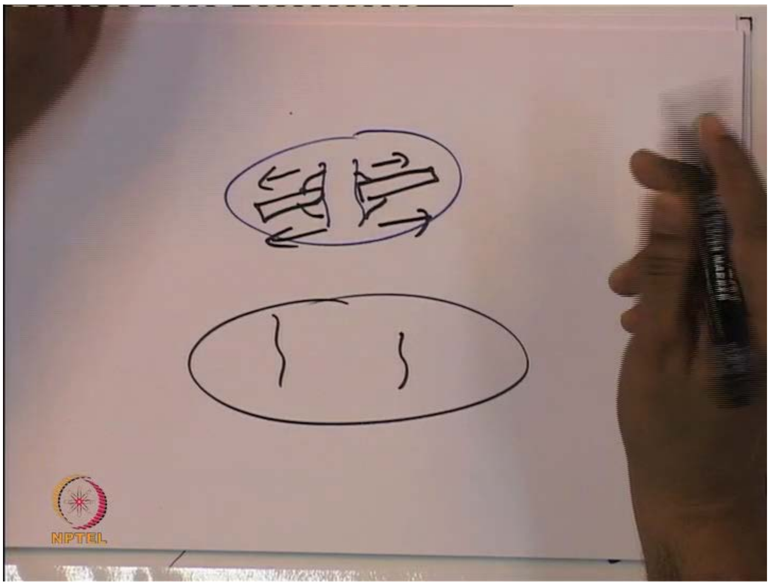

If you imagine, this situation what you have is some cytoskeletal machinery is basically, microtubules and micro tubule connected to this and this generates a force in this direction and takes the chromosomes to both sides. So, this is what typically happens, now, so the micro tubules and the kinetic core, this structure is called kinetic core so they pull the chromosomes in this direction. So what we want is, they exert a force on the chromosomes and pull them in this direction, so essentially what we want is chromosomes being pulled apart in that direction

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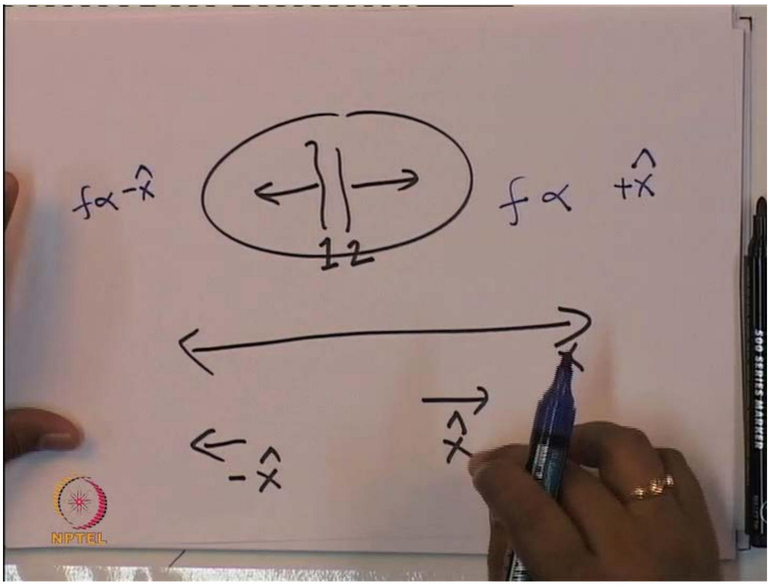


So, we have this chromosomes and we want to have force in this direction does not matter how the forces exerted say the details does not matter we are not worried about it but, do you want to write down so, what do we do, can you write down an expression for forces on chromosome 1 and chromosome 2. So, let me call this chromosome 1 and let me call this chromosome 2 and is written, you may assume that the magnitude of the force is constant that means the force does not change with time, it is always the same force.

And you need to specify, the direction using vector notion, you need to specify is going in this direction and this direction, vector notation. So let us first think of direction so let us say this is your X axis, so this direction, along this increasing X axis, this direction is plus X and this direction is minus X. So this is plus X and this is minus X, so the force along this has to be proportional to some constant times plus X and here it has to be minus X. So this is clear from this.

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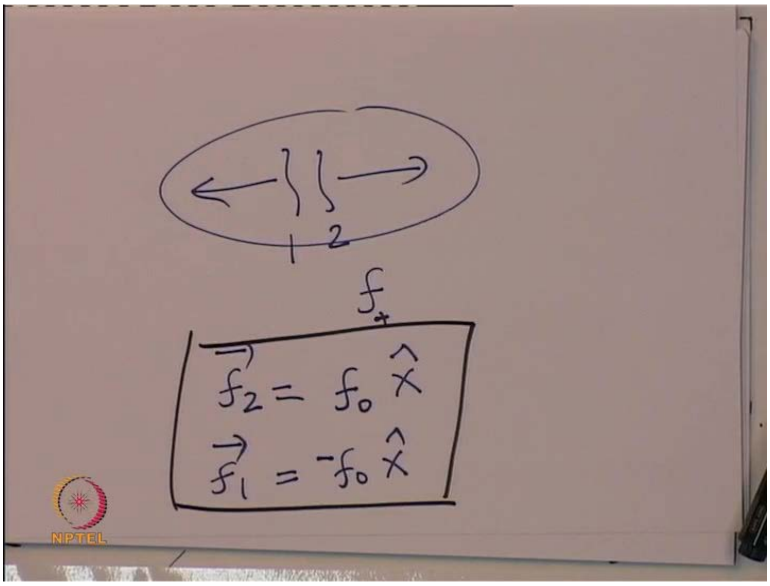


So, this is the basic idea. So the force along this direction has to be plus X and so, let us think of it, so, this is f along the plus direction or a force. So, we call this, chromosomes 1 and this is 2. So force and chromosome 2, which is force on this chromosome is some f 0 which is the constant, we can assumed as a constant X cap and f 1. So, this is a vector force 1 minus is also a vector and this is also some f 0 but minus X cap. So, there we have minus sign. So, one can write f 2 and f 1 this particular simple forms, f 0 X cap and minus f 0 X cap and assuming that the forces is constant now you can make the force time dependent.

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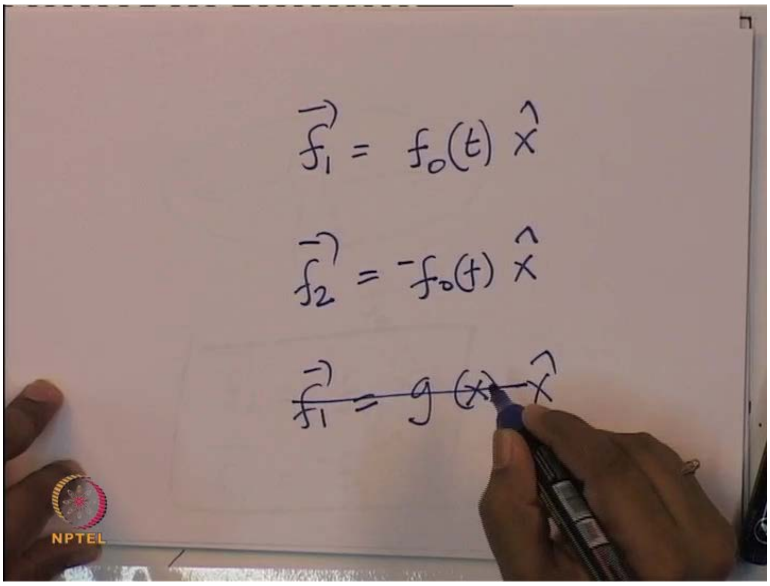

If you want, then you can write f 1 is some f 0, it is a function of time, but along X cap f 2 f 0 of t minus X cap. Now you need not even have this f 0 and f 0 both, you can have this has some function, so f 1 force on 1, so f 0 is just a notation does not mean anything else. So you can call this some function, g of X g of t it can be g of X and t in principle along X cap.

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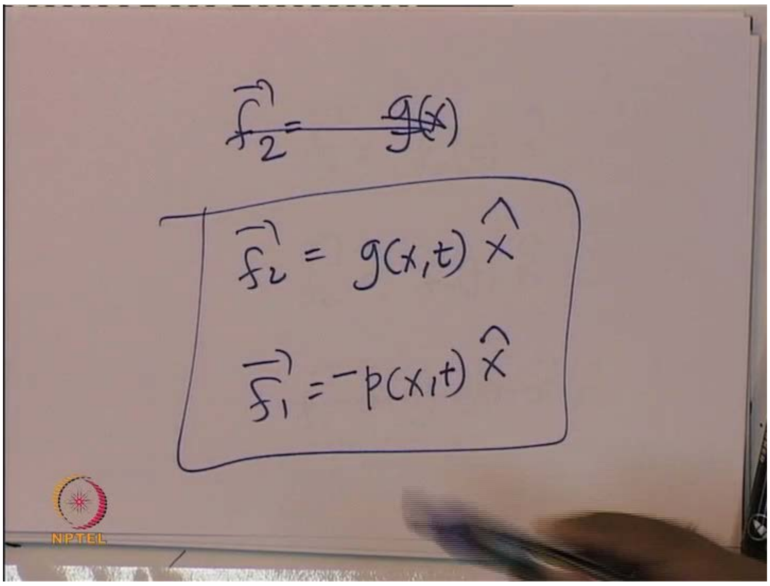


So, little more general I can write f 2 is g of X comma t it can depended on both position and time and you can have it along the X cap similarly, f 1 can be some other function which is p of X, t sum number p sum function p, but along the minus m. So, this can be little more general and the g can have interesting forms and so is p so both f g and p can have interesting functional forms depending on the situation but, the question here is just give you the idea that you have to put this X cap and minus X cap and you can actually put.

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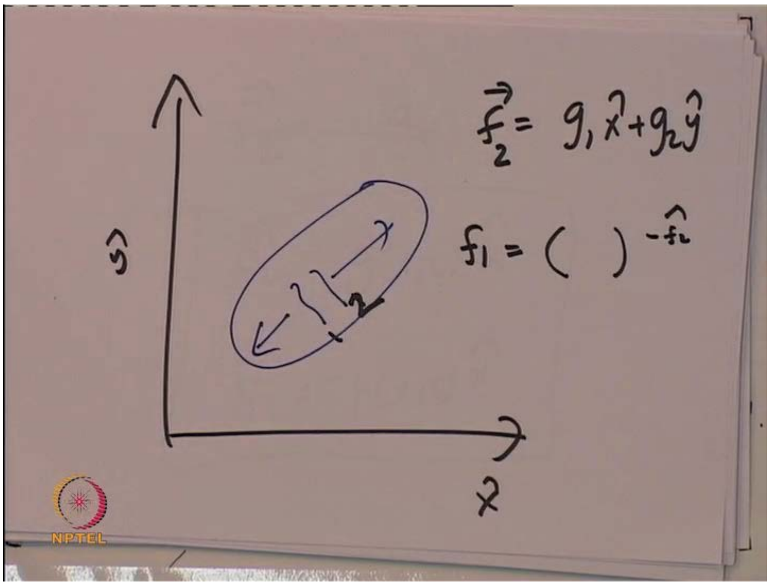


Whatever way you want you can say that. My X axis, my cell is oriented in this particular direction and the chromosomes are being pulled in this direction without loss of generality I can take this as X axis. Now, if you do not want that you can take this as X axis and this as y axis and you can write, this is one if, this is 1 and this is 2, you can write f 2 is some function of so let us say g 1 along X cap and g 2 along y cap some component will X cap component will y cap but, f 1 have to be just minus of it could be it has to be just opposite direction so then you have to make sure that direction of this is just opposite to this.

So, this has to be typically you can write this in the opposite direction so the direction of this, is basically given by this divided by unit vector of this, so you can write some other function but, minus of f 2 cap in the opposite direction of this f 2. So in this way, you can write it whatever way you want, but the simple way is something we discussed here which is just, this is the some general case which is in a simpler but, by general enough case. So, now, this is the question on force on chromosomes.

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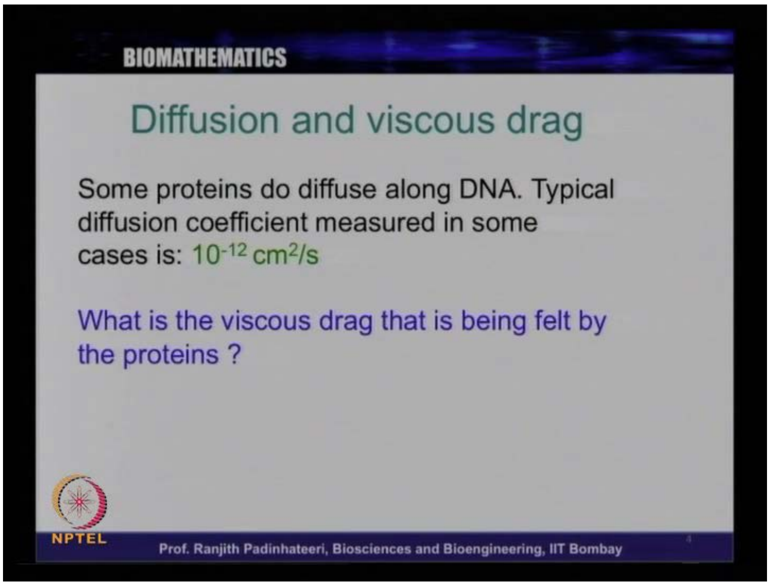


Now, let us look at the next question on diffusion and viscous drag. So something we learned is about diffusion equation. So our one aim here is to familiarize you with some number also, so let us look at the questions here, so the question is some proteins do defuse along DNA typical diffusion co-efficient measured in some cases 10 power minus 12 centimeter square per second. So, this is the typical diffusion coefficient. The question is, what is the viscous drag that is being felt by the proteins.

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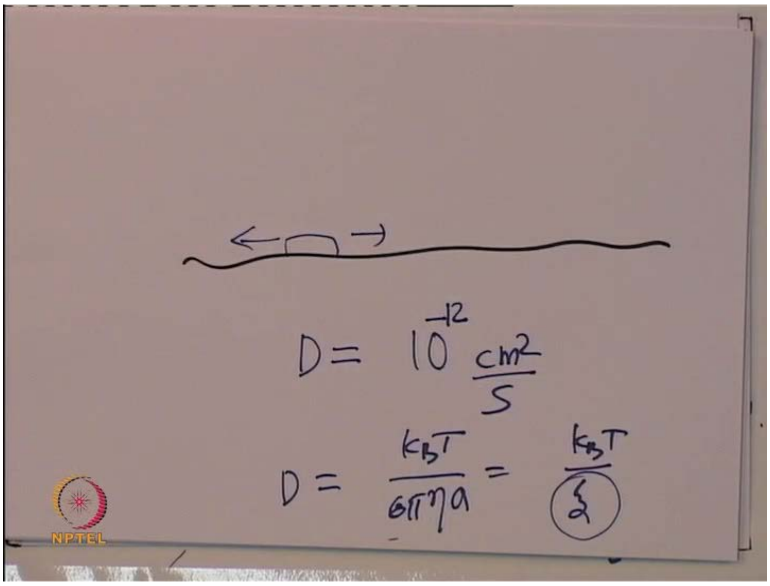

So, the question is like you have DNA and some proteins that is bound on the DNA they can defuse along DNA and the diffusion coefficient which we learnt turns out approximate number is 10 power minus 12 centimeter square per second. Now, the question is, what is the viscous drag?

So, we know, we have learnt the Einstein's famous relation, we said that, D is K B T by 6 pi eta a so this part we can call viscous drag, so, if you want you can write K p t by zeta and this is basically the viscous drag. So, whatever the bottom 6 pi eta a is vicious drag, so we know D we have to just calculate this and of course, we now K B and T. So, it is a very simple thing just applying the formula, just applying the Einstein relation but, given the diffusion coefficient we can compute this quality which is 6 pi eta a.

So, now let us calculate it lets do this before that let us look at this number once more 10 power minus 12 center meter square per second. What is it base pair, because the centimeter on the DNA does not make any sense, the unit of DNA typically DNA length is said or mentioned in the units of base pair, so how do we convert. What is this diffusion coefficient, if we convert that into base pair, so let us think about it a minute.

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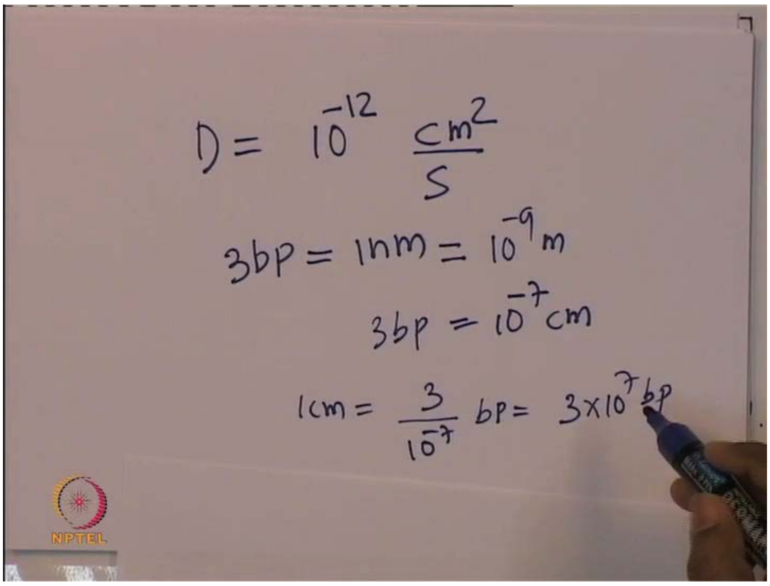


So, 1 centimeter how do you convert D we know, its 10 power minus 12 centimeter square per second so it turns out that 3 base pair is 1 nanometer that is the rough scale 1 base per is basically 0.333 nanometer actually, 3 base per is 1 nanometer so, 1 base per is about 1 by 3 nanometer now what is 1 this implies.

So, 1 nanometer actually means, 10 power minus 9 meter and 1 meter is 10 power 2 centimeter, so this is 10 power minus 7 centimeter is basically 1 nanometer, so 1 nanometer is 10 power minus 7 centimeter or 3 base pair is basically 10 power minus 7 centimeter. So in other words 1 centimeter is 3 by 10 power minus 7 base pair, which is 3 into 10 power 7 base pair this is 1 centimeter, so 1 centimeter is about 3 into 10 power 7 base pair. So now let us substitute this here, so what do we get, instead of 1 centimeter let us substitute 3 into 10 power 7 base pair.

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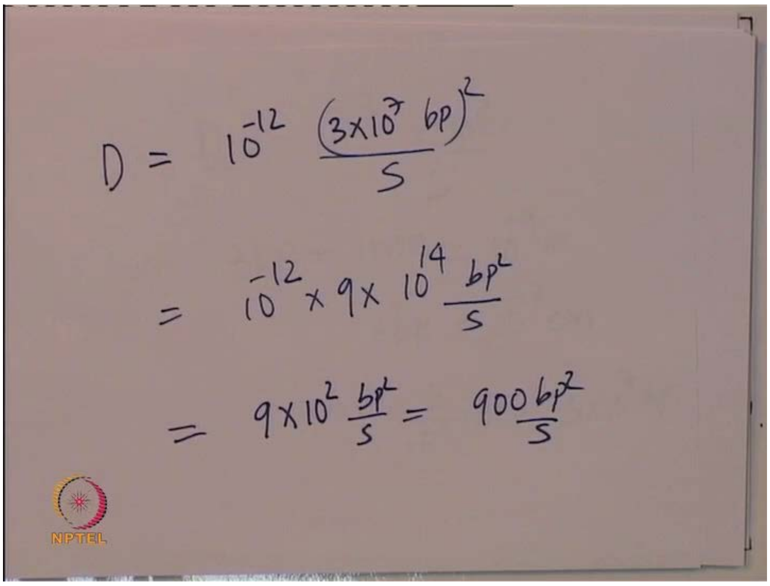


So D is equal to 10 power minus 12, 3 into 10 power 7 base pair square per second. So this is 10 power minus 12 3 into 3 is 9, 10 power 7 into 10 power 14 base pair square per second so this is basically 10 power 9 into 10 power 2 base pair square per second so this is about 900 base pair square per second is the typical diffusion coefficient of a protein. It is a nonspecific protein, going along nonspecific sites of DNA. So this is basically, the answer, so now let us convert this and calculate the viscous drag.

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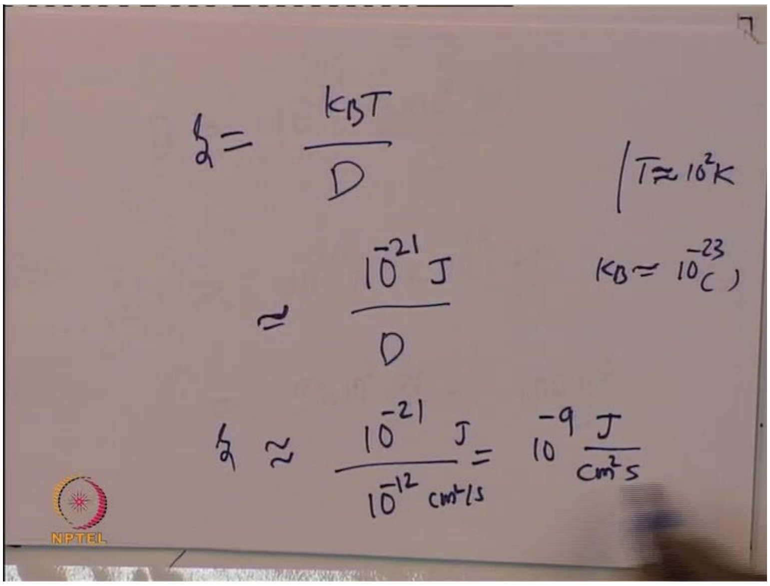


So, viscous drag zeta is basically K B T by D. Now, what is K B T, 1 K B T is actually about 10 power 21 joule for when t is equal to, about 10 power 2 Kelvin, this is roughly 10 power 21 joule and so, let me put an approximate notation here, because I am approximating K B T into 10 power minus 21 joule because K B is 6 something into 10 power minus 23 in SI units.

So, this is 10 power minus 21 joule divided by D and D we said is basically in unit of centimeter square per second. We can write it in terms of even base pair seconds so this is approximately 10 power minus 21 into 10 power minus 12 centimeter square per second into joule, so this is basically 10 power minus 9 joule by centimeter square second, so this is that and you can convert this in to meter, so base pair square second in all that, so but, this is the viscous drag, that you have to calculate approximately. So this is the answer to this question but, how do we calculate diffusion coefficient so, we will see next question and see how to calculate diffusion coefficient.

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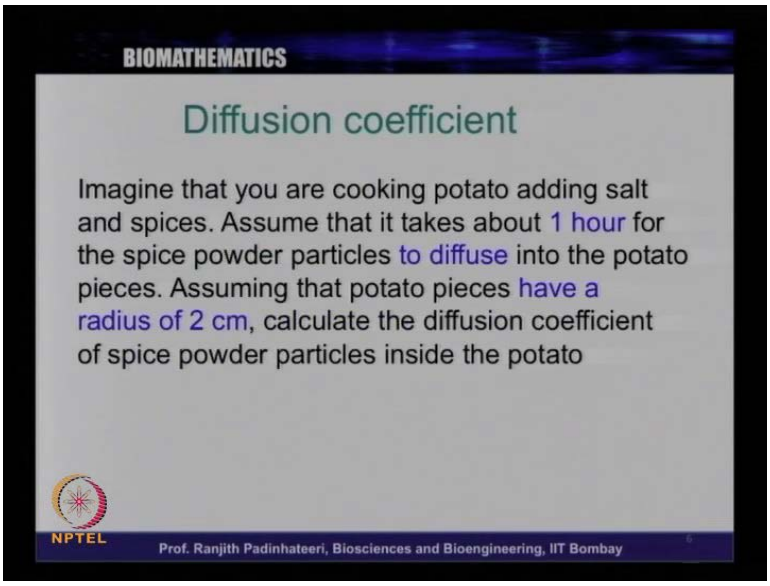


So, let us look at this question on calculating diffusion coefficient, so this is our next question, so imagine that you are cooking potato, adding salt and spices this is something that you do every day and assume that it takes about 1 hour for the all the spice powder particle to defuse into the potato pieces, assuming that potato pieces have a radius of 2 centimeter calculate the diffusion coefficient of spice powder particles inside the potato.

This is some example of which we can think basically, to trained to allow you to think that something that we see in everyday life can be thought of as a mathematical problem and then you can apply this. The idea that we learnt from day to day life like something is applicable in the things that we see in everyday life, something like cooking potato. So, that is what we are thinking about here.

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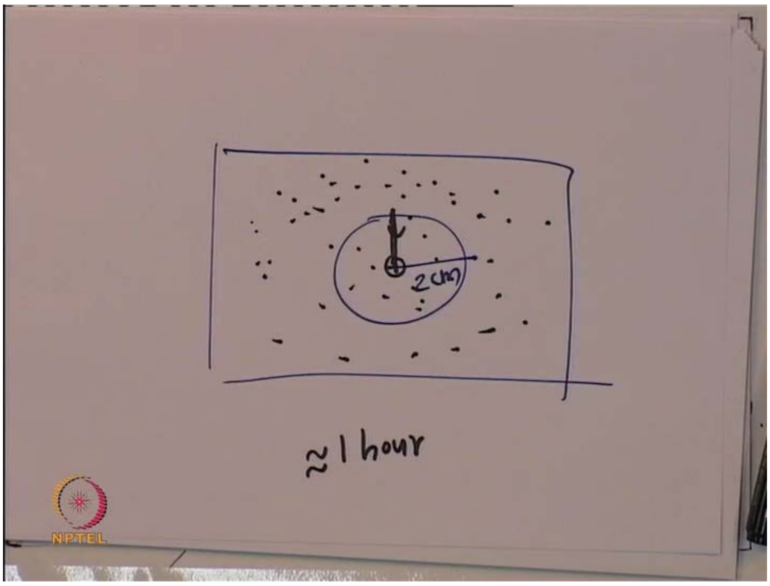


So let us think about it. So, you have a potato and said that the radius is about 2 centimeter and your putting spices, salt, etcetera into cook the potato so all this spice particles will defuse and it will reach here so it has to travel a distance of this much and reach to the core. So that everywhere come nicely salted and the spice reaches everywhere and it says that, it takes about 1 hour for it to completely defuse and reach here then what should be the diffusion coefficient. You have to know only this much how long it really takes for it to defuse and if it takes about 1 hour then we can run estimate the diffusion coefficient based on this information

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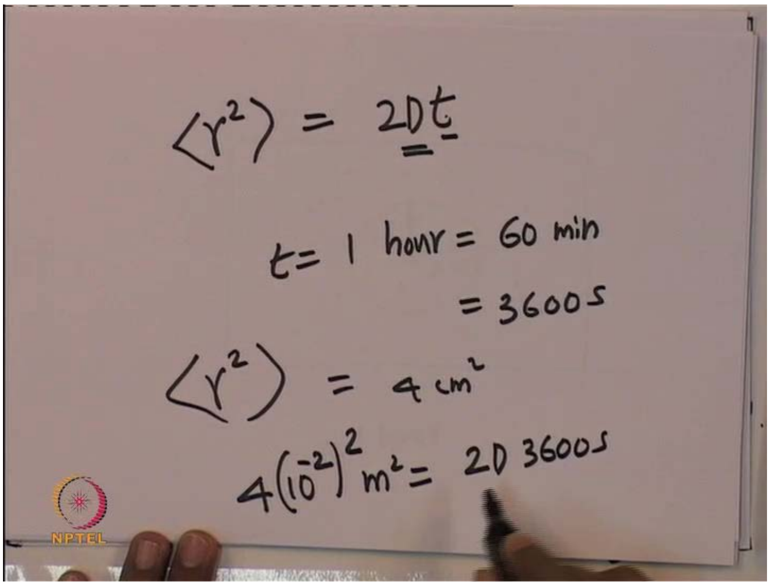

So let us do this calculation, this is began based on this relation that the distance like r square average the average r m s distance is 2 D t in 1 D let us do imagine 1 dimension thing first so its 2 D t so, now we do not know D but we know t, so t is 1 hour, which is basically, 60 minutes which is 3600 second. Now, what is r square average the typical distance is 2 centimeter, so square of the distance average has to be about 4 centimeter square. So this is the r square average, this is about 4 centimeter square so from this, we know t we do not know D, but we know this, so let us substitute 4 centimeter square, so centimeter, let us convert into meter so 10 power minus 2 meter is equal to 2 into D into 3600 second.

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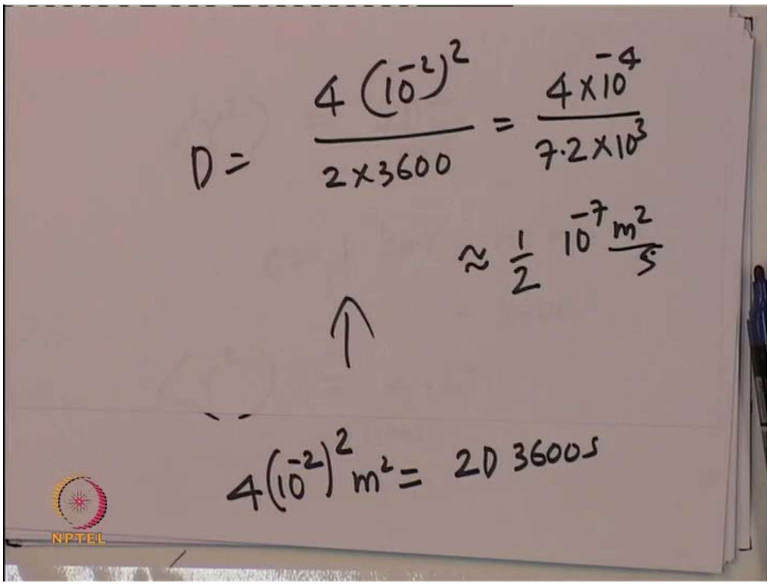


So from this, D is basically equal to 4 into 10 power minus 2 whole square divided by 2 into 3600. So, this is the answer, so you can write 4 into 10 power minus 4 this is 3600 into 2, you can say it is 7.2 into 10 power 3. So this is approximately 4 by 7.2 is about half 10 power 3 into 10 power minus 4 is 10 power minus 7, and the units here is basically meter square per second. So this is the diffusion coefficient of this spice partially, if it takes 1 hour to diffusion reach the middle of the potato. So, this is also to give you some idea, about some numbers, so this is one question, another question about diffusion is something that we will discuss in the next question. So, this is again a simple question.

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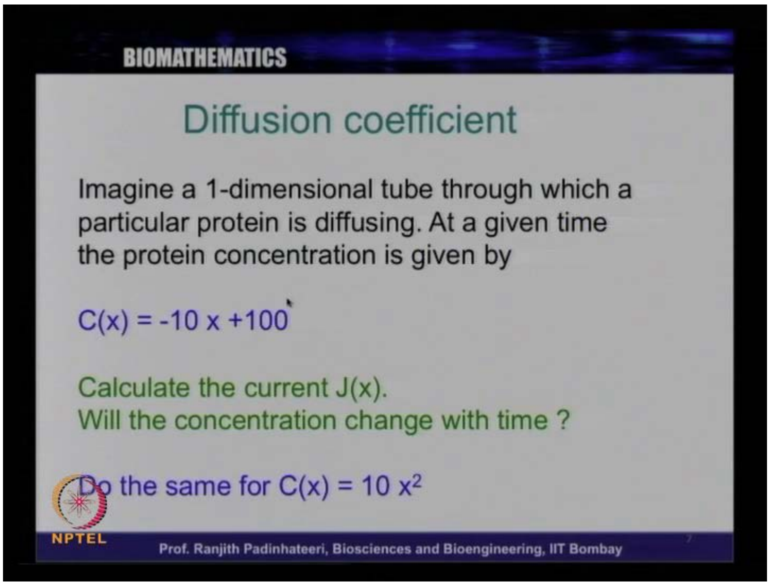


So, next question let us imagine, a 1 dimensional tube through which a particular protein is diffusing, so imagine a 1 dimensional tube through which, at a given time the protein concentration is given by this, calculate the current? Will the concentration change with time? You have to answer this, 2 questions and do the same, whatever, we did for the above problem for were the C of X is 10 powers X square.

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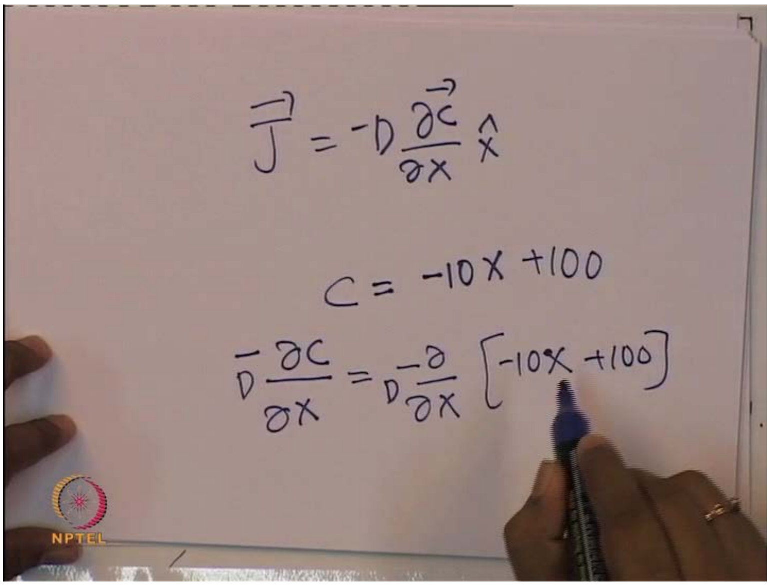


So, the first question is, calculate the current, so we now we learnt that the current J is basically D del C by del X cap so this is the minus sign, so this is the definition of J now let us look, we have C is equal to minus 10 X plus 100 del C by del X we have to calculate from this, so which is minus del by del X of minus 10 X, so minus del C by del X which is what we want and there is D here so I can multiply with D so D del C y del X minus 10 X plus 100. So minus and minus becomes plus

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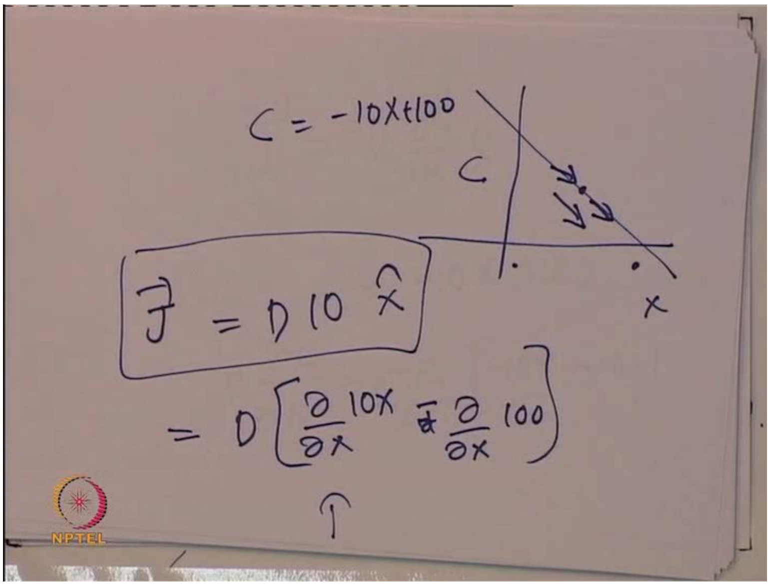


So, this is equal to D into del by del X of 10 X minus del by del X of 100 this is 0 because derivative of constant is 0 and this is basically just 10, so this is D into 10, so 10 D is the current, so there is a current in the X direction which is basically 10 D, so this is basically, the current along the X direction this, we can think of by plotting this function so, basically we are asked to use the function which is minus 10 X plus 100.

If you plot this, the function will look something like this, minus 10 X plus 100, so the slope is negative so, that means the concentration here is less compared to the concentration here, this is C was X so if the concentration is down here it should just flow in this way. But, will at any point, the concentration change with time, the answer we know physically is that, it will not because of whatever comes in will flow out because the flow is constant everywhere flow is just a constant D in to 10.

This is our J current or flow, so it is just D is a constant, 10 is the constant, so flow is a constant flow everywhere, so whatever, comes in will go out, so whatever, the answer we get will be, that the del C by del t is 0. So, that is the next part of the question, will the concentration change with time?

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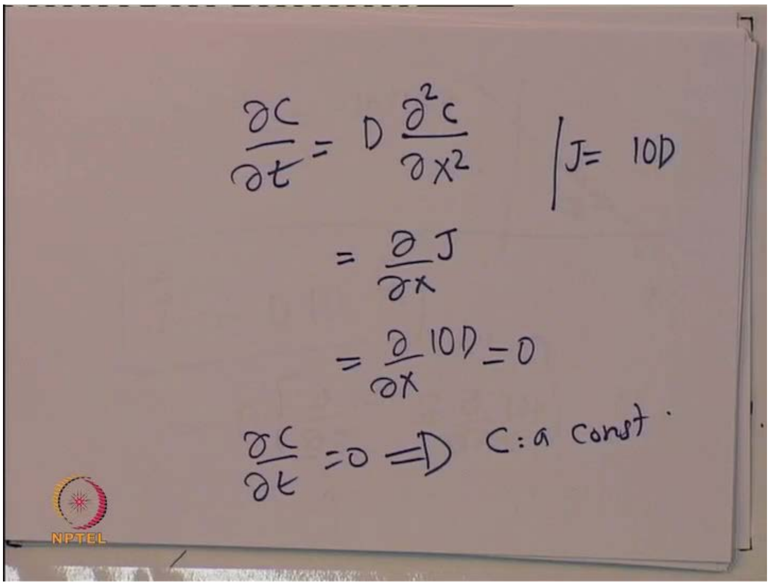


So, the answer to this question is that the change in concentration with del C by del t is basically D del square C by del X Square. So, in other words, we have to just do del by del X of the J we got, so the J we got, as 10 D and derivative of 10 D is basically 0, so del C by del t is 0 which implies, that C is a constant C will not change with time. So, the answer to this question, will the concentration change with time? The answer is no, the concentration will not change with time because J is the constant current or del C by del t is 0. So therefore, C will be a constant, now the next question is again, you can see something, here at the bottom, do the same for C of X is equals 10 X Square. Calculate the current and calculate the size in concentration for 10 X Square. So let us do this.

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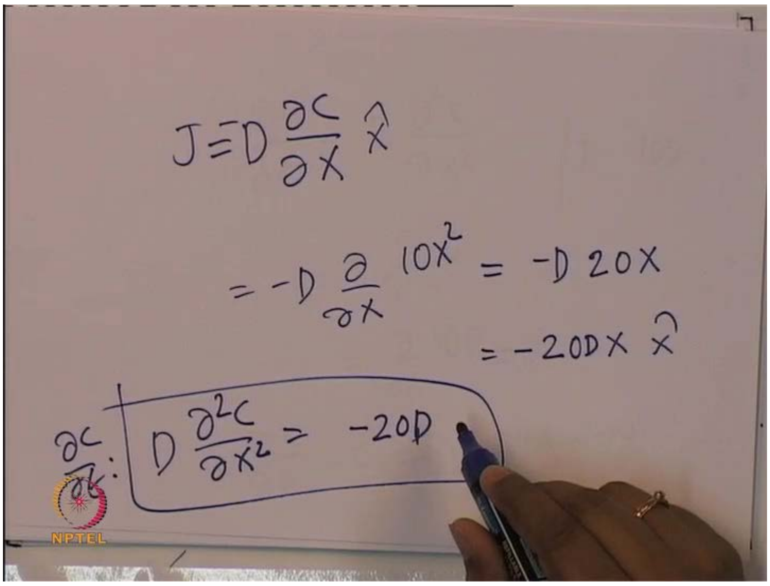


So, the current, we saw is, del C by del X which is, J X cap, so then if you do this, minus D del by del X of C is basically, 10 X square. So, this is basically derivative of 10 X square is 20 X. So, this is minus 20 dx  cap, so this is the answer and if, you calculate the second derivative del square C by del X square with D is one more derivative of this, so the answer will be minus 20 D, so this is actually some non-zero value therefore, this is del C by del t will be some number non-zero. So, in the second case, the answer is a non-zero current and the concentrate will change with time here the concentration will not change with time.

So, we can think about this, we can do this in detail, if you want you should do this, carefully I just quickly showed you the answer. Now, what we should do is basically, to calculate integral of a interesting function, that we learnt call Gaussian function or a normal distribution had this function called Gaussian function. So how do we calculate the integral of Gaussian function, is one question that, I want to discuss in tutorial, so let us discuss this, because this has a lot of ramifications, lot of times.

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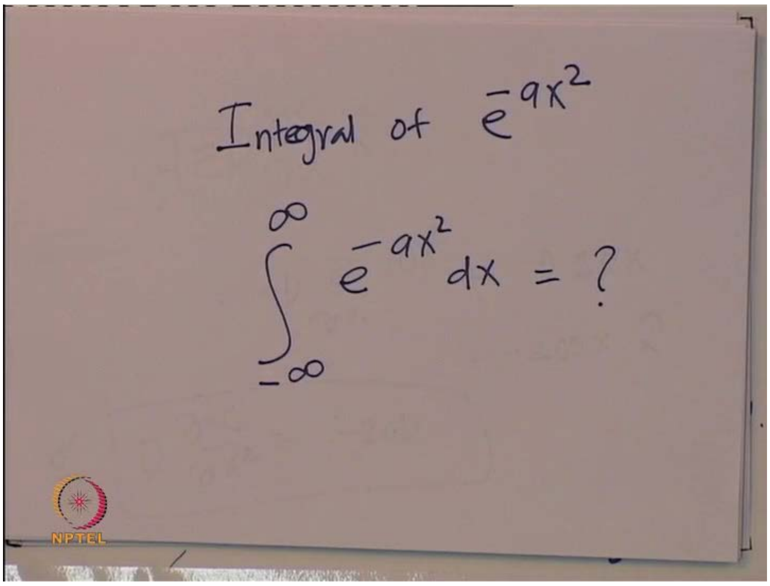


So, what we want to calculate the integral of e power minus a X square, this is the Gaussian function, so what we want to calculate is, integral minus infinity to infinity e power minus a X square d x, This is the Gaussian function and we even discus the answer but, how do we get the answer, whatever is the question. How do we do this, see you have to do in a special way, so I will discuss this and answer is used very much in many cases in probability and so on and so forth. So, let us do this carefully, and learn how to do this.

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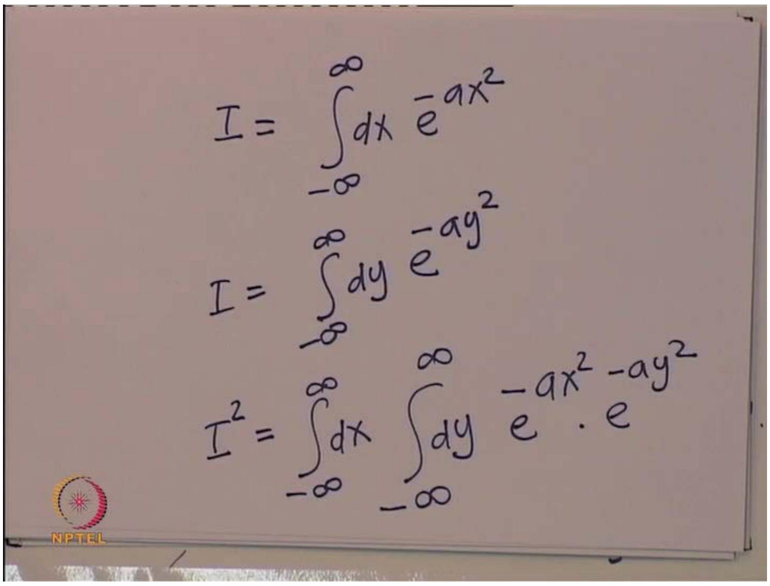

So, the answer we want is basically I is dx  e power minus a X square minus infinity to infinity, now X is the variable here, I could as well used y, instead of X, I could also write this and the same thing to write. If X is just a variable I could use y z or any thing I want. So, let me use y, this also will give you the same answer this and this both have the same answer because I just change the variable X is just a dummy variable and I could change to any variable I want, instead of X, I could use z y k whatever I want, and therefore, I am writing it, as dy   power minus y square, now I multiply this 2, so that I get I square product, which is basically minus infinity to infinity dx  minus infinity to infinity dy   e power minus a X square into e power minus a y square. This is what we have, and there is some reason for me to right like this, so I can write this, as e power a minus a in to X square plus y square.

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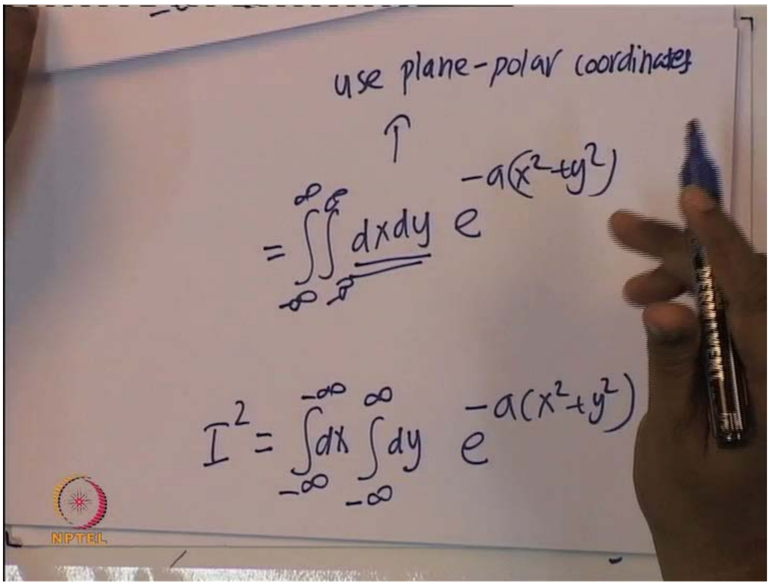


So, I can write, this part of it as I square is dx  d y, e power minus a into x square plus y square. So this is basically, a double integral, dx  dy   e power minus a x square plus y square now. This integral is overall a plane x and y, this is integral plane. So, anything in a plane, can be either described by Cartesian coordinates x and y or you can also use, plane polar coordinates, that means r and theta instead of x and y so if I use plane polar coordinates r and theta.

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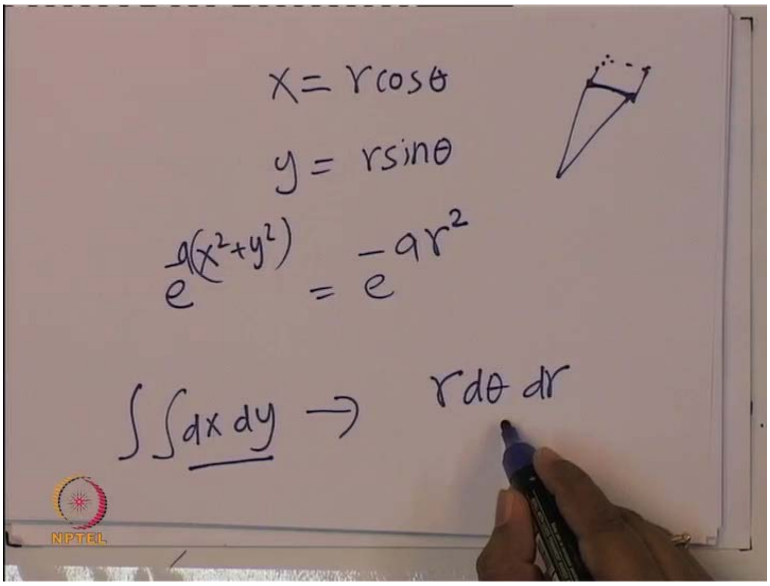


What do I get, is that how do I do that, I can substitute X is equal to r Cos theta and y is equal to r sin theta so e power minus x square plus y square times a, will be e power minus a into X r square Cos square plus r square sin square theta is just r square. So, it will be e power minus a r square, then we have an integral, which is dx  and d y, so how do we convert this into plane polar, this is the area element, this area under, in Cartesian coordinate, the area under polar coordinate is basically, r d theta dr. So r d theta is basically, if we have a radius r d theta is the area in this way, and d theta and r changes in this way, so this is basically, an area element in plane polar coordinate, r d theta dr. So dx  dy   will go to r d theta dr and x square plus y square will go to r square.

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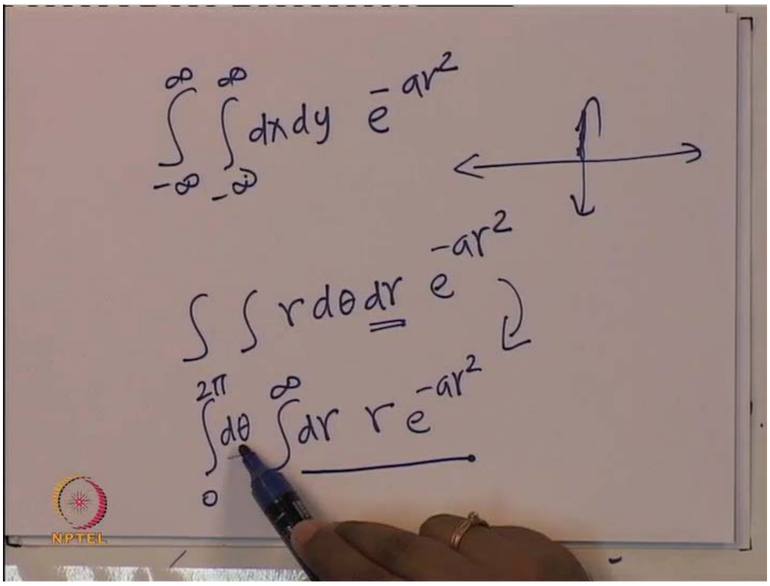

So this integral will essentially we can rewrite, so with the integral, we have now is basically, integral minus infinity to infinity dx dy, e power minus a r square but, dx dy we convert this into r d theta d r, e power minus a r square now look, think about it, now we have to decide what is the limits here, when it was x and y the limits were minus infinity to infinity what does it mean, X will go from all over here to all way minus infinity to plus infinity, y will also go from minus infinity to plus infinity throughout this plane.

So, this integrally throughout the plane, now in the plane polar coordinate r is the radius, and you cannot have negative radius, so r integral will go only from 0 to infinity. So you have 0 to infinity dr and theta is the angle, this will go from 0 to 2 pi, 0 to 2 pi theta and 0 to infinity r and then you have r, e power minus a r square. So if you substitute this has became this and we just rewrote everything, and just put the right limit so instead of minus infinity to infinity here r will go from 0 to infinity and theta will go from 0 to 2 pi,

So, then this will complete any plane, for example, any circle or anything the function in 2 d, so this 2 integrals are equivalent, so you can as well, do this integral. Now when we try to do this integral, let us try to do this integral carefully. Now look at this, this part is actually independent of theta, so it is only dr r and e power minus a r square, so this is independent of theta, therefore, this theta integral can be adjust only this part, so I can do this theta integral 0 to 2 pi d theta is just integral, d theta is just d theta and if you apply the limit 0 to 2 pi this become just 2 pi.

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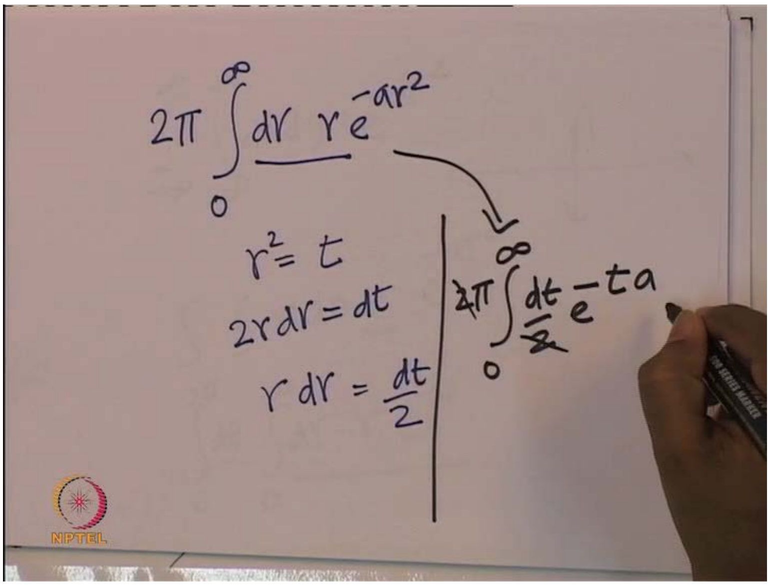


So, this part of the integral is just 2 pi, so let me write 2 pi then you have integral 0 to infinity d r, e power minus a r square. So, how do we do this, 0 to infinity dr r e power minus a r square so this can be done by taking r square, is something, so let us take r square is equal to t, so then I take derivative both side 2 r dr is d t I take derivative both sides, so 2 r dr is d t which implies that r dr is d t by 2, so instead of the r dr here, I can substitute d t by 2 and instead, of r square here, I can substitute t.

So, if I do this substitution, this become 2 pi integral 0 to infinity r dr is basically d t by 2 and e power minus r square is basically r square, t is and there is an a e square minus a t, so there is d t here, d t by 2, so this 2 and this 2 cancel you have pi integral d t e power minus a t, 0 to infinity this is an easy integral to do.

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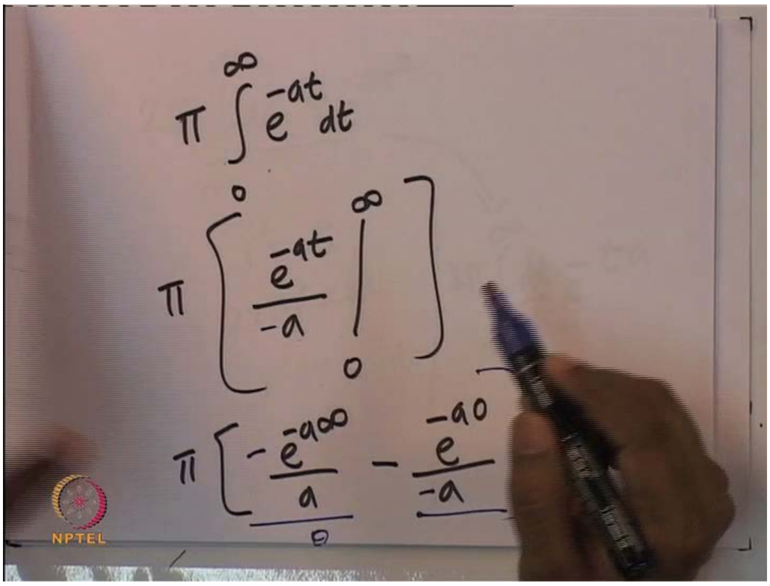


So, what we end up is basically, pi into 0 to infinity e power minus a t d t, this is an easy integral that we all know, which is basically, e power minus a t by a is the integral of this e power minus a t in the limit 0 to infinity and you have pi here. Now, if you apply limits to this, which is e power minus a t, if you apply this limits, what you get pi into e power minus a into infinity, so e power minus a into infinity divided by a minus e power minus a into 0 by a. So, e power minus infinity to anything is 0, so this term is 0, so this part basically 0 and what we have is just this part, so e power 0 is 1. So, what we get is basically, pi into this is 0 minus and there is another minus sign because e power minus a by minus a, there is a minus sign extra because when you integrate, e power minus a t by minus a is what the integral is.

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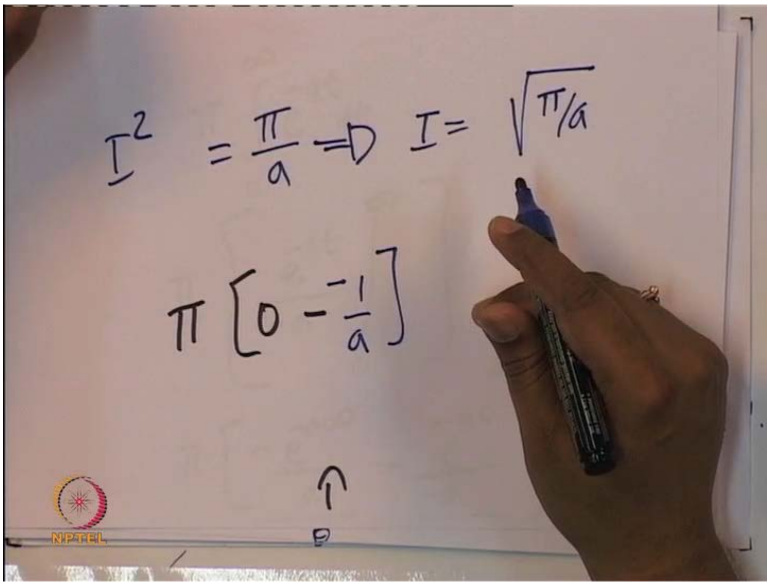


And therefore, a minus sign 0 minus 1 by a, so the answer is basically pi by a, so I square of the integral is pi by a.

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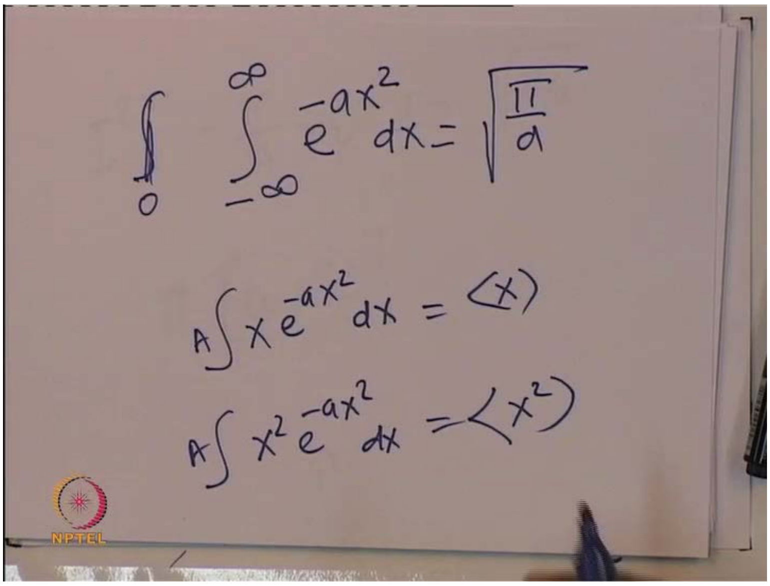


This implies that I is basically root of pi by a. So this is a well known result, that the integral of Gaussian integral minus infinity to infinity e power minus a X square dx is root of pi by a. This is a well-known, simple result to understand but, it is very useful result many times in statistics and many other contexts the integral of a Gaussian function.

So, this will come in many occasions. So, once you know this, you can calculate many other interesting integrals like integral X e power minus a X square dx  integral X square e power minus a X square dx  and so on and so forth. So, this, if you multiply, if this taken as a distribution and with a constant a this will become X average and X square average, so while we discus X average X square average, we discussed some part of it. This might also come again but, this integrals are very useful, once we know that e power minus a X square integral is root of pi by a. So let us quickly look at, how to do this integral which is X square e power minus a X square.

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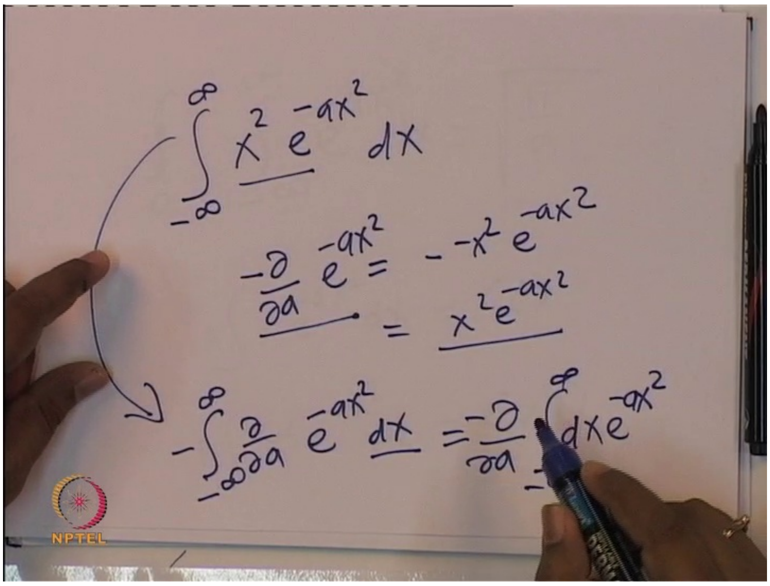


So, what we want to do is that, integral minus infinity to infinity x square minus e by x square dx  now. If you look at this carefully, for a minute, as we said before, this can be written as minus del by del a of e power minus a x square is nothing but, minus this is minus x square e power minus a x square, so this is x square e power minus a x square, minus del by del a of e power minus a x square is this, so then, this is what we want here, so instead of this I can write this, so let me write this, minus infinity to infinity minus del by del a of e power minus a X square dx.

So, this has been rewritten, in this particular fashion, and this is derivative with respect to a and integral with respect to x, so they are independent, so we can exchange, that means I can do first the integral, and then the derivative. And the answer to this integral we know, which is root of pi by a. So we have to just find the derivative of root of pi by a.

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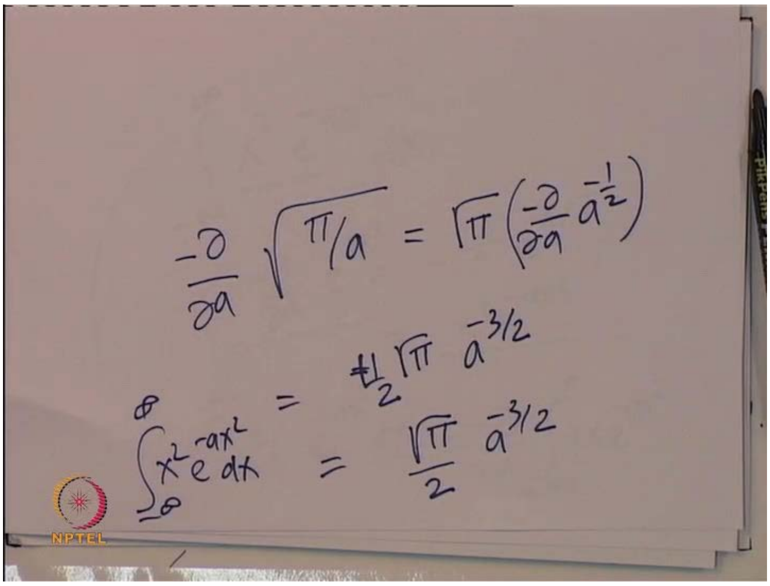


So, what we finally need to do is to just to find the derivative of minus del by del a of root of pi by a, this all we have to know, to get this particular integral, so then, we get the answer. So, this is basically root of pi into minus del by del a into a power minus half. So basically plus half root pi into this half a power minus 3 by 2. So, this could be the answer, this is the plus sign here.

So, this basically root pi by 2 into a power minus 3 by 2. So, this is basically, X square e power minus a X square integral d x, so this is minus infinity to infinity. So, all this is applicable, we just wrote the result of the Gaussian integral as root of pi by a, and you get this particular answer. So, with these many questions, that we discussed few couple of integrals and some vector notation and diffusion and so on. We covered some quick important points about some tutorials, and again I will give you a set of problems as tutorials in the write up. So, with this tutorial we will stop today’s discussion, and we will continue the rest of it, other lectures in the next class bye.